Marius Coman

CONJECTURES ON PRIMES AND FERMAT PSEUDOPRIMES, MANY BASED ON SMARANDACHE FUNCTION

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INTRODUCTION

It is always difficult to talk about arithmetic, because those who do not know what is about, nor do they understand in few sentences, no matter how inspired these might be, and those who know what is about, do no need to be told what is about. For those hwo don't know yet, I will appeal to a comparison. Have you seen the movie "We're no angels" with Robert de Niro and Sean Penn? At a turning point, a character from the movie desperately needed help and look through the pockets of clothes for something that he could use. He found nothing. This branch of mathematics, arithmetic, is well known as the least prolific branch of mathematics in the field of material applications, it will not help you going to the moon or invent the atomic bomb. But, on the other side, you don't need any laboratory or suitcases or jacket pockets to possess or wear them after you. Arithmetic is the branch of mathematics that you keep it in you're soul and you're mind not in you're suitcase or laptop. It also will not help you to gain money (unless you will prove Fermat's last theorem without using complex numbers or you will prove Beal's conjecture which is unlikely) but it will give you something more important than that: an occupation on the train when you are going to the funeral of an aunt of third degree. No, I was kidding, if you allow me; it will give you an accession to a world equally rich in special symbols and in special people. One of these special people is Florentin Smarandache, who has a large contribution in number theory, including the very important Smarandache function and few hundred sequences, series, constants, theorems and conjectures.

Part One of this book of collected papers aims to show new applications of Smarandache function in the study of some well known classes of numbers, like prime numbers, Poulet numbers, Carmichael numbers, Sophie Germain primes etc. Beside the well known notions of number theory, we defined in these papers the following new concepts: "Smarandache-Coman divisors of order k of a composite integer n with m prime factors", "Smarandache-Coman congruence on primes", "Smarandache-Germain primes", "Coman-Smarandache criterion for primality", "Smarandache-Korselt criterion", "Smarandache-Coman constants".

Part Two of this book brings together several articles regarding primes, submitted by the author to the preprint scientific database Vixra. Apparently heterogeneous, these articles have, objectively speaking, a thing in common: they are all directed toward the same goal – discovery of new ordered patterns in the "undisciplined" set of the prime numbers, using the same means – the old and reliable integer numbers. Subjectively speaking, these papers have, of course, another thing in common: the patterns coming from the very mathematical thoughts and obsessions of the author himself: such "mathematical thoughts and obsessions" are: trying to find correspondends of the patterns found in sequences of Fermat pseudoprimes (a little more "disciplined" class of numbers than the class of primes) in sequences of prime numbers; trying to find chains of consecutive or exclusive primes defined by a recurrent formula; trying to show the importance of classification of primes in eight essential subsets (the sequences of the form 30*k + d, where d has the values 1, 7, 11, 13, 17, 19, 23, 29) etc.

This collection of articles seeks to expand the knowledge on some well known classes of primes, like for instance Sophie Germain primes, but also to define new classes of primes, like for instance "ACPOW chains of primes", or classes of integers directly related to primes, like for instance "chameleonic numbers".

At least in one article, namely "On an iterative operation on positive composite integers which probably always conducts to a prime", the author rediscovered a notion already known; in the referred case is about the sequence of primes reached by the operation of iterative replacement of a number by the concatenation of its prime factors (see the sequence A037272 in

OEIS). We chose to still keep the article in this collection of papers because it treats, beside the primes, some issues about Fermat pseudoprimes.

Finally, one last remark which reflects the author’s belief: if were invented many classes of numbers beside integers, this does not necessarily mean that everything it could be said about integers…it was already said.

SUMMARY

Part One.
Conjectures on primes and pseudoprimes based on Smarandache function

1. The Smarandache-Coman divisors of order k of a composite integer n with m prime factors

Abstract. We will define in this paper the Smarandache-Coman divisors of order k of a composite integer n with m prime factors, a notion that seems to have promising applications, at a first glance at least in the study of absolute and relative Fermat pseudoprimes, Carmichael numbers and Poulet numbers.

Definition 1:

We call *the set of Smarandache-Coman divisors of order 1 of a composite positive integer n with m prime factors*, $n = d_1*d_2*...*d_m$, where the least prime factor of n, d_1, is greater than or equal to 2, the set of numbers defined in the following way:
$SCD_1(n) = \{S(d_1 – 1), S(d_2 – 1), ..., S(d_m – 1)\}$, where S is the Smarandache function.

Examples:

1. The set of SC divisors of order 1 of the number 6 is {S(2 – 1), S(3 – 1)} = {S(1), S(2)} = {1, 2}, because 6 = 2*3;
2. $SCD_1(429)$ = {S(3 – 1), S(11 – 1), S(13 – 1)} = {S(2), S(10), S(12)} = {2, 5, 4}, because 429 = 3*11*13.

Definition 2:

We call *the set of Smarandache-Coman divisors of order 2 of a composite positive integer n with m prime factors*, $n = d_1*d_2*...*d_m$, where the least prime factor of n, d_1, is greater than or equal to 3, the set of numbers defined in the following way:
$SCD_2(n) = \{S(d_1 – 2), S(d_2 – 2), ..., S(d_m – 2)\}$, where S is the Smarandache function.

Examples:

1. The set of SC divisors of order 2 of the number 21 is {S(3 – 2), S(7 – 2)} = {S(1), S(5)} = {1, 5}, because 21 = 3*7;
2. $SCD_2(2429)$ = {S(7 – 2), S(347 – 2)} = {S(5), S(345)} = {5, 23}, because 2429 = 7*347.

Definition 3:

We call *the set of Smarandache-Coman divisors of order k of a composite positive integer n with m prime factors*, $n = d_1*d_2*…*d_m$, where the least prime factor of n, d_1, is greater than or equal to k + 1, the set of numbers defined in the following way:
$SCD_k(n) = \{S(d_1 – k), S(d_2 – k), …, S(d_m – k)\}$, where S is the Smarandache function.

Examples:

1. The set of SC divisors of order 5 of the number 539 is {S(7 – 5), S(11 – 5)} = {S(2), S(6)} = {2, 3}, because 539 = 7^2*11;
2. $SCD_6(221)$ = {S(13 – 6), S(17 – 6)} = {S(7), S(11)} = {7, 11}, because 221 = 13*17.

Comment:

We obviously defined the sets of numbers above because we believe that they can have interesting applications, in fact we believe that they can even make us re-think and re-consider the Smarandache function as an instrument to operate in the world of number theory: while at the beginning its value was considered to consist essentially in that to be a criterion for primality, afterwards the Smarandache function crossed a normal process of substantiation, so it was constrained to evolve in a relatively closed (even large) circle of equalities, inequalities, conjectures and theorems concerning, most of them, more or less related concepts. We strongly believe that some of the most important applications of the Smarandache function are still undiscovered. We were inspired in defining the Smarandache-Coman divisors by the passion for Fermat pseudoprimes, especially for Carmichael numbers and Poulet numbers, by the Korselt's criterion, one of the very few (and the most important from them) instruments that allow us to comprehend Carmichael numbers, and by the encouraging results we easily obtained, even from the first attempts to relate these two types of numbers, Fermat pseudoprimes and Smarandache numbers.

Smarandache-Coman divisors of order 1 of the 2-Poulet numbers:

(See the sequence A214305 in OEIS, posted by us, for a list with Poulet numbers with two prime factors)

$SCD_1(341)$ = {S(11 – 1), S(31 – 1)} = {S(10), S(30)} = {5, 5};
$SCD_1(1387)$ = {S(19 – 1), S(73 – 1)} = {S(18), S(72)} = {6, 6};
$SCD_1(2047)$ = {S(23 – 1), S(89 – 1)} = {S(22), S(88)} ={11, 11};
$SCD_1(2701)$ = {S(37 – 1), S(73 – 1)} = {S(36), S(72)} = {6, 6};
$SCD_1(3277)$ = {S(29 – 1), S(113 – 1)} = {S(28), S(112)} = {7, 7};
$SCD_1(4033)$ = {S(37 – 1), S(109 – 1)} = {S(36), S(108)} = {6, 9};
$SCD_1(4369)$ = {S(17 – 1), S(257 – 1)} = {S(16), S(256)} = {6, 10};
$SCD_1(4681)$ = {S(31 – 1), S(151 – 1)} = {S(30), S(150)} = {5, 10};
$SCD_1(5461)$ = {S(43 – 1), S(127 – 1)} = {S(42), S(126)} = {7, 7};
$SCD_1(7957)$ = {S(73 – 1), S(109 – 1)} = {S(72), S(108)} = {6, 9};
$SCD_1(8321)$ = {S(53 – 1), S(157 – 1)} = {S(52), S(156)} = {13, 13}.

Comment:

It is notable how easily are obtained interesting results: from the first 11 terms of the 2-Poulet numbers sequence checked there are already foreseen few patterns.

Open problems:

1. Is for the majority of the 2-Poulet numbers the case that the two Smarandache-Coman divisors of order 1 are equal, as for the seven from the eleven numbers checked above?
2. Is there an infinity of 2-Poulet numbers for which the set of SCD of order 1 is equal to {6, 6}, the case of Poulet numbers 1387 and 2701, or with {6, 9}, the case of Poulet numbers 4033 and 7957?

Smarandache-Coman divisors of order 2 of the 2-Poulet numbers:

$SCD_2(341)$ = {S(11 – 2), S(31 – 2)} = {S(9), S(29)} = {6, 29};
$SCD_2(1387)$ = {S(19 – 2), S(73 – 2)} = {S(17), S(71)} ={17, 71};
$SCD_2(2047)$ = {S(23 – 2), S(89 – 2)} = {S(21), S(87)} = {7, 29};

$SCD_2(2701)$ = {S(37 – 2), S(73 – 2)} = {S(35), S(71)} = {7, 71};
$SCD_2(3277)$ = {S(29 – 2), S(113 – 2)} = {S(27), S(111)} = {9, 37};
$SCD_2(4033)$ = {S(37 – 2), S(109 – 2)} = {S(35), S(107)} = {7, 107};
$SCD_2(4369)$ = {S(17 – 2), S(257 – 2)} = {S(15), S(255)} = {5, 17};
$SCD_2(4681)$ = {S(31 – 2), S(151 – 2)} ={S(29), S(149)} = {29, 149};
$SCD_2(5461)$ = {S(43 – 2), S(127 – 2)} = {S(41), S(125)} = {41, 15};
$SCD_2(7957)$ = {S(73 – 2), S(109 – 2)} ={S(71), S(107)} = {71, 107};
$SCD_2(8321)$ = {S(53 – 2), S(157 – 2)} = {S(52), S(156)} = {17, 31}.

Comment:
In the case of SCD of order 2 of the 2-Poulet numbers there are too foreseen few patterns.

Open problems:
1. Is for the majority of the 2-Poulet numbers the case that the two Smarandache-Coman divisors of order 2 are both primes, as for the eight from the eleven numbers checked above?
2. Is there an infinity of 2-Poulet numbers for which the set of SCD of order 2 is equal to {p, p + 20*k}, where p prime and k positive integer, the case of Poulet numbers 4033 and 4681?

Smarandache-Coman divisors of order 1 of the 3-Poulet numbers:
(See the sequence A215672 in OEIS, posted by us, for a list with Poulet numbers with two prime factors)

$SCD_1(561)$ = $SCD_1(3*11*17)$ = {S(2), S(10), S(16)} = {2, 5, 6};
$SCD_1(645)$ = $SCD_1(3*5*43)$ = {S(2), S(4), S(42)} = {2, 4, 7};
$SCD_1(1105)$ = $SCD_1(5*13*17)$ = {S(4), S(12), S(16)} = {4, 4, 6};
$SCD_1(1729)$ = $SCD_1(7*13*19)$ = {S(6), S(12), S(18)} = {3, 4, 6};
$SCD_1(1905)$ = $SCD_1(3*5*127)$ = {S(2), S(4), S(126)} = {2, 4, 7};
$SCD_1(2465)$ = $SCD_1(5*17*29)$ = {S(4), S(16), S(28)} = {4, 6, 7};
$SCD_1(2821)$ = $SCD_1(7*13*31)$ = {S(6), S(12), S(30)} = {3, 4, 5};
$SCD_1(4371)$ = $SCD_1(3*31*47)$ = {S(2), S(30), S(46)} = {2, 5, 23};
$SCD_1(6601)$ = $SCD_1(7*23*41)$ = {S(6), S(22), S(40)} = {3, 11, 5};
$SCD_1(8481)$ = $SCD_1(3*11*257)$ = {S(2), S(10), S(256)} = {2, 5, 10};
$SCD_1(8911)$ = $SCD_1(7*19*67)$ = {S(6), S(18), S(66)} = {3, 19, 67}.

Open problems:
1. Is there an infinity of 3-Poulet numbers for which the set of SCD of order 1 is equal to {2, 4, 7}, the case of Poulet numbers 645 and 1905?
2. Is there an infinity of 3-Poulet numbers for which the sum of SCD of order 1 is equal to 13, the case of Poulet numbers 561 (2 + 5 + 6 = 13), 645 (2 + 4 + 7 = 13), 1729 (3 + 4 + 6 = 13), 1905 (2 + 4 + 7 = 13) or is equal to 17, the case of Poulet numbers 2465 (4 + 6 + 7 = 17) and 8481 (2 + 5 + 10 = 17)?
3. Is there an infinity of Poulet numbers for which the sum of SCD of order 1 is prime, which is the case of the eight from the eleven numbers checked above? What about the sum of SCD of order 1 plus 1, the case of Poulet numbers 2821 (3 + 4 + 5 + 1 = 13) and 4371 (2 + 5 + 23 + 1 = 31) or the sum of SCD of order 1 minus 1, the case of Poulet numbers 1105 (4 + 4 + 6 – 1 = 13), 2821 (3 + 4 + 5 – 1 = 11) and 4371 (2 + 5 + 23 – 1 = 29)?

2. Seventeen sequences of Poulet numbers characterized by a certain set of Smarandache-Coman divisors

Abstract. In a previous article I defined the Smarandache-Coman divisors of order k of a composite integer n with m prime factors and I sketched some possible applications of this concept in the study of Fermat pseudoprimes. In this paper I make few conjectures about few possible infinite sequences of Poulet numbers, characterized by a certain set of Smarandache-Coman divisors.

Conjecture 1:

There is an infinity of 2-Poulet numbers which have the set of SC divisors of order 1 equal to {p, p}, where p is prime.

The sequence of this 2-Poulet numbers is: 341, 2047, 3277, 5461, 8321, 13747, 14491, 19951, 31417, ... (see the lists below).

Conjecture 2:

There is an infinity of 2-Poulet numbers which have the set of SC divisors of order 2 equal to {p, p + 20*k}, where p is prime and k is non-null integer.

The sequence of this 2-Poulet numbers is: 4033, 4681, 10261, 15709, 23377, 31609, ... (see the lists below).

Conjecture 3:

There is an infinity of 2-Poulet numbers which have the set of SC divisors of order 2 equal to {a, b}, where a + b + 1 is prime.

The sequence of this 2-Poulet numbers is: 1387, 2047, 2701, 3277, 4369, 4681, 8321, 13747, 14491, 18721, 31417, 31609, ... (see the lists below).
Note: This is the case of twelve from the first twenty 2-Poulet numbers.

Conjecture 4:

There is an infinity of 2-Poulet numbers which have the set of SC divisors of order 2 equal to {a, b}, where a + b – 1 is prime.

The sequence of this 2-Poulet numbers is: 4033, 8321, 10261, 13747, 14491, 15709, 19951, 23377, 31417, ... (see the lists below).

Conjecture 5:

There is an infinity of 2-Poulet numbers which have the set of SC divisors of order 2 equal to {a, b}, where a + b – 1 and a + b + 1 are twin primes.

The sequence of this 2-Poulet numbers is: 13747, 14491, 23377, 31417, ... (see the lists below).

Conjecture 6:

There is an infinity of pairs of 2-Poulet numbers which have the set of SC divisors of order 2 equal to {a, b}, respectively to {c, d}, where a + b = c + d and a, b, c, d are primes.

Such pair of 2-Poulet numbers is: (4681, 7957), because 29 + 149 = 71 + 107 = 178.

Conjecture 7:

There is an infinity of pairs of 2-Poulet numbers which have the set of SC divisors of order 2 equal to {a, b}, respectively to {c, d}, where a + b + 1 = c + d – 1.

Such pairs of 2-Poulet numbers are:
(3277, 8321), because 9 + 37 + 1 = 17 + 31 - 1 = 47;
(19951, 5461), because 23 + 31 + 1 = 41 + 15 – 1 = 55.

Conjecture 8:

There is an infinity of 2-Poulet numbers which have the set of SC divisors of order 6 equal to {p, q}, where abs{p – q} = 6*k, where p and q are primes and k is non-null positive integer.

The sequence of this 2-Poulet numbers is:
1387, 2047, 2701, 3277, 4033, 4369, 7957, 13747, 14491, 15709, 23377, 31417, 31609, ... (see the lists below).

Note: This is the case of thirteen from the first twenty 2-Poulet numbers.

Conjecture 9:

There is an infinity of 2-Poulet numbers which have the set of SC divisors of order 6 equal to {a, b}, where abs{a – b} = p and p is prime.

The sequence of this 2-Poulet numbers is: 341, 4681, 10261, ... (see the lists below).

Conjecture 10:

There is an infinity of 2-Poulet numbers which have the set of SC divisors of order 6 equal to {p, q}, where one from the numbers p and q is prime and the other one is twice a prime.

The sequence of this 2-Poulet numbers is: 341, 4681, 5461, 10261, ... (see the lists below).

Conjecture 11:

There is an infinity of 3-Poulet numbers which have the set of SC divisors of order 1 equal to {a, b, c}, where a + b + c is prime and a, b, c are primes.

The sequence of this 2-Poulet numbers is: 561, 645, 1729, 1905, 2465, 6601, 8481, 8911, 10585, 12801, 13741, ... (see the lists below).

Note: This is the case of eleven from the first twenty 2-Poulet numbers.

Conjecture 12:

There is an infinity of 3-Poulet numbers which have the set of SC divisors of order 1 equal to {a, b, c}, where a + b + c - 1 and a + b + c + 1 are twin primes.

The sequence of this 3-Poulet numbers is: 2821, 4371, 16705, 25761, 30121, ... (see the lists below)

Conjecture 13:

There is an infinity of 3-Poulet numbers which have the set of SC divisors of order 1 equal to {n, n, n}.

Such 3-Poulet number is 13981.

Conjecture 14:

There is an infinity of 3-Poulet numbers which have the set of SC divisors of order 2 equal to {5, p, q}, where p and q are primes and q = p + 6*k, where k is non-null positive integer.

Such 3-Poulet numbers are:
1729, because $SCD_2(1729)$ = {5, 11, 17} and 17 = 11 + 6*1;
2821, because $SCD_2(2821)$ = {5, 11, 29} and 29 = 11 + 6*3;
6601, because $SCD_2(6601)$ = {5, 7, 13} and 13 = 7 + 6*1;
13741, because $SCD_2(13741)$ = {5, 11, 149} and 149 = 11 + 6*23;
15841, because $SCD_2(15841)$ = {5, 29, 71} and 71 = 29 + 6*7;
30121, because $SCD_2(30121)$ = {5, 11, 329} and 329 = 11 + 6*53.

Conjecture 15:

There is an infinity of Poulet numbers divisible by 15 which have the set of SC divisors of order 1 equal to {2, 4, 7, n_1, ..., n_i}, where n_1, ..., n_i are non-null positive integers and i > 0.

The sequence of this 3-Poulet numbers is: 18705, 55245, 72855, 215265, 831405, 1246785, ... (see the lists below)

Conjecture 16:

There is an infinity of Poulet numbers divisible by 15 which have the set of SC divisors of order 1 equal to {2, 4, 23, n_1, ..., n_i}, where n_1, ..., n_i are non-null positive integers and i > 0.

The sequence of this 3-Poulet numbers is: 62745, 451905, ... (see the lists below)

Conjecture 17:

There is an infinity of Poulet numbers which are multiples of any Poulet number divisible by 15 which has the set of SC divisors of order 1 equal to {2, 4, n_1, ..., n_i}, where $n_1 = n_2 =...= n_i = 7$ and i > 0.

Examples:

The Poulet number 645 = 3*5*43, having $SCD_1(645) = \{2, 4, 7\}$, has the multiples the Poulet numbers 18705, 72885, which have $SCD_1 = \{2, 4, 7, 7\}$.

The Poulet number 1905 = 3*5*127, having $SCD_1(1905) = \{2, 4, 7\}$, has the multiples 55245, 215265 which have $SCD_1 = \{2, 4, 7, 7\}$.

(see the sequence A215150 in OEIS for a list of Poulet numbers divisible by smaller Poulet numbers)

List of SC divisors of order 1 of the first twenty 2-Poulet numbers:

(see the sequence A214305 that I submitted to OEIS for a list of 2-Poulet numbers)

$SCD_1(341) = \{S(11 – 1), S(31 – 1)\} = \{S(10), S(30)\} = \{5, 5\}$;
$SCD_1(1387) = \{S(19 – 1), S(73 – 1)\} = \{S(18), S(72)\} = \{6, 6\}$;
$SCD_1(2047) = \{S(23 – 1), S(89 – 1)\} = \{S(22), S(88)\} = \{11, 11\}$;
$SCD_1(2701) = \{S(37 – 1), S(73 – 1)\} = \{S(36), S(72)\} = \{6, 6\}$;
$SCD_1(3277) = \{S(29 – 1), S(113 – 1)\} = \{S(28), S(112)\} = \{7, 7\}$;
$SCD_1(4033) = \{S(37 – 1), S(109 – 1)\} = \{S(36), S(108)\} = \{6, 9\}$;
$SCD_1(4369) = \{S(17 – 1), S(257 – 1)\} = \{S(16), S(256)\} = \{6, 10\}$;
$SCD_1(4681) = \{S(31 – 1), S(151 – 1)\} = \{S(30), S(150)\} = \{5, 10\}$;
$SCD_1(5461) = \{S(43 – 1), S(127 – 1)\} = \{S(42), S(126)\} = \{7, 7\}$;
$SCD_1(7957) = \{S(73 – 1), S(109 – 1)\} = \{S(72), S(108)\} = \{6, 9\}$;
$SCD_1(8321) = \{S(53 – 1), S(157 – 1)\} = \{S(52), S(156)\} = \{13, 13\}$;
$SCD_1(10261) = \{S(31 – 1), S(331 – 1)\} = \{S(30), S(330)\} = \{5, 11\}$;
$SCD_1(13747) = \{S(59 – 1), S(233 – 1)\} = \{S(58), S(232)\} = \{29, 29\}$;
$SCD_1(14491) = \{S(43 – 1), S(337 – 1)\} = \{S(42), S(336)\} = \{7, 7\}$;
$SCD_1(15709) = \{S(23 – 1), S(683 – 1)\} = \{S(22), S(682)\} = \{11, 31\}$;
$SCD_1(18721) = \{S(97 – 1), S(193 – 1)\} = \{S(96), S(192)\} = \{8, 8\}$;
$SCD_1(19951) = \{S(71 – 1), S(281 – 1)\} = \{S(70), S(280)\} = \{7, 7\}$;
$SCD_1(23377) = \{S(97 – 1), S(241 – 1)\} = \{S(96), S(240)\} = \{8, 6\}$;
$SCD_1(31417) = \{S(89 – 1), S(353 – 1)\} = \{S(88), S(352)\} = \{11, 11\}$;
$SCD_1(31609) = \{S(73 – 1), S(433 – 1)\} = \{S(72), S(432)\} = \{6, 9\}$.

List of SC divisors of order 2 of the first twenty 2-Poulet numbers:

(see the sequence A214305 that I submitted to OEIS for a list of 2-Poulet numbers)

$SCD_2(341) = \{S(11 – 2), S(31 – 2)\} = \{S(9), S(29)\} = \{6, 29\}$;
$SCD_2(1387) = \{S(19 – 2), S(73 – 2)\} = \{S(17), S(71)\} = \{17, 71\}$;
$SCD_2(2047) = \{S(23 – 2), S(89 – 2)\} = \{S(21), S(87)\} = \{7, 29\}$;
$SCD_2(2701) = \{S(37 – 2), S(73 – 2)\} = \{S(35), S(71)\} = \{7, 71\}$;
$SCD_2(3277) = \{S(29 – 2), S(113 – 2)\} = \{S(27), S(111)\} = \{9, 37\}$;
$SCD_2(4033) = \{S(37 – 2), S(109 – 2)\} = \{S(35), S(107)\} = \{7, 107\}$;
$SCD_2(4369) = \{S(17 – 2), S(257 – 2)\} = \{S(15), S(255)\} = \{5, 17\}$;
$SCD_2(4681) = \{S(31 – 2), S(151 – 2)\} = \{S(29), S(149)\} = \{29, 149\}$;
$SCD_2(5461) = \{S(43 – 2), S(127 – 2)\} = \{S(41), S(125)\} = \{41, 15\}$;
$SCD_2(7957) = \{S(73 – 2), S(109 – 2)\} = \{S(71), S(107)\} = \{71, 107\}$;
$SCD_2(8321) = \{S(53 – 2), S(157 – 2)\} = \{S(51), S(155)\} = \{17, 31\}$;
$SCD_2(10261) = \{S(31 – 2), S(331 – 2)\} = \{S(29), S(329)\} = \{29, 47\}$;

$SCD_2(13747)$ = {S(59 – 2), S(233 – 2)} = {S(57), S(231)} = {19, 11};
$SCD_2(14491)$ = {S(43 – 2), S(337 – 2)} = {S(41), S(335)} = {41, 67};
$SCD_2(15709)$ = {S(23 – 2), S(683 – 2)} = {S(21), S(681)} = {7, 227};
$SCD_2(18721)$ = {S(97 – 2), S(193 – 2)} = {S(95), S(191)} = {19, 191};
$SCD_2(19951)$ = {S(71 – 2), S(281 – 2)} = {S(69), S(279)} = {23, 31};
$SCD_2(23377)$ = {S(97 – 2), S(241 – 2)} = {S(95), S(239)} = {19, 239};
$SCD_2(31417)$ = {S(89 – 2), S(353 – 2)} = {S(87), S(351)} = {29, 13};
$SCD_2(31609)$ = {S(73 – 2), S(433 – 2)} = {S(71), S(431)} = {71, 431}.

List of SC divisors of order 6 of the first twenty 2-Poulet numbers:
(see the sequence A214305 that I submitted to OEIS for a list of 2-Poulet numbers)

$SCD_6(341)$ = {S(11 – 6), S(31 – 6)} = {S(5), S(25)} = {5, 10};
$SCD_6(1387)$ = {S(19 – 6), S(73 – 6)} = {S(13), S(67)} = {13, 67};
$SCD_6(2047)$ = {S(23 – 6), S(89 – 6)} = {S(17), S(83)} = {17, 83};
$SCD_6(2701)$ = {S(37 – 6), S(73 – 6)} = {S(31), S(67)} = {31, 67};
$SCD_6(3277)$ = {S(29 – 6), S(113 – 6)} = {S(23), S(107)} = {23, 107};
$SCD_6(4033)$ = {S(37 – 6), S(109 – 6)} = {S(31), S(103)} = {31, 103};
$SCD_6(4369)$ = {S(17 – 6), S(257 – 6)} = {S(11), S(251)} = {11, 251};
$SCD_6(4681)$ = {S(31 – 6), S(151 – 6)} = {S(25), S(145)} = {10, 29};
$SCD_6(5461)$ = {S(43 – 6), S(127 – 6)} = {S(37), S(121)} = {37, 22};
$SCD_6(7957)$ = {S(73 – 6), S(109 – 6)} = {S(67), S(103)} = {67, 103};
$SCD_6(8321)$ = {S(53 – 6), S(157 – 6)} = {S(47), S(151)} = {47, 151};
$SCD_6(10261)$ = {S(31 – 6), S(331 – 6)} = {S(25), S(325)} = {10, 13};
$SCD_6(13747)$ = {S(59 – 6), S(233 – 6)} = {S(53), S(227)} = {53, 227};
$SCD_6(14491)$ = {S(43 – 6), S(337 – 6)} = {S(37), S(331)} = {37, 331};
$SCD_6(15709)$ = {S(23 – 6), S(683 – 6)} = {S(17), S(677)} = {17, 677};
$SCD_6(18721)$ = {S(97 – 6), S(193 – 6)} = {S(91), S(187)} = {13, 17};
$SCD_6(19951)$ = {S(71 – 6), S(281 – 6)} = {S(65), S(275)} = {13, 11};
$SCD_6(23377)$ = {S(97 – 6), S(241 – 6)} = {S(91), S(235)} = {13, 47};
$SCD_6(31417)$ = {S(89 – 6), S(353 – 6)} = {S(83), S(347)} = {83, 347};
$SCD_6(31609)$ = {S(73 – 6), S(433 – 6)} = {S(67), S(427)} = {67, 61}.

List of SC divisors of order 1 of the first twenty 3-Poulet numbers:
(see the sequence A215672 that I submitted to OEIS for a list of 3-Poulet numbers)

$SCD_1(561)$ = SCD_1(3*11*17) = {S(2), S(10), S(16)} = {2, 5, 6};
$SCD_1(645)$ = SCD_1(3*5*43) = {S(2), S(4), S(42)} = {2, 4, 7};
$SCD_1(1105)$ = SCD_1(5*13*17) = {S(4), S(12), S(16)} = {4, 4, 6};
$SCD_1(1729)$ = SCD_1(7*13*19) = {S(6), S(12), S(18)} = {3, 4, 6};
$SCD_1(1905)$ = SCD_1(3*5*127) = {S(2), S(4), S(126)} = {2, 4, 7};
$SCD_1(2465)$ = SCD_1(5*17*29) = {S(4), S(16), S(28)} = {4, 6, 7};
$SCD_1(2821)$ = SCD_1(7*13*31) = {S(6), S(12), S(30)} = {3, 4, 5};
$SCD_1(4371)$ = SCD_1(3*31*47) = {S(2), S(30), S(46)} = {2, 5, 23};
$SCD_1(6601)$ = SCD_1(7*23*41) = {S(6), S(22), S(40)} = {3, 11, 5};
$SCD_1(8481)$ = SCD_1(3*11*257) = {S(2), S(10), S(256)} = {2, 5, 10};
$SCD_1(8911)$ = SCD_1(7*19*67) = {S(6), S(18), S(66)} = {3, 19, 67};
$SCD_1(10585)$ = SCD_1(5*29*73) = {S(4), S(28), S(72)} = {4, 7, 6};

$SCD_1(12801) = SCD_1(3*17*251) = \{S(2), S(16), S(250)\} = \{2, 6, 15\}$;
$SCD_1(13741) = SCD_1(7*13*151) = \{S(6), S(12), S(150)\} = \{3, 4, 10\}$;
$SCD_1(13981) = SCD_1(11*31*41) = \{S(10), S(30), S(40)\} = \{5, 5, 5\}$;
$SCD_1(15841) = SCD_1(7*31*73) = \{S(6), S(30), S(72)\} = \{3, 5, 6\}$;
$SCD_1(16705) = SCD_1(5*13*257) = \{S(4), S(12), S(256)\} = \{4, 4, 10\}$;
$SCD_1(25761) = SCD_1(3*31*277) = \{S(2), S(30), S(276)\} = \{2, 5, 23\}$;
$SCD_1(29341) = SCD_1(13*37*61) = \{S(12), S(36), S(60)\} = \{4, 6, 5\}$;
$SCD_1(30121) = SCD_1(7*13*331) = \{S(6), S(12), S(330)\} = \{3, 4, 11\}$.

List of SC divisors of order 2 of the first twenty 3-Poulet numbers:
(see the sequence A215672 that I submitted to OEIS for a list of 3-Poulet numbers)

$SCD_2(561) = SCD_1(3*11*17) = \{S(1), S(9), S(15)\} = \{1, 6, 5\}$;
$SCD_2(645) = SCD_1(3*5*43) = \{S(1), S(3), S(41)\} = \{1, 3, 41\}$;
$SCD_2(1105) = SCD_1(5*13*17) = \{S(3), S(11), S(15)\} = \{3, 11, 5\}$;
$SCD_2(1729) = SCD_1(7*13*19) = \{S(5), S(11), S(17)\} = \{5, 11, 17\}$;
$SCD_2(1905) = SCD_1(3*5*127) = \{S(1), S(3), S(125)\} = \{1, 3, 15\}$;
$SCD_2(2465) = SCD_1(5*17*29) = \{S(3), S(15), S(27)\} = \{3, 5, 9\}$;
$SCD_2(2821) = SCD_1(7*13*31) = \{S(5), S(11), S(29)\} = \{5, 11, 29\}$;
$SCD_2(4371) = SCD_1(3*31*47) = \{S(1), S(29), S(45)\} = \{1, 29, 6\}$;
$SCD_2(6601) = SCD_1(7*23*41) = \{S(5), S(21), S(29)\} = \{5, 7, 13\}$;
$SCD_2(8481) = SCD_1(3*11*257) = \{S(1), S(9), S(255)\} = \{1, 6, 17\}$;
$SCD_2(8911) = SCD_1(7*19*67) = \{S(5), S(17), S(65)\} = \{5, 17, 13\}$;
$SCD_2(10585) = SCD_1(5*29*73) = \{S(3), S(27), S(71)\} = \{3, 9, 71\}$;
$SCD_2(12801) = SCD_1(3*17*251) = \{S(1), S(15), S(249)\} = \{1, 5, 83\}$;
$SCD_2(13741) = SCD_1(7*13*151) = \{S(5), S(11), S(149)\} = \{5, 11, 149\}$;
$SCD_2(13981) = SCD_1(11*31*41) = \{S(9), S(29), S(39)\} = \{6, 29, 13\}$;
$SCD_2(15841) = SCD_1(7*31*73) = \{S(5), S(29), S(71)\} = \{5, 29, 71\}$;
$SCD_2(16705) = SCD_1(5*13*257) = \{S(3), S(111), S(255)\} = \{3, 11, 17\}$;
$SCD_2(25761) = SCD_1(3*31*277) = \{S(1), S(29), S(275)\} = \{1, 29, 11\}$;
$SCD_2(29341) = SCD_1(13*37*61) = \{S(11), S(35), S(59)\} = \{11, 7, 59\}$;
$SCD_2(30121) = SCD_1(7*13*331) = \{S(5), S(11), S(329)\} = \{5, 11, 329\}$.

List of SC divisors of order 1 of the first ten Poulet numbers divisible by 3 and 5:
(see the sequence A216364 that I submitted to OEIS for a list of Poulet numbers divisible by 15)

$SCD_1(645) = SCD_1(3*5*43) = \{2, 4, 7\}$;
$SCD_1(1905) = SCD_1(3*5*127) = \{2, 4, 7\}$;
$SCD_1(18705) = SCD_1(3*5*29*43) = \{2, 4, 7, 7\}$;
$SCD_1(55245) = SCD_1(3*5*29*127) = \{2, 4, 7, 7\}$;
$SCD_1(62745) = SCD_1(3*5*47*89) = \{2, 4, 23, 11\}$;
$SCD_1(72855) = SCD_1(3*5*43*113) = \{2, 4, 7, 7\}$;
$SCD_1(215265) = SCD_1(3*5*113*127) = \{2, 4, 7, 7\}$;
$SCD_1(451905) = SCD_1(3*5*47*641) = \{2, 4, 23, 8\}$;
$SCD_1(831405) = SCD_1(3*5*43*1289) = \{2, 4, 7, 23\}$;
$SCD_1(1246785) = SCD_1(3*5*43*1933) = \{2, 4, 7, 23\}$.

3. The Smarandache-Coman congruence on primes and four conjectures on Poulet numbers based on this new notion

Abstract. In two previous articles I defined the Smarandache-Coman divisors of order k of a composite integer n with m prime factors and I made few conjectures about few possible infinite sequences of Poulet numbers, characterized by a certain set of Smarandache-Coman divisors. In this paper I define a very related notion, the Smarandache-Coman congruence on primes, and I also make five conjectures regarding Poulet numbers based on this new notion.

Definition 1:

We define in the following way *the Smarandache-Coman congruence on primes*: we say that *two primes p and q are congruent sco n* and we note p ≡ q(sco n) if S(p – n) = S(q – n) = k, where n is a positive non-null integer and S is the Smarandache function (obviously k is also a non-null integer). We also may say that k is equal to p sco n respectively k is also equal to q sco n and note k = p sco n = q sco n.

Note:

The notion of *Smarandache-Coman congruence* is very related with the notion of *Smarandache-Coman divisors*, which we defined in previous papers in the following way (Definitions 2-4):

Definition 2:

We call *the set of Smarandache-Coman divisors of order 1 of a composite positive integer n with m prime factors*, $n = d_1*d_2*...*d_m$, where the least prime factor of n, d_1, is greater than or equal to 2, the set of numbers defined in the following way: $SCD_1(n) = \{S(d_1 – 1), S(d_2 – 1), ..., S(d_m – 1)\}$, where S is the Smarandache function.

Definition 3:

We call *the set of Smarandache-Coman divisors of order 2 of a composite positive integer n with m prime factors*, $n = d_1*d_2*...*d_m$, where the least prime factor of n, d_1, is greater than or equal to 3, the set of numbers defined in the following way: $SCD_2(n) = \{S(d_1 – 2), S(d_2 – 2), ..., S(d_m – 2)\}$, where S is the Smarandache function.

Examples:

1. The set of SC divisors of order 1 of the number 6 is $SCD_1(6) = \{S(2 – 1), S(3 – 1)\} = \{S(1), S(2)\} = \{1, 2\}$;
2. The set of SC divisors of order 2 of the number 21 is $SCD_2(21) = \{S(3 – 2), S(7 – 2)\} = \{S(1), S(5)\} = \{1, 5\}$.

Definition 4:

We call *the set of Smarandache-Coman divisors of order k of a composite positive integer n with m prime factors*, $n = d_1*d_2*…*d_m$, where the least prime factor of n, d_1, is greater than or equal to k + 1, the set of numbers defined in the following way: $SCD_k(n) = \{S(d_1 – k), S(d_2 – k), …, S(d_m – k)\}$, where S is the Smarandache function.

Note:

As I said above, in two previous articles I applied the notion of *Smarandache-Coman divisors* in the study of Fermat pseudoprimes; now I will apply the notion of *Smarandache-Coman congruence* in the study of the same class of numbers.

Conjecture 1:

There is at least one non-null positive integer n such that the prime factors of a Poulet number P, where P is not divisible by 3 or 5 and also P is not a Carmichael number, are, all of them, congruent sco n.

Verifying the conjecture:

(for the first five Poulet numbers not divisible by 3 or 5; see the sequence A001567 in OEIS for a a list of these numbers; see also the sequence A002034 for the values of Smarandache function)

: For P = 341 = 11*31, we have S(11 – 1) = S(31 – 1) = 5, so the prime factors 11 and 31 are congruent sco 1, which is written 11 ≡ 31(sco 1), or, in other words, 11 sco 1 = 31 sco 1 = 5; we also have S(11 – 7) = S(31 – 7) = 4, so 11 ≡ 31(sco 7);

: For P = 1387 = 19*73, we have S(19 – 1) = S(73 – 1) = 6, so the prime factors 19 and 73 are congruent sco 1, or, in other words, 6 is equal to 19 sco 1 and also with 73 sco 1;

: For P = 2047 = 23*89, we have S(23 – 1) = S(89 – 1) = 11, so the prime factors 19 and 73 are congruent sco 1;

: For P = 2701 = 37*73, we have S(37 – 1) = S(73 – 1) = 6, so the prime factors 19 and 73 are congruent sco 1;

: For P = 3277 = 29*113, we have S(29 – 1) = S(113 – 1) = 7, so the prime factors 29 and 113 are congruent sco 1.

Note:

If the conjecture doesn't hold in this form might be considered only the 2-Poulet numbers not divisible by 3 or 5.

Conjecture 2:

There is at least one non-null positive integer n such that, for all the prime factors (d_1, d_2, ..., d_{k-1}) beside 3 of a k-Poulet number P divisible by 3 and not divisible by 5 is true that there exist the primes q_1, q_2, ..., q_n (not necessarily distinct) such that q_1 = d_1 sco n, q_2 = d_2 sco n, ..., q_{k-1} = d_{k-1} sco n.

Verifying the conjecture:

(for the first four Poulet numbers divisible by 3 and not divisible by 5)

: For P = 561 = 3*11*17, we have 7 = 11 sco 4 and 13 = 17 sco 4;
: For P = 4371 = 3*31*47, we have 31 = 7 sco 3 and 47 = 11 sco 3;
: For P = 8481 = 3*11*257, we have 11 = 7 sco 4 and 257 = 23 sco 4;
: For P = 12801 = 3*17*251, we have 17 = 5 sco 2 and 251 = 83 sco 2.

Conjecture 3:

There is at least one non-null positive integer n such that, for all the prime factors (d_1, d_2, ..., d_{k-1}) beside 5 of a k-Poulet number P divisible by 5 and not divisible by 3 is true that there exist the primes q_1, q_2, ..., q_n (not necessarily distinct) such that $q_1 = d_1$ sco n, $q_2 = d_2$ sco n, ..., $q_{k-1} = d_{k-1}$ sco n.

Verifying the conjecture:

(for the first four Poulet numbers divisible by 5 and not divisible by 3)

: For P = 1105 = 5*13*17, we have 13 = 11 sco 2 and 17 = 5 sco 2;

: For P = 10585 = 5*29*73, we have 29 = 13 sco 3 and 73 = 7 sco 3;

: For P = 11305 = 5*7*17*19, we have 7 = 5 sco 2, 17 = 5 sco 2 and 19 = 17 sco 2;

: For P = 41665 = 5*13*641, we have 13 = 11 sco 2 and 641 = 71 sco 2.

Conjecture 4:

There is at least one non-null positive integer n such that, for all the prime factors (d_1, d_2, ..., d_k) of a k-Poulet number P not divisible by 3 or 5 is true that there exist the primes q_1, q_2, ..., q_n (not necessarily distinct) such that $q_1 = d_1$ sco n, $q_2 = d_2$ sco n, ..., $q_k = d_k$ sco n.

Note:

In other words, because we defined the Smarandache-Coman congruence only on primes, we can say that for any set of divisors d_1, d_2, ..., d_k of a k-Poulet number P not divisible by 3 or 5 there exist a non-null positive integer n such that for any d_i (where i from 1 to k) can be defined a Smarandache-Coman congruence $d_i \equiv q_i$(sco n).

References:

1. Coman, Marius, *The math encyclopedia of Smarandache type notions*, Educational publishing, 2013;
2. Coman, Marius, *Two hundred conjectures and one hundred and fifty open problems about Fermat pseudoprimes*, Educational publishing, 2013.

4. Sequences of primes that are congruent sco n

Abstract. In a previous article I defined the Smarandache-Coman congruence on primes. In this paper I present few sequences of primes that are congruent sco n.

Note:

I will first present again the notion of *Smarandache-Coman congruence*, which is very related with the notion of *Smarandache-Coman divisors*, which I also defined in a previous paper.

Definition:

We define in the following way *the Smarandache-Coman congruence on primes*: we say that *two primes p and q are congruent sco n* and we note p ≡ q(sco n) if S(p – n) = S(q – n) = k, where n is a positive non-null integer and S is the Smarandache function (obviously k is also a non-null integer). We also may say that k is equal to p sco n respectively k is also equal to q sco n and note k = p sco n = q sco n.

Note:

Because, of course, S(3 – 1) = 2 and S(3 – 2) = 1, there is no other prime that are congruent sco n to 3. Also there is no other prime to be congruent sco n to 5 so we start the sequences with the prime 7.

Note:

I will consider only the primes 7, 11, 13, 17 and 19 and the primes congruent sco n to them less than 1000 and, because I didn't yet study deeply all the implications of this new notion, I shall restrain myself from any comments or conjectures.

The sequence of primes congruent to 7 sco 2 (= 5):

(n = 2 is obviously the only possible n for such a congruence)

: 17.

The sequence of primes congruent to 11 sco 4 (= 7):

: 23, 37, 107, 317.

The sequence of primes congruent to 13 sco 2 (= 11):

: 79, 101, 167, 233, 277, 827.

The sequence of primes congruent to 13 sco 6 (= 7):

: 41.

The sequence of primes congruent to 13 sco 8 (= 5):

: 11, 23.

The sequence of primes congruent to 17 sco 4 (= 13):

: 43, 199, 277, 397, 421, 433, 659, 719, 823, 977.

The sequence of primes congruent to 17 sco 6 (= 11):

: 61, 83, 281, 797.

The sequence of primes congruent to 17 sco 10 (= 7):
: 31, 73.

The sequence of primes congruent to 19 sco 2 (= 17):
: 53, 181, 223, 257, 359, 461, 521, 563, 937.

The sequence of primes congruent to 19 sco 6 (= 13):
: 71, 97, 137, 149, 331, 461.

The sequence of primes congruent to 19 sco 8 (= 11):
: 41, 173, 239, 283, 347, 503, 701.

The sequence of primes congruent to 19 sco 12 (= 7):
: 47.

The sequence of primes congruent to 19 sco 14 (= 5):
: 29.

References:

1. Coman, Marius, *The Smarandache-Coman divisors of order k of a composite integer n with m prime factors*, Vixra;
2. Coman, Marius, *Seventeen sequences of Poulet numbers characterized by a certain set of Smarandache-Coman divisors*, Vixra.
3. Coman, Marius, *The Smarandache-Coman congruence on primes and four conjectures on Poulet numbers based on this new notion*, Vixra.

5. Five conjectures on Sophie Germain primes and Smarandache function and the notion of Smarandache-Germain primes

Abstract. In this paper I define a new type of pairs of primes, id est the Smarandache-Germain pairs of primes, notion related to Sophie Germain primes and also to Smarandache function, and I conjecture that for all pairs of Sophie Germain primes but a definable set of them there exist corespondent pairs of Smarandache-Germain primes. I also make a conjecture that attributes to the set of Sophie Germain primes but a definable subset of them a corespondent set of smaller primes, id est Coman-Germain primes.

Conjecture 1:

For any pair of Sophie Germain primes $[p_1, p_2]$ with the property that $S(p_1 - 1)$ is prime, where S is the Smarandache function, we have a corresponding pair of primes $[S(p_1 - 1), S(p_2 - 1)]$, which we named it Smarandache-Germain pair of primes, with the property that between the primes $q_1 = S(p_1 - 1)$ and $q_2 = S(p_2 - 1)$ there exist the following relation: $q_2 = n*q_1 + 1$, where n is non-null positive integer.

Note:

For a list of Sophie Germain primes see the sequence A005384 in OEIS. For the values of Smarandache function see the sequence A002034 in OEIS.

Verifying the Conjecture 1:
(for the first 26 pairs of Sophie Germain primes)

: For [2, 5] we have S(2 – 1) = 1, not prime;
: For [3, 7] we have S(3 – 1) = 2, not odd prime;
: For [5, 11] we have S(5 – 1) = 4, not prime;
: For [11, 23] we have [S(10), S(22)] = [5, 11]
and 5*2 + 1 = 11;
: For [23, 47] we have [S(22), S(46)] = [11, 23]
and 11*2 + 1 = 23;
: For [29, 59] we have [S(28), S(58)] = [7, 29]
and 7*4 + 1 = 29;
: For [41, 83] we have [S(40), S(82)] = [5, 41]
and 5*8 + 1 = 11;
: For [53, 107] we have [S(52), S(106)] = [13, 53]
and 13*4 + 1 = 53;
: For [83, 167] we have [S(82), S(166)] = [41, 83]
and 41*2 + 1 = 83;
: For [89, 179] we have [S(88), S(178)] = [11, 89]
and 11*8 + 1 = 89;
: For [113, 227] we have [S(112), S(226)] = [7, 113]
and 7*16 + 1 = 113;
: For [131, 263] we have [S(130), S(262)] = [13, 131]
and 13*10 + 1 = 131;
: For [173, 347] we have [S(172), S(346)] = [43, 173]
and 43*4 + 1 = 173;
: For [179, 359] we have [S(178), S(358)] = [89, 179]

and 89*2 + 1 = 179;
: For [191, 383] we have [S(190), S(382)] = [19, 191]
and 19*10 + 1 = 191;
: For [233, 467] we have [S(232), S(466)] = [29, 233]
and 29*8 + 1 = 233;
: For [239, 479] we have [S(238), S(478)] = [17, 239]
and 17*14 + 1 = 239;
: For [251, 503] we have S(250 – 1) = 15, not prime;
: For [281, 563] we have [S(280), S(562)] = [7, 281]
and 7*40 + 1 = 281;
: For [293, 587] we have [S(292), S(586)] = [73, 293]
and 73*4 + 1 = 293;
: For [359, 719] we have [S(358), S(718)] = [179, 359]
and 179*2 + 1 = 359;
: For [419, 839] we have [S(418), S(838)] = [19, 419]
and 19*22 + 1 = 419;
: For [431, 863] we have [S(430), S(862)] = [43, 431]
and 43*10 + 1 = 431;
: For [443, 887] we have [S(442), S(886)] = [17, 443]
and 17*26 + 1 = 443;
: For [491, 983] we have S(491 – 1) = 14, not prime;
: For [509, 1019] we have [S(508), S(1018)] = [127, 509]
and 127*4 + 1 = 509.

Conjecture 2:

There exist an infinity of Smarandache-Germain pairs of primes.

Note:

It can be seen that $q_2 = S(p_2 – 1) = p_1$ and also n is often a power of the number 2, so I make a new conjecture:

Conjecture 3:

For any p Sophie Germain prime with the property that S(p – 1) is prime, where S is the Smarandache function, one of the following two statements is true:

1. there exist m non-null positive integer such that (p – 1)/(2^m) = q, where q is prime, q ≥ 5;
2. there exist n prime and m non-null positive integer such that (p – 1)/(n*2^m) = q, where q is prime, q ≥ 5.

Note: we call the primes q from the first statement Coman-Germain primes of the first degree; we call the primes q from the second statement Coman-Germain primes of the second degree.

Verifying the Conjecture 3:
(for the first 21 Sophie Germain primes with the property showed)

The first statement:
: For p = 11, 23, 83, 179 we have m = 1
and q = 5, 11, 41, 89;
: For p = 29, 53, 173, 293, 509 we have m = 2
and q = 7, 13, 43, 73, 127;

: For p = 41, 89, 233 we have m = 3
and q = 5, 11, 29;
: For p = 113 we have m = 4
and q = 7.

The second statement:
: For p = 131, 191, 431 we have (m, n) = (1, 5)
and q = 13, 19, 43;
: For p = 239 we have (m, n) = (1, 7)
and q = 17;
: For p = 281 we have (m, n) = (3, 5)
and q = 7;
: For p = 419 we have (m, n) = (1, 11)
and q = 19;
: For p = 443 we have (m, n) = (1, 13)
and q = 17.

Conjecture 4:

There exist an infinity of Coman-Germain primes of the first degree.

Conjecture 5:

There exist an infinity of Coman-Germain primes of the second degree.

Notes:

We have the following sequence of Smarandache-Germain pairs of primes:
[5, 11], [11, 23], [7, 29], [5, 41], [13, 53], [41, 83], [11, 89], [7, 113], [13, 131], [43, 173], [89, 179], [19, 191], [29, 233], [17, 239], [7, 281], [73, 293], [179, 359], [19, 419], [43, 431], [17, 443], [127, 509] (...).

We have the following sequence of Coman-Germain primes of the first degree:
5, 11, 7, 5, 13, 41, 11, 7, 13, 43, 89, 29, 73, 179, 127 (...).

We have the following sequence of Coman-Germain primes of the second degree:
13, 19, 17, 7, 19, 43, 17 (...).

6. Two conjectures which generalize the conjecture on the infinity of Sophie Germain primes

Abstract. In a previous paper ("Five conjectures on Sophie Germain primes and Smarandache function and the notion of Smarandache-Germain primes") I defined two notions: the Smarandache-Germain pairs of primes and the Coman-Germain primes of the first and second degree. The few conjectures that I made on these particular types of primes inspired me to make two other conjectures regarding two sets of primes that are generalizations of the set of Sophie Germain primes. And, based on the observation of the first few primes from these two possible infinite sets of primes, I also made a conjecture regarding the primes q of the form q = p*2^n + 31 = r*2^m + 3, where p, r are primes an m, n are non-null positive integers.

Conjecture 1:

There exist an infinity of primes q of the form q = p_1*p_2*...*p_m*2^n + 1, where p_1, p_2, ..., p_m are odd distinct primes, for any n non-null natural integer respectively for any m non-null natural integer. We call this type of primes *Coman-Germain primes of the first kind*.

Note:

For [n, m] = [1, 1] the conjecture is the same with the conjecture on the infinity of Sophie Germain primes, *i.e.* the primes of the form q = 2*p + 1.

The first three primes of this form for few values of [n, m]:

1. For [n, m] = [2, 1] the primes q are of the form 4*p + 1;
   the sequence of these primes is: 13, 29, 43, ...;
2. For [n, m] = [3, 1] the primes q are of the form 8*p + 1;
   the sequence of these primes is: 41, 89, 137, ...;
3. For [n, m] = [1, 2] the primes q are of the form 2*p_1*p_2 + 1;
   the sequence of these primes is: 31, 43, 67, ...;
4. For [n, m] = [2, 2] the primes q are of the form 4*p_1*p_2 + 1;
   the sequence of these primes is: 61, 157, 229, ...;
5. For [n, m] = [3, 2] the primes q are of the form 8*p_1*p_2 + 1;
   the sequence of these primes is: 281, 409, 457, ...;
6. For [n, m] = [1, 3] the primes q are of the form 2*p_1*p_2*p_3 + 1;
   the sequence of these primes is: 211, 331, 571 (...).

Conjecture 2:

There exist an infinity of primes r of the form r = 2*(p_1*p_2*...*p_m*2^n + 1) + 1, where p_1, p_2, ..., p_m are odd distinct primes, for any n non-null natural integer respectively for any m non-null natural integer. We call this type of primes *Coman-Germain primes of the second kind*.

The first three primes of this form for few values of [n, m]:

1. For [n, m] = [1, 1] the primes q are of the form 2*(2*p + 1) + 1 = 4*p + 3;
   the sequence of these primes is: 23, 31, 47, ...;
2. For [n, m] = [2, 1] the primes q are of the form 2*(4*p + 1) + 1 = 8*p + 3;

the sequence of these primes is: 43, 59, 107, ...;

3. For [n, m] = [3, 1] the primes q are of the form 2*(8*p + 1) + 1 = 16*p + 3; the sequence of these primes is: 83, 179, 211, ...;
4. For [n, m] = [1, 2] the primes q are of the form $2*(2*p_1*p_2 + 1) + 1 = 4*p_1*p_2 + 3$; the sequence of these primes is: 223, 263, 311, ...;
5. For [n, m] = [2, 2] the primes q are of the form $2*(4*p_1*p_2 + 1) + 1 = 8*p_1*p_2 + 3$; the sequence of these primes is: 283, 443, 523 (...).

Conjecture 3:

There exist an infinity of primes q of the form q = p*2^n + 31 that can be also written as q = r*2^m + 3, where n, m are non-null positive integers and p, r odd primes.

The first three primes of this form for few values of [n, m]:

1. For [n, m] = [1, 1] we have q = 2*p + 31 = 2*r + 3:
   : q = 37 = 2*3 + 31 = 2*17 + 3, so [p, r] = [3, 17];
   : q = 41 = 2*5 + 31 = 2*19 + 3, so [p, r] = [5, 19];
   : q = 89 = 2*29 + 31 = 2*43 + 3, so [p, r] = [29, 43].
2. For [n, m] = [2, 3] we have q = 4*p + 31 = 8*r + 3:
   : q = 43 = 4*3 + 31 = 8*5 + 3, so [p, r] = [3, 5];
   : q = 59 = 4*7 + 31 = 8*7 + 3, so [p, r] = [7, 7];
   : q = 107 = 4*19 + 31 = 8*13 + 3, so [p, r] = [19, 13].
3. For [n, m] = [2, 4] we have q = 4*p + 31 = 16*r + 3:
   : q = 83 = 4*13 + 31 = 16*5 + 3, so [p, r] = [13, 5];
   : q = 179 = 4*37 + 31 = 16*11 + 3, so [p, r] = [37, 11];
   : q = 467 = 4*109 + 31 = 16*29 + 3, so [p, r] = [109, 29].

Annex

The two conjectures from my previous paper mentioned in Abstract where I defined the *Smarandache-Germain pairs of primes* and the *Coman-Germain primes of the first and second degree*:

Conjecture 1:

For any pair of Sophie Germain primes $[p_1, p_2]$ with the property that $S(p_1 – 1)$ is prime, where S is the Smarandache function, we have a corresponding pair of primes $[S(p_1 – 1), S(p_2 – 1)]$, which we named it Smarandache-Germain pair of primes, with the property that between the primes $q_1 = S(p_1 – 1)$ and $q_2 = S(p_2 – 1)$ there exist the following relation: $q_2 = n*q_1 + 1$, where n is non-null positive integer.

Conjecture 2:

For any p Sophie Germain prime with the property that S(p – 1) is prime, where S is the Smarandache function, one of the following two statements is true:

3. there exist m non-null positive integer such that (p – 1)/(2^m) = q, where q is prime, $q \geq 5$;
4. there exist n prime and m non-null positive integer such that (p – 1)/(n*2^m) = q, where q is prime, $q \geq 5$.

Note: we call the primes q from the first statement *Coman-Germain primes of the first degree*; we call the primes q from the second statement *Coman-Germain primes of the second degree*.

7. An ordered set of certain seven numbers that results constantly from a recurrence formula based on Smarandache function

Abstract. Combining two of my favorite topics of study, the recurrence relations and the Smarandache function, I discovered a very interesting pattern: seems like the recurrent formula f(n) = S(f(n – 2)) + S(f(n – 1)), where S is the Smarandache function and f(1), f(2) are any given different non-null positive integers, leads every time to a set of seven values (i.e. 11, 17, 28, 24, 11, 15, 16) which is then repeating infinitely.

Conjecture:

The recurrent formula f(n) = S(f(n – 2)) + S(f(n – 1)), where S is the Smarandache function, leads every time to the set of seven consecutive values {11, 17, 28, 24, 11, 15, 16}, set which is then repeating infinitely, for any given different non-null positive integers f(1), f(2).

Verifying the conjecture for few pairs [f(1), f(2)]

For [f(1), f(2)] = [1, 2]:

: f(3) = S(1) + S(2) = 3; f(4) = S(2) + S(3) = 5;
: f(5) = S(3) + S(5) = 8; f(6) = S(5) + S(8) = 9;
: f(7) = S(8) + S(9) = 10; f(8) = S(9) + S(10) = 11;
: f(9) = S(10) + S(11) = 16; f(10) = S(11) + S(10) = 17;
: f(11) = S(16) + S(17) = 23; f(12) = S(17) + S(23) = 40;
: f(13) = S(23) + S(40) = 28; f(14) = S(40) + S(28) = 12;
: f(15) = S(28) + S(12) = 11; f(16) = S(12) + S(11) = 15;
: f(17) = S(11) + S(15) = 16; f(18) = S(15) + S(16) = 11;
: f(19) = S(16) + S(11) = 17; f(20) = S(11) + S(17) = 28;
: f(21) = S(17) + S(28) = 24; f(22) = S(28) + S(24) = 11;
: f(23) = S(24) + S(11) = 15; f(24) = S(11) + S(15) = 16
(...)

For [f(1), f(2)] = [7, 13]:

: f(3) = S(7) + S(13) = 20; f(4) = S(13) + S(20) = 18;
: f(5) = S(20) + S(18) = 11; f(6) = S(18) + S(11) = 17
(...)

For [f(1), f(2)] = [531, 44]:

: f(3) = S(531) + S(44) = 70; f(4) = S(44) + S(70) = 18;
: f(5) = S(70) + S(18) = 13; f(6) = S(18) + S(13) = 19;
: f(7) = S(13) + S(19) = 32; f(8) = S(19) + S(32) = 27;
: f(9) = S(32) + S(27) = 17; f(10) = S(27) + S(17) = 26;
: f(11) = S(17) + S(26) = 30; f(12) = S(26) + S(30) = 18;
: f(13) = S(30) + S(18) = 11; f(14) = S(18) + S(19) = 17
(...)

For [f(1), f(2)] = [5, 11]:

: f(3) = S(5) + S(11) = 16; f(4) = S(11) + S(16) = 17;
(...)
: f(12) = 11; f(13) = 17
(...)

For [f(1), f(2)] = [341, 561]:

: f(3) = S(341) + S(561) = 48; f(4) = S(561) + S(48) = 23;
: f(5) = S(48) + S(23) = 29; f(6) = S(23) + S(29) = 52;
: f(7) = S(29) + S(52) = 42; f(8) = S(52) + S(42) = 20;
: f(9) = S(42) + S(20) = 12; f(10) = S(20) + S(12) = 9;
: f(11) = S(12) + S(9) = 10; f(12) = S(9) + S(10) = 11;
(...)
: f(22) = 11; f(23) = 17
(...)

For [f(1), f(2)] = [49, 121]:

: f(3) = S(49) + S(121) = 35; f(4) = S(121) + S(35) = 29;
: f(5) = S(35) + S(29) = 36; f(6) = S(29) + S(36) = 35;
: f(7) = S(36) + S(35) = 13; f(8) = S(35) + S(13) = 20;
: f(9) = S(13) + S(20) = 18; f(10) = S(20) + S(18) = 11;
: f(11) = S(18) + S(11) = 17 (...)

Open problems

I. Is there any exception to this apparent rule?
II. Is there a finite or infinite set of exceptions?
III. Is there a superior limit for n such that eventually f(n) = 11 and f(n + 1) = 17?
IV. Is the obtaining of a constant repeating set of values a characteristic of other recurrent formulas based similarly on the Smarandache function, having three or more terms?

8. A recurrent formula inspired by Rowland's formula and based on Smarandache function which might be a criterion for primality

Abstract. Studying the two well known recurrent relations with the exceptional property that they generate only values which are equal to 1 or are odd primes, id est the formula which belongs to Eric Rowland and the one that belongs to Benoit Cloitre, I managed to discover a formula based on Smarandache function, from the same family of recurrent relations, which, instead to give a prime value for any input, seems to give the same value, 2, if and only if the value of the input is an odd prime; also, for any value of input different from 1 and different from an odd prime, the value of output is equal to n + 1. I name this relation the Coman-Smarandache criterion for primality and the exceptions from this rule, if they exist, Coman-Smarandache pseudoprimes.

Introduction

The Rowland's formula was first noticed in 2003 summer camp NKS (New Kind of Science) organized by Wolfram Science and was subsequently proved to be true (transformed in theorem) by one of the participants in this camp, Eric Rowlands, who also conjectured that all odd primes can be generated by this formula. This formula (theorem) is:
: Let be the following recurrence relation: f(1) = 7, and, for n ≥ 2, f(n) = f(n – 1) + gcd[n, f(n – 1)]; then, the formula g(n) = f(n) - f(n – 1) has the exceptional property that it's result can be only a value which is equal to 1 or to an odd prime. The first values of g(n) are (see the sequence A132199 in OEIS): 1, 1, 1, 5, 3, 1, 1, 1, 1, 11, 3, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 23, 3, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1 (...). The first primes resulting from this formula are (see the sequence A137613 in OEIS): 5, 3, 11, 3, 23, 3, 47, 3, 5, 3, 101, 3, 7, 11, 3, 13, 233, 3, 467, 3, 5, 3, 941, 3, 7, 1889 (...).

French mathematician Benoit Cloitre further found a similar formula:
: Let f(1) = 1, and, for n ≥ 2, f(n) = f(n – 1) + lcm[n, f(n – 1)]; then, the formula g(n) = f(n)/f(n – 1) – 1 has also, as result, only a value which is equal to 1 or to an odd prime.

Conjecture 1

Let f(1) = 1 and f(n) = S(f(n – 1)) + lcm[n, S(f(n – 1))], where S is the Smarandache function and lcm the least common multiple. Then the value of the function g(n) = f(n)/S(f(n – 1)) is equal to 2 if and only if n is an odd prime.

Conjecture 2

The value of the function g(n), defined in Conjecture 1, is g(n) = n + 1 for n different from 1 and n different from odd primes.

Verifying the conjectures
(up to n = 17)

: f(2) = 1 + lcm[2, 1] = 3; then g(2) = 3/1 = 3;
: **f(3)** = 3 + lcm[3, 3] = 6; then g(3) = 6/3 = **2**;
: f(4) = 3 + lcm[4, 3] = 15; then g(4) = 15/3 = 5;
: **f(5)** = 5 + lcm[5, 5] = 10; then g(5) = 10/5 = **2**;

: f(6) = 5 + lcm[6, 5] = 35; then g(6) = 35/5 = 7;
: **f(7)** = 7 + lcm[7, 7] = 14; then g(7) = 14/7 = **2**;
: f(8) = 7 + lcm[8, 7] = 63; then g(8) = 63/7 = 9;
: f(9) = 7 + lcm[9, 7] = 70; then g(9) = 70/7 = 10;
: f(10) = 7 + lcm[10, 7] = 77; then g(10) = 77/7 = 11;
: **f(11)** = 11 + lcm[11, 11] = 22; then g(11) = 22/11 = **2**;
: f(12) = 11 + lcm[12, 11] = 143; then g(12) = 143/11 = 13;
: **f(13)** = 13 + lcm[13, 13] = 26; then g(13) = 26/13 = **2**;
: f(14) = 13 + lcm[14, 13] = 195; then g(14) = 195/13 = 15;
: f(15) = 13 + lcm[15, 13] = 208; then g(15) = 208/13 = 16;
: f(16) = 13 + lcm[16, 13] = 221; then g(16) = 221/13 = 17;
: **f(17)** = 17 + lcm[17, 17] = 17; then g(17) = 34/17 = **2.**

Note

The function g(n) = f(n)/S(f(n – 1)) – 1, where f(n) = f(n – 1) + lcm[n, f(n – 1)] might also be interesting to study as a prime generating formula, as it gives prime values (i.e. 5, 17, 23, 191, 383) for the following consecutive values of n: 4, 5, 6, 7, 8; however, for n = 9 the value obtained is a semiprime and for n = 10 is not even obtained an integer value, because m is not always divisible by S(m) so f(n), which is always divisible by f(n – 1), is not always divisible by S(f(n – 1)).

References:

4. Rowland, Eric, *A simple prime-generating recurrence*;
5. Peterson, Ivars, *A new formula for generating primes*;
6. Shevelev, Vladimir, *Generalizations of the Rowland Theorem*.

9. The Smarandache-Korselt criterion, a variant of Korselt's criterion

Abstract. Combining two of my favourite objects of study, the Fermat pseudoprimes and the Smarandache function, I was able to formulate a criterion, inspired by Korselt's criterion for Carmichael numbers and by Smarandache function, which seems to be necessary (though not sufficient as the Korselt's criterion for absolute Fermat pseudoprimes) for a composite number (without a set of probably definable exceptions) to be a Fermat pseudoprime to base two.

Conjecture:

Any Poulet number, without a set of definable exceptions, respects either the Korselt's criterion (case in which it is a Carmichael number also) either *the Smarandache-Korselt criterion*.

Definition:

A composite odd integer n = $d_1*d_2*...*d_n$, where d_1, d_2, ..., d_n are its prime factors, is said that respects *the Smarandache-Korselt criterion* if n – 1 is divisible by $S(d_i – 1)$, where S is the Smarandache function and $1 \leq i \leq n$.

Note:
A Carmichael number not always respects *the Smarandache-Korselt criterion*: for instance, in the case of the number 561 = 3*11*17, 560 it is divisible by S(3 – 1) = 2 and by S(11 – 1) = 5 but is not divisible by S(17 – 1) = 6; in the case of the number 1729 = 7*13*19, 1728 it is divisible by S(6) = 3, S(12) = 4 and S(18) = 6.

Verifying the conjecture:
(for the first five Poulet numbers and for two bigger consecutive numbers which are not Carmichael numbers also):

: For P = 341 = 11*31, P – 1 = 340 is divisible by S(10) = 5 and S(30) = 5;
: For P = 645 = 3*5*43, P – 1 = 644 is divisible by S(2) = 2, S(4) = 4 and S(42) = 7;
: For P = 1387 = 19*73, P – 1 = 1386 is divisible by S(18) = 6 and S(72) = 6;
For P = 1905 = 3*5*127, P – 1 = 1904 is divisible by S(2) = 2, S(4) = 4 and S(42) = 7;
: For P = 2047 = 23*89, P – 1 = 2046 is divisible by S(22) = 11 and S(88) = 11;
: For P = 2701 = 37*73, P – 1 = 2700 is divisible by S(36) = 6 and S(72) = 6;
(...)

: For 999855751441 = 774541*1290901, P – 1 is divisible by S(774540) = 331 and S(1290900) = 331;
: For P = 999857310721 = 2833*11329*31153, P – 1 is divisible by S(2832) = 59 and S(11328) = 59 and S(31152 = 59).

Comment:
One exception that we met (which probably is part of a set of definable exceptions) is the Poulet number P = 999828475651 = 191*4751*1101811; indeed, P – 1 is not divisible by S(1101810) = 1933, and P is not a Carmichael number.

10. Few interesting sequences obtained by recurrence and based on Smarandache function

Abstract. In one of my previous papers, "An ordered set of certain seven numbers that results constantly from a recurrence formula based on Smarandache function", combining two of my favorite topics of study, the recurrence relations and the Smarandache function, I discovered that the formula f(n) = S(f(n – 2)) + S(f(n – 1)), where S is the Smarandache function and f(1), f(2) are any given different non-null positive integers, seems to lead every time to a set of seven values (i.e. 11, 17, 28, 24, 11, 15, 16) which is then repeating infinitely. In this paper I show few other interesting patterns based on recurrence and Smarandache function and I define the Smarandache-Coman constants.

Conjecture 1:

The recurrent formula f(n) = S(f(n – 3)) + S(f(n – 2)) + S(f(n – 1)), where S is the Smarandache function, leads every time to the set of twelve consecutive values {31, 49, 52, 58, 56, 49, 50, 31, 55, 52, 55, 35}, set which is then repeating infinitely, for any given different non-null positive integers f(1), f(2), f(3).

Verifying the conjecture for [f(1), f(2), f(3)] = [1, 2, 3]:

: f(4) = S(1) + S(2) + S(3) = 6;
: f(5) = S(2) + S(3) + S(6) = 8;
: f(6) = S(3) + S(6) + S(8) = 10;
: f(7) = S(6) + S(8) + S(10) = 12;
: f(8) = S(8) + S(10) + S(12) = 13;
: f(9) = S(10) + S(12) + S(13) = 22;
: f(10) = S(12) + S(13) + S(22) = 28;
: f(11) = S(13) + S(22) + S(28) = **31**;
: f(12) = S(22) + S(28) + S(31) = **49**;
: f(13) = S(28) + S(31) + S(49) = **52**;
: f(14) = S(31) + S(49) + S(52) = 58;
: f(15) = S(49) + S(52) + S(58) = 56;
: f(16) = S(52) + S(58) + S(56) = 49;
: f(17) = S(58) + S(56) + S(49) = 50;
: f(18) = S(56) + S(49) + S(50) = 31;
: f(19) = S(49) + S(50) + S(31) = 55;
: f(20) = S(50) + S(31) + S(55) = 52;
: f(21) = S(31) + S(55) + S(52) = 55;
: f(22) = S(55) + S(52) + S(55) = 35;
: f(23) = S(52) + S(55) + S(35) = **31**;
: f(24) = S(55) + S(35) + S(31) = **49**;
: f(25) = S(35) + S(31) + S(49) = **52**;
(...)

Note:

It can be seen that f(26) = f(14) and the sequence becomes cyclic.

Open problems:

: Is there any exception to this apparent rule?

: Is there a finite or infinite set of exceptions?

: Is there (in case that conjecture is true) a superior limit for n such that eventually f(n) = 31, f(n + 1) = 49 and f(n + 2) = 52?

Conjecture 2:

For any k integer, $k \geq 2$, the function f(n) = S(f(n – k)) + S(f(n – k + 1)) +...+ S(f(n – 2)) + S(f(n – 1)) leads to a number of k consecutive values of f(n) from which the sequence of values of f(n) is repeating infinitely, for any given different non-null positive integers f(1), f(2),..., f(k), where f(1) < f(2) < ...< f(k); we name these values the *Smarandache-Coman constants*:

: for k = 2, the set of *Smarandache-Coman constants* obviously contains two elements, i.e. 11 and 17;

: for k = 3, the set of *Smarandache-Coman constants* obviously contains three elements, i.e. 31, 49 and 52.

Open problem:

: Which is the *Smarandache-Coman set of constants* for k = 4? But for k = 5, k = 6 etc.?

Conjecture 3:

The recurrent formula f(n) = abs{S(f(n – 1)) - S(f(n – 2)) + S(f(n – 3)) - S(f(n – 4))}, where S is the Smarandache function, leads every time to f(m) = 0 for a certain m, for any given different non-null positive integers f(1), f(2), f(3), f(4), where f(1) < f(2) < f(3) < f(4).

Verifying the conjecture for [f(1), f(2), f(3), f(4)] = [1, 2, 3, 4]:

: f(5) = abs{S(4) – S(3) + S(2) - S(1)} = 2;
: f(6) = abs{S(2) - S(4) + S(3) - S(2)} = 1;
: f(7) = abs{S(1) - S(2) + S(4) - S(3)} = 0.

Verifying the conjecture for [f(1), f(2), f(3), f(4)] = [7, 11, 125, 1729]:

: f(5) = abs{S(1729) – S(125) + S(11) - S(7)} = 8;
: f(6) = abs{S(8) - S(1729) + S(125) - S(11)} = 11;
: f(7) = abs{S(11) - S(8) + S(1729) - S(125)} = 7;
: f(8) = abs{S(7) – S(11) + S(8) - S(1729)} = 19;
: f(9) = abs{S(19) - S(7) + S(11) - S(8)} = 19;
: f(10) = abs{S(19) - S(19) + S(7) - S(11)} = 4;
: f(11) = abs{S(4) – S(19) + S(19) - S(7)} = 3;
: f(12) = abs{S(3) - S(4) + S(19) - S(19)} = 1;
: f(13) = abs{S(1) - S(3) + S(4) - S(19)} = 17;
: f(14) = abs{S(17) – S(1) + S(3) - S(4)} = 15;
: f(15) = abs{S(15) - S(17) + S(1) - S(3)} = 14;
: f(16) = abs{S(14) - S(15) + S(17) - S(1)} = 18;

: f(17) = abs{S(18) – S(14) + S(15) - S(17)} = 13;
: f(18) = abs{S(13) - S(18) + S(14) - S(15)} = 9;
: f(19) = abs{S(9) - S(13) + S(18) - S(14)} = 8;
: f(20) = abs{S(8) – S(9) + S(13) - S(18)} = 5;
: f(21) = abs{S(5) - S(8) + S(9) - S(13)} = 6;
: f(22) = abs{S(6) - S(5) + S(8) - S(9)} = 4;
: f(23) = abs{S(4) – S(6) + S(5) - S(8)} = 2;
: f(24) = abs{S(2) - S(4) + S(6) - S(5)} = 4;
: f(25) = abs{S(4) - S(2) + S(4) - S(6)} = 3;
: f(26) = abs{S(3) – S(4) + S(2) - S(4)} = 3;
: f(27) = abs{S(3) - S(3) + S(4) - S(2)} = 2;
: f(28) = abs{S(2) - S(3) + S(3) - S(4)} = 2;
: f(29) = abs{S(2) – S(2) + S(3) - S(3)} = 0.

Open problems:

: Is there any exception to this apparent rule?
: Is there a finite or infinite set of exceptions?
: Is there (in case that conjecture is true) a superior limit for m such that eventually f(m) = 0?

Reference:

Coman, Marius, *An ordered set of certain seven numbers that results constantly from a recurrence formula based on Smarandache function*, Vixra.

11. A very simple formula which conducts to large primes and products of very few primes

Abstract. Obviously, like everyone fond of arithmetic, I always dreamed to discover formulas to generate only primes; unfortunately, during the time, I dropped somewhat to find this Holy Grail. I found out that there are formulas that generate only primes, like Rowland's formula, but often these formulas haven't the desired impact, because, for instance, the value of the numbers used as "input" is larger than the one of the primes obtained as "output" and so on. In this paper I present a very simple formula based on Smarandache function, which, using primes of a certain form, conducts often to larger primes and products of very few primes and I also make four conjectures.

Observation:

Let p be a prime of the form 29 + 72*k, where k is positive integer; then the number n = p + 4*S(p + 1), where S is the Smarandache function, is often a prime, a square of a prime or a product of very few primes.

We have the following values of n for the first consecutive twenty-four values of p, *i.e.* 29, 101, 173, 317, 389, 461, 677, 821, 1109, 1181, 1613, 1901, 1973, 2333, 2477, 2549, 2621, 2693, 2837, 2909, 3413, 3557, 3701, 3917:

: n = 29 + 4*S(30) = 49 = 7^2;
: n = 101 + 4*S(102) = 169 = 13^2;
: n = 173 + 4*S(174) = 289 = 17^2;
: n = 317 + 4*S(318) = 529 = 23^2;
: n = 389 + 4*S(390) = 441 = 3^2*7^2;
: n = 461 + 4*S(462) = 505 = 5*101;
: n = 677 + 4*S(678) = 1129 prime;
: n = 821 + 4*S(822) = 1369 = 37^2;
: n = 1109 + 4*S(1110) = 1257 = 3*419;
: n = 1181 + 4*S(1182) = 1969 = 11*179;
: n = 1613 + 4*S(1614) = 2689 prime;
: n = 1901 + 4*S(1902) = 3169 prime;
: n = 1973 + 4*S(1974) = 2161 prime;
: n = 2333 + 4*S(2334) = 3889 prime;
: n = 2477 + 4*S(2478) = 2713 prime;
: n = 2549 + 4*S(2550) = 2617 prime;
: n = 2621 + 4*S(2622) = 2713 prime;
: n = 2693 + 4*S(2694) = 4489 = 67^2;
: n = 2837 + 4*S(2838) = 3009 = 3*17*59;
: n = 2909 + 4*S(2910) = 3297 = 3*7*157;
: n = 3413 + 4*S(3414) = 5689 prime;
: n = 3557 + 4*S(3558) = 7^2*11^2;
: n = 3701 + 4*S(3702) = 6169 = 31*199;
: n = 3917 + 4*S(3918) = 6529 prime.

Note the interesting fact that 2477 + 4*S(2478) = 2621 + 4*S(2622) = 2713.

We have the following values of n for six larger, consecutive, values of p:

: n = 720000000101 + 4*57443753 = 720229775113 prime;
: n = 720000000677 + 4*2876801 = 720011507881 = 769*936295849;
: n = 720000001037 + 4*15222631 = 720060891561 prime;
: n = 720000001901 + 4*120000000317 = 11*109*1000834031;
: n = 720000002261 + 4*120000000377 = 7*66947*2560661;
: n = 720000002333 + 4*120000000389 = 1200000003889 prime.

Note the interesting fact that the numbers (720000001901 + 1)/6, (720000002261 + 1)/6 and (720000002333 + 1)/6 are primes, respectively they are equal to 120000000317, 120000000377 and 120000000389.

Conjecture 1:
There is an infinity of primes p of the form p = 29 + 72*k, where k is positive integer.

Conjecture 2:
There is an infinity of primes q of the form q = (p + 1)/6, where p is prime of the form p = 29 + 72*k, where k is positive integer.

Conjecture 3:
There is an infinity of primes p for which the number p + 4*S(p + 1) is prime.

Example for Conjecture 3: 17 + S(18) = 23 prime.

Conjecture 4:
There is an infinity of primes p for which the number p + 4*S(p + 1) is equal to q + 4*S(q + 1), where q is a distinct prime from p and S is the Smarandache function.

Example for Conjecture 4: for p = 19 we have q = 23; indeed, 19 + 4*S(20) = 23 + 4*S(24) = 39.

Part Two.
Conjectures on some well known and less known types of primes

1. On a new type of recurrent sequences of primes - ACPOW chains

Abstract. An interesting type of recurrent sequences of primes which could eventually lead to longer chains of successive primes than known Cunningham chains or CPAP's . Few conjectures including a stronger version of Legendre's conjecture and one regarding the Fermat primes. A classification of the set of primes.

I was studying the numbers of the form b = 9 + 6*(10*a + 1), where 10*a + 1 prime, when I noticed that, for many of these numbers b and for many consecutive values of c, the number b – 2^c is a prime.

For instance, for a = 1 we get b = 75 and 75 – 2 = 73 (prime), 75 – 2^2 = 71 (prime), 75 – 2^3 = 67 (prime), 75 – 2^4 = 59 (prime), 75 – 2^5 = 43 (prime), 75 – 2^6 = 11 (prime), 75 – 2^7 = -53 (prime in absolute value), 75 – 2^8 = -181 (prime in absolute value).

This is how I discovered the potential of the recurrent sequences of primes of the type p_0 + 2^x = p_1, p_1 + 2^(x – 1) = p_2 ,..., p_{x-1} + 2 = p_x, where p_0, p_1, ..., p_x are integers, primes in absolute value.

Descending ACPOW chains

I so define a *descending proper ACPOW chain* of primes the recurrent sequence of primes that I expose it above.

Note: I name these chains of primes ACPOW-k chains, abbreviation from chains of k primes obtained "adding consecutive powers".

Example: the sequence -181, -53, 11, 43, 59, 67, 71, 73 is a descending ACPOW-8 chain of primes (in absolute value), because -181 + 2^7 = -53, -53 + 2^6 = 11, 11 + 2^5 = 43, 43 + 2^4 = 59, 59 + 2^3 = 67, 67 + 2^2 = 71, 71 + 2 = 73. It can be seen that the numbers of the terms from the sequence is equal to the power of 2 which we add to the first term plus 1.

I also define a *descending improper ACPOW chain* of primes the recurrent sequence of primes of the type p_0 + 2^x = p_1, p_1 + 2^(x – 1) = p_2 ,..., p_{x-y} + 2^y = p_{x-y+1}, where 0 < y < x.

Example: the sequence -1433, -409, 103, 359, 487 is a descending improper ACPOW-5 chain of primes (in absolute value), because -1433 + 2^10 = -409, -409 + 2^9 = 103, 103 + 2^8 = 359, 359 + 2^7 = 487.

Note: I name these chains "improper" because the last term is not equal to antepenultimate one plus 2, but plus a power of 2 bigger than 1 (indeed, in the example above, 487 + 2^6 is no longer prime).

I list below few of these chains that I discovered, with a minimum lengh 5 and maximum length 8 (because of the special nature of the number 1, I listed also the chains that include it):

Descending proper ACPOW chains:
: 1423, 1439, 1447, 1451, 1453;
: -181, -53, 11, 43, 59, 67, 71, 73.

Descending improper ACPOW chains:
: -769, -257, -1, 127, 191, 223, 239;
: -829, -317, -61, 67, 131, 163, 179;
: -1433, -409, 103, 359, 487.

Note: It can be seen the chains I exposed can't be extended neither before the first term nor further than the last one, so they are "complete" (in the same manner a Cunningham chain is "complete"); a descending proper ACPOW chain is complete "to the right" by definition.

Comment: I generalize these chains to the sequences as the ones I described above but allowing beside the powers of 2 the adding with the powers of any even number, in both proper/improper versions.

We note these chains with the abbreviation *ACPOW(m)-k*, meaning "a chain of k primes obtained adding consecutive powers of m". With this notation, an ACPOW-k is equivalent to an ACPOW(2)-k.

Example: The sequence 11, 1307, 1523, 1559 would be a descending improper ACPOW(6)-4 chain, cause 11 + 6^4 = 1307, 1307 + 6^3 = 1523, 1523 + 6^2 = 1559 (and 1559 + 6 is no longer a prime, neither 11 – 6^5 is not a prime in absolute value).

Ascending ACPOW chains

I define an *ascending proper ACPOW chain* of primes the recurrent sequence of primes of the type $p_0 + 2 = p_1$, $p_1 + 2^2 = p_2$,..., $p_{x-1} + 2^x = p_x$.

Note: An ascending proper ACPOW chain is complete (in the sence a Cunningham chain is "complete") "to the left" by definition.

It can be seen that an ascending proper ACPOW-k chain with k > 3 can start only with a prime having last digit 7 (if would be 3, the second term would be divizible by 5, it would be 9 the third term would be divisible by 5, if would be 1 the forth term would be divisible by 5). If the first term is 7, the last digit of the next terms would be 9, 3, 1 and then again 7, 9, 3, 1 repeatedly.

Example: the sequence 17, 19, 23, 31, 47, 79 is an ascending proper ACPOW-6 chain. It can be seen that the numbers of the terms from the sequence is equal to the power of 2 which we add to the last term plus 1.

I also define an *ascending improper ACPOW chain* of primes the recurrent sequence of primes of the type $p_0 + 2^x = p_1$, $p_1 + 2^{(x + 1)} = p_2$,..., $p_{y-1} + 2^{(x + y – 1)} = p_y$.

Note: I named these chains "improper" because the second term is not equal to first one plus 2, but plus a power of 2 bigger than 1. Such a chain is complete to the left if $p_0 – 2^{(x – 1)}$ is not a prime.

It can easily be proved that there are just the following posibilities for an ascending improper ACPOW chain to have the length bigger than 3:
: If the first term has the last digit 1, then the power of 2 added to it must be of the form 4*i; also first term can be only of the form 30*j + 1 (example of such a chain of length 4: 151, 167, 199, 263);
: If the first term has the last digit 3, then the power of 2 added to it must be of the form 4*i + 3; also first term can be only of the form 30*j + 23 (example of such a chain of length 7: 173, 181, 197, 229, 293, 421, 677);
: If the first term has the last digit 7, then the power of 2 added to it must be of the form 4*i + 1; also first term can be only of the form 30*j + 17 (example of such a chain of length 5: 617, 1129, 2153, 4201, 8297);
: If the first term has the last digit 9, then the power of 2 added to it must be of the form 4*i + 2; also first term can be only of the form 30*j + 19.

Comments:
: I also generalize these chains in the same manner: the sequence 11, 17, 53, 269 is an ascending proper ACPOW(6)-4 chain, because 11 + 6 = 17, 17 + 6^2 = 53, 53 + 6^3 = 269;
I used the terms “proper” and “ascending” to exhaustively define the concept, but it would be better that those two terms to be implied and only the terms “improper” and “descending” to be mentioned.

Another generalization of ACPOW chains

Note: I define further more ACPOW chains only for “proper” and “ascending” sences, the implied ones.

I define an *ACPOW(2,n)-k chain* of primes the recurrent sequence of k primes: p_0, p_1 = p_0 + 2^n, p_2 = p_1 + 2^(2*n),..., p_x = p_{x-1} + 2^(x*n).

I define an *ACPOW(m,n)-k chain* of primes the recurrent sequence of k primes: p_0, p_1 = p_0 + m^n, p_2 = p_1 + m^(2*n),..., p_x = p_{x-1} + m^(x*n).

Note: With this notation, an ACPOW-k is equivalent to an ACPOW(2,1)-k.

Examples of ACPOW(2,n)-k chains:

ACPOW(2,3)-4: 29, 37, 101, 613,
where 29 + 2^3 = 37, 37 + 2^(3*2) = 101, 101 + 2^(3*3) = 613;
ACPOW(2,3)-4: 59, 67, 131, 643.

Quasi-ACPOW chains or QACPOW chains

I define a *quasi-ACPOW chain* of primes or a *QACPOW* a proper or improper, ascending or descending ACPOW chain but with the difference that the terms of the series are not necessary primes but primes or squares of primes.

Examples:
: 107, 109, 113, 11^2, 137, 13^2, 233, 19^2, 617, 1129, 2153, 4201, 8297 is a QACPOW-13

(because: 107 + 2 = 109; 109 + 2^2 = 113; 113 + 2^3 = 121; 121 + 2^4 = 137; 137 + 2^5 = 169; 169 + 2^6 = 233; 233 + 2^7 = 361; 361 + 2^8 = 617; 617 + 2^9 = 1129; 1129 + 2^10 = 2153; 2153 + 2^11 = 4201; 4201 + 2^12 = 8297);

: 167, 13^2, 173, 181, 197, 229, 293, 421, 677 is QACPOW-9;
: 227, 229, 233, 241, 257, 17^2, 353 is a QACPOW-7.

Observations about QACPOW chains and the difference between squares of consecutive primes:

: 5^2, 41, 7^2
is a descending QACPOW because 25 + 2^4 = 41, 41 + 2^3 = 49
(incomplete chain, because -199, -71, -7, 5^2, 41, 7^2, 53 is the complete improper chain);

: 7^2, 131, 11^2
is a descending QACPOW(2,3) because 49 + 2^(3*2) = 113, 113 + 2^3 = 121
(incomplete chain, because -463, 7^2, 131, 11^2 is the complete improper chain);

: 11^2, 41, 13^2
is an ascending QACPOW because 121 + 2^4 = 137, 137 + 2^5 = 169
(incomplete chain, because 107, 109, 113, 11^2, 137, 13^2, 233, 19^2, 617, 1129, 2153, 4201, 8297 is the complete proper chain).

Conjecture 1: Between any two squares of consecutive odd primes p_1^2 and p_2^2 there are at least $p_2 - p_1$ prime numbers that can be written as p_1^2 + sum 2^x, where x from i to j, j ≥ i ≥ 1, where x, i, j positive integers.

Check for few first primes:

3^2 + 2 = 11 prime;
3^2 + 2 + 2^2 + 2^3 = 23 prime;
3^2 + 2^2 = 13 prime;
3^2 + 2^3 = 17 prime.

Note: between 3^2 and 5^2 there are 4 such primes.

5^2 + 2 + 2^2 = 31 prime;
5^2 + 2^2 = 29 prime;
5^2 + 2^2 + 2^3 = 37 prime;
5^2 + 2^4 = 41 prime.

Note: between 5^2 and 7^2 there are 4 such primes.

7^2 + 2 + 2^2 + 2^3 + 2^4 = 79 prime;
7^2 + 2^2 = 53 prime;
7^2 + 2^2 + 2^3 = 61 prime;
7^2 + 2^2 + 2^3 + 2^4 + 2^5 = 109 prime;
7^2 + 2^3 + 2^4 = 73 prime;
7^2 + 2^4 + 2^5 = 97 prime;
7^2 + 2^6 = 113 prime.

Note: between 7^2 and 11^2 there are 7 such primes.

11^2 + 2 + 2^2 = 127 prime;
11^2 + 2 + 2^2 + 2^3 + 2^4 = 151 prime;
11^2 + 2^2 + 2^3 + 2^4 = 149 prime;
11^2 + 2^4 = 137 prime.

Note: between 11^2 and 13^2 there are 4 such primes.

23^2 + 2^2 + 2^3 = 541 prime;
23^2 + 2^2 + 2^3 + 2^4 = 557 prime;
23^2 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6 = 653 prime;
23^2 + 2^4 + 2^5 = 577 prime;
23^2 + 2^4 + 2^5 + 2^6 = 641 prime;
23^2 + 2^6 = 593 prime.

Note: between 23^2 and 29^2 there are 6 such primes.

Comment: it's obviously that between two squares of consecutive primes, p_1^2 and p_2^2, there is not necessarily a prime of the form p_1^2 + 2 + 2^2 +...+ 2^h. Example: none of the numbers y = 23^2 + sum 2^x, where y < 29^2 (*i.e.* 531, 535, 543, 559, 591, 655, 783) is prime.

Conjecture 2: Between any two squares of odd primes p^2 and q^2, where p < q, there are at least (q – p)/2 + 4 prime numbers that can be written as p^2 + sum 2^x, where x from i to j, j ≥ i ≥ 1, where x, i, j positive integers.

Notes:
: between 3^2 and 5^2 there are 4 such primes (*i.e.* 11, 13, 17, 23);
: between 3^2 and 7^2 there are 6 such primes (*i.e.* 11, 13, 17, 23, 37, 41);
: between 3^2 and 11^2 there are 8 such primes (*i.e.* 11, 13, 17, 23, 37, 41, 71, 73);
: between 3^2 and 13^2 there are 9 such primes (*i.e.* 11, 13, 17, 23, 37, 41, 71, 73, 137);
: between 3^2 and 17^2 there are 12 such primes (*i.e.* 11, 13, 17, 23, 37, 41, 71, 73, 137, 233, 257, 263);
: between 3^2 and 19^2 there are 12 such primes (*i.e.* 11, 13, 17, 23, 37, 41, 71, 73, 137, 233, 257, 263).

Conjecture 3: Between any two squares of consecutive numbers a^2 and (a + 1)^2 there are at least 2 prime numbers that can be written as a^2 + sum 2^x, where x from i to j, j ≥ i ≥ 1, where x, i, j positive integers, if a is odd or can be written as (a + 1)^2 – sum 2^x, where x from i to j, j ≥ i ≥ 1, where x, i, j positive integers, if a is even.
Comment: this is a stronger version of Legendre's conjecture.

Check for few first consecutive numbers and few randomly chosen ones:

3^2 - 2 = 7 prime;
3^2 – 2^2 = 5 prime.

Note: between 2^2 and 3^2 there are 2 such primes.

3^2 + 2 = 11 prime;

3^2 + 2^2 = 13 prime.

Note: between 3^2 and 4^2 there are 2 such primes.

5^2 - 2 = 23 prime;
5^2 – (2 + 2^2) = 19 prime.

Note: between 4^2 and 5^2 there are 2 such primes.

5^2 + 2 + 2^2 = 31 prime;
5^2 + 2^2 = 29 prime.

Note: between 5^2 and 6^2 there are 2 such primes.

7^2 - 2 = 47 prime;
7^2 – (2 + 2^2) = 43 prime.

Note: between 6^2 and 7^2 there are 2 such primes.

23^2 + 2^2 + 2^3 = 541 prime;
23^2 + 2^2 + 2^3 + 2^4 = 557 prime.

Note: between 23^2 and 24^2 there are 2 such primes.

561^2 + 2 = 314723 prime;
561^2 + 2 + 2^2 +...+ 2^9 = 315743 prime.
561^2 + 2^3 + 2^4 + 2^5 = 314777 prime.
561^2 + 2^7 + 2^8 + 2^9 = 315617 prime.

Note: between 561^2 and 562^2 there are 4 such primes.

Comment: we could introduce the notion "general", and note the new series with *GACPOW(m,f(x))*, for the series such that were already defined but where the exponents of 2 (or of a number m) are not the consecutive numbers 1, 2, 3, (...) neither the consecutive products 1*n, 2*n, 3*n, (...), but consecutive values of a bijective function f(x), where x and f(x) are positive integers.

ACPOW pairs of primes

I name an ACPOW pair of primes two primes of the form:
[p + 2^m – 2, p + 2^n – 2], where p prime and n > m > 0.

Observations:

: for p = 3 the pair becomes [2^m + 1, 2^n + 1] so the only terms of the pair can be only Fermat primes; as the only known Fermat primes are 3, 5, 17, 257, 65537, we have the only known ACPOW pairs of primes having the first term 3: [3,5], [3,17], [3,257], [3,65537]; of course, [5,17], [5,257], [5,65537], [17,257], [17,65537], [257,65537] are also ACPOW pairs of primes;

: for p = 5 the pair becomes [2^m + 3, 2^n + 3] and the first few ACPOW pairs of primes having the first term 5, generated for p = 5, are: [5,7], [5,11], [5,19], [5,67], [5,131], [5,4099];

: for p = 7 the pair becomes [2^m + 5, 2^n + 5] and the first few ACPOW pairs of primes having the first term 7, generated for p = 7, are: [7,13], [7,37], [7,2053], [7,140737488355333], [7,9007199254740997];

: for p = 11 the pair becomes [2^m + 9, 2^n + 9] and the first few ACPOW pairs of primes having the first term 11, generated for p = 11, are: [11,13], [11,17], [11,41], [11,73], [11,137], [11,521], [11,1033], [11,262153], [11,8388617];

: for p = 13 the pair becomes [2^m + 11, 2^n + 11] and the first few ACPOW pairs of primes having the first term 13, generated for p = 13, are: [13,19], [13,43], [13,139], [13,523], [13,32779], [13,8388619].

: for p = 17 the pair becomes [2^m + 15, 2^n + 15] and the first few ACPOW pairs of primes having the first term 17, generated for p = 17, are: [17,19], [17,23], [17,31], [17,47], [17,79], [17,271], [17,1039], [17,2063], [17,4111].

Properties of ACPOW pairs of primes:

(1) First we notice that these pairs seems to be relatively rare for a given p as the value of n is growing (especially for p = 7, from the cases that I considered).

(2) The series of twin primes and the series of cousin primes are subsets of the series of ACPOW pairs of primes.

(3) At the numbers of the form N = 2^k + 9 we noticed something interesting: for k of the form k = 24*h – 1, N has frequently only few prime factors:

N = 2^191 + 9 (where 191 = 8*24 – 1) has just 3 prime factors;
N = 2^215 + 9 (where 215 = 9*24 – 1) has just 3 prime factors;
N = 2^239 + 9 (where 239 = 10*24 – 1) has just 3 prime factors;
N = 2^263 + 9 (where 263 = 11*24 – 1) is a prime number with 80 digits!

(4) If [q,r] and [q + 2,r + 2] are both ACPOW pairs of primes, then we have some interesting values for the numbers r – q – 2, but we didn’t gather enough data to jump with conclusions.

(5) If [q,r] is an ACPOW pair of primes, and both q and r have the last digit 7, the numbers q*r – q - 1 and q*r – r – 1 are often primes or products of few primes; we name these numbers the pair affiliated to an ACPOW pair of such primes (having the last digit 7).

: for [17,257] the affiliated pair is [4111,19*229];
: for [17,65537] is [19*229*241,1114111];
: for [257,65537] is [3079*5449,16842751];
: for [7,67] is [401,461];
: for [7,37] is [13*17,251];
: for [7,9007199254740997] is
[11*13*17*22230849662051,23*2741321512312477];
: for [17,137] is [7*313,2311];

: for [17, 8388617] is [659*203669,7*61*333973].

Conjecture 4: I conjecture (against the heuristic arguments of Hardy and Wright) that are infinite many Fermat primes, that they are all, beside the first two terms, 3 and 5, of the form 10*k + 7, and that the all terms F_n, beginning with the term F_4 = 257, satisfy the relations:

(1) one of the numbers 17*F_n – 18 and 17*F_n – F_n – 1 is a prime number having the last four digits 4111;

(2) the other one of the numbers 17*F_n – 18 and 17*F_n – F_n – 1 is a product of primes as it follows: 19*229 for n = 4, 19*229*241 for n = 5, 19*229*241*p, where p prime, for n = 6 and so on (for F_n corresponds a number of n - 2 primes).

ACPOW chains of the second kind

I define *ACPOW chains of the second kind* the recurrent sequences of primes of the type $p_n = p_{n-1}$ + 2^k, where k has the smaller value for that p_{n-1} + 2^k is a prime.

Examples:

: 3, 5, 7, 11, 13, 17, 19, 23, 31, 47, 79, 83, 211 (...);

: 29, 37, 41, 43, 59, 67, 71, 73, 89, 97, 101, 103 (...);

: 53, 61, 317 (...);

: 127, 131, 139, 1163 (...);

: 137, 139, 1163 (...).

We clasify the set of prime numbers in the following manner:
: a prime is of class ACPOW I if can be written as $p_n = p_{n-1}$ + 2^k, where p_n = 3;
: a prime is of class ACPOW II if can be written as $p_n = p_{n-1}$ + 2^k, where p_n is the first prime that doesn't belong to the first class;
: a prime is of class ACPOW III if can be written as $p_n = p_{n-1}$ + 2^k, where p_n is the first prime that doesn't belong to the first and the second class,
and so on.

Comment: the clasification is not exclusive: a prime can belong to more than one class.

Note: to the end of the article an addenda will cover the division into classes of the first few primes from the first few classes.

We name the *ACPOW problems* the following two questions:
: is there or not an infinite number of such classes?
: is there such a class with a finite number of terms?

Conclusion: we end here for now this analysis, hoping the we highlighted some of the possible applications of ACPOW chains of primes and ACPOW pairs of primes.

ADDENDA

Primes from class ACPOW I

3, 5, 7, 11, 13, 17, 19, 23, 31, 47, 79, 83, 211, 227, 229, 233, 241, 257, 769, 773 (...).

Note: we didn't find a prime of the form 773 + 2^n for n from 1 to 100 (773 + 2^100 is a number with 31 digits).

Primes from class ACPOW II

29, 37, 41, 43, 59, 67, 71, 73, 89, 97, 101, 103, 107, 109, 113, 241, 257 (...).

Note: we can see that from the number 241 class ACPOW II has the same terms with class ACPOW I.

Primes from class ACPOW III

53, 61, 317, 349, 353, 134218081 (...).

Primes from class ACPOW IV

127, 131, 139, 1163, 1171, 1187, 1699, 263843 (...).

Primes from class ACPOW V

137, 139, 1163 (...).

Note: we can see that from the number 139 class ACPOW V has the same terms with class ACPOW IV.

Primes from class ACPOW VI

149, 151, 167, 199, 263, 271, 1048847 (...).

Primes from class ACPOW VII

157, 173, 181, 197, 199, 263 (...).

Note: we can see that from the number 199 class ACPOW VII has the same terms with class ACPOW VI.

Primes from class ACPOW VIII

163, 167, 199, 263 (...).

Note: we can see that from the number 199 class ACPOW VIII has the same terms with class ACPOW VI.

Primes from class ACPOW IX

179, 181, 197 (...).

Note: we can see that from the number 181 class ACPOW IX has the same terms with class ACPOW VII.

Primes from class ACPOW X

191, 193, 197, 199, 263 (...).

Note: we can see that from the number 199 class ACPOW X has the same terms with class ACPOW VI.

Primes from class ACPOW XI

223, 227, 229 (...).

Note: we can see that from the number 227 class ACPOW XI has the same terms with class ACPOW I.

Primes from class ACPOW XII

239, 241, 257 (...).

Note: we can see that from the number 241 class ACPOW XII has the same terms with class ACPOW I.

Primes from class ACPOW XIII

251, 283, 347, 349, 353, 134218081 (...).

Note: we can see that from the number 353 class ACPOW XIII has the same terms with class ACPOW III.

Primes from class ACPOW XIV

269, 271, 1048847 (...).

Note: we can see that from the number 271 class ACPOW XIV has the same terms with class ACPOW VI.

Primes from class ACPOW XV

277, 281, 283, 347 (...).

Note: we can see that from the number 283 class ACPOW XV has the same terms with class ACPOW XIII.

Primes from class ACPOW XVI

293, 421, 677, 709, 773 (...).

Note: we can see that from the number 773 class ACPOW XVI has the same terms with class ACPOW I.

Primes from class ACPOW XVII

307, 311, 313, 317, 349, 353, 134218081 (...).

Note: we can see that from the number 353 class ACPOW XVII has the same terms with class ACPOW III.

Primes from class ACPOW XVIII

331, 347, 349 (...).

Note: we can see that from the number 349 class ACPOW XVIII has the same terms with class ACPOW XIII.

Primes from class ACPOW XIX

337, 353, 134218081 (...).

Note: we can see that from the number 349 class ACPOW XVIII has the same terms with class ACPOW XIII.

Observations:
: the classes I, II, XI, XII, XVI are converging to the same terms, also the classes III, XIII, XV, XVII and XIX, the classes IV and V, the classes VI, VII, VIII, IX, X and XIV;
: some primes seems to highlight as ***convergence primes for classes ACPOW*****:** five classes converge to the prime 353, four to the prime 199, three to the prime 241, two to the primes 139, 181, 227, 271, 283, 349, 773.

2. Few recurrent series based on the difference between succesive primes

Abstract. Despite the development of computer systems, the chains of succesive primes obtained through an iterative formula yet have short lenghts; for instance, the largest known chain of primes in arithmetic progression is an AP-26. We present here few formulas that might lead to interesting chains of primes.

I.

I.1.

I was studying twin primes when I noticed that the greater prime (q) from the pair can be obtained from the lesser one (p) through the formula q = p*(d/2) + d/2 + 1, where d is the difference between the two primes, namely 2 (the formula is obviously equivalent to q = p*1 + 1 + 1 = p + 2). I applied this formula to the following recurrence relation and I observed that it produces interesting results (*i.e.* many succesive primes):

Let be A(n+1) = A(n)*(d/2) + d/2 + 1, where A(0) is any chosen prime and d is the difference between A(m) and the next consecutive prime, if the process of iteration stops to the first composite term A(m) obtained. If the iteration is supposed to continue, the right definition is that d is the difference between A(m) and the smallest prime which is greater than A(m).

I list below few such series and I stop the iteration at the first composite term obtained.

We have for A(0) = 3:
A(1) = 3*1 + 1 + 1 = 5 (because d = (5 - 3)/2 = 1);
A(2) = 5*1 + 1 + 1 = 7 (because d = (7 - 5)/2 = 1);
A(3) = 7*2 + 2 + 1 = 17 (because d = (11 - 7)/2 = 2);
A(4) = 17*1 + 1 + 1 = 19 (because d = (19 - 17)/2 = 1);
A(5) = 19*2 + 2 + 1 = 41 (because d = (23 - 19)/2 = 2);
A(6) = 41*1 + 1 + 1 = 43 (because d = (43 - 41)/2 = 1);
A(7) = 43*2 + 2 + 1 = 89 (because d = (47 - 43)/2 = 2);
A(8) = 97*4 + 4 + 1 = 19^2 (because d = (97 - 89)/2 = 4).

We choose another prime as a starting term, of course not one resulted from the above iterative process (5, 7, 17, 19, 41, 43, 89), that would conduct to same result because the sequence depends only by the last term obtained.

We have for A(0) = 11:
A(1) = 11*1 + 1 + 1 = 13;
A(2) = 13*2 + 2 + 1 = 29;
A(3) = 29*1 + 1 + 1 = 31;
A(4) = 31*3 + 3 + 1 = 97;
A(5) = 97*2 + 2 + 1 = 197;
A(6) = 197*1 + 1 + 1 = 199;
A(7) = 199*6 + 6 + 1 = 1201;
A(8) = 1201*6 + 6 + 1 = 7213;
A(9) = 7213*3 + 3 + 1 = 23*941.

We have for A(0) = 23:
A(1) = 23*3 + 3 + 1 = 73;
A(2) = 73*3 + 3 + 1 = 223;
A(3) = 223*2 + 2 + 1 = 449;
A(4) = 449*4 + 4 + 1 = 1801;
A(5) = 1801*5 + 5 + 1 = 9011;
A(6) = 9011*1 + 1 + 1 = 9013;
A(7) = 9013*8 + 8 + 1 = 37*1949.

Note: We didn't further obtained notable results (long chains of succesive primes) for starting terms 37, 47, 53, 59, 61 so it's clear that the formula doesn't conduct to such results if we choose randomly the starting prime.

I.2.
The starting term of the sequence is allowed to be a prime in absolute value; also the terms of the sequence.

We have for A(0) = -13:
A(1) = (-13)*1 + 1 + 1 = -11;
A(2) = (-11)*2 + 2 + 1 = -19;
A(3) = (-19)*1 + 1 + 1 = -17;
A(4) = (-17)*2 + 2 + 1 = -31;
A(5) = (-31)*1 + 1 + 1 = -29;
A(6) = (-29)*3 + 3 + 1 = -83;
A(7) = (-83)*2 + 2 + 1 = -163;
A(8) = (-163)*2 + 2 + 1 = -5*97.

We have for A(0) = -23:
A(1) = (-23)*2 + 2 + 1 = -43;
A(2) = (-43)*1 + 1 + 1 = -41;
A(3) = (-41)*2 + 2 + 1 = -79;
A(4) = (-79)*3 + 3 + 1 = -233;
A(5) = (-233)*2 + 2 + 1 = -463;
A(6) = (-463)*1 + 1 + 1 = -461;
A(7) = (-461)*2 + 2 + 1 = -919;
A(8) = (-919)*4 + 4 + 1 = -3671;
A(9) = (-3671)*2 + 2 + 1 = -97*227.

I.3.
The starting term of the sequence is a prime of the form 2*k*11 + 1.

We have for A(0) = 67:
A(1) = 67*2 + 2 + 1 = 137;
A(2) = 137*1 + 1 + 1 = 139;
A(3) = 139*5 + 5 + 1 = 701;
A(4) = 701*4 + 4 + 1 = 53^2.

We have for A(0) = 89: A(1) = 89*4 + 4 + 1 = 19^2.

We have for A(0) = 199: A(1) = 199*6 + 6 + 1 = 1201.

We have for A(0) = 331:   A(1) = 331*3 + 3 + 1 = 997.

We have for A(0) = 353:   A(1) = 353*3 + 3 + 1 = 1063.

We have for A(0) = 397:   A(1) = 397*2 + 2 + 1 = 797.

We have for A(0) = 419:   A(1) = 419*2 + 1 + 1 = 421.

We have for A(0) = 463:   A(1) = 463*2 + 2 + 1 = 929.

We have for A(0) = 617:   A(1) = 617*1 + 1 + 1 = 619.

Note: This choice of starting term seems to conduct to interesting but not outstanding results.

I.4.
The starting term of the sequence is allowed to be a square of a prime. The starting value of d is now the difference between the starting term and the smaller prime bigger than this (*i.e.* d = 29 – 25 = 4 for starting term 25) and after that d is the difference between A(m) and the next consecutive prime.

We have for A(0) = 3^2 = 9:
A(1) = 9*1 + 1 + 1 = 11 which is prime.

We have for A(0) = 5^2 = 25:
A(1) = 25*2 + 2 + 1 = 53 which is prime.

We have for A(0) = 7^2 = 49:
A(1) = 49*2 + 2 + 1 = 101 which is prime.

We have for A(0) = 11^2 = 121:
A(1) = 121*3 + 3 + 1 = 367 which is prime;
A(2) = 367*3 + 3 + 1 = 1105 which is Fermat pseudoprime to base 2.

We have for A(0) = 13^2 = 169:
A(1) = 169*2 + 2 + 1 = 341 which is Fermat pseudoprime to base 2.
Note: We didn’t observed further a notable pattern.

I.5.
The starting term of the series is a prime but the value of d is now the difference between A(m) and the second consecutive prime (*e.g.* d = 7 – 3 = 4; d = 11 - 5 = 6; d = 13 – 7 = 6 and so on).

We have for A(0) = 3:
A(1) = 3*2 + 2 + 1 = 9 = 3^2.

We have for A(0) = 5:
A(1) = 5*3 + 3 + 1 = 19;
A(2) = 19*5 + 5 + 1 = 121 = 11^2.

We have for A(0) = 7:

A(1) = 7*3 + 3 + 1 = 25 = 5^2.

We have for A(0) = 11:
A(1) = 11*3 + 3 + 1 = 37.

We have for A(0) = 13:
A(1) = 13*3 + 3 + 1 = 43.
Note: We didn't observed further a notable pattern.

I.6.
The starting term of the series is a prime but the value of d is now the difference between A(m) and the third consecutive prime (*e.g.* d = 11 – 3 = 8; d = 13 - 5 = 8; d = 17 – 7 = 10 and so on).

We have for A(0) = 3: A(1) = 3*4 + 4 + 1 = 17.

We have for A(0) = 5: A(1) = 5*4 + 4 + 1 = 5^2.

We have for A(0) = 7: A(1) = 7*5 + 5 + 1 = 41.

We have for A(0) = 11: A(1) = 11*4 + 4 + 1 = 7^2.

We have for A(0) = 13: A(1) = 13*5 + 5 + 1 = 71.

We have for A(0) = 17: A(1) = 17*6 + 6 + 1 = 109.

We have for A(0) = 19: A(1) = 19*6 + 6 + 1 = 11^2.

We have for A(0) = 23: A(1) = 23*7 + 7 + 1 = 13^2.

We have for A(0) = 29: A(1) = 29*6 + 6 + 1 = 181.

We have for A(0) = 31: A(1) = 31*6 + 6 + 1 = 193.

We have for A(0) = 37: A(1) = 37*5 + 5 + 1 = 191.

We have for A(0) = 41: A(1) = 41*6 + 6 + 1 = 11*23.

We have for A(0) = 43: A(1) = 43*8 + 8 + 1 = 353.

We have for A(0) = 47: A(1) = 43*7 + 7 + 1 = 337.

We have for A(0) = 53: A(1) = 53*7 + 7 + 1 = 379.

We have for A(0) = 59: A(1) = 59*6 + 6 + 1 = 19^2.

We have for A(0) = 61: A(1) = 61*6 + 6 + 1 = 373.

We have for A(0) = 67: A(1) = 67*6 + 6 + 1 = 409.

We have for A(0) = 71: A(1) = 71*6 + 6 + 1 = 433.

We have for A(0) = 73:
A(1) = 73*7 + 7 + 1 = 593;
A(2) = 593*7 + 7 + 1 = 4159;
A(3) = 4159*26 + 27 + 1 = 108161;
A(4) = 108161*15 + 15 + 1 = 1622431;
A(5) = 1622431*20 + 20 + 1 = 32448641;
A(6) = 32448641*19 + 19 + 1 = 21247*29017.

Note: This formula seems also to conduct to interesting results.

II.

We define now another resembling series, based again on the differences between consecutive primes:

Let be A(n+1) = (A(n)+ 1))*(d_1/2) + d_2/2) + 1, where A(0) is any chosen prime, d_1 is the difference between A(m) and the next consecutive prime and d_2 is the difference between A(m) and the second consecutive prime (the definition is made under the presumtion that process of iteration stops to the first composite term obtained).

I list below few such series:

We have for A(0) = 3:
A(1) = 4*(1 + 2) + 1 = 13;
A(2) = 14*(2 + 3) + 1 = 71;
A(3) = 72*(1 + 4) + 1 = 19^2.

We have for A(0) = 5:
A(1) = 6*(1 + 3) + 1 = 5^2.

We have for A(0) = 7:
A(1) = 8*(2 + 3) + 1 = 41;
A(2) = 42*(1 + 3) + 1 = 13^2.

We have for A(0) = 11:
A(1) = 12*(1 + 3) + 1 = 7^2.

We have for A(0) = 23:
A(1) = 24*(3 + 4) + 1 = 13^2.

Note: This formula could also lead to interesting results, but these results still seem to depend on the choice of the starting term.

Conclusion: I believe that especially the first formula is appealing because it is such easy to compute, though the longest chains of primes that I obtained so far using it are just 9 primes long: 11, 13, 29, 31, 97, 197, 199, 1201, 7213 (for starting term 11) and -23, -43, -41, -79, -233, -463, -461, -919, -3671 (for starting term -23). But, on the other side, neither Cunningham chains or AP chains were much more longer before being largely computed.

3. A possible unimportant but sure interesting conjecture about primes

Abstract. Studying, related to Fermat pseudoprimes, my main object of study, the concatenation and the primes of the form n*p – n + 1, where p is also prime, I found incidentally an interesting possible fact about primes. Because the proof or disproof of the conjecture, and also the consequences in the case that is true, are beyond me, I shall limit myself to enunciate the conjecture and give few examples.

Conjecture:
Every prime p that ends in a group of digits that form a prime can be written at least in one way as p = n*q – (n – 1)*r, where n is positive integer, n > 1, and q, r another primes that ends in the same group of digits.

Examples:

The numbers 11, 211, 311, 811, 911, 1511, 1811, 2011, 2111, 2311 are the first ten primes that end in the digits 11, that form a prime.
These primes can be written as:

: 11 = 3*211 – 2*311;
: 211 = 7*811 – 6*911;
: 311 = 6*811 – 5*911;
: 811 = 4*211 – 3*11;
: 911 = 3*311 – 2*11;
: 1511 = 5*311 – 4*11;
: 1811 = 9*211 – 8*11 = 6*311 – 5*11 = 2*911 – 1*11;
: 2011 = 10*211 – 9*11 = 3*811 – 2*211;
: 2111 = 7*311 – 6*11 = 2*1511 – 1*911 = 2*1811 – 1*1511;
: 2311 = 3*911 – 2*211.

The numbers 29, 229, 829, 929, 1129, 1229, 1429, 2029 are the first eight primes that end in the digits 29, that form a prime.
These primes can be written as:

: 29 = 9*829 – 8*929 = 3*829 – 2*1229 = 4*929 – 3*1229;
: 229 = 7*829 – 6*929 = 6*1229 – 5*1429 = 3*1429 – 2*2029;
: 829 = 4*229 – 3*29 = 3*1229 – 4*1429 = 2*1429 – 1*2029;
: 929 = 3*1129 – 2*1229;
: 1129 = 3*929 – 2*829;
: 1229 = 6*229 – 5*29;
: 1429 = 7*229 – 6*29;
: 2029 = 10*229 – 9*29 = 4*1429 – 3*1229 = 3*1429 – 2*1129.

Observation:

Seems that the conjecture above can be extended for primes that end in a group of digits that form not a prime but a square of prime.

Example:

The numbers 3529, 6529, 10529, 21529, 27529, 30529, 33529, 36529 are eight from the first ten primes that end in the digits 529, that form a square of a prime.
These primes can be written as:

: 3529 = 4*21529 – 3*27529 = 5*27529 – 4*33529;
: 6529 = 2*3529 – 1*529 = 5*30529 – 4*36529;
: 10529 = 4*7529 – 3*6529;
: 21529 = 7*3529 – 6*529 = 6*6529 – 5*3529;
: 27529 = 9*3529 – 8*529 = 8*6529 – 7*3529;
: 30529 = 10*3529 – 9*529 = 5*6529 – 4*529;
: 33529 = 10*6529 – 9*3529 = 2*27529 – 1*21529;
: 36529 = 6*6529 – 5*529 = 11*6529 – 10*3529.

Note: Obviously, in this case is admitted for r from the expression p = n*q – (n – 1)*r to be not just a prime but the ending group of digits itself, that form a square of prime.

Observation:

The conjecture above can also be extended for Fermat pseudoprimes that end in a group of digits that form a prime. For instance, the number 1729 is the first absolute pseudoprime that ends in the digits 29, and can be written as 1729 = 2*1429 – 1*1129, where 1429 and 1129 are primes.

4. A formula based on twin primes that generates chains of primes in arithmetic progression

Abstract. I was studying recurrences of the form P(n) = P(n – 1) + 2^k – 2, when incidentally I found a chain of 5 primes in arithmetic progression that satisfy this recurrence (8329, 8839, 9349, 9859, 10369). But, interesting, instead of find easily other chains of primes based on this recurrence, I obtained easily such chains (up to AP-6) defining in other way, based on twin primes, the relation between those 5 primes.

The formula is: p + 2 + 30*m + p*30*n, where m and n are integers, m non-negative and n positive, and p is the smaller prime from a pair of twin primes; giving a constant value to m and consecutive values to n we obtain for the following pairs of twin primes:

For (11, 13): we have p = 11.

We take m = 1. The formula becomes 43 + 330*n. We obtain for n from 14 to 17 a chain of four primes in arithmetic progression (4663, 4993, 5323, 5653) and for n from 35 to 38 another such chain (11593, 11923, 12253, 12583).

We take m = 2. The formula becomes 73 + 330*n. We obtain for n from 5 to 8 a chain of four primes in arithmetic progression (1723, 2053, 2383, 2713).

We take m = 3. The formula becomes 103 + 330*n. We obtain for n from 3 to 6 a chain of four primes in arithmetic progression (1093, 1423, 1753, 2083).

We take m = 4. The formula becomes 133 + 330*n. We obtain for n from 3 to 6 a chain of four primes in arithmetic progression (1123, 1453, 1783, 2113) and for n from 43 to 48 a chain of six primes in arithmetic progression (14323, 14653, 14983, 15313, 15643, 15973).

We take m = 5. The formula becomes 163 + 330*n. We obtain for n from 36 to 39 a chain of four primes in arithmetic progression (12043, 12373, 12703, 13033).

For (17, 19): we have p = 17.

We take m = 0. The formula becomes 19 + 510*n. We obtain for n from 14 to 18 a chain of five primes in arithmetic progression (7159, 7669, 8179, 8689, 9199).

We take m = 1. The formula becomes 49 + 510*n. We obtain for n from 23 to 26 a chain of four primes in arithmetic progression (11779, 12289, 12799, 13309) and for n from 30 to 34 a chain of five primes in arithmetic progression (15349, 15859, 16369, 16879, 17389).

We take m = 2. The formula becomes 79 + 510*n. We obtain for n from 10 to 14 a chain of five primes in arithmetic progression (5179, 5689, 6199, 6709, 7219).

We take m = 3. The formula becomes 109 + 510*n. We obtain for n from 40 to 45 a chain of six primes in arithmetic progression (20509, 21019, 21529, 22039, 22549, 23059).

We take m = 5. The formula becomes 169 + 510*n. We obtain for n from 16 to 20 a chain of five primes in arithmetic progression (8329, 8839, 9349, 9859, 10369).

For (29, 31): we have p = 29.

We take m = 1. The formula becomes 61 + 870*n. We obtain for n from 30 to 34 a chain of five primes in arithmetic progression (26161, 27031, 27901, 28771, 29641).

We take m = 2. The formula becomes 91 + 870*n. We obtain for n from 43 to 46 a chain of four primes in arithmetic progression (37501, 38371, 39241, 40111).

We take m = 4. The formula becomes 151 + 870*n. We obtain for n from 42 to 45 a chain of four primes in arithmetic progression (36691, 37561, 38431, 39301).

We take m = 5. The formula becomes 181 + 870*n. We obtain for n from 40 to 44 a chain of five primes in arithmetic progression (34981, 35851, 36721, 37591, 38461).

For (41, 43): we have p = 41.

We take m = 1. The formula becomes 73 + 1230*n. We obtain for n from 13 to 17 a chain of five primes in arithmetic progression (16063, 17293, 18523, 19753, 20983).
We take m = 5. The formula becomes 193 + 1230*n. We obtain for n from 4 to 7 a chain of four primes in arithmetic progression (5113, 6343, 7573, 8803).

For (59, 61): we have p = 59.

We take m = 0. The formula becomes 61 + 1770*n. We obtain for n from 7 to 11 a chain of five primes in arithmetic progression (12451, 14221, 15991, 17761, 19531).

We take m = 2. The formula becomes 121 + 1770*n. We obtain for n from 17 to 20 a chain of four primes in arithmetic progression (30211, 31981, 33751, 35521) and for n from 25 to 28 another such chain (44371, 46141, 47911, 49681).

We take m = 4. The formula becomes 181 + 1770*n. We obtain for n from 28 to 32 a chain of five primes in arithmetic progression (49741, 51511, 53281, 55051, 56821).

Conclusion: we obtained two AP-6, eight AP-5, twelve AP-4 and many AP-3, considering just first five pairs of twin primes, beside of course the pairs (3,5) and (5,7), values for m up to 5 and values for n up to 50.

5. On an iterative operation on positive composite integers which probably always conducts to a prime

Abstract. By playing with one of my favorite class of numbers, Poulet numbers, and one of my favorite operation, concatenation, I raised to myself few questions that seem interesting, worthy to share. I also conjectured that, reiterating a certain operation which will be defined, eventually for every Poulet number it will be find a corresponding prime. Then I extrapolated the conjecture for all composite positive integers.

Conjecture 1:

For any Poulet number P is defined the following operation which always, eventually, leads to a prime number:

Let P be a Poulet number, $P = p_1*p_2*...*p_n$, where $p_1 \leq p_2 \leq p_3 \leq ... \leq p_n$ are the prime factors of P (not distinct, it can be seen from definition: for instance for the only two Poulet numbers non-squarefree known, the squares of Wieferich primes, we have $P = p_1*p_2$, where $p_1 = p_2$; the reason for writing the number P in this way instead writing $P = p_1$^2 it will be seen further).

Then we consider the number $Q_1 = p_1p_2...p_n$, obtained by concatenation of the numbers that form the ordered set $\{p_1, p_2, p_3, ...,p_n\}$.

(Example: P = 561 = 3*11*17; then Q_1 = 31117)

Then we have the following posibilities: Q is a prime or a composite number; if it is composite (it doesn't matter if is squarefree or not) we reiterate the operation until is obtained a prime number.

(Example: Q_1 = 31117 = 29*29*37; then Q_2 = 292937 = 457*641; Q_3 = 457641 = 3*3*50849; Q_4 = 3350849 = 131*25579; Q_5 = 13125579 = 3*4375193; finally, the number Q_6 = 34375193 is prime)

Note:

Our conjecture is that, reiterating the operation above, from every Poulet number is obtained, eventually, a prime.

Verifying the conjecture for the first few Poulet numbers (beside 561 which was given above as an example):

P = 341 = 11*31;
: 1131 = 3*13*29;
: 31329 = 3*3*59*59;
: 335959 = 13*43*601;
: 1343601 = 3*3*3*7*7109;
: 33377109 = 3*607*18329;
: 360718329 = 3*120239443;
: 3120239443 = 523*5966041;

:	5235966041 = 419*12496339;
:	41912496339 = 3*7*1995833159;
:	371995833159 = 3*3*19*1459*1491031;
:	331914591491031 = 3*3*709*145949*356399;
:	33709145949356399 = 2011*16762379885309;
:	201116762379885309 = 3*17*1357927*2904033817;
:	31713579272904033817 = 13*337*7238890498266157;
:	1333772388904982661 57 = 3*13*53*64526966081518271;
:	31353645269660815182 71 = 3037*1032388714839012683;
:	30371032388714839012683 = 3*3*2879*16260571*72084117943;
:	33287916260571720841 17943 = 15765766319*211140490015097;
:	15765766319211140490015097 = 575063*27415720224064390319;
:	57506327415720224064390319 = 32869*17495612101286995 06051
:	32869174956121012869950 6051 = 23*61*1283*1597*30391*3762309931337;
:	23611283159730391376230 9931337 =
3*509*13381*115555862055 09014201651;
:	3509133811155558620550 9014201651 is a prime number.

P = 645 = 3*5*43;
:	3543 = 3*1181;
:	31181 is a prime number.

P = 1105 = 5*13*17;
:	51317 = 7*7331;
:	77331 = 3*149*173;
:	3149173 is a prime number.

P = 1387 = 19*73;
:	1973 is a prime number.

P = 1729 = 7*13*19;
:	71319 = 3*23773;
:	323773 = 199*1627;
:	1991627 = 11*331*547;
:	11331547 = 29*390743;
:	29390743 is a prime number.

P = 1905 = 3*5*127;
:	35127 = 3*3*3*1301;
:	3331301 is a prime number.

P = 2047 = 23*89;
:	2389 is a prime number.

Verifying the conjecture for the two squares of Wieferich primes (because they represent a special case):

P = 1194649 = 1093*1093;
:	10931093 = 73*137*1093;
:	731371093 = 17*223*192923;

: 17223192923 = 2089*8244707;
: 20898244707 = 3*11483606643;
: 311483606643 = 3*3*11*11*286027187;
: 3311286027187 = 3*110370428675729;
: 3110370428675729 = 21977*141528435577;
: 21977141528435577 = 3*11*17351*38382455519;
: 3111735138382455519 = 3*11*11*113899*164113*458599;
: 3111111389916411345 8599 = 359*8666048439878583 1361;
: 3598666048439878583 1361 = 162523*2214250320533 01907;
: 1625232214250320533 01907 is a prime number.

P = 1194649 = 3511*3511;
: 35113511 = 73*137*3511;
: 731373511 = 11*66488501;
: 1166488501 = 53*2687*8191;
: 5326878191 = 653*8157547;
: 6538157547 = 3*67*32528147;
: 36732528147 = 3*7*37*47274811;
: 373747274811 = 3*3*41527474979;
: 3341527474979 is a prime number.

Note:

The numbers P = 1387 = 19*73 and P = 2047 = 23*89 conducted to a prime from the first step: 1973 and 2389 are both primes. These two 2-Poulet numbers have in common the fact that, in both cases, $p_2 = 4*p_1 – 3$; indeed, 73 = 19*4 – 3 and 89 = 4*23 – 3. Another such 2-Poulet number is P = 13747 = 59*233; 59233 is also a prime number.

Conjecture 2:

For any composite positive integer, the operation defined above, always, eventually, leads to a prime number; so, we have the function f defined on the set of composite positive integers with values in the set of prime numbers; the first five values of f are:
: f(4) = 211;
: f(6) = 23;
: f(8) = 3331113965338635107;
: f(9) = 311;
: f(10) = 773.

6. Four conjectures about three subsets of pairs of twin primes

Abstract. In this paper are stated four conjectures about three subsets of pairs of twin primes, i.e. the pairs of the form (p^2 + q – 1, p^2 + q + 1), where p and q are primes (not necessarily distinct), the pairs of the form (p + q – 1, p + q + 1), where p, q and q + 2 are all three primes and the pairs of the form (p^2 + q – 1, p^2 + q + 1), where p, q and q + 2 are all three primes.

Conjecture 1:
Any pair of twin primes (a, b) greater than (29, 31) can be written as (a = p^2 + q – 1, b = p^2 + q + 1), where p and q are primes (not necessarily distinct).

Verifying the conjecture:
(for the first 5 such pairs of twin primes)

For (a, b) = (29, 31), we have:
a = 5^2 + 5 – 1 = 29; b = 5^2 + 5 + 1 = 31.
For (a, b) = (41, 43), we have:
a = 5^2 + 17 – 1 = 41; b = 5^2 + 17 + 1 = 43.
For (a, b) = (59, 61), we have:
a = 7^2 + 11 – 1 = 59; b = 7^2 + 11 + 1 = 61.
For (a, b) = (71, 73), we have:
a = 5^2 + 47 – 1 = 71; b = 2^2 + 47 + 1 = 73;
a = 7^2 + 23 – 1 = 71; b = 7^2 + 23 + 1 = 73.
For (a, b) = (101, 103), we have:
a = 7^2 + 53 – 1 = 101; b = 7^2 + 53 + 1 = 103.

Note:
The conjecture can also be formulated in the following way: For any pair of twin primes (t, t + 2), where t is greater than or equal to 29, there exist two primes a, b such that (t + 1) – a^2 = b.

Verifying the conjecture:
(for the following 5 pairs of twin primes)

: 108 – 25 = 83; 108 – 49 = 59;
: 138 – 25 = 113; 138 – 49 = 89; 138 – 121 = 17;
: 150 – 49 = 101; 150 – 121 = 29;
: 180 – 49 = 131; 180 – 121 = 59; 180 – 169 = 11;
: 192 – 25 = 167; 192 – 121 = 71; 192 – 169 = 23.

Note:
The conjecture is also checked for the next twenty pairs of twin primes, with the lesser term equal to: 197, 227, 239, 269, 281, 311, 347, 419, 431, 462, 521, 569, 599, 617, 641, 659, 809, 821, 827, 857.

Conjecture 2:
There exist an infinity of pairs of primes of the form (p^2 + q – 1, p^2 + q + 1), where p and q are primes (not necessarily distinct).

Note:
This conjecture is equivalent with the Conjecture about the infinity of twin primes if the Conjecture 1 from above is true, in such case being representative for the entire set of the pairs of twin primes or may state something different (but still implying the infinity of the pairs of twin primes) if the Conjecture 1 is not true, in such case being representative for an infinite subset of the set of the pairs of twin primes. The two conjectures below also implies the infinity of the pairs of twin primes.

Conjecture 3:
There exist an infinity of pairs of primes (p, q), where p + 2 and q + 2 are also primes, such that p – q + 1 = t, where t is also prime.

Examples:

: 227 – 197 + 1 = 31, so (p, q, t) = (227, 197, 31);
: 239 – 227 + 1 = 13, so (p, q, t) = (239, 227, 13).

Note:
The conjecture can also be formulated in the following way: There exist an infinity of pairs of primes (p, p + 2), such that p = t + q – 1, where t is prime and the numbers q and q + 2 are also primes.

Conjecture 4:
There exist an infinity of pairs of primes (p, q), where p + 2 and q + 2 are also primes, such that p – q + 1 = t^2, where t is also prime.

Examples:

: 569 – 521 + 1 = 7^2, so (p, q, t) = (569, 521, 7);
: 851 – 857 + 1 = 5^2, so (p, q, t) = (851, 857, 5).

Note:
The conjecture can also be formulated in the following way: There exist an infinity of pairs of primes (p, p + 2), such that p = t^2 + q – 1, where t is prime and the numbers q and q + 2 are also primes.

7. Twenty-four conjectures about “the eight essential subsets of primes”

Abstract. In this paper are made twenty-four conjectures about eight subsets of prime numbers, i.e. the primes of the form 30*k + 1, 30*k + 7, 30*k + 11, 30*k + 13, 30*k + 17, 30*k + 19, 30*k + 23 respectively 30*k + 29. Because we strongly believe that this classification of primes can have many applications, we refered in the title of this paper to these subsets of primes as to “the eight essential subsets of primes”. The conjectures state that each from these eight sets of primes has an infinity of terms and also that each one of them can be entirely defined with a recurrent formula starting from just three given terms.

I.

Conjecture 1:
The sequence a(n), as it will be defined below, has in infinity of terms that are prime numbers.

The sequence a(n), where n non-null positive integer, is defined in the following way:

: a(1) = 31, a(2) = 61, a(3) = 151 [the first three terms of the sequence are the smallest three primes of the form 30*k + 1];

: a(n) is the smallest prime greater than a(n – 1) that can be written as a(n) = a(i) + a(j) – 1, where 1 ≤ i ≤ j < n (if such prime exists for any n, as the conjecture states).

Conjecture 2:
Any prime of the form 30*k + 1 is a term of the sequence a(n) as it is defined by Conjecture 1 [in other words, the sequence a(n) is the same with the sequence of the primes of the form 30*k + 1].

Verifying the conjecture 2:
(for the primes of the form 30*k + 1 up to 421)

: a(4) = a(1) + a(3) – 1 = 181;
: a(5) = a(2) + a(3) – 1 = 211;
: a(6) = a(1) + a(5) – 1 = a(2) + a(4) – 1 = 241;
: a(7) = a(1) + a(6) – 1 = a(2) + a(5) – 1 = 271;
: a(8) = a(2) + a(7) – 1 = a(3) + a(4) – 1 = 331;
: a(9) = a(3) + a(7) – 1 = a(5) + a(5) – 1 = 421.

Conjecture 3:
If the Conjecture 2 doesn’t hold, than is true at least that any prime of the form 30*k + 1 is a term of a sequence a(n) that can be defined as follows:

: a(1), a(2), a(3) are three distinct primes of the form 30*k + 1;

: a(n) is the smallest prime greater than a(n – 1) that can be written as a(n) = a(i) + a(j) – 1, where 1 ≤ i ≤ j < n.

II.

Conjecture 4:
The sequence a(n), as it will be defined below, has in infinity of terms that are prime numbers.

The sequence a(n), where n non-null positive integer, is defined in the following way:

: a(1) = 37, a(2) = 67, a(3) = 97;
: a(n) is the smallest prime greater than a(n – 1) that can be written as a(n) = a(i) + a(j) – 7, where 1 ≤ i ≤ j < n (if such prime exists for any n, as the conjecture states).

Conjecture 5:

Any prime of the form 30*k + 7, where k > 0, is a term of the sequence a(n) as it is defined by Conjecture 4 [in other words, the sequence a(n) is the same with the sequence of the primes of the form 30*k + 7, for k > 0].

Verifying the conjecture 5:
(for the primes of the form 30*k + 7 up to 367)

: a(4) = a(1) + a(3) – 7 = a(2) + a(2) – 7 = 127;
: a(5) = a(1) + a(4) – 7 = a(2) + a(3) – 7 = 157;
: a(6) = a(4) + a(5) – 7 = 277;
: a(7) = a(1) + a(6) – 7 = a(5) + a(5) – 7 = 307;
: a(8) = a(1) + a(7) – 7 = a(2) + a(6) – 7 = 337;
: a(9) = a(1) + a(8) – 7 = a(2) + a(7) – 7 = 367.

Conjecture 6:

If the Conjecture 5 doesn't hold, than is true at least that any prime of the form 30*k + 7, where k > 0, is a term of a sequence a(n) that can be defined as follows:
: a(1), a(2), a(3) are three distinct primes of the form 30*k + 7;
: a(n) is the smallest prime greater than a(n – 1) that can be written as a(n) = a(i) + a(j) – 7, where 1 ≤ i ≤ j < n.

III.

Conjecture 7:
The sequence a(n), as it will be defined below, has in infinity of terms that are prime numbers.

The sequence a(n), where n non-null positive integer, is defined in the following way:

: a(1) = 41, a(2) = 71, a(3) = 101;
: a(n) is the smallest prime greater than a(n – 1) that can be written as a(n) = a(i) + a(j) – 11, where 1 ≤ i ≤ j < n (if such prime exists for any n, as the conjecture states).

Conjecture 8:

Any prime of the form 30*k + 11, where k > 0, is a term of the sequence a(n) as it is defined by Conjecture 7 [in other words, the sequence a(n) is the same with the sequence of the primes of the form 30*k + 11, for k > 0].

Verifying the conjecture 8:
(for the primes of the form 30*k + 11 up to 367)

: a(4) = a(1) + a(3) – 11 = a(2) + a(2) – 11 = 131;
: a(5) = a(2) + a(4) – 11 = a(3) + a(3) – 11 = 191;
: a(6) = a(2) + a(5) – 11 = a(4) + a(4)- 11 = 251;
: a(7) = a(1) + a(6) – 11 = a(3) + a(5) – 11 = 281;
: a(8) = a(1) + a(7) – 11 = a(2) + a(6) – 11 = 311;
: a(9) = a(3) + a(8) – 1 = a(4) + a(7) – 1 = 401.

Conjecture 9:

If the Conjecture 8 doesn't hold, than is true at least that any prime of the form 30*k + 11, where k > 0, is a term of a sequence a(n) that can be defined as follows:
: a(1), a(2), a(3) are three distinct primes of the form 30*k + 11;
: a(n) is the smallest prime greater than a(n – 1) that can be written as a(n) = a(i) + a(j) – 11, where 1 ≤ i ≤ j < n.

IV.

Conjecture 10:

The sequence a(n), as it will be defined below, has in infinity of terms that are prime numbers.

The sequence a(n), where n non-null positive integer, is defined in the following way:
: a(1) = 43, a(2) = 73, a(3) = 103;
: a(n) is the smallest prime greater than a(n – 1) that can be written as a(n) = a(i) + a(j) – 13, where 1 ≤ i ≤ j < n (if such prime exists for any n, as the conjecture states).

Conjecture 11:

Any prime of the form 30*k + 13, where k > 0, is a term of the sequence a(n) as it is defined by Conjecture 10 [in other words, the sequence a(n) is the same with the sequence of the primes of the form 30*k + 13, for k > 0].

Conjecture 12:

If the Conjecture 11 doesn't hold, than is true at least that any prime of the form 30*k + 13, where k > 0, is a term of a sequence a(n) that can be defined as follows:
: a(1), a(2), a(3) are three distinct primes of the form 30*k + 13;
: a(n) is the smallest prime greater than a(n – 1) that can be written as a(n) = a(i) + a(j) – 13, where 1 ≤ i ≤ j < n.

V.

Conjecture 13:

The sequence a(n), as it will be defined below, has in infinity of terms that are prime numbers.

The sequence a(n), where n non-null positive integer, is defined in the following way:
: a(1) = 47, a(2) = 107, a(3) = 137;
: a(n) is the smallest prime greater than a(n – 1) that can be written as a(n) = a(i) + a(j) – 17, where 1 ≤ i ≤ j < n (if such prime exists for any n, as the conjecture states).

Conjecture 14:

Any prime of the form 30*k + 17, where k > 0, is a term of the sequence a(n) as it is defined by Conjecture 13 [in other words, the sequence a(n) is the same with the sequence of the primes of the form 30*k + 17, for k > 0].

Conjecture 15:

If the Conjecture 14 doesn't hold, than is true at least that any prime of the form 30*k + 17, where k > 0, is a term of a sequence a(n) that can be defined as follows:
: a(1), a(2), a(3) are three distinct primes of the form 30*k + 17;
: a(n) is the smallest prime greater than a(n – 1) that can be written as a(n) = a(i) + a(j) – 17, where $1 \le i \le j < n$.

VI.

Conjecture 16:
The sequence a(n), as it will be defined below, has in infinity of terms that are prime numbers.

The sequence a(n), where n non-null positive integer, is defined in the following way:
: a(1) = 79, a(2) = 109, a(3) = 139;
: a(n) is the smallest prime greater than a(n – 1) that can be written as a(n) = a(i) + a(j) – 19, where $1 \le i \le j < n$ (if such prime exists for any n, as the conjecture states).

Conjecture 17:

Any prime of the form 30*k + 19, where k > 0, is a term of the sequence a(n) as it is defined by Conjecture 16 [in other words, the sequence a(n) is the same with the sequence of the primes of the form 30*k + 19, for k > 0].

Conjecture 18:

If the Conjecture 17 doesn't hold, than is true at least that any prime of the form 30*k + 19, where k > 0, is a term of a sequence a(n) that can be defined as follows:
: a(1), a(2), a(3) are three distinct primes of the form 30*k + 19;
: a(n) is the smallest prime greater than a(n – 1) that can be written as a(n) = a(i) + a(j) – 19, where $1 \le i \le j < n$.

VII.

Conjecture 19:
The sequence a(n), as it will be defined below, has in infinity of terms that are prime numbers.

The sequence a(n), where n non-null positive integer, is defined in the following way:
: a(1) = 53, a(2) = 83, a(3) = 113;
: a(n) is the smallest prime greater than a(n – 1) that can be written as a(n) = a(i) + a(j) – 23, where $1 \le i \le j < n$ (if such prime exists for any n, as the conjecture states).

Conjecture 20:

Any prime of the form 30*k + 23, where k > 0, is a term of the sequence a(n) as it is defined by Conjecture 19 [in other words, the sequence a(n) is the same with the sequence of the primes of the form 30*k + 23, for k > 0].

Conjecture 21:

If the Conjecture 20 doesn't hold, than is true at least that any prime of the form 30*k + 23, where k > 0, is a term of a sequence a(n) that can be defined as follows:
: a(1), a(2), a(3) are three distinct primes of the form 30*k + 23;

: a(n) is the smallest prime greater than a(n – 1) that can be written as a(n) = a(i) + a(j) – 23, where 1 ≤ i ≤ j < n.

VIII.

Conjecture 22:
The sequence a(n), as it will be defined below, has in infinity of terms that are prime numbers.

The sequence a(n), where n non-null positive integer, is defined in the following way:
: a(1) = 59, a(2) = 89, a(3) = 149;
: a(n) is the smallest prime greater than a(n – 1) that can be written as a(n) = a(i) + a(j) – 29, where 1 ≤ i ≤ j < n (if such prime exists for any n, as the conjecture states).

Conjecture 23:
Any prime of the form 30*k + 29, where k > 0, is a term of the sequence a(n) as it is defined by Conjecture 22 [in other words, the sequence a(n) is the same with the sequence of the primes of the form 30*k + 29, for k > 0].

Conjecture 24:
If the Conjecture 23 doesn't hold, than is true at least that any prime of the form 30*k + 29, where k > 0, is a term of a sequence a(n) that can be defined as follows:
: a(1), a(2), a(3) are three distinct primes of the form 30*k + 29;
: a(n) is the smallest prime greater than a(n – 1) that can be written as a(n) = a(i) + a(j) – 29, where 1 ≤ i ≤ j < n.

8. Three conjectures about an infinity of subsets of integers, each with possible infinite terms primes or squares of primes

Abstract. In my previous paper «Twenty-four conjectures about "the eight essential subsets of primes"» are made three conjectures about each one from the following eight subsets: the primes of the form 30*k + 1, 30*k + 7, 30*k + 11, 30*k + 13, 30*k + 17, 30*k + 19, 30*k + 23 respectively 30*k + 29. The conjectures from that paper state that each from these eight sets of primes has an infinity of terms and also that each one of them can be entirely defined with a recurrent formula starting from just three given terms. In this paper are generalized the three conjectures for an infinity of subsets, each having possibly an infinity of terms which are primes or squares of primes, subsets of integers of the form 2*p(1)*p(2)*...*p(m)*k + d, where p(1), p(2), ..., p(m) are the first m odd primes, k is a non-null positive integer and d an odd positive integer satisfying certain conditions.

Conjecture 1:

The sequence a(n), as it will be defined below, has an infinity of terms that are primes or squares of primes.

The sequence a(n), where n non-null positive integer, is defined in the following way:

: a(1) = q_1, a(2) = q_2, a(3) = q_3, where q_1, q_2 and q_3 are the first three integers which are primes or squares of primes of the form $2*p_1*p_2*...*p_m*k + d$, where p_1, p_2, ..., p_m are the first m odd primes, k is a non-null positive integer and d is equal to 1 or is equal to any odd positive integer which satisfies the following two conditions: d is co-prime to any of the primes p_1, p_2, ..., p_m and d < $2*p_1*p_2*...*p_m$.

: a(n) is the smallest prime or square of prime greater than a(n – 1) that can be written as a(n) = a(i) + a(j) – d, where 1 ≤ i ≤ j < n (if such prime or square of prime exists for any n, as the conjecture states).

Conjecture 2:

Any prime or square of prime of the form $2*p_1*p_2*...*p_m*k + d$ is a term of the sequence a(n) as it is defined by Conjecture 1 [in other words, the sequence a(n) is the same with the sequence of the integers which are primes or squares of primes of the form $2*p_1*p_2*...*p_m*k + d$, where k > 0].

Conjecture 3:

If the Conjecture 2 doesn't hold, than is true at least that any prime or square of prime of the form $2*p_1*p_2*...*p_m*k + d$ is a term of a sequence a(n) that can be defined as follows:

: a(1), a(2), a(3) are three distinct integers which are primes or squares of primes of the form $2*p_1*p_2*...*p_m*k + d$, where k > 0;

: a(n) is the smallest integer which is prime or square of prime greater than a(n – 1) that can be written as a(n) = a(i) + a(j) – d, where 1 ≤ i ≤ j < n.

Verifying the Conjecture 2 for m = 1:
(for first few terms)

For m = 1 we have $p_1 = p_m = 3$ and only two possible values for d (because d must be odd, smaller than 2*3 = 6 and also co-prime with 3), *i.e.* d = 1 and d = 5.

I.

For d = 1 the first three integers which are primes or squares of primes of the form 6*k + 1, where k > 0, are 7, 13 and 19:

: Conjecture 1 states that there exist an infinity of integers of the form 6*k + 1 which are primes or squares of primes;

: Conjecture 2 states that any prime or square of prime of the form 6*k + 1 can be defined starting from the primes 7, 13 and 19 with the formula from Conjecture 1. The next six integers of this form which are primes or squares of primes are 5^2 = 1 + 19 – 1 = 13 + 13 – 1; 31 = 13 + 19 – 1; 37 = 7 + 31 – 1; 43 = 7 + 37 – 1, 7^2 = 7 + 43 – 1 = 13 + 37 – 1 = 19 + 31 - 1; 61 = 19 + 43 – 1 = 31 + 31 – 1.

II.

For d = 5 the first three integers which are primes or squares of primes of the form 6*k + 5, where k > 0, are 11, 17 and 23:

: Conjecture 1 states that there exist an infinity of integers of the form 6*k + 5 which are primes or squares of primes;

: Conjecture 2 states that any prime or square of prime of the form 6*k + 5 can be defined starting from the primes 11, 17 and 23 with the formula from Conjecture 1. The next six integers of this form which are primes or squares of primes are 29 = 17 + 17 – 5; 41 = 23 + 23 – 5; 47 = 11 + 41 – 5 = 23 + 29 – 5; 53 = 11 + 47 – 5 = 17 + 41 – 5 = 29 + 29 – 5; 59 = 11 + 53 – 5 = 17 + 47 – 5 = 23 + 41 – 5; 71 = 17 + 59 – 5 = 23 + 53 – 5 = 29 + 47 – 5.

Verifying the Conjecture 2 for m = 2:

For m = 2 we have $p_1 = 3$ and $p_2 = p_m = 5$ and the subsets of integers of the form 30*k + 1, 30*k + 7, 30*k + 11, 30*k + 13, 30*k + 17, 30*k + 19, 30*k + 23; for these eight subsets of integers we treated a more strict form of the three conjectures from above in the previous paper «Twenty-four conjectures about "the eight essential subsets of primes"».

Verifying the Conjecture 2 for m = 3:
(for d = 1, d = 11 and the first few terms)

For m = 3 we have $p_1 = 3$, $p_2 = 5$ and $p_3 = p_m = 7$ and the subsets of primes of the form 210*k + 1, 210*k + 11, 210*k + 13, 210*k + 17, 210*k + 19, 210*k + 23, 210*k + 29, 210*k + 31, 210*k + 37, 210*k + 41, 210*k + 43, 210*k + 47, 210*k + 53, 210*k + 59, 210*k + 61, 210*k + 67, 210*k + 71, 210*k + 73, 210*k + 79, 210*k + 83, 210*k + 89, 210*k + 97, 210*k + 101, 210*k + 103, 210*k + 107, 210*k + 109, 210*k + 13, 210*k + 121, 210*k + 127, 210*k + 131, 210*k + 137, 210*k + 139, 210*k + 143, 210*k + 149, 210*k + 151, 210*k + 157, 210*k + 163, 210*k + 167, 210*k + 169, 210*k + 173, 210*k + 179, 210*k + 181, 210*k + 187, 210*k + 191, 210*k + 193, 210*k + 197, 210*k + 199, 210*k + 209.

: Conjecture 1 states that there exist an infinity of integers of each of these forms which are primes or squares of primes.

: Conjecture 2 states that any prime or square of prime of this form can be defined with the formula from Conjecture 1 starting from the three integers which are primes or squares of primes of the respective form, considering k > 0.

> : The first three integers which are primes or squares of primes of the form 210*k + 1, where k > 0, are 211, 421 and 631; the following next six integers which are primes or squares of primes of this form are 29^2 = 21 + 631 – 1 = 421 + 421 – 1; 1051 = 421 + 631 – 1 = 211 + 29^2 – 1; 1471 = 421 + 1051 – 1; 41^2 = 211 + 1471 – 1 = 631 + 1051 – 1; 2311 = 631 + 41^2 – 1; 2521 = 211 + 2311 – 1 = 1051 + 1471 – 1.
>
> : The first three integers which are primes or squares of primes of the form 210*k + 11, where k > 0, are 431, 641 and 1061; the following next six integers which are primes or squares of primes of this form are 1481 = 431 + 1061 – 11; 1901 = 431 + 1481 – 11; 2111 = 641 + 1481 – 11; 2531 = 431 + 2111 – 11; 2741 = 641 + 2111 – 11; 3371 = 641 + 2741 – 11.

9. Pairs of primes or pseudoprimes that generate an infinity of primes or pseudoprimes via a certain recurrence relation

Abstract. In this paper are made five conjectures about a type of pairs of primes respectively Fermat pseudoprimes which have the property to generate an infinity of primes respectively Fermat pseudoprimes via a recurrence formula that will be defined in this paper; we name the pairs with this property Coman pairs of primes respectively Coman pairs of pseudoprimes. Because it is easy to show that two given primes respectively pseudoprimes do not form such a pair and it is very difficult to prove that they form such a pair, the correct expression about two odd primes (or pseudoprimes) p, q, where p = 30*k + d and q = 30*h + d, where k, h are non-null positive integers and d has the values 1, 7, 11, 13, 17, 19, 23, 29, is that the pair (p,q) is not a Coman pair respectively that the pair (p,q) is a possible Coman pair of primes (or pseudoprimes).

Definition 1:

We call the pair of odd primes (p, q), where p = 30*k + d and q = 30*h + d, where k, h are non-null positive integers and d has the values 1, 7, 11, 13, 17, 19, 23, 29, a *[possible] Coman pair of primes* if the sequence a(n), as it will be defined below, has [possibly] an infinity of terms that are prime numbers.

The sequence a(n), where n non-null positive integer, is defined in the following way:
: a(1) = p, a(2) = q;
: a(3) is the smallest number, different from p and q, which is prime from the following three ones: p + q – d, 2*p – d and 2*q - d;
: a(n) is the smallest prime greater than a(n – 1) that can be written as a(n) = a(i) + a(j) – d, where 1 ≤ i ≤ j < n.

Note: the definition implies that the one from the numbers p + q – d, 2*p – d and 2*q - d is prime and that for any n, n ≥ 4, there exist i, j, where 1 ≤ i ≤ j < n, such that a(n) = a(i) + a(j) – d, where a(n), a(i) and a(j) are all three primes.

Examples:

The pair of primes (37, 67) is a possible *Coman pair of primes* because:
: a(3) = 37 + 67 – 7 = 97 is prime;
: a(4) = 37 + 97 – 7 = 67 + 67 = 127 is prime;
: a(5) = 37 + 127 – 7 = 67 + 97 – 7 = 157 is prime;
: a(6) = 127 + 157 – 7 = 277 is prime.
: a(7) = 37 + 277 – 7 = 157 + 157 – 7 = 307 is prime.
(...)

The pair of primes (97, 127) is not a possible *Coman pair of primes* because 2*97 – 7 = 187 is not prime, 2*127 – 7 = 247 is not prime and also 97 + 127 – 7 = 217 is not prime.

Definition 2:

We call a *Coman pair of primes* a *Coman strict pair of primes* if a(3) = p + q – d and i ≠ j, in other words a(i) ≠ a(j).

Examples:

The pair of primes (37, 67) is a possible *Coman strict pair of primes* because:
: a(3) = 37 + 67 – 7 = 97 is prime;
: a(4) = 37 + 97 – 7 = 127 is prime;
: a(5) = 37 + 127 – 7 = 67 + 97 – 7 = 157 is prime;
: a(6) = 127 + 157 – 7 = 277 is prime;
: a(7) = 37 + 277 – 7 is prime.
(...)

The pair of primes (37, 157) is not a possible *Coman strict pair of primes* because 37 + 157 – 7 = 187 is not prime, but is a possible Coman pair of primes, because a(3) = 37 + 37 – 7 = 67 is prime, a(4) = 67 + 67 – 7 = 127 is prime (...).

Definition 3:

We call the pair of odd primes (p, q) a *[possible] generalized Coman pair of primes to base b*, where b is a non-null integer, if the sequence a(n), as it will be defined below, has [possibly] an infinity of terms that are prime numbers.

The sequence a(n), where n non-null positive integer, is defined in the following way:
: a(1) = p, a(2) = q;
: a(3) is the smallest number, different from p and q, which is prime from the following three ones: p + q – b, 2*p – b and 2*q - b;
: a(n) is the smallest prime greater than a(n – 1) that can be written as a(n) = a(i) + a(j) – b, where 1 ≤ i ≤ j < n.

Note: the definition implies that the one from the numbers p + q – b, 2*p – b and 2*q - b is prime and that for any n, n ≥ 4, there exist i, j, where 1 ≤ i ≤ j < n, such that a(n) = a(i) + a(j) – b, where a(n), a(i) and a(j) are all three primes.

Examples:

The pair of primes (7, 13) is a possible *generalized Coman pair of primes to base 1* because:
: a(3) = 7 + 13 – 1 = 19 is prime;
: a(4) = 13 + 19 – 1 = 31 is prime;
: a(5) = 7 + 31 – 1 = 19 + 19 – 1 = 37 is prime;
: a(6) = 7 + 37 – 1 = 13 + 31 – 1 = 43 is prime;
: a(7) = 19 + 43 – 1 = 31 + 31 – 1 = 61 is prime.
(...)

The pair of primes (11, 17) is a possible *generalized Coman pair of primes to base -1* because:
: a(3) = 11 + 11 + 1 = 23 is prime;
: a(4) = 11 + 17 + 1 = 29 is prime;
: a(5) = 11 + 29 + 1 = 41 is prime;
: a(6) = 17 + 29 + 1 = 47 is prime;
: a(7) = 11 + 41 + 1 = 53 is prime;
: a(8) = 11 + 47 + 1 = 59 is prime.
(...)

Definition 4:

We call a *generalized Coman pair of primes* a *generalized Coman strict pair of primes* if a(3) = p + q – b and i ≠ j, in other words a(i) ≠ a(j).

Note that the term a(n) = 23 is not a term of the *generalized Coman strict pair of primes* (11, 17), but is a term of the *generalized Coman pair of primes* (11, 17).

Definition 5:

We call the pair (p, q) of odd Fermat pseudoprimes to the same base b, where p = 30*k + d and q = 30*h + d, where k, h are non-null positive integers and d has the values 1, 7, 11, 13, 17, 19, 23, 29, a *[possible] Coman pair of pseudoprimes* if the sequence a(n), as it will be defined below, has [possibly] an infinity of terms that are pseudoprimes to the same base b.

The sequence a(n), where n non-null positive integer, is defined in the following way:
: a(1) = p, a(2) = q;
: a(3) is the smallest number, different from p and q, which is prime or Fermat pseudoprime to base b from the following three ones: p + q – d, 2*p – d and 2*q - d;
: a(n) is the smallest number which is prime or pseudoprime to base b greater than a(n – 1) that can be written as a(n) = a(i) + a(j) – d, where 1 ≤ i ≤ j < n.

Note: the definition implies that one from the numbers p + q – d, 2*p – d and 2*q - d is a prime or a pseudoprime to base b and that for any n, n ≥ 4, there exist i, j, where 1 ≤ i ≤ j < n, such that a(n) = a(i) + a(j) – d, where a(n), a(i) and a(j) are primes or pseudoprimes to base b, and, finally, that there is an infinite subset of the set a(n), namely a(m), whose terms are all of them pseudoprimes to base b.

Examples:

The pair of Fermat pseudoprimes to base 2 (2701, 2821), where 2701 = 30*90 + 1 and 2821 = 30*94 + 1, is a possible *Coman pair of pseudoprimes* because:
: a(3) = 2701 + 2821 – 1 = 5521 is prime;
: a(4) = 2821 + 2821 – 1 = 5641 is prime;
: a(5) = 2701 + 5521 – 1 = 8221 is prime;
: a(6) = 5521 + 8221 – 1 = 13741 is Fermat pseudoprime to base 2.
(...)

Conjecture 1:

There is an infinity of Coman pairs of primes.

Conjecture 2:

There is an infinity of Coman strict pairs of primes.

Conjecture 3:

There is an infinity of generalized Coman pairs of primes to any base b, where b is non-null integer.

Conjecture 4:

There is an infinity of generalized Coman strict pairs of primes to any base b, where b is non-null integer.

Conjecture 5:

There exist pairs of Poulet numbers which are Coman pairs of pseudoprimes.

References:

7. Coman, Marius, *Twenty-four conjectures about "the eight essential subsets of primes"*, Vixra;
8. Coman, Marius, *Three conjectures about an infinity of subsets of integers, each with possible infinite terms primes or squares of primes*, Vixra.

10. Three conjectures about semiprimes inspired by a recurrent formula involving 2-Poulet numbers

Abstract. Studying the relation between the two prime factors of a 2-Poulet number I found an interesting recurrent formula involving these numbers that seems to lead often to a value which is semiprime; based on this observation I made three conjectures about semiprimes.

Observation 1

We take a pair of 2-Poulet numbers which have a common prime factor, as for instance the pair [P_1 = 341 = 11*31, P_2 = 4681 = 31*151] or the pair [P_1 = 1387 = 19*73, P_2 = 2701 = 37*73] and apply on it the recurrent formula $P_n = P_{n-2} + \gcd((P_{n-2} - 1), (P_{n-1} - 1))$.

In the case [P_1, P_2] = [341, 4681] we have:

: P_3 = 341 + gcd(340, 4680) = 361;
: P_4 = 4681 + gcd(4680, 360) = 5041;
: P_5 = 361 + gcd(360, 5040) = 721;
: P_6 = 5041 + gcd(5040, 720) = 5761;
: P_7 = 721 + gcd(720, 5760) = 1441;
: P_8 = 5761 + gcd(5760, 1440) = 7201;
: P_9 = 1441 + gcd(1440, 7200) = 2881;
: P_{10} = 7201 + gcd(7200, 2880) = 8641;
: P_{11} = 2881 + gcd(2880, 8640) = 5761;
: P_{12} = 8641 + gcd(8640, 5760) = 11521;
: P_{13} = 5760 + gcd(5760, 11520) = 11521.

Starting from P_{14}, we will have $P_{14} = P_{15} = 2*(P_{12} - 1) + 1$, $P_{16} = P_{17} = 2*(P_{14} - 1) + 1$ and so on.

In the case [P_1, P_2] = [1387, 2701] we have:

: P_3 = 1387 + gcd(340, 4680) = 1405;
: P_4 = 2701 + gcd(4680, 360) = 2809;
: P_5 = 1405 + gcd(360, 5040) = 2809;

Starting from P_6, we will have $P_6 = P_7 = 2*(P_4 - 1) + 1$ = 5617, $P_8 = P_9 = 2*(P_{12} - 1) + 1$ = 11233, $P_{10} = P_{11} = 2*(P_8 - 1) + 1$ = 22465, $P_{12} = P_{13} = 2*(P_{10} - 1) + 1$ = 44929, $P_{14} = P_{15} = 2*(P_{12} - 1) + 1$ = 89857, $P_{16} = P_{17} = 2*(P_{14} - 1) + 1$ = 179713 and so on.

It can be seen that many of the values of the terms P_i are semiprimes: 361 = 19*19, 5041 = 71*71, 721 = 7*103, 5761 = 7*823, 1441 = 11*131, 7201 = 19*379, 2881 = 43*67, 11521 = 41*281, 1405 = 5*281, 2809 = 53*53, 5617 = 41*137, 11233 = 47*239, 22465 = 5*4493, 44929 = 179*251, 89857 = 59*1523, 179713 = 29*6197.

More than that, between the two distinct prime factors p and q from many of the semiprimes obtained above there exist the relation q – p + 1 = n, where n is a prime or a square of a prime:

: 103 – 7 + 1 = 97;
: 131 – 11 + 1 = 121 = 11^2;
: 379 – 19 + 1 = 361 = 19^2;
: 67 – 43 + 1 = 25 = 5^2;
: 281 – 41 + 1 = 41;
: 281 – 5 + 1 = 277;
: 67 – 43 + 1 = 25 = 5^2;
: 137 – 41 + 1 = 97;
: 239 – 47 + 1 = 193;
: 4493 – 5 + 1 = 4489 = 67^2
: 251 – 179 + 1 = 73.

A very interesting thing it happens even if between p and q there is not the relation from the preceding paragraph; in many of these cases q – p + 1 = n, where n is a semiprime whose two prime factors admit themselves the relation showed:

: 823 – 7 + 1 = 817 = 19*43 and 43 – 19 + 1 = 25 = 5^2;
: 1523 – 59 + 1 = 1465 = 5*293 and 293 – 5 + 1 = 17^2;
: 6197 – 29 + 1 = 6169 = 31*199 and 199 – 31 + 1 = 13^2.

Observation 2

We also observed that the iterative formula $a_{n+1} = 2*(a_n – 1) + 1$, where a_1 is a square of a prime minus nine, seems likewise to often conduct to primes, power of primes or semiprimes with the characteristics of those from Observation 1.

For a_1 = 7^2 – 9 = 40 we obtain the following sequence:
: 79, 157, 313, 625, 1249, 2497, 4993, 9985, 19969, 39937, 79873, 159745, 319489, 638977, 1277953 (...), where:
: 79, 157, 313, 1249, 4993, 39937, 79873, 319489, 638977 are primes;
: 625 = 5^4 is a power of prime;
: 2497 = 11*227; 227 – 11 + 1 = 217 = 7*31; 31 – 7 + 1 = 25 = 5^2;
: 9985 = 5*1997; 1997 – 5 + 1 = 1993 prime;
: 19969 = 19*1051; 1051 – 19 + 1 = 1033 prime.
: 1277953 = 101*12653; 12653 – 101 + 1 = 12553 prime.

Conjecture 1

For any odd prime n there exist an infinity of pairs of odd primes [p, q] such that q – p + 1 = n.

Conjecture 2

For any semiprime p_1*q_1, where p_1 and q_1 are odd distinct primes, there exist an infinity of pairs of odd primes $[p_2, q_2]$ such that $q_2 – p_2 + 1 = p_1*q_1$.

Conjecture 3

For any odd prime n there exist an infinity of pairs of odd primes $[p_i, q_i]$, for any i from 1 to infinite, such that:

: $q_1 – p_1 + 1 = n$;
: $q_2 – p_2 + 1 = p_1*q_1$;
: $q_3 – p_3 + 1 = p_2*q_2$;
(...)
: $q_i – p_i + 1 = p_{i-1}*q_{i-1}$.

Note:

This is an interesting way to construct (possible) infinite sequences of semiprimes p_i*q_i, starting from a given prime and considering, for instance, the smallest p_i for which the relations from Conjecture 3 are verified. For instance, in the conditions mentioned, we take n = 13. We have:

: $p_1 = 5$ because is the smallest prime such that $n – 1 + p_1 = q_1$ is prime, so $q_1 = 13 – 1 + 5 = 17$;
: $p_2 = 5$ because is the smallest prime such that $p_1*q_1 – 1 + p_2 = q_2$ is prime, so $q_2 = 5*17 – 1 + 5 = 89$;
: $p_3 = 5$ because is the smallest prime such that $p_2*q_2 – 1 + p_3 = q_3$ is prime, so $q_3 = 5*89 – 1 + 5 = 449$;
: $p_4 = 7$ because is the smallest prime such that $p_3*q_3 – 1 + p_4 = q_4$ is prime, so $q_4 = 5*449 – 1 + 7 = 2251$ (...).

We obtained the following sequence of semiprimes p_i*q_i: 85, 445, 2245, 15757 (...).

11. Two conjectures on primes and a conjecture on Fermat pseudoprimes to base two

Abstract. I treated the 2-Poulet numbers in many papers already but they continue to be a source of inspiration for me; in this paper I make a conjecture on primes inspired by the relation between the two prime factors of a 2-Poulet number and I also make a conjecture on Fermat pseudoprimes to base two.

Conjecture 1 (on primes):

For any prime p, p ≥ 7, there exist an infinity of primes q, q > p, such that the number r = (q – 1)/(p – 1) is a natural number. In other words, for any such prime p there exist an infinity of natural numbers r such that q = r*p – p + 1 is prime.

Conjecture 2 (on primes):

For any prime p, p ≥ 7, there exist an infinity of primes q, q > p, such that the number r = (q – 1)/(p – 1) is a rational but not natural number. In other words, for any such prime p there exist an infinity of rational but not natural numbers r such that q = r*p – p + 1 is prime.

Conjecture 3 (on 2-Poulet numbers):

For any 2-Poulet number $P = d_1*d_2$, where $d_2 > d_1$, the following statement is true: the number $r = (d_2 – 1)/(d_1 – 1)$ is a rational number.

Verifying the conjecture 3:

(For the first seventy-five 2-Poulet numbers)

Note:

In the column I are listed the first seventy-five 2-Poulet numbers, in the column II are listed the cases when $r = (d_2 – 1)/(d_1 – 1)$ is a natural number (put it in other way, the cases when $d_2 = r*d_1 – r + 1$) and in the column III are listed the cases when $r = (d_2 – 1)/(d_1 – 1)$ is a rational but not natural number.

I.	II.	III.
1 341 = 11*31	(d2 = 3*d1 – 2)	
2 1387 = 19*73	(d2 = 4*d1 – 3)	
3 2047 = 23*89	(d2 = 4*d1 – 3)	
4 2701 = 37*73	(d2 = 2*d1 – 1)	
5 3277 = 29*113	(d2 = 4*d1 – 3)	
6 4033 = 37*109	(d2 = 2*d1 – 1)	
7 4369 = 17*257	(d2 = 16*d1 – 15)	
8 4681 = 31*151	(d2 = 5*d1 – 4)	
9 5461 = 43*127	(d2 = 3*d1 – 2)	
10 7957 = 73*109		(d2 – 1)/(d1 – 1) = 3/2
11 8321 = 53*157	(d2 = 3*d1 – 2)	
12 10261 = 31*331	(d2 = 11*d1 – 10)	

13 13747 = 59*233	(d2 = 4*d1 – 3)	
14 14491 = 43*337	(d2 = 8*d1 – 7)	
15 15709 = 23*683	(d2 = 31*d1 – 30)	
16 18721 = 97*193	(d2 = 2*d1 – 1)	
17 19951 = 71*281	(d2 = 4*d1 – 3)	
18 23377 = 97*241		(d2 – 1)/(d1 – 1) = 5/2
19 31417 = 89*353	(d2 = 4*d1 – 3)	
20 31609 = 73*433	(d2 = 6*d1 – 5)	
21 31621 = 103*307	(d2 = 3*d1 – 2)	
22 35333 = 89*397		(d2 – 1)/(d1 – 1) = 9/2
23 42799 = 127*337		(d2 – 1)/(d1 – 1) = 8/3
24 49141 = 157*313	(d2 = 2*d1 – 1)	
25 49981 = 151*331	(d2 = 2*d1 – 1)	
26 60701 = 101*601	(d2 = 6*d1 – 5)	
27 60787 = 89*683		(d2 – 1)/(d1 – 1) = 31/4
28 65077 = 59*1103	(d2 = 19*d1 – 18)	
29 65281 = 97*673	(d2 = 7*d1 – 6)	
30 80581 = 61*1321	(d2 = 22*d1 – 21)	
31 83333 = 167*499	(d2 = 3*d1 – 2)	
32 85489 = 53*1613	(d2 = 31*d1 – 30)	
33 88357 = 149*593	(d2 = 4*d1 – 3)	
34 90751 = 151*601	(d2 = 4*d1 – 3)	
35 104653 = 229*457	(d2 = 2*d1 – 1)	
36 123251 = 59*2089	(d2 = 36*d1 – 35)	
37 129889 = 193*673		(d2 – 1)/(d1 – 1) = 7/2
38 130561 = 137*953	(d2 = 7*d1 – 6)	
39 150851 = 251*601		(d2 – 1)/(d1 – 1) = 12/5
40 162193 = 241*673		(d2 – 1)/(d1 – 1) = 14/5
41 164737 = 257*641		(d2 – 1)/(d1 – 1) = 5/2
42 181901 = 101*1801	(d2 = 18*d1 – 17)	
43 188057 = 89*2113	(d2 = 24*d1 – 23)	
44 194221 = 167*1163	(d2 = 7*d1 – 6)	
45 196093 = 157*1249	(d2 = 8*d1 – 7)	
46 215749 = 79*2731	(d2 = 35*d1 – 34)	
47 219781 = 271*811	(d2 = 3*d1 – 2)	
48 220729 = 103*2143	(d2 = 21*d1 – 20)	
49 226801 = 337*673	(d2 = 2*d1 – 1)	
50 233017 = 43*5419	(d2 = 129*d1 – 128)	
51 241001 = 401*601		(d2 – 1)/(d1 – 1) = 3/2
52 249841 = 433*577		(d2 – 1)/(d1 – 1) = 4/3
53 253241 = 157*1613		(d2 – 1)/(d1 – 1) = 31/3
54 256999 = 233*1103		(d2 – 1)/(d1 – 1) = 19/4
55 264773 = 149*1777	(d2 = 12*d1 – 11)	
56 271951 = 151*1801	(d2 = 12*d1 – 11)	
57 275887 = 263*1049	(d2 = 4*d1 – 3)	
58 280601 = 277*1013		(d2 – 1)/(d1 – 1) = 11/3
59 282133 = 307*919	(d2 = 3*d1 – 2)	
60 294271 = 103*2857	(d2 = 28*d1 – 27)	
61 318361 = 241*1321		(d2 – 1)/(d1 – 1) = 11/2
62 357761 = 131*2731	(d2 = 21*d1 – 20)	

63 390937 = 313*1249 (d2 = 4*d1 – 3)
64 396271 = 223*1777 (d2 = 8*d1 – 7)
65 422659 = 163*2593 (d2 = 16*d1 – 15)
66 435671 = 191*2281 (d2 = 12*d1 – 11)
67 443719 = 167*2657 (d2 = 16*d1 – 15)
68 452051 = 251*1801 (d2 – 1)/(d1 – 1) = 36/5
69 458989 = 277*1657 (d2 = 6*d1 – 5)
70 481573 = 337*1429 (d2 – 1)/(d1 – 1) = 17/4
71 486737 = 233*2089 (d2 = 9*d1 – 8)
72 489997 = 157*3121 (d2 = 20*d1 – 19)
73 513629 = 293*1753 (d2 = 6*d1 – 5)
74 514447 = 359*1433 (d2 = 4*d1 – 3)
75 556169 = 457*1217 (d2 – 1)/(d1 – 1) = 8/3

Comment:

It can be seen that are already outlined few subsets of 2-Poulet numbers, such the following ones:

: 2-Poulet numbers $P = d_1*d_2$ for which $r = (d_2 – 1)/(d_1 – 1)$ is of the form r = p^m/2^n, where p odd prime and m, n positive integers; such numbers are: 7957, 23377, 35333, 60787, 129889, 164737, 241001, 256999, 318361, 481573 (...);

: 2-Poulet numbers $P = d_1*d_2$ for which $r = (d_2 – 1)/(d_1 – 1)$ is of the form r = n/3, where n positive integer; such numbers are: 42799, 249841, 253241, 280601, 556169 (...);

: 2-Poulet numbers $P = d_1*d_2$ for which $r = (d_2 – 1)/(d_1 – 1)$ is of the form r = n/5, where n positive integer; such numbers are: 150851, 162193, 452051 (...).

12. An interesting property shared by a set of primes

Abstract. In this paper I present a property of a set of primes, interesting not because it has a value for distinguish primes from odd composites, because there are such numbers which also have this property, but because it seems to split the set of primes into two classes – the primes that have this property and the primes that have not this property – containing primes in surprisingly equal proportion.

Note:

We name the primes p that have the property that can be written as p = m*s – m + 1, where s is the sum of their digits and m is a non-null positive integer, primes of class I and the primes that haven't it primes of class II.

Example:

: 19 is a prime of class I because it can be written as 19 = m*s – m + 1, where s = 10 and m = 2;

: 23 is a prime of class II because it can't be written as 23 = m*5 – m + 1, where m non-null positive integer.

Conjecture:

Given any positive integer N large enough (probably the condition that N > 78 is sufficient), let a be the number of primes of the class I less than or equal to N and b the number of primes of the class II less than or equal to N; let also be r the largest value from a/b and b/a; then r < 2.

Primes of class I:

(that have this property)
2, 3, 5, 7, 11, 13, 19, 31, 37, 41, 43, 61, 71, 73, 101, 103, 113, 127, 137, 151, 157, 163, 181, 191, 193, 199, 211, 223, 229, 239, 241, 271, 281 (...).

The corresponding values of m:
1, 1, 1, 1, 10, 4, 2, 10, 4, 10, 7, 10, 10, 8, 100, 34, 16, 14, 34, 7, 13, 18, 20, 19, 16, 11, 70, 37, 19, 40, 30, 28 (...).

Primes of class II:

(that have not this property)
17, 23, 29, 47, 53, 59, 67, 79, 83, 89, 97, 107, 109, 131, 139, 149, 167, 173, 179, 197, 227, 233, 239, 251, 257, 263, 269, 277, 283, 293 (...).

Observation:

It can be seen that, for N = 100, r = 14/11; for N = 200, r = 26/20; for N = 300, r = 33/30.

Note:

Interesting classes of primes can be formed based on this property; such a class contains the primes p that can be written as p = s^2 – s + 1, where s is the sum of their digits; such primes are: 13, 43, 151, 157 (...).

13. The notion of chameleonic numbers, a set of composites that “hide” in their inner structure an easy way to obtain primes

Abstract. In this paper I present the notion of “chameleonic numbers”, a set of composite squarefree numbers not divisible by 2, 3 or 5, having two, three or more prime factors, which have the property that can easily generate primes with a certain formula, other primes than they own prime factors but in an amount proportional with the amount of these ones.

Definition:

We define in the following way a “chameleonic number”: the non-null positive composite squarefree integer C not divisible by 2, 3 or 5 is such a number if the absolute value of the number P – d + 1 is always a prime or a power of a prime, where d is one of the prime factors of C and P is the product of all prime factors of C but d.

Example:

The Hardy-Ramanujan number, 1729 = 7*13*19, is a “chameleonic number” because:

: 7*13 – 19 + 1 = 73, a prime;

: 7*19 – 13 + 1 = 121 = 11^2, a square of a prime;

: 13*19 – 7 + 1 = 241, a prime.

Comment:

Indeed, we obtained using the decomposition in prime factors of the number 1729 with the three prime factors [7, 13, 19] the triplet of primes [11, 73, 241], but this is the defining property of the “chameleonic numbers”; the property that I was talking about in title and in abstract refers to another triplet of primes, obtained with a certain formula. The numbers N = 30*(d – 1) + C, where C is a “chameleonic number” and d one of its prime factors, are often primes, Fermat pseudoprimes or “chameleonic numbers” themselves.

Example:

The Hardy-Ramanujan number, 1729 = 7*13*19, which is also a “chameleonic number”, as it can be seen above, generates with the mentioned formula the following three numbers:

: N_1 = 30*(7 – 1) + 1729 = 23*83, a “chameleonic number” because 83 – 23 + 1 = 61, a prime;

: N_2 = 30*(13 – 1) + 1729 = 2089, a prime;

: N_3 = 30*(19 – 1) + 1729 = 2269, a prime.

Chameleonic semiprimes:

The set of chameleonic numbers with two prime factors is: 77, 91, 119, 133, 143, 161, 187, 203 (...).

Indeed:

: for 77 = 7*11 we have 11 – 7 + 1 = 5, prime;

: for 91 = 7*13 we have 13 – 7 + 1 = 7, prime;

: for 119 = 7*17 we have 17 – 7 + 1 = 11, prime;
: for 133 = 7*19 we have 19 – 7 + 1 = 13, prime;
: for 143 = 11*13 we have 13 – 11 + 1 = 3, prime;
: for 161 = 7*23 we have 23 – 7 + 1 = 17, prime;
: for 187 = 11*17 we have 17 – 11 + 1 = 7, prime;
: for 203 = 7*29 we have 29 – 7 + 1 = 23, prime.
(...)

Duplets of numbers obtained from the chameleonic semiprimes with the formula mentioned above:

: 30*(7 – 1) + 77 = 257, prime;
: 30*(11 – 1) + 77 = 377 = 13*29, a "chameleonic number" because 29 – 13 + 1 = 17, a prime;

: 30*(7 – 1) + 91 = 271, prime;
: 30*(13 – 1) + 91 = 451 = 11*41, a "chameleonic number" because 41 – 11 + 1 = 31, a prime;

: 30*(7 – 1) + 119 = 299 = 13*23, a "chameleonic number" because 23 – 13 + 1 = 11, a prime;
: 30*(17 – 1) + 119 = 599, prime;

: 30*(7 – 1) + 133 = 313, prime;
: 30*(19 – 1) + 133 = 673, prime;

: 30*(11 – 1) + 143 = 443, prime;
: 30*(13 – 1) + 143 = 503, prime;

: 30*(7 – 1) + 161 = 341 = 11*31, a Fermat pseudoprime to base two;
: 30*(23 – 1) + 161 = 821, prime;

: 30*(11 – 1) + 187 = 487, prime;
: 30*(17 – 1) + 187 = 667 = 23*29, a "chameleonic number" because 29 – 23 + 1 = 7, a prime;

: 30*(7 – 1) + 203 = 383, prime;
: 30*(29 – 1) + 203 = 1043 = 7*149, an "extended chameleonic number" because 149 – 7 + 1 = 143, a "chameleonic number" (but we extend only intuitively the definition in this paper).

Note:
Many Fermat pseudoprimes to base two with two prime factors are also chameleonic numbers (see the articles about 2-Poulet numbers posted by us on Vixra).

Chameleonic numbers with three prime factors:

The set of chameleonic numbers with three prime factors is: 1309, 1729, 2233, 2849, 3289 (...).

Indeed:

: for 1309 = 7*11*17 we have:
: 7*11 – 17 + 1 = 61, prime;
: 7*17 – 11 + 1 = 109, prime;
: 11*17 – 7 + 1 = 181, prime;
: for 2233 = 7*11*29 we have:
: 7*11 – 29 + 1 = 49 = 7^2, a square of a prime;
: 7*29 – 11 + 1 = 193, prime;
: 11*29 – 7 + 1 = 313, prime;
: for 2849 = 7*11*37 we have:
: 7*11 – 37 + 1 = 41, prime;
: 7*37 – 11 + 1 = 289 = 17^2, a square of a prime;
: 11*37 – 7 + 1 = 401, prime;
: for 3289 = 11*13*23 we have:
: 11*13 – 23 + 1 = 121 = 11^2, a square of a prime;
: 11*23 – 13 + 1 = 361 = 19^2, a square of a prime;
: 13*23 – 11 + 1 = 289 = 17^2, a square of a prime.

Triplets of numbers obtained from the chameleonic semiprimes with the formula mentioned above:

: 30*(7 – 1) + 1309 = 1489, prime;
: 30*(11 – 1) + 1309 = 1609, prime;
: 30*(17 – 1) + 1309 = 1789, prime;

: 30*(7 – 1) + 2233 = 2413 = 19*127, a “chameleonic number” because 127 – 19 + 1 = 109, a prime;
: 30*(11 – 1) + 2233 = 2533 = 17*149, an “extended chameleonic number” because 149 – 17 + 1 = 133, a a “chameleonic number”;
: 30*(29 – 1) + 2233 = 3073 = 7*439, a “chameleonic number” because 439 – 7 + 1 = 433, a prime;
(...)

Open problems:

I. Are there other interesting properties of the chameleonic numbers?
II. There exist chameleonic numbers with 4, 5, 6 or more prime factors?

14. A very simple but possible important conjecture about primes

Abstract. In this paper I present a conjecture about primes with an extremely simple enunciation, but very interesting despite (or on the contrary, because of) its simplicity.

Conjecture:

Any prime number q, $q \geq 11$, can be written as $q = 3*(p_1 - 1) + p_2$, where p_1 and p_2 are odd primes.

The sequence of the lowest p_1 for which the primes that can be written as $q = 3*(p_1 - 1) + p_2$, where p_1 and p_2 are odd primes (the conjecture above states that all the primes greater than or equal to 11 can be written this way), can be written this way:

: 3, 3, 3, 3, 3, 3, 5, 3, 5, 3, 3, 3, 3, 7, 3, 3, 3, 5, 3, 7, 5, 3, 3, 3, 3, 7, 7, 3, 5, 5, 5, 3, 3, 7, 3, 3, 7, 5, 5, 3, 3, 5, 5, 11, 3, 3, 3, 5, 5, 3, 3, 3, 11, 3, 5, 3, 5, 11, 7, 3, 3, 7, 3, 11, 5, 3, 3, 7, 3, 3, 11, 3, 7, 5, 5, 7, 5, 5, 5, 3, 5, 3, 7, 5, 3, 3, 5, 11, 5 (...)

The corresponding p_2 and q in the sequence above:

: (5,11), (7,13), (11,17), (13,19), (17,23), (23,29), (19,31), (31,37), (29,41), (37,43), (41,47), (47,53), (53,59), (43,61), (61,67), (67,73), (73,79), (71,83), (83,89), (79,97), (89,101), (97,103), (101,107), (103,109), (107,113), (109,127), (113,131), (131,137), (127,139), (137,149), (139,151), (151,157), (157,163), (149,167), (167,173), (173,179), (163,181), (179,191), (181,193), (191,197), (193,199), (199,211), (211,223), (197,227), (223,229), (227,233), (233,239), (229,241), (239,251), (251,257), (257,263), (263,269), (241,271), (271,277), (269,281), (277,283), (281,293), (277,307), (293,311), (307,313), (311,317), (313,331), (331,337), (317,347), (337,349), (347,353), (353,359), (349,367), (367,373), (373,379), (353,383), (383,389), (379,397), (389,401), (397,409), (401,419), (409,421), (419,431), (421,433), (433,439), (431,443), (443,449), (439,457), (449,461), (457,463), (461,467), (467,479), (457, 487), (479, 491), (487, 499).

Note:

Another way to enunciate the conjecture: for any prime p greater than or equal to 11 there exist at least a smaller prime q such that $q = p - 3*n$, where n can be 2, 4, 6, 10, 12, 16, 18, 22, 28 and so on (n + 1 is odd prime).

It is always difficult to talk about arithmetic, because those who do not know what is about, nor do they understand in few sentences, no matter how inspired these might be, and those who know what is about, do no need to be told what is about. Arithmetic is that branch of mathematics that you keep it in you're soul and in you're mind, not in you're suitcase or laptop.

Part One of this book of collected papers aims to show new applications of Smarandache function in the study of some well known classes of numbers, like Sophie Germain primes, Poulet numbers, Carmichael numbers ets. Beside the well-known notions of number theory, we defined in these papers the following new concepts: "Smarandache-Coman divisors of order k of a composite integer n with m prime factors", "Smarandache-Coman congruence on primes", "Smarandache-Germain primes", Coman-Smarandache criterion for primality", "Smarandache-Korselt criterion", "Smarandache-Coman constants".

Part Two of this book brings together several papers on few well known and less known types of primes.

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