The notion of chameleonic numbers, a set of composites that "hide" in their inner structure an easy way to obtain primes

Marius Coman
Bucuresti, Romania
email: mariuscoman13@gmail.com

Abstract. In this paper I present the notion of "chameleonic numbers", a set of composite squarefree numbers not divisible by 2, 3 or 5, having two, three or more prime factors, which have the property that can easily generate primes with a certain formula, other primes than they own prime factors but in an amount proportional with the amount of these ones.

Definition:

We define in the following way a "chameleonic number": the non-null positive composite squarefree integer C not divisible by 2, 3 or 5 is such a number if the absolute value of the number P – d + 1 is always a prime or a power of a prime, where d is one of the prime factors of C and P is the product of all prime factors of C but d.

Example:

The Hardy-Ramanujan number, 1729 = 7*13*19, is a "chameleonic number" because:

: 7*13 – 19 + 1 = 73, a prime;
: 7*19 – 13 + 1 = 121 = 11^2, a square of a prime;
: 13*19 – 7 + 1 = 241, a prime.

Comment:

Indeed, we obtained using the decomposition in prime factors of the number 1729 with the three prime factors [7, 13, 19] the triplet of primes [11, 73, 241], but this is the defining property of the "chameleonic numbers"; the property that I was talking about in title and in abstract refers to another triplet of primes, obtained with a certain formula. The numbers N = 30*(d – 1) + C, where C is a "chameleonic number" and d one of its prime factors, are often primes, Fermat pseudoprimes or "chameleonic numbers" themselves.

Example:

The Hardy-Ramanujan number, 1729 = 7*13*19, which is also a "chameleonic number", as it can be seen above, generates with the mentioned formula the following three numbers:

: N_1 = 30*(7 – 1) + 1729 = 23*83, a "chameleonic number" because 83 – 23 + 1 = 61, a prime;
: N_2 = 30*(13 – 1) + 1729 = 2089, a prime;
: N_3 = 30*(19 – 1) + 1729 = 2269, a prime.

Chameleonic semiprimes:

The set of chameleonic numbers with two prime factors is: 77, 91, 119, 133, 143, 161, 187, 203 (...).

Indeed:

: for 77 = 7*11 we have 11 – 7 + 1 = 5, prime;
: for 91 = 7*13 we have 13 – 7 + 1 = 7, prime;
: for 119 = 7*17 we have 17 – 7 + 1 = 11, prime;
: for 133 = 7*19 we have 19 – 7 + 1 = 13, prime;
: for 143 = 11*13 we have 13 – 11 + 1 = 3, prime;
: for 161 = 7*23 we have 23 – 7 + 1 = 17, prime;
: for 187 = 11*17 we have 17 – 11 + 1 = 7, prime;
: for 203 = 7*29 we have 29 – 7 + 1 = 23, prime.
(...)

Duplets of numbers obtained from the chameleonic semiprimes with the formula mentioned above:

: 30*(7 – 1) + 77 = 257, prime;
: 30*(11 – 1) + 77 = 377 = 13*29, a "chameleonic number" because 29 – 13 + 1 = 17, a prime;

: 30*(7 – 1) + 91 = 271, prime;
: 30*(13 – 1) + 91 = 451 = 11*41, a "chameleonic number" because 41 – 11 + 1 = 31, a prime;

: 30*(7 – 1) + 119 = 299 = 13*23, a "chameleonic number" because 23 – 13 + 1 = 11, a prime;
: 30*(17 – 1) + 119 = 599, prime;

: 30*(7 – 1) + 133 = 313, prime;
: 30*(19 – 1) + 133 = 673, prime;

: 30*(11 – 1) + 143 = 443, prime;
: 30*(13 – 1) + 143 = 503, prime;

: 30*(7 – 1) + 161 = 341 = 11*31, a Fermat pseudoprime to base two;
: 30*(23 – 1) + 161 = 821, prime;

: 30*(11 – 1) + 187 = 487, prime;
: 30*(17 – 1) + 187 = 667 = 23*29, a "chameleonic number" because 29 – 23 + 1 = 7, a prime;

: 30*(7 – 1) + 203 = 383, prime;
: 30*(29 – 1) + 203 = 1043 = 7*149, an "extended chameleonic number" because 149 – 7 + 1 = 143, a "chameleonic number" (but we extend only intuitively the definition in this paper).

Note:
Many Fermat pseudoprimes to base two with two prime factors are also chameleonic numbers (see the articles about 2-Poulet numbers posted by us on Vixra).

Chameleonic numbers with three prime factors:
The set of chameleonic numbers with three prime factors is: 1309, 1729, 2233, 2849, 3289 (...).

Indeed:
: for 1309 = 7*11*17 we have:
: 7*11 – 17 + 1 = 61, prime;
: 7*17 – 11 + 1 = 109, prime;
: 11*17 – 7 + 1 = 181, prime;
: for 2233 = 7*11*29 we have:
: 7*11 – 29 + 1 = 49 = 7^2, a square of a prime;
: 7*29 – 11 + 1 = 193, prime;
: 11*29 – 7 + 1 = 313, prime;
: for 2849 = 7*11*37 we have:
: 7*11 – 37 + 1 = 41, prime;
: 7*37 – 11 + 1 = 289 = 17^2, a square of a prime;
: 11*37 – 7 + 1 = 401, prime;
: for 3289 = 11*13*23 we have:
: 11*13 – 23 + 1 = 121 = 11^2, a square of a prime;
: 11*23 – 13 + 1 = 361 = 19^2, a square of a prime;
: 13*23 – 11 + 1 = 289 = 17^2, a square of a prime.

Triplets of numbers obtained from the chameleonic semiprimes with the formula mentioned above:

: 30*(7 – 1) + 1309 = 1489, prime;
: 30*(11 – 1) + 1309 = 1609, prime;
: 30*(17 – 1) + 1309 = 1789, prime;

: 30*(7 – 1) + 2233 = 2413 = 19*127, a "chameleonic number" because 127 – 19 + 1 = 109, a prime;
: 30*(11 – 1) + 2233 = 2533 = 17*149, an "extended chameleonic number" because 149 – 17 + 1 = 133, a a "chameleonic number";
: 30*(29 – 1) + 2233 = 3073 = 7*439, a "chameleonic number" because 439 – 7 + 1 = 433, a prime;
(...)

Open problems:
I. Are there other interesting properties of the chameleonic numbers?
II. There exist chameleonic numbers with 4, 5, 6 or more prime factors?