THERMODYNAMICS OF CHAPLYGIN GAS INTERACTING WITH COLD DARK MATTER

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The main goal of the present work is to investigate the validity of the second law of gravitational thermodynamics in an expanding Gödel-type universe filled with generalized Chaplygin gas interacting with cold dark matter. By assuming the Universe as a thermodynamical system bounded by the apparent horizon, and calculating separately the entropy variation for generalized Chaplygin gas, cold dark matter and for the horizon itself, we obtained an expression for the time derivative of the total entropy. We conclude that the 2nd law of gravitational thermodynamics is conditionally valid in the cosmological scenario where the generalized Chaplygin gas interacts with cold dark matter.

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I. INTRODUCTION

In black hole thermodynamics, the entropy and temperature are proportional to the area of the horizon and the surface gravity at the horizon, respectively[1, 2]. The entropy, temperature and mass satisfy the 1st law of thermodynamics[3] for a black hole. This interesting feature encourages theoretical physicists to find a definition describing the relationship between the black hole thermodynamics and Einstein's field equations. Bekenstein, in 1973, found a relation between the thermodynamics of a black hole and the event horizon[4]. In black hole physics, an event horizon is the measure of entropy which means each horizon corresponds to an entropy. Furthermore, Wang with his collaborators[5], in 2006, obtained interesting results for the first and second laws of thermodynamics. They emphasized that our universe should be non-static, however the usual description of the thermodynamical quantities on the event horizon may be more complicated than in the static spacetime.

Cai and Kim[6] showed, in general relativity, that the Friedman equation can be written in the form of the first law of thermodynamics:

$$-dE = T_A dS_h \tag{1}$$

on the dynamical apparent horizon $\widetilde{r}_A$. Here, Cai and Kim assumed that $T_A = (2\pi\widetilde{r}_A)^{-1}$, $S_h = \pi\widetilde{r}_A^2 G^{-1}$ and dE are the Hawking temperature, the horizon entropy and the amount of the internal energy flow through horizon, respectively[6]. Later, Friedman equation in general relativity was written in another form[7]

$$dE = TdS + WdV \tag{2}$$

at dynamical apparent horizon, where $E = \rho V$ and $W = \frac{1}{2}(\rho - p)$ are the internal energy and work density, respectively.

The Chaplygin gas is one of the famous issues of current interest in modern cosmology. In their interesting paper Kamenshchik *et al.*[8] have considered the Chaplygin gas obeying the following equation of state

$$p^c = -\frac{\Lambda}{\rho^c}, \tag{3}$$

where p^c and ρ^c are respectively pressure and energy density in comoving reference frame, with $\rho^c > 0$; Λ is a positive constant[9]. After this result, a certain interest[10–13] has been raised in literature because of its many interesting and intriguingly unique features[9]. Also, this model is investigated from the field theory points of view[14]. On the other hand, the Chaplygin gas emerges as an effective fluid related to d-branes[15, 16] and can also be defined using the Born-Infeld action[17].

In the present work, we suggest a correspondence between cold dark matter scenario and the Chaplygin gas dark energy model. Type Ia supernovae observations[2, 18–20] indicates that the matter in our universe is dominated by two enigmatic components: dark energy and dark matter. The dark energy is an exotic matter with large negative pressure (above 73 percent) and the dark matter is an invisible matter without pressure (about 23 percent of the universe). The remaining part (about 4 percent) is occupied by some other cosmic matters. It is commonly believed that our universe has a phase transition[21] from decelerating to accelerating and expands with accelerating velocity. This interesting feature of the universe is caused by two mysterious dark components. There are several proposals to be a candidate for dark part of the universe, but still the nature of dark universe is completely unknown[22]. The cosmological constant is the best instrument to identify this nature of the universe, but it causes some other difficulties like fine-tuning and cosmic-coincidence puzzle[23]

The plan of the work is as follows: in then next section, we construct the scenario where generalized Chaplygin gas interacts with cold dark matter. Section III is devoted to study the 2nd law of gravitational thermodynamics for our cosmological model by considering the

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universe as a system bounded by the apparent horizon. Finally, in section IV, we give conclusions and the summary of the obtained results.

II. CHAPLYGIN GAS INTERACTING WITH COLD DARK MATTER

One of the famous solutions of general relativistic field equations with cosmological constant for incoherent matter was obtained by Gödel in 1949[24]. The line-element representation of the expanding Gödel-type spacetimes is written as

$$\begin{aligned} ds^2 &= -dt^2 - 2a\lambda_2 e^{nx}dtdy \\ &\quad +a^2[dx^2 + \lambda_1 e^{2nx}dy^2 + dz^2], \end{aligned} \tag{4}$$

here the scale factor a depends on t. When $\lambda_1 = 1$ and $\lambda_2 = n = 0$, the expanding Gödel-type model reduces to the flat Friedmann-Robetson-Walker universe. Thus the expanding Gödel-type universe is the generalization of the flat Friedmann-Robetson-Walker model. By considering the line-element (4), the metric tensor $g_{\mu\nu}$ is written as

$$\begin{aligned} g_{\mu\nu} &= -\delta^0_\mu\delta^0_\nu + a^2\delta^1_\mu\delta^1_\nu + \lambda_1 a^2 e^{2nx}\delta^2_\mu\delta^2_\nu + a^2\delta^3_\mu\delta^3_\nu \\ &\quad -a\lambda_2 e^{nx}[\delta^0_\mu\delta^2_\nu + \delta^2_\mu\delta^0_\nu], \end{aligned} \tag{5}$$

and its inverse $g^{\mu\nu}$ is

$$\begin{aligned} g^{\mu\nu} &= -\frac{\lambda_1}{\lambda_1+\lambda_2^2}\delta^\mu_0\delta^\nu_0 + \frac{1}{a^2}\delta^\mu_1\delta^\nu_1 + \frac{e^{-2nx}}{a^2(\lambda_1+\lambda_2^2)}\delta^\mu_2\delta^\nu_2 \\ &\quad +\frac{1}{a^2}\delta^\mu_3\delta^\nu_3 - \frac{\lambda_2 e^{-nx}}{a^2(\lambda_1+\lambda_2^2)}[\delta^0_\mu\delta^2_\nu + \delta^2_\mu\delta^0_\nu]. \end{aligned} \tag{6}$$

The surviving components of the Christoffel symbols

$$2\Gamma^\alpha_{\mu\nu} = g^{\alpha\beta}(\partial_\mu g_{\beta\nu} + \partial_\nu g_{\beta\mu} - \partial_\beta g_{\mu\nu}) \tag{7}$$

are

$$\Gamma^0_{00} = \frac{\dot{a}\lambda_2^2}{(\lambda_1+\lambda_2^2)a}, \qquad \Gamma^0_{10} = \Gamma^0_{01} = \frac{n\lambda_2^2}{2(\lambda_1+\lambda_2^2)}, \tag{8}$$

$$\Gamma^0_{11} = \Gamma^0_{33} = \frac{\lambda_1 a\dot{a}}{\lambda_1+\lambda_2^2}, \qquad \Gamma^0_{20} = \Gamma^0_{02} = -\frac{\dot{a}\lambda_1\lambda_2 e^{nx}}{\lambda_1+\lambda_2^2}, \tag{9}$$

$$\Gamma^0_{21} = \Gamma^0_{12} = -\frac{na\lambda_1\lambda_2 e^{nx}}{2(\lambda_1+\lambda_2^2)}, \qquad \Gamma^0_{22} = \frac{a\dot{a}\lambda_1^2 e^{2nx}}{\lambda_1+\lambda_2^2}, \tag{10}$$

$$\Gamma^1_{10} = \Gamma^1_{01} = \Gamma^3_{30} = \Gamma^3_{03} = \frac{\dot{a}}{a}, \qquad \Gamma^1_{20} = \Gamma^1_{02} = \frac{n\lambda_2 e^{nx}}{2a} \tag{11}$$

$$\Gamma^1_{22} = -n\lambda_1 e^{2nx}, \qquad \Gamma^2_{00} = -\frac{\dot{a}\lambda_2 e^{-nx}}{a^2(\lambda_1+\lambda_2^2)}, \tag{12}$$

$$\Gamma^2_{10} = \Gamma^2_{01} = -\frac{n\lambda_2 e^{-nx}}{2a^2(\lambda_1+\lambda_2^2)}, \qquad \Gamma^2_{11} = -\frac{\dot{a}\lambda_2 e^{-nx}}{\lambda_1+\lambda_2^2}, \tag{13}$$

$$\Gamma^2_{20} = \Gamma^2_{02} = \frac{\dot{a}\lambda_1}{a(\lambda_1+\lambda_2^2)}, \qquad \Gamma^2_{21} = \Gamma^2_{12} = \frac{n(2\lambda_1+\lambda_2^2)}{2(\lambda_1+\lambda_2^2)}, \tag{14}$$

$$\Gamma^2_{22} = \frac{\lambda_1\lambda_2\dot{a}e^{nx}}{\lambda_1+\lambda_2^2}, \qquad \Gamma^2_{33} = \frac{\dot{a}\lambda_2 e^{-nx}}{\lambda_1+\lambda_2^2}. \tag{15}$$

Next, the non-vanishing components of the Ricci tensor

$$R_{\alpha\beta} = \partial_\rho\Gamma^\rho_{\beta\alpha} - \partial_\beta\Gamma^\rho_{\rho\alpha} + \Gamma^\rho_{\rho\lambda}\Gamma^\lambda_{\beta\alpha} - \Gamma^\rho_{\beta\lambda}\Gamma^\lambda_{\rho\alpha} \tag{16}$$

are given as

$$R_{00} = \frac{(n^2+4\dot{a}^2)\lambda_2^2 - 2a\ddot{a}(3\lambda_1+2\lambda_2^2)}{2a^2(\lambda_1+\lambda_2^2)}, \tag{17}$$

$$R_{10} = R_{01} = \frac{n\dot{a}\lambda_2^2}{a(\lambda_1+\lambda_2^2)}, \tag{18}$$

$$R_{11} = \frac{4\dot{a}^2\lambda_1 - n^2(2\lambda_1+\lambda_2^2) + 2\lambda_1 a\ddot{a}}{2(\lambda_1+\lambda_2^2)}, \tag{19}$$

$$R_{20} = R_{02} = \frac{\lambda_2 e^{nx}}{2a(\lambda_1+\lambda_2^2)}(n^2\lambda_2^2 - 4\lambda_1\dot{a}^2 - 2\lambda_1 a\ddot{a}), \tag{20}$$

$$R_{21} = R_{12} = -\frac{n\dot{a}\lambda_1\lambda_2 e^{nx}}{\lambda_1+\lambda_2^2}, \tag{21}$$

$$R_{22} = -\frac{\lambda_1 e^{2nx}}{2(\lambda_1+\lambda_2^2)}[n^2(2\lambda_1+3\lambda_2^2) - 4\lambda_1\dot{a}^2 - 2\lambda_1 a\ddot{a}], \tag{22}$$

$$R_{33} = \frac{\lambda_1}{\lambda_1+\lambda_2^2}(2\dot{a}^2 + a\ddot{a}). \tag{23}$$

Hence, we find the Ricci (curvature) scalar as

$$R = g^{\mu\nu}R_{\mu\nu} = \frac{12\dot{a}^2\lambda_1 - n^2(4\lambda_1+3\lambda_2^2) + 12\lambda_1 a\ddot{a}}{2a^2(\lambda_1+\lambda_2^2)}. \tag{24}$$

Next, the energy-momentum tensor is defined as

$$T^\mu_\nu = T^{\mu(c)}_\nu + T^{\mu(m)}_\nu, \tag{25}$$

where the superscripts c ad m denote Chaplygin gas and cold dark matter, respectively, and the corresponding energy-momentum tensors are given as follows:

$$T^{\mu(c)}_\nu = (-\rho^c, p^c, p^c, p^c), \tag{26}$$

$$T^{\mu(m)}_\nu = (-\rho^m, p^m, p^m, p^m). \tag{27}$$

In this section, we consider the generalized Chaplygin gas when there is an interaction between generalized Chaplygin gas energy density ρ^c and a cold dark matter with $\omega^m = 0$.

General relativistic field equations (with assuming $8\pi G = 1$ and $c = 1$)

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = -T_{\mu\nu}, \tag{28}$$

for the line-element (4) leads to the following system of equations (for details check appendix)

$$\begin{aligned}\rho^c + \rho^m &= \frac{4(3\lambda_1 + 2\lambda_2^2)\dot{a}^2 - n^2(4\lambda_1 + \lambda_2^2)}{(\lambda_1 + \lambda_2^2)a^2} \\ &- \frac{2\lambda_2^2 \ddot{a}}{(\lambda_1 + \lambda_2^2)a},\end{aligned} \tag{29}$$

$$\frac{n^2\lambda_2^2 - 4\lambda_1(\dot{a}^2 + 2a\ddot{a})}{4(\lambda_1 + \lambda_2^2)} = -p^c, \tag{30}$$

$$\frac{3n^2\lambda_1\lambda_2^2 e^{2nx}}{4(\lambda_1 + \lambda_2^2)} + \frac{\lambda_1^2(\dot{a}^2 + 2a\ddot{a})e^{2nx}}{\lambda_1 + \lambda_2^2} = p^c, \tag{31}$$

$$\frac{n^2(3\lambda_2^2 + 4\lambda_1)}{4(\lambda_1 + \lambda_2^2)} - \frac{\lambda_1(\dot{a}^2 + 2a\ddot{a})}{\lambda_1 + \lambda_2^2} = -p^c, \tag{32}$$

where, for the scale factor a, an overdot means the first and double overdot means the second derivative with respect to time. The continuity equations, $T^{\mu\nu}_{;\nu} = 0$, for Chaplygin gas and Cold Dark Matter, are

$$\dot{\rho}^c + 3\frac{\dot{a}}{a}(1 + \omega^c)\rho^c = -\Sigma, \tag{33}$$

$$\dot{\rho}^m + 3\frac{\dot{a}}{a}\rho^m = \Sigma, \tag{34}$$

Here, the interaction is defined by the quantity $\Sigma = \Gamma\rho^c$[25]. Also, $\Sigma < 0$ corresponds to energy transfer from the cold dark matter sector to the other constituent. On the other hand, in case $\Sigma > 0$, there is an energy transfer from Chaplygin gas sector to cold dark matter sector. This is a decaying of the Chaplygin gas component into cold dark matter with the decay rate Γ. We take a ratio of two energy densities as $r = \rho^m/\rho^c$. Next, we define the mean expansion rate as an average Hubble rate $H = \frac{\dot{a}}{a}$. After following Ref. [26], if we assume

$$\omega^c_{\text{eff}} = \omega^c + \frac{\Gamma}{3H}, \qquad \omega^m_{\text{eff}} = \frac{-\Gamma}{3rH}, \tag{35}$$

then the continuity equations can be given in their standard form

$$\dot{\rho}^c + 3H(1 + \omega^c_{\text{eff}})\rho^c = 0, \tag{36}$$

$$\dot{\rho}^m + 3H(1 + \omega^m_{\text{eff}})\rho^m = 0. \tag{37}$$

In the generalized Chaplygin gas approach[17], the equation of state[27], $p = \frac{-\Lambda}{\rho}$ where Λ is a positive constant, is generalized to $p = \frac{-\Lambda}{\rho^\alpha}$. By considering this relation, one can find

$$\omega^c = \frac{p^c}{\rho^c} = \frac{-\Lambda}{(\rho^c)^{\alpha+1}}. \tag{38}$$

Hence, we have the effective equation of state for the generalized Chaplygin gas as

$$\omega^c_{\text{eff}} = \frac{\Gamma}{3H} - \frac{\Lambda}{(\rho^c)^{\alpha+1}}. \tag{39}$$

Here as in Ref. [5], the decay rate is given by

$$\Gamma = 3Hb^2(1 + r) \tag{40}$$

with the coupling constant b^2.

III. THE SECOND LAW OF GRAVITATIONAL THERMODYNAMICS

In this section, we discuss the validity of the generalized second law of thermodynamics in the expanding Gödel-type spacetime bounded by an apparent horizon with size $\widetilde{r}_A$ which coincides with the Hubble horizon in the case of a flat geometry, i.e. $\widetilde{r}_A = \frac{1}{H}$. The first law of thermodynamics gives

$$T_A dS = pdV + dE, \qquad dS = \frac{pdV + dE}{T_A}, \tag{41}$$

where T_A, S, E and p are the temperature, entropy, internal energy and pressure of the system, respectively. The corresponding entropies will become

$$dS^c = \frac{p^c dV + dE^c}{T_A}, \qquad dS^m = \frac{dE^m}{T_A}, \tag{42}$$

where p^c, p^m, E^c and E^m are the pressures and internal energies of Chaplygin gas and cold dark matter, respectively. Also, we assume that the system is in equilibrium, which implies that all the components of the system have the same temperature[4]. Thermodynamical quantities are related to the cosmological quantities by the following definitions

$$p^c = \omega^c_{\text{eff}}\rho^c, \tag{43}$$

$$E^c = \sqrt{\lambda_1} e^{nx} a^3 \rho^c, \qquad E^m = \sqrt{\lambda_1} e^{nx} a^3 \rho^m, \tag{44}$$

where $V = \sqrt{\lambda_1} e^{nx} a^3$ is the volume of the system containing all the matter. On the other hand, the entropy of the horizon is defined as $S_h = \frac{kA}{4}$, where $A = 4\pi\widetilde{r}_A^2$ is the surface area of the black hole and k is the Boltzmann constant. So that, we find

$$S_h = k\pi\widetilde{r}_A^2, \tag{45}$$

$$\dot{S}_h = 2k\pi\widetilde{r}_A\dot{\widetilde{r}}_A. \tag{46}$$

Also, the time derivation of equation (42) yields

$$\dot{S}^c = \frac{1}{T_A}\left(3p^c\dot{a}a^2\sqrt{\lambda_1}e^{nx} + \dot{E}^c\right). \tag{47}$$

$$\dot{S}^m = \frac{\dot{E}^m}{T_A}. \tag{48}$$

At this point, we have to connect the temperature of the fluids T, which is equal to that of the apparent horizon T_A, with the geometry of the universe. The temperature of the horizon in terms of its radius is described by[28–31]

$$T_A = \frac{1}{2\pi\widetilde{r}_A}. \tag{49}$$

Now by making use of equations (36), (37), (43), (44) and (46), we obtain

$$\dot{S}_{total} = \dot{S}_h + \dot{S}^c + \dot{S}^m = -2k\pi\frac{\dot{H}}{H^3} + \frac{2\pi V}{K}\Gamma\rho^c. \tag{50}$$

The second term given in equation (50) may be interpreted as entropy production term due to the interaction between the Chaplygin gas and cold dark matter.

Furthermore, by using equation (29), we calculate

$$\begin{aligned} 4(3\lambda_1 + \lambda_2^2)H^2 &= (\lambda_1 + \lambda_2^2)(\rho^c + \rho^m) \\ &+ (4\lambda_1 + \lambda_2^2)\frac{n^2}{a^2} - 2\lambda_2^2\frac{\ddot{a}}{a}, \end{aligned} \tag{51}$$

and by making use of this result and equations (36)-(40) we obtain

$$\begin{aligned} \dot{S}_{total} &= 6\pi\sqrt{\lambda_1}e^{nx}a^3b^2(1+r)\rho^c \\ &+ \frac{k\pi(\lambda_1+\lambda_2^2)}{4(3\lambda_1+\lambda_2^2)H^4}\left[3H\left(\rho^c + \rho^m - \frac{\Lambda}{(\rho^c)^\alpha}\right)\right. \\ &\qquad + \frac{2Hn^2(4\lambda_1+\lambda_2^2)}{(\lambda_1+\lambda_2^2)a^2} \\ &\qquad \left. - \frac{2\lambda_2^2}{\lambda_1+\lambda_2^2}\left(\frac{\dot{\ddot{a}}}{a} - H\frac{\ddot{a}}{a}\right)\right], \end{aligned} \tag{52}$$

with

$$3\lambda_1 + \lambda_2^2 \geq 0. \tag{53}$$

Also, equation (52) and $\dot{S}_t \geq 0$ implies that

$$\begin{aligned} 3H\left(\rho^c + \rho^m - \frac{\Lambda}{(\rho^c)^\alpha}\right) &\geq \frac{2\lambda_2^2}{\lambda_1+\lambda_2^2}\left(\frac{\dot{\ddot{a}}}{a} - H\frac{\ddot{a}}{a}\right) \\ &- \frac{2Hn^2(4\lambda_1+\lambda_2^2)}{(\lambda_1+\lambda_2^2)a^2} \\ &+ \frac{8H^3V\Gamma}{k}\frac{(3\lambda_1+\lambda_2^2)}{\lambda_1+\lambda_2^2}\rho^c. \end{aligned} \tag{54}$$

Now, we consider some interesting cases involving the condition on different parameters.

- *Case I.* If $3\lambda_1 + \lambda_2^2 = 0$:

In this case, Hubble parameter H and $\dot{S}_{total}$ tend to infinity from results (51) and (52), respectively. This situation may happen for very large time ($t \to \infty$), i.e. when the expansion rate is very high. Here, all the useable energy in the Universe will be converted into unuseable form of energy. The stage is also known, in literature, as the heat death of the system and the heat death of our universe is one of the suggested fates. If the case occurs, then all the thermodynamic free energy will be diminished from our universe and motion or life cannot continue any more. In other words, the entropy of the universe will reach its maximum value.

- *Case II.* If $3\lambda_1 + \lambda_2^2 > 0$:

Here, the validity of the 2nd law of gravitational thermodynamics depends on b^2, Λ, α, metric potentials and the equation of state parameters. If we remove anisotropy in the metric potentials (by choosing $n = \lambda_2 = 0$ and $\lambda_1 = 1$), the validity depends on b^2, Λ, α and the equation of state parameters. However, the 2nd law of gravitational thermodynamics is conditionally valid.

- *Case III.* When $\lambda_1 = -\lambda_2^2$:

Under this condition, equation (52) gives that

$$T_A\dot{S}_{total} = 6\pi\lambda_2e^{nx}a^3b^2(1+r)\rho^c. \tag{55}$$

Also, if we assume that there is no interaction between the Chaplygin gas and cold dark matter, then the Chaplygin gas and cold dark matter are separately conserved. And, we get

$$T_A\dot{S}_{total} = 0. \tag{56}$$

It means the validity of the 2nd law holds for all time in this case. Besides, the result corresponds to a reversible adiabatic expansion of our universe.

- *Case IV.* If we choose $n = \lambda_2 = 0$ and $\lambda_1 = 1$

In this case, the line-element (4) reduces to the following form

$$ds^2 = -dt^2 + a^2[dx^2 + dy^2 + dz^2], \tag{57}$$

which is known as the flat Friedmann-Robertson-Walker spacetime. Consequently, equation (52) leads to

$$\dot{S}_{total} = 6\pi a^3b^2(\rho^c + \rho^m) + \frac{k\pi}{4H^3}\left(\rho^c + \rho^m - \frac{\Lambda}{(\rho^c)^\alpha}\right). \tag{58}$$

Next, after ignoring the interaction between the Chaplygin gas and cold dark matter, it follows that

$$\dot{S}_{total} = \frac{k\pi}{4H^3}\left(\rho^c + \rho^m - \frac{\Lambda}{(\rho^c)^\alpha}\right). \tag{59}$$

Hence $\dot{S}_{total} \geq 0$ for $\rho^c + \rho^m \geq \frac{\Lambda}{(\rho^c)^\alpha} \geq 0$. From this point of view, one can say that the 2nd law of gravitational thermodynamics holds for all time in this case if the null energy condition for the considered matter is satisfied. It is considerable to mention here that this result extends the investigation of Mubasher et al. [32]. The authors proved the validity of the generalized 2nd law of thermodynamics for the flat Friedmann-Robertson-Walker spacetime with a similar scenario. Hence, we conclude that the generalized 2nd law of thermodynamics is conditionally valid in the Gödel universe with generalized Chaplygin gas.

IV. CONCLUSIONS

In the present work, we investigated the cosmological scenario where the generalized Chaplygin gas interacts with cold dark matter and examined the validity of the second law of gravitational thermodynamics. By assuming our universe as a thermodynamical system bounded by the apparent horizon, and calculating separately the entropy variation for the generalized Chaplygin gas, cold dark matter and for the horizon itself, we obtained an expression for the time derivative of the total entropy of the Gödel universe. We have discussed four special cases for its validity. For a particular value of $3\lambda_1 + \lambda_2^2 = 0$, the case provides a general validity for the 2nd law of gravitational thermodynamics. In this case, the heat death of our niverse will take place due to infinite expansion and all types of motion or life cannot continue any more. Next, the second case $3\lambda_1 + \lambda_2^2 > 0$ gives the conditional validity of the 2nd law. In the third case, i.e. $\lambda_1 = -\lambda_2^2$, we see that the validity of the 2nd law holds for all time. At this stage the result describes a reversible adiabatic expansion of the Universe. In the fourth case, the Gödel universe reduces to the flat Friedmann-Robertson-Walker spacetime and the general validity of 2nd law of thermodynamics depends upon the condition $\rho^c + \rho^m \geq \frac{\Lambda}{(\rho^c)^\alpha} \geq 0$. This result extends the investigation of Mubasher *et al.*: the authors proved the validity of the 2nd law of gravitational thermodynamics for the flat Friedmann-Robertson-Walker spacetime with a similar scenario. We conclude that the 2nd law of gravitational thermodynamics is conditionally valid in the scenario where the generalized Chaplygin gas interacts with cold dark matter.

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