International J.Math. Combin. Vol.4 (2009), 63-74

Independent Complementary Distance Pattern Uniform Graphs

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Abstract: A graph $G = (V, E)$ is called to be *Smarandachely uniform k-graph* for an integer $k \geq 1$ if there exists $M_1, M_2, \cdots, M_k \subset V(G)$ such that $f_{M_i}(u) = \{d(u, v) : v \in M_i\}$ for $\forall u \in V(G) - M_i$ is independent of the choice of $u \in V(G) - M_i$ and integer $i,\ 1 \leq i \leq k$. Each such set $M_i,\ 1 \leq i \leq k$ is called a CDPU set [6, 7]. Particularly, for $k = 1$, a Smarandachely uniform 1-graph is abbreviated to a *complementary distance pattern uniform* graph, i.e., CDPU graphs. This paper studies independent CDPU graphs.

Key Words: Smarandachely uniform k-graph, complementary distance pattern uniform, independent CDPU.

AMS(2000): 05C22.

§1. Introduction

For all terminology and notation in graph theory, not defined specifically in this paper, we refer the reader to Harary [4]. Unless mentioned otherwise, all the graphs considered in this paper are simple, self-loop-free and finite.

Let $G = (V, E)$ represent the structure of a chemical molecule. Often, a topological index (TI), derived as an invariant of G, is used to represent a chemical property of the molecule. There are a number of TIs based on distance concepts in graphs [5] and some of them could be designed using distance patterns of vertices in a graph. There are strong indications in the literature cited above that the notion of CDPU sets in G could be used to design a class of TIs that represent certain stereochemical properties of the molecule.

Definition 1.1([6]) *Let $G = (V, E)$ be a (p, q) graph and M be any non-empty subset of $V(G)$. Each vertex u in G is associated with the set $f_M(u) = \{d(u, v) : v \in M\}$, where $d(u, v)$ denotes the usual distance between u and v in G, called the M-distance pattern of u.*

[1]Received Oct.9, 2009. Accepted Nov. 24, 2009.

A graph $G = (V, E)$ is called to be Smarandachely uniform k-graph for an integer $k \geq 1$ if there exists $M_1, M_2, \cdots, M_k \subset V(G)$ such that $f_{M_i}(u) = \{d(u,v) : v \in M_i\}$ for $\forall u \in V(G) - M_i$ is independent of the choice of $u \in V(G) - M_i$ and integer i, $1 \leq i \leq k$. Each such set M_i, $1 \leq i \leq k$ is called a CDPU set. Particularly, for $k = 1$, a Smarandachely uniform 1-graph is abbreviated to a complementary distance pattern uniform graph, i.e., CDPU graphs. The least cardinality of the CDPU set is called the CDPU number denoted by $\sigma(G)$.

The following are some of the results used in this paper.

Theorem 1.2([7]) *Every connected graph has a CDPU set.*

Definition 1.3([7]) *The least cardinality of CDPU set in G is called the* CDPU number *of G, denoted $\sigma(G)$.*

Remark 1.4([7]) Let G be a connected graph of order p and let $(e_1, e_2, \ldots, e_k)$ be the non decreasing sequence of eccentricities of its vertices. Let M consists of the vertices with eccentricities $e_1, e_2, \ldots, e_{k-1}$ and let $|V - M| = p - m$ where $|M| = m$. Then $\sigma(G) \leqslant m$, since all the vertices in $V - M$ have $f_M(v) = \{1, 2, \ldots, e_{k-1}\}$.

Theorem 1.5([7]) *A graph G has $\sigma(G) = 1$ if and only if G has at least one vertex of full degree.*

Corollary 1.6([7]) *For any positive integer n, $\sigma(G + K_m) = 1$.*

Theorem 1.7([7]) *For any integer n, $\sigma(P_n) = n - 2$.*

Theorem 1.8([7]) *For all integers $a_1 \geq a_2 \geq \cdots \geq a_n \geq 2$, $\sigma(K_{a_1, a_2, \ldots, a_n}) = n$.*

Theorem 1.9([7]) *$\sigma(C_n) = n - 2$, if n is odd and*
$\sigma(C_n) = n/2$, if $n \geq 8$ is even. Also $\sigma(C_4) = \sigma(C_6) = 2$.

Theorem 1.10([7]) *If $\sigma(G_1) = k_1$ and $\sigma(G_2) = k_2$, then $\sigma(G_1 + G_2) = min(k_1, k_2)$.*

Theorem 1.11([7]) *Let T be a CDPU tree. Then $\sigma(T) = 1$ if and only if T is isomorphic to P_2, P_3 or $K_{1,n}$.*

Theorem 1.12([7]) *The central subgraph of a maximal outerplanar graph has CDPU number* 1 *or* 3.

Remark 1.13([7]) For a graph G which is not self centered, $\max f_M(v) = \mathrm{diam}(G) - 1$.

Theorem 1.14([7]) *The shadow graph of a complete graph K_n has exactly two $\sigma(K_n)$ disjoint CDPU sets.*

The following were the problems identified by B. D. Acharya [6, 7].

Problem 1.15 *Characterize graphs G in which every minimal CDPU-set is independent.*

Problem 1.16 *What is the maximum cardinality of a minimal CDPU set in G.*

Problem 1.17 *Determine whether every graph has an independent CDPU-set.*

Problem 1.18 *Characterize minimal CDPU-set.*

Fig.1 following depicts an independent CDPU graph.

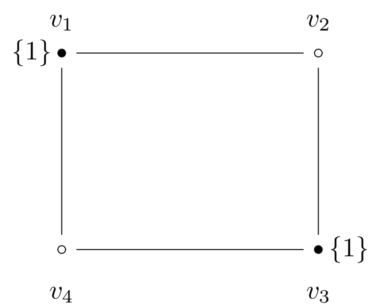

Fig.1: An independent CDPU graph with $M = \{v_2, v_4\}$

§2. Main Results

Definition 2.1 *A graph G is called an Independent CDPU graph if there exists an independent CDPU set for G.*

Following two observations are immediate.

Observations 2.2 Complete graphs are independent CDPU.

Observations 2.3 Star graph $K_{1,n}$ is an Independent CDPU graph.

Proposition 2.4 *C_n with n even is an Independent CDPU graph.*

Proof Let C_n be a cycle on n vertices and $V(C_n) = \{v_1, v_2, \ldots, v_n\}$, where n is even. Choose M as the set of alternate vertices on C_n, say, $\{v_2, v_4, \ldots, v_n\}$. Then,
$f_M(v_i) = \{1, 3, 5, \ldots, m-1\}$ for $i = 1, 3, \ldots, n-1$, if $C_n = 2m$ and m is even and
$f_M(v_i) = \{1, 3, 5, \ldots, m\}$, for $i = 1, 3, \ldots, n-1$ if $C_n = 2m$ and m odd. Therefore, $f_M(v_i)$ is identical depending on whether m is odd or even. Hence, the alternate vertices $\{v_2, v_4, \ldots, v_n\}$ forms a CDPU set M. Also all the vertices in M are non-adjacent. Hence C_n, n even is an independent CDPU graph. □

Theorem 2.5 *A cycle C_n is an independent CDPU graph if and only if n is even.*

Proof Let C_n be a cycle on n vertices. Suppose n is even. Then from Proposition 2.4, C_n is an independent CDPU graph.

Conversely, suppose that C_n is an independent CDPU graph. That is, there exist vertices in M such that every pair of vertices are non adjacent. We have to prove that n is even. Suppose n is odd. Then from Theorem 1.9, $\sigma(C_n) = n - 2$, which implies that $|M| \geq n - 2$.

But from n vertices, we cannot have $n-2$ (or more) vertices which are non-adjacent. □

Theorem 2.6 *A graph G which contains a full degree vertex is an independent CDPU.*

Proof Let G be a graph which contains a full degree vertex v. Then, from Theorem 1.5, G is CDPU with CDPU set $M=\{v\}$. Also M is independent. Therefore, G is an independent CDPU. □

Remark 2.7 If the CDPU number of a graph G is 1, then clearly G is independent CDPU.

Theorem 2.8 *A complete n-partite graph G is an independent CDPU graph for any n.*

Proof Let $G=K_{a_1,a_2,\ldots,a_n}$ be a complete n-partite graph. Then, $V(G)$ can be partitioned into n subsets $V_1,V_2,\ldots,V_n$ where $|V_1|=a_1,|V_2|=a_2,\ldots,|V_n|=a_n$. Take all the vertices from the partite set, say, V_i of $K_{a_1,a_2,\ldots,a_n}$ to constitute the set M. Since each element of a partite set is non-adjacent to the other vertices in it and is adjacent to all other partite sets, we get, $f_M(u)=\{1\},\forall u\in V(K_{a_1,a_2,\ldots,a_n})-M$. Hence, the complete n-partite graph G is an independent CDPU graph for any n. □

Corollary 2.9 *Complete n-partite graphs have n distinct independent CDPU sets.*

Proof Let $G=K_{a_1,a_2,\ldots,a_n}$ be a complete n-partite graph. Then, $V(G)$ can be partitioned into n subsets $V_1,V_2,\ldots,V_n$ where $|V_1|=a_1,|V_2|=a_2,\ldots,|V_n|=a_n$. Take M_1 as the vertices corresponding to the partite set V_1, M_2 as the vertices corresponding to the partite set V_2, $\ldots$, M_i corresponds to the vertices of the partite set V_i, $\ldots$, M_n corresponds to the vertices of the partite set V_n. Then from Theorem 2.8, each $M_i, 1\le i\le n$ form a CDPU set. Hence there are n distinct CDPU sets. □

Theorem 2.10 *A path P_n is an independent CDPU graph if and only if $n=2,3,4,5$.*

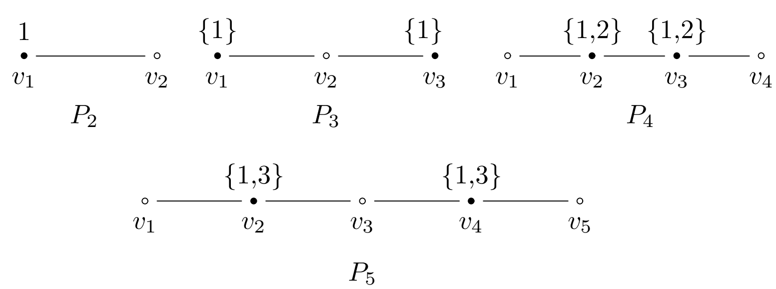

Fig.2: An independent CDPU paths

Proof Let P_n be a path on n vertices and $V(P_n)=\{v_1,v_2,\ldots,v_n\}$. When $n=2$ and 3, P_2 and P_3 contains a vertex of full degree and hence from Theorem 2.6, P_2 and P_3 are independent

CDPU. When $n = 4$, take $M = \{v_1, v_4\}$. Then $f_M(v_2) = f_M(v_3) = \{1, 2\}$, whence M is independent CDPU. When $n = 5$, let $V(G) = \{v_1, v_2, \ldots, v_5\}$ and choose $M = \{v_1, v_3, v_5\}$. Then, $f_M(v_2) = f_M(v_4) = \{1, 3\}$. Hence, P_5 is an independent CDPU graph.

Conversely, suppose that P_n is an independent CDPU graph. That is, there exists a CDPU set M such that no two of the vertices are adjacent. From n vertices, we can have at most $\frac{n}{2}$ or $\frac{n+1}{2}$ vertices which are non adjacent. From Theorem 1.7, $\sigma(P_n) = n - 2, n \geq 3$. When $n \geq 6$, we cannot choose a CDPU set M such that $n - 2$ vertices are non-adjacent. Hence P_n is independent CDPU only for $n = 2, 3, 4$ and 5. $\square$

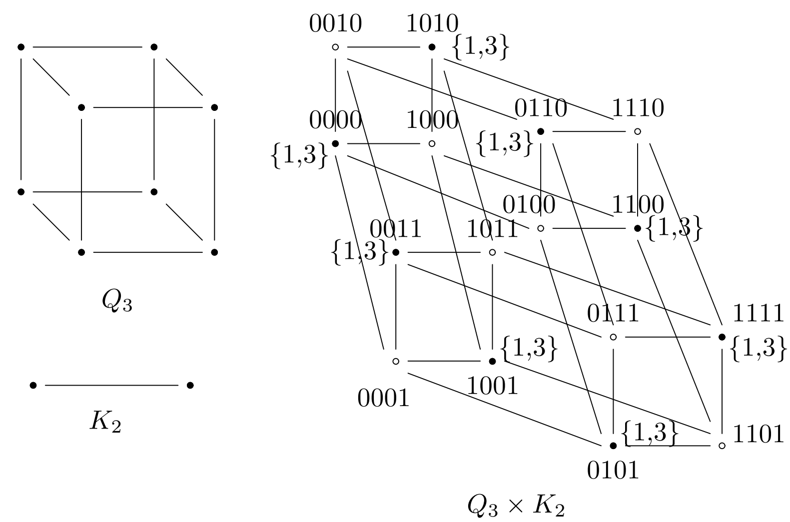

Fig.3 : Q_4

Theorem 2.11 *n-cube Q_n is an independent CDPU graph with $|M| = 2^{n-1}$.*

Proof We have $Q_n = K_2 \times Q_{n-1}$ and has 2^n vertices which may be labeled $a_1 a_2 \ldots a_n$, where each a_i is either 0 or 1. Also two points in Q_n are adjacent if their binary representations differ at exactly one place. Take M as the set of all vertices whose binary representation differ at two places. Clearly the vertices in M are non adjacent and also maximal. We have to check whether M is CDPU. For let $M = \{v_1, v_3, \ldots, v_{2^n - 1}\}$. Consider a vertex v_i which does not belong to M. Clearly v_i is adjacent to a vertex v_j in M. Hence $1 \in f_M(v_i)$. Then, since v_j is in M, v_j is adjacent to a vertex v_k not in M. Hence 2 does not belong to $f_M(v_i)$. Since v_k is not an element of M and v_k is adjacent to a vertex v_l in M, $3 \in f_M(v_i)$. Proceeding in the same manner, we get $f_M(v_i) = \{1, 3, \ldots, n - 1\}$. Hence Q_n is independent CDPU with $|M| = \frac{2^n}{2}$. $\square$

Theorem 2.12 *Ladder $P_n \times K_2$ is an independent CDPU graph if and only if $n \leq 4$.*

Proof First we have to prove that $P_n \times K_2$ is an independent CDPU graph for $n \leq 4$.

When $n = 2$, take $M = \{v_2, v_4\}$, so that $f_M(v_i) = \{1\}$ for $i = 1, 3$.

When $n=3$, take $M=\{v_1,v_4\}$, so that $f_M(v_i)=\{1,2\}$, for $i=2,4,6$.

When $n=4$, take $M=\{v_1,v_3,v_5,v_7\}$, so that $f_M(v_i)=\{1,3\}$ for $i=2,4,6,8$. Therefore, $P_n\times K_2$ is an independent CDPU graph for $n\leq 4$.

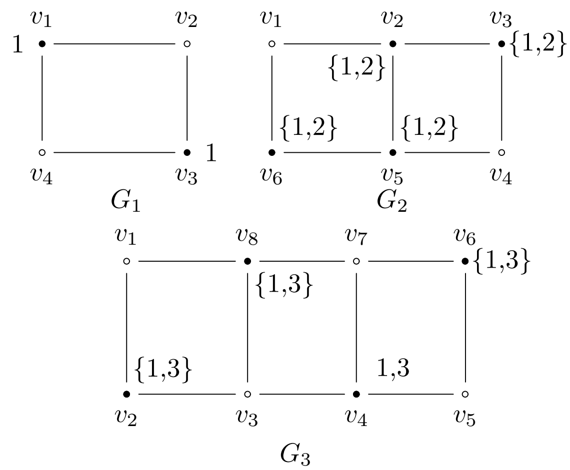

Fig.4: $P_n\times K_2$ for $n\leq 4$

Conversely, suppose that $P_n\times K_2$ is an independent CDPU graph. We have to prove that $n\leq 4$. If possible, suppose $n=k\geq 5$. In $P_n\times K_2$, since the number of vertices is even, and the vertices in $P_n\times K_2$ forms a Hamiltonian cycle, then the only possibility of M to be an independent CDPU set is to choose M as the set of all alternate vertices of the Hamiltonian cycle. Clearly, in this case M is a maximal independent set. Denote $M_1=\{v_1,v_3,\ldots,v_{2n-1}\}$ and $M_2=\{v_2,v_4,\ldots,v_{2n}\}$. Consider $M_1=\{v_2,v_4,\ldots,v_i,\ldots,v_{2n}\}$.

Case 1 n is odd.

In this case, $f_{M_1}(v_1)=\{1,3,\ldots,n\}$. Since n is odd we have two central vertices, say, v_i and v_j in $P_n\times K_2$. Since v_i and v_j are of the same eccentricity and M_1 is a maximal independent set, v_j does not belong to M_1. Then, $f_{M_1}(v_j)=\{1,3,\ldots,\frac{n+1}{2}\}$.

Thus, $f_{M_1}(v_1)\neq f_{M_1}(v_j)$. Hence M_1 is not a CDPU.

Case 2 n is even.

In this case, $f_{M_1}(v_1)=\{1,3,\ldots,n-1\}$. Since n is even, there are four central vertices v_i,v_j,v_k,v_l in $P_n\times K_2$. Clearly the graph induced by $T=\{v_i,v_j,v_k,v_l\}$ is a cycle on four vertices. Since M_1 is maximal and consists of the alternate vertices of $P_n\times K_n$, v_j,v_l should necessarily be outside M_1. Thus, $f_{M_1}(v_j)=\{1,3,\ldots,\frac{n}{2}\}$.

Thus, $f_{M_1}(v_1)\neq f_{M_1}(v_j)$. Hence M_1 is not a CDPU.

Therefore $P_n\times K_2$ is not independent CDPU for $n\geq 5$. Hence the theorem. □

Theorem 2.13 *If G_1 and G_2 are independent CDPU graphs, then G_1+G_2 is also an independent CDPU graph.*

Proof Since G_1 and G_2 are independent CDPU graphs, there exist $M_1 \subset V(G_1)$ and $M_2 \subset V(G_2)$ such that no two vertices in M_1 (and in M_2) are adjacent. Now, in G_1+G_2, every vertex of G_1 is adjacent to every vertices of G_2. Then clearly, independent CDPU set M_1 of G_1 (or M_2 of G_2) is an independent CDPU set for $G_1 + G_2$. Hence the theorem. □

Remark 2.14 If G_1 and G_2 are independent CDPU graphs, then the cartesian product $G_1 \times G_2$ need not have an independent CDPU set. But $G_i \times G_i$ is independent CDPU for $i = 1, 2$ as illustrated in Fig.5.

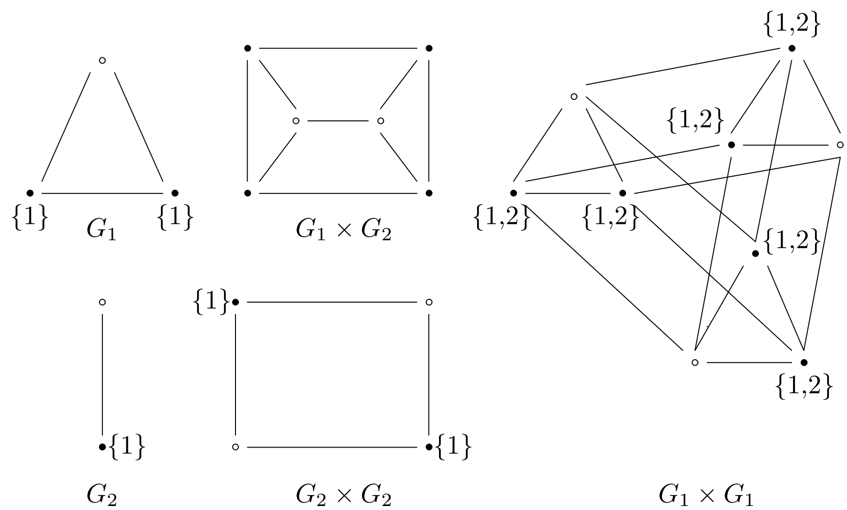

Fig.5

Definition 2.15 *An independent set that is not a proper subset of any independent set of G is called maximal independent set of G. The number of vertices in the largest independent set of G is called the independence number of G and is denoted by $\beta(G)$.*

§3. Independence CDPU Number

The least cardinality of the independent cdpu set in G is called the independent CDPU number of G, denoted by $\sigma_i(G)$. In general, for an independent CDPU graph, $\sigma_i(G) \leq \beta(G)$, where $\beta(G)$ is the independence number of G.

Theorem 3.1 *If G is an independent CDPU graph with n vertices, then $r(G) \leq \sigma_i(G) \leq \lceil \frac{n}{2} \rceil$, where $r(G)$ is the radius of G.*

Proof We have, $\beta(G) \leq \lceil \frac{n}{2} \rceil$ and hence $\sigma_i(G) \leq \lceil \frac{n}{2} \rceil$. Now we prove that $r(G) \leq \sigma_i(G)$. Suppose $r(G) = k$. Then, there are vertices with eccentricities $k, k+1, k+2, \ldots, d$, where d is the diameter of G. Let v be the central vertex of G and $e = uv$. Since the central vertex v of a graph on $n(\geq 3)$ vertices cannot be a pendant vertex, there exists a vertex w which is adjacent to v. Hence, w is of eccentricity $k+1$. Also u is of eccentricity $k+1$. By a similar argument there exists at least two vertices each of eccentricity $k+1, k+2, \ldots, d$. Hence, the CDPU set should necessarily consists of all vertices with eccentricity $k, k+1, k+2, \ldots, d-1$. Thus, $\sigma(G) \geq 1 + \{2+2+\ldots(d-1-k)times\} \geq k$. Whence, $\sigma_i(G) \geq r(G)$. Therefore, $r(G) \leq \sigma_i(G) \leq \lceil \frac{n}{2} \rceil$. □

Theorem 3.2 *A graph G has $\sigma_i(G) = 1$ if and only if G has at least one vertex of full degree.*

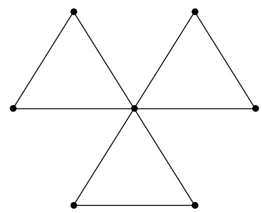

Fig.6: A graph with $\sigma_i(G) = 1$

Proof Suppose that G has one vertex v_i with full degree. Take $M = \{v_i\}$. Then $f_M(u) = \{1\}$, for every $u \in V - M$. Also M is independent. Hence $\sigma_i(G) = 1$.

Conversely, suppose that G is a graph with $\sigma_i(G) = 1$. That is, there exists an independent set M which contains only one vertex v_i which is a CDPU set of G. Also, $\sigma_i(G) = 1$ implies, v_i is adjacent to all other vertices. Hence v_i is a vertex with full degree. □

Corollary 3.3 *The independent CDPU number of a complete graph is* 1.

Corollary 3.4 *If M is the maximal independent set of a graph G with $|M| = 1$, then G is an independent CDPU.*

Proof The result follows since M is a maximal independent set and $|M| = 1$, there is a vertex v of full degree. □

Theorem 3.5 *Peterson Graph is an independent CDPU graph with $\sigma_i(G) = 4$.*

Proof Let G be a Peterson Graph with $V(G) = \{v_1, v_2, \ldots, v_{10}\}$. Let M be such that M contains two non adjacent vertices from the outer cycle and two non-adjacent vertices from the inner cycle. Let it be $\{v_3, v_5, v_6, v_7\}$. Clearly, M is a maximal independent set of G. Also $f_M(v_i) = \{1, 2\}$, for every $i = 1, 2, 4, 8, 9, 10$. Thus, M is a CDPU set of G. Hence, G is an independent CDPU graph with $\sigma_i(G) \leq 4$. To prove that $\sigma_i(G) = 4$, it is enough to prove that the deletion of any vertex from M does not form a CDPU set. For, let $M_1 = \{v_3, v_5, v_7\}$. Then, $f_M(v_i) = \{1, 2\}$, for $i = 1, 2, 4, 8, 9, 10$ and $f_M(v_6) = \{2\}$. Hence M_1 cannot be a CDPU set for G. Thus $\sigma_i(G) = 4$. □

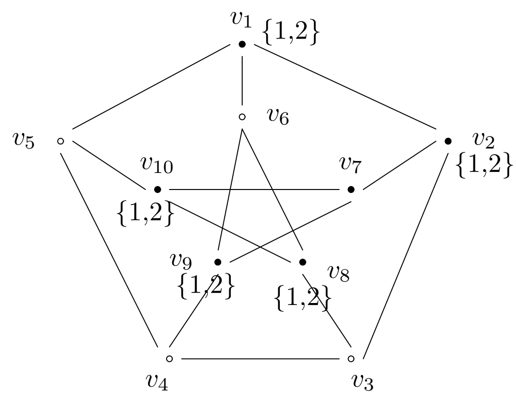

Fig.7

Theorem 3.6 *Shadow graphs of K_n are independent CDPU with $|M| = n$.*

Proof Let $v_1, v_2, \ldots, v_n$ be the vertices of K_n and $v'_1, v'_2, \ldots, v'_n$ be the corresponding shadow vertices. Clearly, $M = \{v'_1, v'_2, \ldots, v'_n\}$ is a maximal independent set of $S(K_n)$. Also, from Theorem 1.14, M forms a CDPU set. Hence $|M| = n$. $\square$

Definition 3.7 *A set of points which covers all the lines of a graph G is called a point cover for G. The smallest number of points in any point cover for G is called its point covering number and is denoted by $\alpha_0(G)$.*

It is natural to rise the following question by definition:

Does there exist any connection between the point covering for a graph and independent CDPU set?

Proposition 3.8 *If $\alpha_0(G) = 1$, then $\sigma_i(G) = 1$*

Proof Since $\alpha_0(G) = 1$, we have to cover every edges by a single vertex. This implies that there exists a vertex of full degree. Hence from Theorem 3.2, $\sigma_i(G) = 1$. $\square$

Remark 3.9 The converse of Proposition 3.8 need not be true. Note that in Figure 6, $\sigma_i(G) = 1$, but $\alpha_0(G) = 6$.

Theorem 3.10 *The central subgraph $< C(G) >$ of a maximal outerplanr graph G is an independent CDPU graph with $\sigma_i(G) = 1, 2$ or 3.*

Proof Fig.8 depicts all the central subgraphs of maximal outerplanr graph [3]. Since G_1, G_2, G_3, G_4, G_5 have a full degree vertex, those graphs are independent CDPU and $\sigma_i(G_j) = 1$, for $j = 1, 2, 3, 4, 5$.

In G_6, let $M = \{v_1, v_4\}$. Then, $f_M(v_i) = \{1, 2\}$, for every $v_i \in V - M$. Since M is independent, G_6 is independent CDPU and $\sigma_i(G_6) = 2$.

In G_7, let $M = \{v_1, v_3, v_5\}$. Then, $f_M(v_i) = \{1, 2\}$ for every $v_i \in V - M$. Hence, G_7 is independent CDPU with $\sigma_i(G_7) = 3$. □

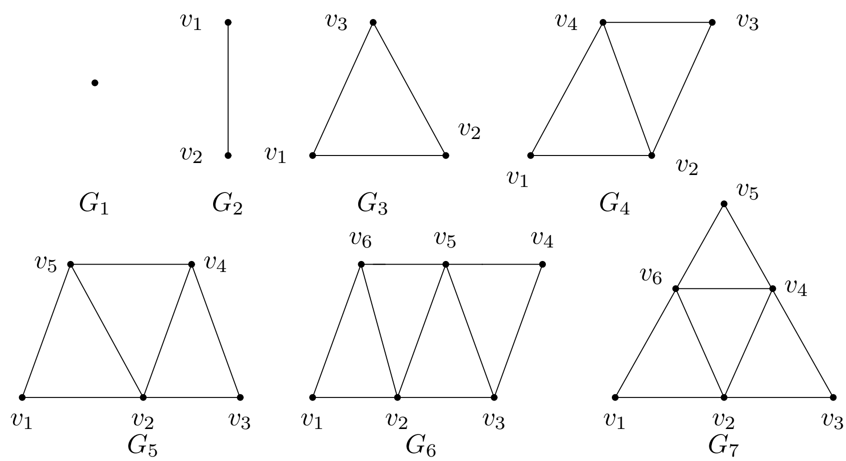

Fig.8: Central subgraphs of a maximal outerplanar graph

Theorem 3.11 *The independent CDPU number of an even cycle* $C_n, n \geq 8$ *is* $\frac{n}{2}$.

Proof From Proposition 2.4, the alternate vertices of the even cycle constitute the independent CDPU set. As already proved, removal of any vertex from M does not give a cdpu set. Hence, $\sigma_i(C_n) = \frac{n}{2}$. □

Remark 3.12 $\sigma_i(C_6) = 2$.

Theorem 3.13 *For all integers* $a_1 \geq a_2 \geq \cdots \geq a_n \geq 2, \sigma_i(K_{a_1,a_2,\ldots,a_n}) = min\{a_1, a_2, \ldots, a_n\}$.

Proof From Theorem 2.8 and Corollary 2.9, all the n partite sets form an independent CDPU set. Hence the independent CDPU number is the minimum of all $a_i's$. □

Theorem 3.14 *If* $\sigma_i(G_1) = k_1$ *and* $\sigma_i(G_2) = k_2$*, then* $\sigma_i(G_1 + G_2) = min.\{k_1, k_2\}$.

Proof From Theorem 2.13, either M_1 or M_2 is an independent cdpu set for $G_1 + G_2$. Also $\sigma_i(G_1 + G_2)$ is the minimum among M_1 and M_2. □

Theorem 3.15 *If* G_1 *and* G_2 *are independent CDPU cycles with* $n, m(\geq 4)$ *vertices respectively, then* $G_1 \times G_2$ *is independent CDPU with* $|M| = \frac{mn}{2}$.

Proof Since G_1 has n vertices and G_2 has m vertices, then $G_1 \times G_2$ has mn vertices. Without loss of generality, assume that $m > n$. In the construction of $G_1 \times G_2$, G_2 is drawn n times and then the corresponding adjacency is given according as the adjacency in G_1. Since G_2 is an independent CDPU cycle, from Theorem 3.11, $\sigma_i(G_2) = \frac{m}{2}$. Therefore in $G_1 \times G_2$ there are $\frac{mn}{2}$ vertices in the CDPU set. □

Remark 3.16 In Theorem 3.15, if any one of G_1 or G_2 is C_3, then $|M| = n$, since $\sigma_i(C_3) = 1$.

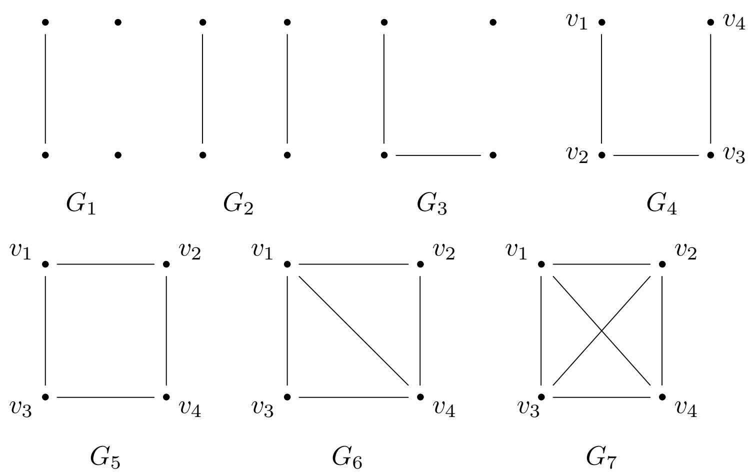

Fig.9: Graphs whose subdivision graphs are bipartite complementary

Theorem 3.17 *The connected graphs, whose subdivision graphs are bipartite complementary are independent CDPU.*

Proof Fig.9 depicts the seven graphs whose subdivision graphs are bipartite self-complementary [2]. In $G_4, M_1 = \{v_1, v_2\}$ gives $f_{M_1}(v_3) = f_{M_1}(v_4) = \{1, 2\}$.

In $G_5, M_2 = \{v_1, v_4\}$ gives $f_{M_2}(v_3) = f_{M_2}(v_2) = \{1\}$.

In $G_6, M_3 = \{v_2, v_3\}$ gives $f_{M_3}(v_1) = f_{M_3}(v_4) = \{1\}$.

In $G_7, M_4 = \{v_1\}$ gives $f_{M_4}(v_2) = f_{M_4}(v_3) = f_{M_4}(v_4) = \{1\}$. Hence M_1, M_2, M_3, M_4 are independent CDPU sets. Thus the connected graphs G_4, G_5, G_6 and G_7 are independent CDPU. □

§4. Conclusion and Scope

As already stated in the introduction, the concept under study has important applications in the field of Chemistry. The study is interesting due to its applications in Computer Networks and Engineering, especially in Control System. In a closed loop control system, signal flow graph representation is used for gain analysis. So in certain control systems specified by certain characteristics, we can find out M, a set consisting of two vertices such that one vertex will be the take off point and other vertex will be the summing point.

Following are some problems that are under investigation:

1. Characterize independent CDPU trees.
2. Characterize unicyclic graphs which are independent CDPU.
3. What is the independent CDPU number for a generalized Peterson graph.
4. What are those classes of graphs with $r(G) = \sigma_i(G)$, where $r(G)$ is the radius of G.

Acknowledgements

The authors are deeply indebted to Professor Dr.B.D.Acharya, SRC-IIIDMS, University of Mysore, Mysore-560 005, India for sparing his valuable time in sharing his many incisive thoughts to propel our vigorous discussion on the content of this paper. They are thankful to the Department of Science & Technology, Government of India for supporting this research under the project No. SR/S4/MS:277/06.

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