THE DYNAMICS OF LIGHT IN TELEPARALLEL BIANCHI-TYPE I UNIVERSE

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In the present study, using the Fourier analyze method and considering the Bianchi-type I spacetime, we investigate the dynamics of photon in the torsion gravity, and show that the free-space Maxwell equations give the same results. Furthermore, we also discuss the harmonic oscillator behavior of the solutions.

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1. Introduction

The Duffin-Kemmer-Petiau(DKP) equation is an eligible relativistic wave equation that describes spin-0 and spin-1 bosons with the advantage over standard relativistic equations[1]. A detailed investigation of DKP equation can be found in Refs. [2, 3, 4]. Next, Akhiezer and Berestetskii[5], in 1965, discussed an application of the DKP field to scalar QED. More recently, there have been new interests in DKP theory: it has been applied to QCD by Gribov[6] and to covariant Hamiltonian dynamics by Kanatchikov[7]. On the other hand, it has been studied in curved spacetime[1], discussed in casual approach[8] and investigated with 5D Galilean covariance[9]. There also have been given detailed proofs of the equivalance between DKP and Klein-Gordon fields[10], and some points regarding DKP interaction with electromagnetic field[11].

The torsion gravity (or teleparallel gravity) is an alternative approach to gravitation and corresponds to a gauge theory for the translation group based on Weitzenböck geometry[12]. In this theory, gravitation is attributed to torsion[13] which plays the role of a force[14], and the curvature tensor vanishes identically. The interesting place of torsion gravity is that, due to its gauge structure, it can reveal a more convenient approach to consider some specific problems.

The wave equation for spin-0 and spin-1 bosons in torsion gravity is defined as[15]

$$\left\{ i\beta^{\mu}\left(\partial_{\mu}-\frac{1}{2}K_{\mu\alpha\beta}S^{\alpha\beta}\right)-m\right\}\Psi=0. \tag{1}$$

Here $K_{\mu\alpha\beta}$ and $S^{\alpha\beta}$ are the torsion tensor and spin tensor, respectively. Next, the β-matrices obey the following algebraic relations

$$\beta^{(a)}\beta^{(b)}\beta^{(c)}+\beta^{(c)}\beta^{(b)}\beta^{(a)}=\beta^{(a)}\eta^{(b)(c)}+\beta^{(c)}\eta^{(b)(a)}. \tag{2}$$

Here $a,b,c=0,1,2,3$, and η^{ab} is the metric tensor of Minkowski spacetime with signature $(+,-,-,-)$. The Latin alphabet will be used to indicate Minkowski indexes, while Riemann-Cartan indexes will be indicated by Greek letters.

The DKP equation is very similar to Dirac's equation but the algebraic properties of β^{a} matrices, which have no inverses, make it more difficult to deal with. These matrices are given by the definition:

$$\beta^{\mu}=\gamma^{\mu}\otimes\mathrm{I}+\mathrm{I}\otimes\gamma^{\mu}, \tag{3}$$

and they are related to flat Minkowski spacetime as $\beta^{\mu}(x)=h^{\mu}_{(i)}\widetilde{\beta}^{(i)}$ with a tetrad frame that satisfies

$$g_{\mu\nu}=h^{(i)}_{\mu}h^{(j)}_{\nu}\eta_{(i)(j)}. \tag{4}$$

In relativistic quantum mechanics, the counterpart of the Maxwell equations can be described by taking zero-mass limit of the DKP equation. Unal, in 1997, showed that the wave equation of massless spin-1 particle in flat space-time is equivalent to free space Maxwell equations[16]. Then, Unal and Sucu solved the general relativistic massless-DKP (mDKP) equations in Robertson-Walker space-time written in spherical coordinates[17]. By using the same technique, in Einstein's theory of general relativity, the mDKP equation had been solved for various spacetimes and showed the mDKP equation is equivalent to free space Maxwell equations[18, 19, 20, 21]. In the method, the β-matrices are written as a direct product of Pauli spin matrices with unit matrix and this definition leads to a spinor which is related to complex combination of the electric and magnetic fields. On the other hand, the quantum mechanical solution is important to discuss the wave-particle duality of electromagnetic fields, since the particle nature of the electromagnetic field can be analyzed only by a quantum mechanical equation. Furthermore, the mDKP equation removes the unavoidable usage of $(3+1)$D spacetime splitting formalism for the Maxwell equations[22].

The mDKP equation in torsion gravity is given as

$$i\beta^{\mu}\left(\partial_{\mu}-\frac{1}{2}K_{\mu\alpha\beta}S^{\alpha\beta}\right)\Psi=0, \tag{5}$$

where β^{μ} are now:

$$\beta^{\mu}=\sigma^{\mu}\otimes\mathrm{I}+\mathrm{I}\otimes\sigma^{\mu}, \tag{6}$$

with $\sigma^\mu = (I, \overrightarrow{\sigma})$. Next, the spin tensor can be defined as

$$4S^{\mu\nu} = \{\beta^\mu, \beta^\nu\}, \tag{7}$$

and the torsion tensor is written as

$$K_\mu^{(b)(a)} = -K_\mu^{(a)(b)} = h^{\alpha(a)} h^{\beta(b)} K_{\mu\alpha\beta}, \tag{8}$$

or

$$K_\mu^{(b)(a)} = h^{\nu(b)} \left\{ \Gamma^\alpha_{\mu\nu} h^{(a)}_\alpha - \partial_\mu h^{(a)}_\nu \right\}, \tag{9}$$

with Christoffel symbols

$$\Gamma^\alpha_{\mu\nu} = \frac{1}{2} g^{\alpha\beta} (\partial_\mu g_{\beta\nu} + \partial_\nu g_{\beta\mu} - \partial_\beta g_{\mu\nu}). \tag{10}$$

In the present work, we investigate the behavior of the massless spin-1 particles by examining mDKP equation in the Bianchi-type I universe in teleparallel gravity. The work is organized as follow: in the next section investigate the mDKP equation in torsion gravity explicitly and obtain its second order form for a given geometry. In section 3, we discuss the free-space Maxwell equations the Bianchi-type I universe. In section 4, we find oscillating frequency of the photon. Finally, we discuss our results.

2. Massless spin-1 particles in torsion gravity

The line-element of the Bianchi-type I universe is

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 dy^2 + C^2 dz^2, \tag{11}$$

where A, B and C are functions of t alone and these expansion factors could be determined via field equations in Einstein's theory of general relativity or torsion gravity. We know that the non-trivial tetrad field induces a torsion gravity structure on spacetime which is directly related to the presence of the gravitational field. Using the relation (4), we obtain the tetrad components:

$$h^a{}_\mu = \delta^a_0 \delta^0_\mu + A \delta^a_1 \delta^1_\mu + B \delta^a_2 \delta^2_\mu + C \delta^a_3 \delta^3_\mu \tag{12}$$

and its inverse is

$$h_a{}^\mu = \delta^0_a \delta^\mu_0 + A^{-1} \delta^1_a \delta^\mu_1 + B^{-1} \delta^2_a \delta^\mu_2 + C^{-1} \delta^3_a \delta^\mu_3 \tag{13}$$

The line-element given by eqn. (11) can be reduced to the flat Friedmann-Robertson-Walker line element in a special case. Defining $A = B = C = T(t)$ and transforming the line-element (11) from t, x, y, z coordinates to the spherical coordinates, we obtain

$$ds^2 = dt^2 - T^2(t)[dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)], \tag{14}$$

according to:

$$\begin{aligned} x &= r\sin\theta\cos\phi, \\ y &= r\sin\theta\sin\phi, \\ z &= r\cos\theta. \end{aligned} \tag{15}$$

The Friedmann-Robertson-Walker spacetime has received considerable interests in the relativistic cosmology. Maybe one of the most important features of this model is, as predicted by inflation[23, 24, 25], the flatness which agrees with the observed cosmic microwave background radiation.

By using the definition of Christoffel symbols given by the equation (10), the non-vanishing components are found as

$$\Gamma^0_{11} = A\dot{A}, \qquad \Gamma^0_{22} = B\dot{B}, \tag{16}$$

$$\Gamma^0_{33} = C\dot{C}, \qquad \Gamma^1_{01} = \Gamma^1_{10} = \frac{\dot{A}}{A}, \tag{17}$$

$$\Gamma^2_{02} = \Gamma^2_{20} = \frac{\dot{B}}{B}, \qquad \Gamma^3_{03} = \Gamma^3_{30} = \frac{\dot{C}}{C}, \tag{18}$$

where a dot indicates the derivative with respect to t. Thence, the surviving components of the torsion tensor are obtained as

$$K_1^{(0)(1)} = -K_1^{(1)(0)} = -\dot{A}, \tag{19}$$

$$K_2^{(0)(2)} = -K_2^{(2)(0)} = -\dot{B}, \tag{20}$$

$$K_3^{(0)(3)} = -K_3^{(3)(0)} = -\dot{C}, \tag{21}$$

or in another form we find

$$K_{110} = K_{101} = A\dot{A}, \tag{22}$$

$$K_{220} = K_{202} = B\dot{B}, \tag{23}$$

$$K_{330} = K_{303} = C\dot{C}. \tag{24}$$

By making use of eqn. (6) we get

$$\beta^0 = 2(I \otimes I), \tag{25}$$

$$\beta^1 = \sigma^1 \otimes I + I \otimes \sigma^1, \tag{26}$$

$$\beta^2 = \sigma^2 \otimes I + I \otimes \sigma^2, \tag{27}$$

$$\beta^3 = \sigma^3 \otimes I + I \otimes \sigma^3. \tag{28}$$

Now, we obtain the mDKP equation as:

$$\Big[\beta^0 \partial_t + \beta^1 (\partial_x + \dot{A} A S^{01}) + \beta^2 (\partial_y + \dot{B} B S^{02}) + \beta^3 (\partial_z + \dot{C} C S^{03})\Big] \Psi = 0, \tag{29}$$

where

$$\beta^{(0)} = \widetilde{\beta}^{(0)}, \qquad \beta^{(1)} = \frac{1}{A} \widetilde{\beta}^{(1)},$$
$$\beta^{(2)} = \frac{1}{B} \widetilde{\beta}^{(2)}, \qquad \beta^{(3)} = \frac{1}{C} \widetilde{\beta}^{(3)}, \tag{30}$$

and

$$S^{01}=\frac{1}{A}\widetilde{S}^{(01)},\qquad S^{02}=\frac{1}{B}\widetilde{S}^{(02)},$$
$$S^{03}=\frac{1}{C}\widetilde{S}^{(03)}. \tag{31}$$

Then, eqn. (29) can be re-written as

$$\begin{aligned}[2(I\otimes I)\partial_t &+ \frac{1}{A}(\sigma^{(1)}\otimes I+I\otimes\sigma^{(1)})(\partial_x+\dot{A}\widetilde{S}^{01})\\ &+\frac{1}{B}(\sigma^{(2)}\otimes I+I\otimes\sigma^{(2)})(\partial_y+\dot{B}\widetilde{S}^{02})\\ &+\frac{1}{C}(\sigma^{(3)}\otimes I+I\otimes\sigma^{(3)})(\partial_z+\dot{C}\widetilde{S}^{03})]\Psi=0.\end{aligned} \tag{32}$$

Next, after using the standard representation of Pauli spin matrices, and defining the 4-component wave-function as

$$\Psi=\begin{pmatrix}\Xi_0\\ \Xi_1\\ \Xi_2\\ \Xi_3\end{pmatrix}. \tag{33}$$

Eqn. (32) gives the following equations

$$\left(2\partial_t+\frac{2}{C}\partial_z+\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{2\dot{C}}{C}\right)\Xi_0 +\left(\frac{1}{A}\partial_x-\frac{i}{B}\partial_y\right)(\Xi_1+\Xi_2)+\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right)\Xi_3=0, \tag{34}$$

$$\left(\frac{1}{A}\partial_x+\frac{i}{B}\partial_y\right)\Xi_0+\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}\right)\Xi_1 +\left(2\partial_t+\frac{\dot{A}}{A}+\frac{\dot{B}}{B}\right)\Xi_2+\left(\frac{1}{A}\partial_x-\frac{i}{B}\partial_y\right)\Xi_3=0, \tag{35}$$

$$\left(\frac{1}{A}\partial_x+\frac{i}{B}\partial_y\right)\Xi_0+\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}\right)\Xi_2 +\left(2\partial_t+\frac{\dot{A}}{A}+\frac{\dot{B}}{B}\right)\Xi_1+\left(\frac{1}{A}\partial_x-\frac{i}{B}\partial_y\right)\Xi_3=0, \tag{36}$$

$$\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right)\Xi_0+\left(\frac{1}{A}\partial_x+\frac{i}{B}\partial_y\right)(\Xi_1+\Xi_2) +\left(2\partial_t-\frac{2}{C}\partial_z+\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{2\dot{C}}{C}\right)\Xi_3=0. \tag{37}$$

Here, one can see that we have $\Xi_1=\Xi_2$. Then, by making use of this relation, we find

$$\left(\partial_t+\frac{1}{C}\partial_z+\frac{\dot{A}}{2A}+\frac{\dot{B}}{2B}+\frac{\dot{C}}{C}\right)\Xi_0 +\left(\frac{1}{A}\partial_x-i\frac{1}{B}\partial_y\right)\Xi_1+\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right)\Xi_3=0, \tag{38}$$

$$\left(\frac{1}{A}\partial_x+\frac{i}{B}\partial_y\right)\Xi_0+2\left(\partial_t+\frac{\dot{A}}{A}+\frac{\dot{B}}{B}\right)\Xi_1 +\left(\frac{1}{A}\partial_x-\frac{i}{B}\partial_y\right)\Xi_3=0, \tag{39}$$

$$\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right)\Xi_0+\left(\frac{1}{A}\partial_x+\frac{i}{B}\partial_y\right)\Xi_1 +\left(\partial_t-\frac{1}{C}\partial_z+\frac{\dot{A}}{2A}+\frac{\dot{B}}{2B}+\frac{\dot{C}}{C}\right)\Xi_3=0. \tag{40}$$

After defining the following new wave functions

$$\begin{aligned}\Xi_0&=-\Omega^1+i\Omega^2,\\ \Xi_1&=\Xi_2=\Omega^3,\\ \Xi_3&=\Omega^1+i\Omega^2\end{aligned} \tag{41}$$

Eqns. (38), (39) and (40) transform into another forms:

$$\left(\partial_t+\frac{\dot{A}}{A}+\frac{\dot{C}}{C}\right)\Omega^2+\frac{i}{C}\partial_z\Omega^1-\frac{i}{A}\partial_x\Omega^3=0 \tag{42}$$

$$\left(\partial_t+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)\Omega^1-\frac{i}{C}\partial_z\Omega^2+\frac{i}{B}\partial_y\Omega^3=0 \tag{43}$$

$$\left(\partial_t+\frac{\dot{A}}{A}+\frac{\dot{B}}{B}\right)\Omega^3+\frac{i}{A}\partial_x\Omega^2-\frac{i}{B}\partial_y\Omega^1=0 \tag{44}$$

After this step, now we perform the following Fourier transformation,

$$\Omega^m(\overrightarrow{k},t)=\frac{1}{8\pi^3}\int e^{i\overrightarrow{k}.\overrightarrow{x}}F^m(\overrightarrow{x},t)d^3x, \tag{45}$$

where $m=1,2,3$. Hence, we find

$$\left(\partial_t+\frac{\dot{A}}{A}+\frac{\dot{C}}{C}\right)F^2-\frac{k_3}{C}F^1+\frac{k_1}{A}F^3=0, \tag{46}$$

$$\left(\partial_t+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)F^1+\frac{k_3}{C}F^2-\frac{ik_2}{B}F^3=0, \tag{47}$$

$$\left(\partial_t+\frac{\dot{A}}{A}+\frac{\dot{B}}{B}\right)F^3-\frac{k_1}{A}F^2+\frac{k_2}{B}F^1=0. \tag{48}$$

By defining

$$F^m=\frac{R^m}{\sqrt{-g}}H^m, \tag{49}$$

with $R^1=A$, $R^2=B$ and $R^3=C$, we get

$$\dot{H}^1+\frac{Bk_3}{AC}H^2-\frac{Ck_2}{AB}H^3=0, \tag{50}$$

$$\dot{H}^2-\frac{Ak_3}{BC}H^1+\frac{Ck_1}{AB}H^3=0, \tag{51}$$

$$\dot{H}^3 - \frac{Bk_1}{AC}H^2 + \frac{Ak_2}{BC}H^1 = 0. \tag{52}$$

We can write these equations in a general form[26];

$$\dot{H^m} = \frac{1}{\sqrt{-g}}\sum_{n=1}^{3}\varepsilon^{mnl}k_l(R^n)^2H^n. \tag{53}$$

To solve this equation exactly, for suitable symmetry, we can use spherical coordinates. From this point of view, H^m should be written in terms of spherical coordinates. The components of k in spherical coordinates are defined as

$$\begin{aligned} k_1 &= k\sin\theta\cos\varphi, \\ k_2 &= k\sin\theta\sin\varphi, \\ k_3 &= k\cos\theta, \end{aligned} \tag{54}$$

or in matrix representations

$$\begin{pmatrix} \sin\theta\cos\varphi & \sin\theta\sin\varphi & \cos\theta \\ \cos\theta\cos\varphi & \cos\theta\sin\varphi & -\sin\theta \\ -\sin\varphi & \cos\varphi & 0 \end{pmatrix} = \begin{pmatrix} \frac{k_1}{k} & \frac{k_2}{k} & \frac{k_3}{k} \\ \frac{k_3k_1}{kk_\perp} & \frac{k_3k_2}{kk_\perp} & -\frac{k_\perp}{k} \\ -\frac{k_2}{k_\perp} & \frac{k_1}{k_\perp} & 0 \end{pmatrix}, \tag{55}$$

where

$$k_\perp^2 = k_1^2 + k_2^2. \tag{56}$$

On the other hand, we define also

$$\begin{pmatrix} H^r \\ H^\theta \\ H^\varphi \end{pmatrix} = \begin{pmatrix} \frac{k_1}{k} & \frac{k_2}{k} & \frac{k_3}{k} \\ \frac{k_3k_1}{kk_\perp} & \frac{k_3k_2}{kk_\perp} & -\frac{k_\perp}{k} \\ -\frac{k_2}{k_\perp} & \frac{k_1}{k_\perp} & 0 \end{pmatrix} \begin{pmatrix} H^1 \\ H^2 \\ H^3 \end{pmatrix}. \tag{57}$$

Ergo we have

$$\begin{aligned} \dot{H}^r &= 0, \\ \dot{H}^\theta &= -\left(\frac{k_3^2 - k_\perp^2}{kk_\perp}\right)\dot{H}^3, \\ \dot{H}^\varphi &= -\frac{k_2}{k_\perp}\dot{H}^1 + \frac{k_1}{k_\perp}\dot{H}^2, \end{aligned} \tag{58}$$

and we obtain

$$\dot{H}^\theta = -k\left(\alpha H^\theta + \beta H^\varphi\right) \tag{59}$$

$$\dot{H}^\varphi = k\left(\gamma H^\theta + \alpha H^\varphi\right) \tag{60}$$

where

$$\sqrt{-g}\alpha = \frac{k_1k_2k_3}{kk_\perp^2}(B^2 - A^2), \tag{61}$$

$$\sqrt{-g}\beta = \frac{1}{k_\perp^2}(A^2k_2^2 + B^2k_1^2). \tag{62}$$

Here the parameter γ is determined by

$$\gamma = \frac{\Lambda^2}{\beta k^2} + \frac{\alpha^2}{\beta}, \tag{63}$$

with

$$\Lambda^2 = \sum_i\left(\frac{k_i}{R^i}\right)^2. \tag{64}$$

Λ^2 is a generalization of the dispersion relation $\omega^2 = |k|^2$. After eliminating H^φ from equations (59) and (60), we obtain the following second order differential equation:

$$\ddot{H}^\theta - \frac{\dot{\beta}}{\beta}\dot{H}^\theta + \left[\Lambda^2 + k\beta\partial_0\left(\frac{\alpha}{\beta}\right)\right]H^\theta = 0. \tag{65}$$

In an explicit form, this result can be re-written as

$$\begin{aligned} &\ddot{H}^\theta - \left[\left(\frac{\dot{A}}{B} - \frac{A\dot{B}}{B^2} - \frac{A\dot{C}}{BC}\right)k_2^2 \right. \\ &\left. + \left(\frac{\dot{B}}{A} - \frac{B\dot{A}}{A^2} - \frac{B\dot{C}}{AC}\right)k_1^2\right]\left[\frac{Ak_2^2}{B} + \frac{Bk_1^2}{A}\right]^{-1}\dot{H}^\theta \\ &+ \left\{\frac{k_1^2}{A^2} + \frac{k_2^2}{B^2} + \frac{k_3^2}{C^2} + \frac{2k_1k_2k_3}{ABCk_\perp^2}\left[B\dot{B}\right.\right. \\ &\left.\left. - A\dot{A} - \frac{(B^2 - A^2)(A\dot{A}k_2^2 + B\dot{B}k_1^2)}{A^2k_2^2 + B^2k_1^2}\right]\right\}H^\theta = 0. \end{aligned} \tag{66}$$

We mentioned before that if we define

$$A(t) = B(t) = C(t) = T(t), \tag{67}$$

the Bianchi-type I model transforms into the flat Friedmann-Robertson-Walker spacetime. Hence, under this limit, eqn. (66) is reduced to

$$T^2\ddot{H}^\theta + T\dot{T}\dot{H}^\theta + (k_1^2 + k_2^2 + k_3^2)H^\theta = 0. \tag{68}$$

Now, by defining conformal time as

$$\frac{\partial}{\partial t} = \frac{1}{T}\frac{\partial}{\partial\eta}, \tag{69}$$

we get

$$\frac{\partial^2H^\theta}{\partial\eta^2} + (k_1^2 + k_2^2 + k_3^2)H^\theta = 0. \tag{70}$$

It is easy to see that this equation has the following solution,

$$H^\theta = Me^{\mp i\sqrt{k_1^2+k_2^2+k_3^2}\eta} \tag{71}$$

where M is a normalization constant.

3. The free-space Maxwell equations in the Bianchi-type I universe

The interaction of electromagnetic and gravitational fields is described by the Maxwell equations in a given background and source. In the absence of an electromagnetic source these equations are written as

$$\frac{1}{\sqrt{-g}}(\sqrt{-g}F^{\mu\nu})_{,\nu} = 0, \tag{72}$$

and

$$F_{\mu\nu,\sigma} + F_{\sigma\mu,\nu} + F_{\nu\sigma,\mu} = 0, \tag{73}$$

where $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$. Here we investigate the Maxwell equations for the line-element given by eqn. (11) to show the correspondence between the mDKP equation in torsion gravity and the Maxwell equations. The covariant and covariant field strengths, $F^{\mu\nu}$ and $F_{\mu\nu}$, in the general coordinates are

$$\begin{aligned}
F^{01} &= \frac{1}{A}E^{(1)}, \qquad F_{01} = -AE^{(1)},\\
F^{02} &= \frac{1}{B}E^{(2)}, \qquad F_{02} = -BE^{(2)},\\
F^{03} &= \frac{1}{C}E^{(3)}, \qquad F_{03} = -CE^{(3)},\\
F^{12} &= \frac{1}{AB}B^{(3)}, \qquad F_{12} = ABB^{(3)},\\
F^{13} &= \frac{-1}{AC}B^{(2)}, \qquad F_{13} = -ACB^{(2)},\\
F^{23} &= \frac{1}{BC}B^{(1)}, \qquad F_{23} = BCB^{(1)},
\end{aligned} \tag{74}$$

where $E^{(i)}$ and $B^{(i)}$ are the components of the electric and magnetic fields in the local Lorentz frame. In terms of these components, Maxwell equations can be expanded as

$$\partial_x \begin{pmatrix} E^{(1)} \\ B^{(1)} \end{pmatrix} + \partial_y \begin{pmatrix} E^{(2)} \\ B^{(2)} \end{pmatrix} + \partial_z \begin{pmatrix} E^{(3)} \\ B^{(3)} \end{pmatrix} = 0, \tag{75}$$

$$[BC\partial_t + \partial_t(BC)] \begin{pmatrix} B^{(1)} \\ -E^{(1)} \end{pmatrix} + C\partial_y \begin{pmatrix} E^{(3)} \\ B^{(3)} \end{pmatrix} - B\partial_z \begin{pmatrix} E^{(2)} \\ B^{(2)} \end{pmatrix} = 0, \tag{76}$$

$$[AC\partial_t + \partial_t(AC)] \begin{pmatrix} B^{(2)} \\ E^{(2)} \end{pmatrix} - C\partial_x \begin{pmatrix} E^{(3)} \\ -B^{(3)} \end{pmatrix} + A\partial_z \begin{pmatrix} E^{(1)} \\ -B^{(1)} \end{pmatrix} = 0, \tag{77}$$

$$[AB\partial_t + \partial_t(AB)] \begin{pmatrix} B^{(3)} \\ -E^{(3)} \end{pmatrix} - A\partial_y \begin{pmatrix} E^{(1)} \\ B^{(1)} \end{pmatrix} + B\partial_x \begin{pmatrix} E^{(2)} \\ B^{(2)} \end{pmatrix} = 0. \tag{78}$$

By defining a complex spinor Ξ:

$$\Xi = \begin{pmatrix} \Xi^1 \\ \Xi^2 \\ \Xi^3 \end{pmatrix} = \begin{pmatrix} E^{(1)} + iB^{(1)} \\ E^{(2)} + iB^{(2)} \\ E^{(3)} + iB^{(3)} \end{pmatrix}, \tag{79}$$

and making a suitable Fourier transformation:

$$\Xi^m(\overrightarrow{k}, t) = \frac{1}{(2\pi)^3} \int d^3x e^{i\overrightarrow{k}.\overrightarrow{x}} \Im^m(\overrightarrow{x}, t), \tag{80}$$

for the spinor form of the Maxwell equations we find

$$k_1\Im^1 + k_2\Im^2 + k_3\Im^3 = 0, \tag{81}$$

$$\left(\partial_t + \frac{\dot{A}}{A} + \frac{\dot{C}}{C}\right)\Im^2 - \frac{1}{C}k_3\Im^1 + \frac{1}{A}k_1\Im^3 = 0, \tag{82}$$

$$\left(\partial_t + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right)\Im^1 + \frac{1}{C}k_3\Im^2 - \frac{1}{B}k_2\Im^3 = 0, \tag{83}$$

$$\left(\partial_t + \frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right)\Im^3 - \frac{1}{A}k_1\Im^2 + \frac{1}{B}k_2\Im^1 = 0. \tag{84}$$

To eliminate $\frac{\dot{A}}{A} + \frac{\dot{C}}{C}$, $\frac{\dot{B}}{B} + \frac{\dot{C}}{C}$ and $\frac{\dot{A}}{A} + \frac{\dot{B}}{B}$ terms, we can define:

$$\Im^i = \frac{R^i}{\sqrt{-g}}\Re^i, \qquad (i = 1, 2, 3) \tag{85}$$

with $R^1 = A$, $R^2 = B$ and $R^3 = C$. Thence, we get

$$\dot{\Re}^1 + \frac{B}{AC}k_3\Re^2 - \frac{C}{AB}k_2\Re^3 = 0 \tag{86}$$

$$\dot{\Re}^2 - \frac{A}{BC}k_3\Re^1 + \frac{C}{AB}k_1\Re^3 = 0 \tag{87}$$

$$\dot{\Re}^3 - \frac{B}{AC}k_1\Re^2 + \frac{A}{BC}k_2\Re^1 = 0 \tag{88}$$

These three equations are exactly the same results as obtained torsion gravity (see eqns. (50), (51) and (52)).

4. The oscillation region

A general method to find the frequency spectrum is to impose the condition on functions which are the solutions of differential equation. The functions must be bounded for all values as usually done in quantum mechanics, this procedure gives the quantization of frequency. Since we didn't solve the mDKP equation exactly, we will restrict ourselves to discuss how we can obtain the oscillation region of the photon.

The general method for obtaining the oscillation region is to write the differential equation that does not include the first derivative and simulate this to the second order differential equation that describes the harmonic oscillator. Here we define

$$H^\theta(t) = \beta^{\frac{1}{2}}(t)\Phi(t). \tag{89}$$

Next, by making use of this definition in eqn. (65), we obtain

$$\ddot{\Phi}(t) + w^2(t)\Phi(t) = 0, \tag{90}$$

where

$$w^2 = \frac{\ddot{\beta}}{2\beta} - \frac{3}{4}\left(\frac{\dot{\beta}}{\beta}\right)^2 + \Lambda^2 + k\beta\partial_0\left(\frac{\alpha}{\beta}\right). \tag{91}$$

Hence the oscillation region is:

$$-\sqrt{\frac{\ddot{\beta}}{2\beta}-\frac{3}{4}\left(\frac{\dot{\beta}}{\beta}\right)^2+\Lambda^2+k\beta\partial_0\left(\frac{\alpha}{\beta}\right)}<\omega$$
$$<\sqrt{\frac{\ddot{\beta}}{2\beta}-\frac{3}{4}\left(\frac{\dot{\beta}}{\beta}\right)^2+\Lambda^2+k\beta\partial_0\left(\frac{\alpha}{\beta}\right)}. \quad (92)$$

5. Conclusions

In this work we mainly focused on the Duffin-Kemmer-Petiau theory in torsion gravity for the generalized Bianchi-type I universe. We have obtained a second order relativistic wave-equation that describes massless spin-1 particles coupled to the gravitational field. The method of separation of variables and the Fourier transformation has been used due to the symmetry of the generalized Bianchi-type I spacetime. Also, the oscillatory behavior of the result has been discussed. Furthermore, we show that the massless Duffin-Kemmer-Petiau equation in torsion gravity and the free-space Maxwell equations agree with each other and give the same results. This interesting feature strongly motivates us to use the massless Duffin-Kemmer-Petiau equation in torsion gravity to investigate the dynamics of light. Another motivation is that the results obtained can be used to discuss the Photon production in some special spacetime models.

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