On A Conjecture By Russo

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The Smarandache Square-Partial-Digital Subsequence(SSPDS) is the sequence of square integers which can be partitioned so that each element of the partition is a perfect square[1][2][3]. For example, 3249 is in SSPDS since 3249 can be partitioned into $324 = 18^2$ and $9 = 3^2$.

The first terms of the sequence are:

49, 144, 169, 361, 441, 1225, 1369, 1444, 1681, 1936, 3249, 4225, 4900, 11449, 12544, 14641, ...

where the square roots are

7, 12, 13, 19, 21, 35, 37, 38, 41, 44, 57, 65, 70, 107, 112, 121, ...

this sequence is assigned the identification code A048653[4].

L. Widmer examined this sequence and posed the following question[2]:

Is there a sequence of three or more consecutive integers whose squares are in SPDS?

For the purposes of this examination, we will assume that 0 is not a perfect square. For example, the number 90 will not be considered as a number that can be partitioned into two perfect squares. Furthermore, elements of the partition are not allowed to have leading zeros. For example, 101 cannot be partitioned into perfect squares.

Russo[5] considered this question and concluded that the only additional solution to the Widmer question up to 3.3E+9 was

n	n^2	Partition
12225	149450625	1,4,9,4,50625
12226	149475076	1,4,9,4,75076
12227	149499529	1,4,9,4,9,9,529

and made the following conjecture:

There are no four consecutive integers whose squares are in SSPDS.

The purpose of this short paper is to present several additional solutions to the Widmer question as well as a counterexample to the Russo conjecture.

A computer program was written in the language Delphi Ver. 4 and run for all numbers n, where $n \leq 100{,}000{,}000$ and the following ten additional solutions were found

n	n^2	Partition
376779	141962414841	1, 4, 1, 9, 6241, 4, 841
376780	141963168400	1, 4, 196, 3168400
376781	141963921961	1, 4, 196392196,1

n	n^2	Partition
974379	949414435641	9, 4, 9, 4, 1, 4, 4356, 4, 1
974380	949416384400	9,4,9,4,16,384400
974381	949418333161	9, 4, 9, 4, 1833316, 1

n	n^2	Partition
999055	998110893025	9, 9, 81, 1089, 3025
999056	998112891136	9, 9, 81, 1, 289, 1, 1, 36
999057	998114889249	9, 9, 81, 1, 4, 889249

n	n^2	Partition
999056	998112891136	9, 9, 81, 1, 289, 1, 1, 36
999057	998114889249	9, 9, 81, 1, 4, 889249
999058	998116887364	9, 9, 81, 16, 887364

n	n^2	Partition
2000341	4001364116281	400, 1, 36, 4, 116281
2000342	4001368116964	400, 1, 36, 81, 16, 9, 64
2000343	4001372117649	400,1,3721, 1764,9

n	n^2	Partition
2063955	4259910242025	4, 25, 9, 9, 1024, 2025
2063956	4259914369936	4, 25, 9, 9, 1, 4, 36, 9, 9, 36
2063957	4259918497849	4, 25, 9, 9, 1849, 784, 9

n	n^2	Partition
2083941	4342810091481	43428100, 9, 1, 4, 81
2083942	4342814259364	434281, 4, 25, 9, 36, 4
2083943	4342818427249	434281, 842724, 9

n	n^2	Partition
4700204	22091917641616	2209, 1, 9, 1764, 16, 16
4700205	22091927042025	2209, 1, 9, 2704, 2025
4700206	22091936442436	2209, 1, 9, 36, 4, 42436

n	n^2	Partition
5500374	30254114139876	3025, 4, 1, 1, 4, 139876
5500375	30254125140625	3025, 4, 1, 25, 140625
5500376	30254136141376	3025, 4, 1, 36, 141376

n	n^2	Partition
80001024	6400163841048576	6400, 16384, 1048576
80001025	6400164001050625	6400, 1, 6400, 1050625
80001026	6400164161052676	6400, 1, 64, 16, 1052676

n	n^2	Partition
92000649	8464119416421201	8464, 1, 1, 9, 4, 16, 421201
92000650	8464119600422500	8464, 1, 19600, 4, 22500
92000651	8464119784423801	8464, 1, 1, 9, 784, 423801

Pay particular attention to the four consecutive numbers 999055, 999056, 999057 and 999058. These four numbers are a counterexample to the conjecture by Russo.

Given the frequency of three consecutive integers whose squares are in SSPDS, the following conjecture is made:

There are an infinite number of three consecutive integer sequences whose squares are in SSPDS.

In terms of larger sequences, the following conjecture also appears to be a safe one:

There is an upper limit to the length of consecutive integer sequences whose squares are in SSPDS.

We close with an unsolved question:

What is the length of the largest sequence of consecutive integers whose squares are in SSPDS?

References

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[5] F. Russo, "On An Unsolved Question About the Smarandache Square-Partial-Digital Subsequence" *http://www.gallup.unm.edu/~smarandache/russo1.htm.*