CALCULATING THE SMARANDACHE FUNCTION FOR POWERS OF A PRIME

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Introduction

The Smarandache function is an integer function, S, of an integer variable, n. S is the smallest integer such that S! is divisible by n. If the prime factorisation of n is known

$$n = \prod m_i^{p_i}$$

where the p_i are primes then it has been shown that

$$S(n) = \mathrm{Max}\left(S\left(m_i^{p_i}\right)\right)$$

so a method of calculating S for prime powers will be useful in calculating S(n).

The inverse function

It is easier to start with the inverse problem. For a given

prime, p, and a given value of S, a multiple of p, what is the maximum power, m, of p which is a divisor of S! ? If we consider the case p=2 then all even numbers in the factorial contribute a factor of 2, all multiples of 4 contribute another, all multiples of 8 yet another and so on.

$$m = S\ \mathrm{DIV}2 + (S\ \mathrm{DIV}2)\mathrm{DIV}2 + ((S\ \mathrm{DIV}2)\mathrm{DIV}2)\mathrm{DIV}2 + \ldots$$

In the general case

$$m = S\ \mathrm{DIV}p + (S\ \mathrm{DIV}p)\mathrm{DIV}p + ((S\ \mathrm{DIV}p)\mathrm{DIV}p)\mathrm{DIV}p + \ldots$$

The series terminates by reaching a term equal to zero. The Pascal program at the end of this paper contains a function invSpp to calculate this function.

Using the inverse function

If we now look at the values of S for succesive powers of a prime, say p=3,

m	1	2	3	4	5	6	7	8	9	10 ...
	*	*		*	*	*		*	*	*
S(3^m)	3	6	9	9	12	15	18	18	21	24 ...

where the asterisked values of m are those found by the inverse function, we can see that these latter determine the points after which S increases by p. In the Pascal program the procedure tabsmarpp fills an array with the values of S for successive powers of a prime.

The Pascal program

The program tests the procedure by accepting a prime input from the keyboard, calculating S for the first 1000 powers, reporting the time for this calculation and entering an endless loop of accepting a power value and reporting the corresponding S value as stored in the array.

The program was developed and tested with Acornsoft ISO-Pascal on a BBC Master. The function 'time' is an extension to standard Pascal which delivers the timelapse since last reset in centi-seconds. On a computer with a 65C12 processor running at 2 MHz the 1000 S values are calculated in about 11 seconds, the exact time is slightly larger for small values of the prime.

```
program TestabSpp(input,output);
var t,p,x: integer;
Smarpp:array[1..1000] of integer;

function invSpp(prime,smar:integer):integer;
var m,x:integer;
begin
m:=0;
x:=smar;
repeat
x:=x div prime;
m:=m+x;
until x<prime;
invSpp:=m;
end; {invSpp}
```

```
  procedure tabsmarpp(prime,tabsize:integer);
  var i,s,is:integer;
  exit:boolean;
  begin
  exit:=false;
  i:=1;
  is:=1;
  s:=prime;
  repeat
  repeat
  Smarpp[i]:=s;
  i:=i+1;
  if i>tabsize then exit:=true;
  until (i>is) or exit;
  s:=s+prime;
  is:=invSpp(prime,s);
  until exit;
  end; {tabsmarpp}

  begin
  read(p);
  t:=time;
  tabsmarpp(p,1000);
  writeln((time-t)/100);
  repeat
  read(x);
writeln('Smarandache for ',p,' to power ',x,' is ',Smarpp[x]);
  until false;
  end. {testabspp}
```