A FORMULA OF THE SMARANDACHE FUNCTION

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Abstract. In this paper we give a formula expressing the Smarandache function S(n) by means of n without using the factorization of n.

For any positive integer n, let S (n) denote the Smarandache function of n. Then we have

(1) $$S(n) = \min\{ a \mid a \in N,\ n \mid a! \},$$

(See [1]). In this paper we give a formula of S(n) without using the factorization of n as follows:

Theorem. For any positive integer n, we have

(1) $$S(n) = n+1 - \left[\sum_{k=1}^{n} n^{-(n \sin (k! \pi / n))^2} \right]$$

Proof. Let a = S(n). It is an obvious fact that $1 \leq a \leq n$. We see from (1) that

(2) $$n \mid k!, \quad k = a, a+1, \dots, n.$$

It implies that

$$(4) \qquad n^{-(n \sin (k! \pi / n))^2} = n^0 = 1, \qquad k = a, a+1, \dots, n.$$

On the other hand, since $n \nmid k!$ for $k = 1, \dots, a-1$, we have $\sin (k!\pi/n) \neq 0$ and

$$(5) \qquad \left(n \sin \frac{k! \pi}{n}\right)^2 \geq \left(n \sin \frac{\pi}{n}\right)^2 > 1, \quad k = 1, \dots, a-1.$$

Hense, by (5), we get

$$(6) \qquad 0 < n^{-(n \sin (k! \pi/ n))^2} < 1/n, \quad k = 1, \dots, a-1.$$

Therefore, by (4) and (6), we obtain

$$(7) \qquad n+1-a < \sum_{k=1}^{n} n^{-(n \sin (k! \pi/n))^2} < n+1-a+(a-1)/n < n+2-a.$$

Thus, by (7), we get (1) immediately. The theorem is proved.

Reference

1. F Smarandache, A function in the number theory, Smarandache function J. 1 (1990), No.1, 3 - 17.