# A resolution of the Okamoto-Nolen-Schiffer Anomaly

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**Abstract**

Half a century old Okamoto-Nolen-Schiffer anomaly, has not been properly resolved as of now. We revive here, an old model of Sacks of a phenomenological exchange potential, which uses Majorana exchange potential idea to obtain the electric charge switching back and forth between a neutron-proton pair, thereby producing a current along the line joining them. Thus a modified charge current is required to fulfil a new continuity equation. This introduces a new term, completely missed out so far, in the binding energy difference in the mirror nuclei. Thus the above anomaly is resolved consistently within a completely nuclear physics framework.

**Keywords:** Okamoto-Nolen-Schiffer anomaly, Majorana exchange potential, mirror nuclei

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The Okamoto-Nolen-Schiffer anomaly is half a century old problem in nuclear physics [1,2,3]. As of today, inspite of intense amount of work, there is no consensus as to what it means. All approaches, firstly within conventional nuclear physics domain and next beyond it, within QCD and quark models etc, have been attempted.

The Okamoto-Nolen-Schiffer anomaly is the discrepancy between experiment and theory for the binding energy difference in mirror nuclei. Let us express it in terms of the quantity $\Delta E$.

$$\Delta E = M_{Z_>} - M_{Z_<} + \delta_{np} \tag{1}$$

$M_{Z_>}$ and $M_{Z_<}$ is the atomic mass of the larger and the smaller charge nuclei. $\delta_{np} = 0.782$ MeV is the n-p atomic mass difference. Experimentally $\Delta E$ is 0.764, 3.54, 7.28 and 18.83 MeV for $^3H$, $^{17}O$, $^{41}Ca$, and $^{208}Pb$ respectively. The best theoretical calculations of the energy difference $(\Delta E)_{theory}$ betwenn mirror nuclei are found to be 5-10 percent smaller than $(\Delta E)_{expt}$. The difference $(\Delta E)_{expt}$ - $(\Delta E)_{theory}$ for heavy nuclei is about 900 KeV.

To understand this problem, we revive here an old model of Sachs [4] of a phenomenological exchange potential. We know that the Majorana potential is the space exchange potential. Let us define it as [4, p. 60]

$$V = \frac{1}{2} \sum_{j,k\; j \neq k} J(\vec{x}_j - \vec{x}_k) P_{jk} \tag{2}$$

where J is a simple attractive function and P is the space exchange operator. Note that the full attractive potential produced by J is felt by nucleons in the same state only. It is known that there is no classical analogue of the Majorana potential. We get a better feel of the nature of the potential by considering the electric current and the charge density. Define charge density as

$$\rho(\vec{r}) = \sum_{k=1,A} e_k \int {\psi^2}_{r_k=r} d^{3(A-1)}\vec{r} \tag{3}$$

Here integration is over all 3(A-1) nucleon configuration space of all the particles other than the k-th. $e_k$ is the charge of the particle-k. A sum over all spins is also implied.

The equation of continuity is

$$div\vec{S} + \frac{\partial\rho}{\partial t} = 0 \tag{4}$$

The current density $\vec{S}$ is defined such that the above continuity equation holds good. For ordinary forces used in nuclear physics, both in the isopin Generalized Pauli Exclusion Principle formalism or the ordinary neutron-proton formalism, this holds true. This current given by

$$\vec{S}_0(\vec{r}) = \frac{\hbar}{2Mi}\sum_k e_k \int \{\psi^* grad_k \psi - \psi grad_k \psi^*\}_{r_k=r} d^{3(A-1)}\vec{r} \tag{5}$$

satisfies the continuity equation by virtue of the time dependent Schroedinger equation.

Problem arises, however if the potentail in the Schrondinger equation involves a space exchange operator. $\vec{S}_0$ does not satisfy the above continuity equation anymore [4,5]. A satisfactory current $\vec{S}$ is obtained [4,5.6] only by adding an appropriate quantity $\vec{S}_x$ to $\vec{S}_0$ as

$$\vec{S} = \vec{S}_0 + \vec{S}_x \tag{6}$$

where $\vec{S}_x$ is the space exchange current. They suggest a simple framework to extend this structure as [4,5,6]

$$\vec{S}_x(\vec{r}) = -\frac{ie}{\hbar}\langle\sum_{\pi\nu} \vec{r}_{\pi\nu}[\int_0^1 d\alpha\delta(\vec{r} - \vec{r}_\pi - \alpha\vec{r}_{\pi\nu})]J(\vec{r}_{\pi\nu})P_{\pi\nu}\rangle \tag{7}$$

The indices $\pi, \nu$ are labels for proton and neutron. Note that this cotribution comes only from points lying on a line connecting a proton and a neutron. This current flows on a filament connecting unlike particles. This may be interpreted as the electric charge switching back and forth between a neutron-proton pair, thereby producing a current along the line joining them [4. p. 62, 5,6].

Now without much ado, we right away have the solution of the Okamoto-Nolen-Schiffer anomaly given in eqn, (1) above. We assume that the total nuclear potential should include the above Majorana exchange potential. This brings in a new space exchange current $\vec{S}_x$ whch has been missing so far. So we add a term in eqn. (1) to take account of this discrepancy

$$\Delta E = M_{Z_>} - M_{Z_<} + \delta_{np} + \epsilon_{Sachs} \tag{8}$$

where $\epsilon_{Sachs}$ ( named after R. G. Sachs ) is a new parameter taking account of the earstwhile missing space-exchange current. We fix it to account for the above anomaly. And this is it!

## References


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