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On Near Mean Graphs

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Abstract: Let $G = (V, E)$ be a graph with p vertices and q edges and let $f : V(G) \to \{0, 1, 2, \ldots, q-1, q+1\}$ be an injection. The graph G is said to have a *near mean labeling* if for each edge, there exist an induced injective map $f^* : E(G) \to \{1, 2, \ldots, q\}$ defined by

$$f^*(uv) = \begin{cases} \dfrac{f(u)+f(v)}{2} & \text{if } f(u)+f(v) \text{ is even,} \\ \dfrac{f(u)+f(v)+1}{2} & \text{if } f(u)+f(v) \text{ is odd.} \end{cases}$$

We extend this notion to *Smarandachely near m-mean labeling* (as in [9]) if for each edge $e = uv$ and an integer $m \geq 2$, the induced Smarandachely m-labeling f^* is defined by

$$f^*(e) = \left\lceil \frac{f(u)+f(v)}{m} \right\rceil.$$

A graph that admits a Smarandachely near mean m-labeling is called *Smarandachely near m-mean graph*. The graph that admits a near mean labeling is called a near mean graph (NMG). In this paper, we proved that the graphs $P_n, C_n, K_{2,n}$ are near mean graphs and $K_n (n > 4)$ and $K_{1,n} (n > 4)$ are not near mean graphs.

Key Words: Labeling, near mean labeling, near mean graph, Smarandachely near m-labeling, Smarandachely near m-mean graph.

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§1. Introduction

By a graph, we mean a finite simple and undirected graph. The vertex set and edge set of a graph G denoted are by $V(G)$ and $E(G)$ respectively. Let $f : V(G) \to \{0, 1, 2, \ldots, q-1, q+1\}$ be an injection. The graph G is said to have a *near mean labeling* if for each edge, there exist an induced injective map $f^* : E(G) \to \{1, 2, \ldots, q\}$ defined by

$$f^*(uv) = \begin{cases} \dfrac{f(u)+f(v)}{2} & \text{if } f(u)+f(v) \text{ is even,} \\ \dfrac{f(u)+f(v)+1}{2} & \text{if } f(u)+f(v) \text{ is odd.} \end{cases}$$

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and an integer $m \geq 2$, the induced Smarandachely m-labeling f^* is defined by

$$f^*(e) = \left\lceil \frac{f(u)+f(v)}{m} \right\rceil .$$

A graph that admits a Smarandachely near mean m-labeling is called *Smarandachely near m-mean graph*. A path P_n is a graph of length $n-1 \cdot K_n$ and C_n are complete graph and cycle with n vertices respectively. Terms and notations not used here are as in [2].

§2. Preliminaries

The mean labeling was introduced in [3]. Let G be a (p,q) graph. In [4], we proved that the graphs Book B_n, Ladder L_n, Grid $P_n \times P_n$, Prism $P_m \times C_3$ and $L_n \odot K_1$ are near mean graphs. In [5], we proved that *Join* of graphs, $K_2 + mK_1, K_n^1 + 2K_2$, $S_m + K_1P_n + 2K_1$ and double fan are near mean graphs. In [6], we proved Family of trees, Bi-star, Sub-division Bi-star $P_m \ominus 2K_1, P_m \ominus 3K_1$, $P_m \ominus K_{1,4}$ and $P_m \ominus K_{1,3}$ are near mean graphs. In [7], special class of graphs triangular snake, quadrilateral snake, C_n^+, $S_{m,3}, S_{m,4}$, and parachutes are proved as near mean graphs. In [8], we proved the graphs armed and double armed crown of C_3 and C_4 are near mean graphs. In this paper we proved that the graphs $P_n, C_n, K_{2,n}$ are near mean graphs and $K_n (n > 4)$ and $K_{1,n} (n > 4)$ are not near mean graphs.

§3 Near Mean Graphs

Theorem 3.1 *The path P_n is a near mean graph.*

Proof Let P_n be a path of n vertices with $V(P_n) = \{u_1, u_2, \ldots, u_n\}$ and $E(P_n) = \{(u_iu_{i+1})/i = 1, 2, \ldots, n-1\}$. Define $f : V(P_n) \to \{0, 1, 2, \ldots, n-1, n+1\}$ by

$$f(u_i) = i - 1, 1 \leq i \leq n$$
$$f(u_n) = n + 1.$$

Clearly, f is injective. It can be verified that the induced edge labeling given by $f^*(u_iu_{i+1}) = i(1 \leq i \leq n)$ are distinct. Hence, P_n is a near mean graph. □

Example 3.2 A near mean labeling of P_4 is shown in Figure 1.

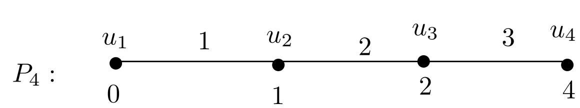

Figure 1: P_4

Theorem 3.3 $K_n, (n > 4)$ *is not a near mean graph.*

Proof Let $f : V(G) \to \{0, 1, 2, \ldots, q-1, q+1\}$. To get the edge label 1 we must have either 0 and 1 as vertex labels or 0 and 2 as vertex labels.

In either case 0 must be label of some vertex. In the same way to get edge label q, we must have either $q-1$ and $q+1$ as vertex labels or $q-2$ and $q+1$ as vertex labels. Let u be a vertex whose veretx label 0.

Case i To get the edge label q. Assign vertex labels $q-1$ and $q+1$ to the vertices w and x and respectively.

Subcase a. Let v be a vertex whose vertex label be 2, then the edges vw and ux get the same label.

Subcase b. Let v be a vertex whose vertex label be 1.

Then the edges vw and ux get the same label when q is odd. Similarly, when q is even, the edges uw and vw get the same label as well the edges ux and vx get the same label.

Case ii. To get the edge label q assign the vertex label $q-2$ and $q+1$ to the vertices w and x respectively.

Subcase a. Let v be the vertex whose vertex label be 1.

As $n > 4$, to get edge label 2, there should be a vertex whose vertex label is either 3 or 4. Let it be z (say). When vertex label of z is 3, the edges ux and wz have the same label also the edges uz and vz get the same edge label. When the vertex label of z is 4, the edges vx and wz have the same label.

Subcase b. Let v be a vertex whose vertex label 2.

As $n > 4$, to get edge label 2, there should be a vertex, say z whose vertex label is either 3 or 4. When vertex label of z is 3, the edges ux and wz get the same label. Suppose the vertex label of z is 4.

If q is even then the edges ux and wz have the same label. If q is odd then the edges vw and ux have the same label. Hence $K_n (n \geq 5)$ is not a near mean graph. □

Remark 3.4 *K_2, K_3 and K_4 are near mean graphs.*

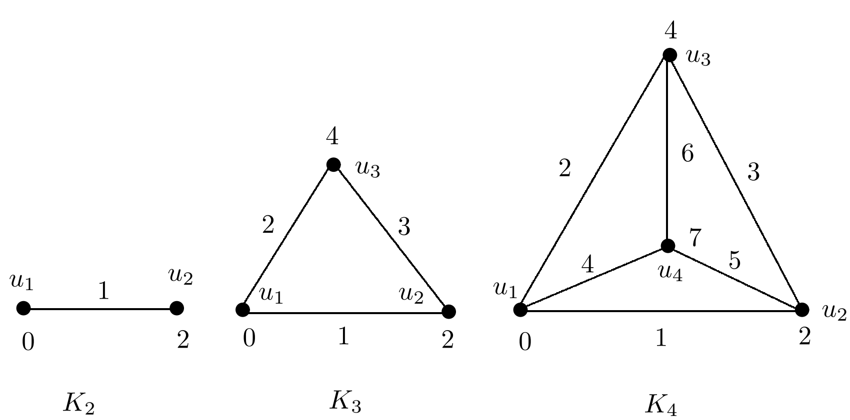

Figure 2: K_2, K_3, K_4

Theorem 3.5 *A cycle C_n is a near mean graph for any integer $n \geq 1$.*

Proof Let $V(C_n) = (u_1, u_2, u_3, \ldots, u_n, u_1\}$ and $E(C_n) = \{[(u_i u_{i+1}) : 1 \leq i \leq n-1] \cup (u_1 u_n)\}$.

Case i Let n be even, say $n = 2m$.

Define $f : V(C_n) \to \{0, 1, 2, \ldots, 2m, 2m+2\}$ by

$$f(u_i) = i - 1, 1 \leq i \leq m.$$
$$f(u_{m+j}) = m + j, 1 \leq i < m.$$
$$f(u_n) = 2m + 1.$$

Clearly f is injective. The set of edge labels of C_n is $\{1, 2, \ldots, q\}$.

Case ii. Let n be odd, say $n = 2m+1$.

Define $f : V(C_n) \to \{0, 1, 2, \ldots, 2m-1, 2m+1\}$ by

$$f(u_i) = i - 1, 1 \leq i \leq m$$
$$f(u_{m+j}) = m + j, 1 \leq j \leq m.$$
$$f(u_{2m+1}) = 2m + 2.$$

Clearly f is injective. The set of edge labels of C_n is $\{1, 2, \ldots, q\}$. □

Example 3.6 A near mean labeling of C_6 and C_7 is shown in Figure 3.

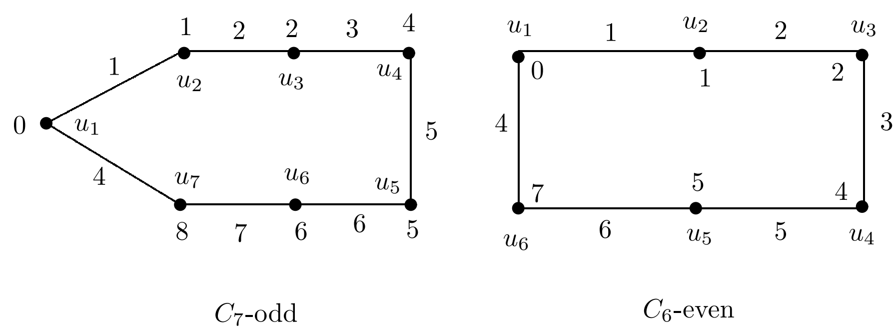

Figure 3: C_6, C_7

Theorem 3.7 *$K_{1,n} (n > 4)$ is not a near mean graph.*

Proof Let $V(K_{1,n}) = \{u, v_i : 1 \leq i \leq n\}$ and $E(K_{1,n}) = \{(uv_i) : 1 \leq i \leq n\}$. To get the edge label 1, either 0 and 1 (or) 0 and 2 are assigned to u and v_i for some i. In either case 0 must be label of some vertex.

Suppose if $f(u) = 0$, then we can not find an edge label q. Suppose if $f(v_1) = 0$, then either $f(u) = 1$ or $f(u) = 2$.

Case i. Let $f(u) = 1$.

To get edge label q, we need the following possibilities either $q-1$ and $q+1$ or $q-2$ and $q+1$. If $f(u) = 1$, it is possible only when q is either 2 or 3. But $q > 4$, so it is not possible to get edge value q.

Case ii. Let $f(u) = 2$.

As in Case i, if $f(u) = 2$ and if one of the edge value is q, then the value of q is either 3 or 4. From both the cases it is not possible to get the edge value q, when $q > 4$.

Hence, $K_{1,n}(n > 5)$ is not a near mean graph. □

Remark 3.8 *$K_{1,n}, n \leq 4$ is a near mean graph. For example, one such a near mean labeling is shown in Figure* 4.

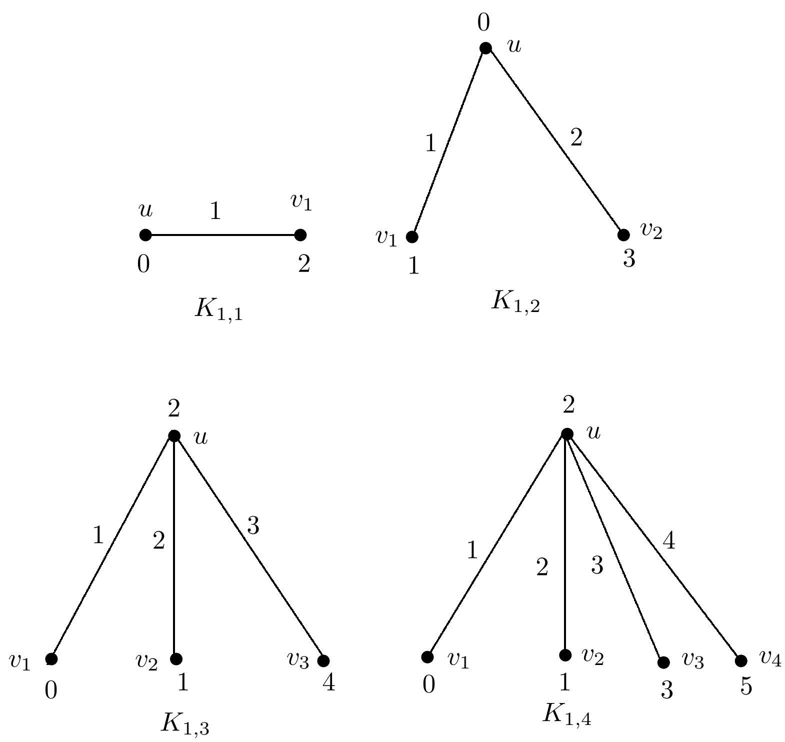

Figure 4: $K_{1,n}, n \leq 4$

Theorem 3.9 *$K_{2,n}$ admits near mean graph.*

Proof Let (V_1, V_2) be the bipartition of $V(K_{2,n})$ with $V_1 = \{u_1 u_2\}$ and $V_2 = \{v_1, v_2, \ldots, v_n\}$. $E(K_{2,n}) = \{(u_1 v_i), (u_2 v_i) : 1 \leq i \leq n\}$.

Define an injective map $f : V(K_{2,n}) \to \{0, 1, 2, \ldots, 2n-1, 2n+1\}$ by

$$f(u_1) = 1$$
$$f(u_2) = 2n + 1$$
$$f(v_i) = 2(i-1), 1 \leq i \leq n.$$

Then, it can be verified $f^*(u_1 v_i) = i, 1 \leq i \leq n$, $f^*(u_2 v_i) = n + i, 1 \leq i \leq n$ and the edge values are distinct. Hence, $K_{2,n}$ is a near mean graph. □

Example 3.10 A near mean labeling of $K_{2,4}$ is shown in Figure 5.

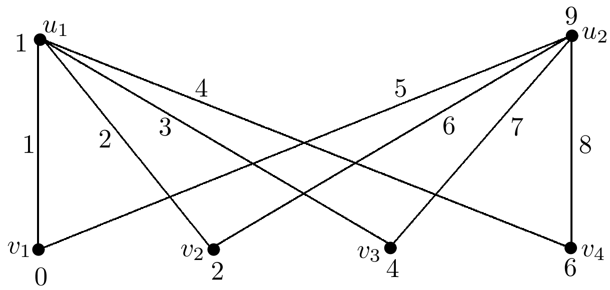

Figure 5: $K_{2,4}$

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