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Smarandache Breadth Pseudo Null Curves in Minkowski Space-time

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Abstract A regular curve with more than 2 breadths in Minkowski 3-space is called a *Smarandache Breadth Curve* [8]. In this short paper, we adapt notion of Smarandache breadth curves to Pseudo null curves in Minkowski space-time and study a special case of Smarandache breadth curves. Some characterizations of Pseudo null curves of constant breadth in Minkowski space-time are presented.

Key Words: Minkowski space-time, pseudo null curves, Smarandache breadth curves, curves of constant breadth.

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§1. Introduction

Curves of constant breadth were introduced by L. Euler [4]. In [6], some geometric properties of plane curves of constant breadth are given. And, in another work [7], these properties are studied in the Euclidean 3-Space E^3. Moreover, M. Fujivara [5] had obtained a problem to determine whether there exist space curve of constant breadth or not, and he defined "*breadth*" for space curves and obtained these curves on a surface of constant breadth. In [1], this kind curves are studied in four dimensional Euclidean space E^4.

A regular curve with more than 2 breadths in Minkowski 3-space is called a *Smarandache Breadth Curve*. In this paper, we adapt Smarandache breadth curves to pseudo null curves in Minkowski space-time. We investigate position vector of simple closed pseudo null curves and give some characterizations in the case of constant breadth. We used the method of [7], [8].

§2. Preliminaries

To meet the requirements in the next sections, here, the basic elements of the theory of curves in the space E_1^4 are briefly presented (A more complete elementary treatment can be found in [2]).

Minkowski space-time E_1^4 is an Euclidean space E^4 provided with the standard flat metric given by

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$$g = -dx_1^2 + dx_2^2 + dx_3^2 + dx_4^2,$$

where (x_1, x_2, x_3, x_4) is a rectangular coordinate system in E_1^4.

Since g is an indefinite metric, recall that a vector $v \in E_1^4$ can have one of the three causal characters; it can be space-like if $g(v,v) > 0$ or $v = 0$, time-like if $g(v,v) < 0$ and null (light-like) if $g(v,v)$=0 and $v \neq 0$. Similarly, an arbitrary curve $\alpha = \alpha(s)$ in E_1^4 can be locally be space-like, time-like or null (light-like), if all of its velocity vectors $\alpha'(s)$ are respectively space-like, time-like or null. Also, recall the norm of a vector v is given by $\|v\| = \sqrt{|g(v,v)|}$. Therefore, v is a unit vector if $g(v,v) = \pm 1$. Next, vectors v, w in E_1^4 are said to be orthogonal if $g(v,w) = 0$. The velocity of the curve $\alpha(s)$ is given by $\|\alpha'(s)\|$. And $\alpha(s)$ is said to be parametrized by arclength function s, if $g(\alpha'(s), \alpha'(s)) = \pm 1$.

Denote by $\{T(s), N(s), B_1(s), B_2(s)\}$ the moving Frenet frame along the curve $\alpha(s)$ in the space E_1^4. Then T, N, B_1, B_2 are, respectively, the tangent, the principal normal, the first binormal and the second binormal vector fields. Recall that space-like curve with space-like first binormal and null principal normal with null second binormal is called a pseudo null curve in Minkowski space-time. Let $\alpha = \alpha(s)$ be a pseudo unit speed null curve in E_1^4. Then the following Frenet equations are given in [3]:

$\alpha = \alpha(s)$ is a pseudo null curve. Then we can write that

$$\begin{bmatrix} T' \\ N' \\ B_1' \\ B_2' \end{bmatrix} = \begin{bmatrix} 0 & \kappa & 0 & 0 \\ 0 & 0 & \tau & 0 \\ 0 & \sigma & 0 & -\tau \\ -\kappa & 0 & -\sigma & 0 \end{bmatrix} \begin{bmatrix} T \\ N \\ B_1 \\ B_2 \end{bmatrix} \tag{1}$$

where T, N, B_1 and B_2 are mutually orthogonal vectors satisfying equations

$$g(T,T) = 1, g(B_1,B_1) = 1, g(N,N) = g(B_2,B_2) = 0, g(N,B_2) = 1,$$
$$g(T,N) = g(T,B_1) = g(T,B_2) = g(N,B_1) = g(B_1,B_2) = 0.$$

And here, κ, τ and σ are first, second and third curvature of the curve α, respectively. And, a pseudo null curve's first curvature κ can take only two values: 0 when α is a straight line or 1 in all other cases. In the rest of the paper, we shall assume $\kappa = 1$ at every point.

In the same space, authors, in [3], gave a characterization with the following theorem.

Theorem 2.1 *Let $\alpha = \alpha(s)$ be a pseudo null unit speed curve with curvatures $\kappa = 1$, $\tau \neq 0$ and $\sigma \neq 0$ for each $s \in I \subset R$. Then, α lies on the hyperbolic sphere (H_0^3), if and only if $\dfrac{\sigma}{\tau} = constant < 0$.*

§3. Smarandache breadth pseudo null curves in $\mathbf{E}_1^4$

In this section, first, we adapt the notion of Smarandache breadth curves to the space E_1^4 with the following definition.

Definition 3.1 *A regular curve with more than 2 breadths in Minkowski space-time is called a Smarandache breadth curve.*

Let $\varphi = \varphi(s)$ be a Smarandache Breadth pseudo null curve. Moreover, let us suppose $\varphi = \varphi(s)$ simple closed pseudo null curve in the space E_1^4. These curves will be denoted by (C). The normal plane at every point P on the curve meets the curve at a single point Q other than P. We call the point Q the opposite point of P. We consider a curve in the class Γ as in [5] having parallel tangents T and T^* in opposite directions at the opposite points φ and φ^* of the curve. A simple closed pseudo null curve having parallel tangents in opposite directions at opposite points can be represented with respect to Frenet frame by the equation

$$\varphi^* = \varphi + m_1 T + m_2 N + m_3 B_1 + m_4 B_2, \tag{2}$$

where $m_i(s)$, $1 \leq i \leq 4$ are arbitrary functions and φ and φ^* are opposite points. Differentiating both sides of (2) and considering Frenet equations, we have

$$\begin{aligned} \frac{d\varphi^*}{ds} = T^* \frac{ds^*}{ds} = (\frac{dm_1}{ds} - m_4 + 1)T + (\frac{dm_2}{ds} + m_1 + m_3\sigma)N + \\ (\frac{dm_3}{ds} + m_2\tau - m_4\sigma)B_1 + (\frac{dm_4}{ds} - m_3\tau)B_2. \end{aligned} \tag{3}$$

We know that $T^* = -T$ and if we call ϕ as the angle between the tangent of the curve (C) at point $\varphi(s)$ with a given fixed direction and consider $\frac{d\phi}{ds} = \kappa = 1 = \frac{d\phi}{ds^*} = \kappa^*$, since $ds = ds^*$. Then, we get the following system of ordinary differential equations:

$$\begin{aligned} m_1' &= m_4 - 2 \\ m_2' &= -m_1 - m_3\sigma \\ m_3' &= m_4\sigma - m_2\tau \\ m_4' &= m_3\tau \end{aligned} \tag{4}$$

Using system (4), we have the following differential equation with respect to m_1 as

$$\frac{d}{ds}\left[\frac{1}{\tau}\frac{d}{ds}(\frac{1}{\tau}\frac{d^2m_1}{ds^2})\right] - \frac{\sigma}{\tau}\frac{d^2m_1}{ds^2} - \frac{d}{ds}\left[\frac{\sigma}{\tau}(\frac{dm_1}{ds} + 2)\right] - m_1 = 0. \tag{5}$$

Corollary 3.2 *The differential equation of fourth order with variable coefficients* (5) *is a characterization for* φ^**. Via its solution, position vector of a simple closed pseudo null curve can be determined.*

However, a general solution of (5) has not yet been found. If the distance between opposite points of (C) and (C^*) is constant, then, due to null frame vectors, we may express

$$\|\varphi^* - \varphi\| = m_1^2 + 2m_2m_4 + m_3^2 = l^2 = constant. \tag{6}$$

Hence, we write

$$m_1\frac{dm_1}{ds} + m_2\frac{dm_4}{ds} + m_4\frac{dm_2}{ds} + m_3\frac{dm_3}{ds} = 0. \tag{7}$$

Considering system (4), we obtain

$$m_1 = 0. \tag{8}$$

Since, we have, respectively

$$\begin{aligned} m_2 &= s + c \\ m_3 &= 0 \\ m_4 &= 2 \end{aligned} \tag{9}$$

Using obtained equations and considering $(4)_2$, we have $\dfrac{\sigma}{\tau} = \dfrac{s+c}{2}$. Thus, we immediately arrive at the following results.

Corollary 3.3 *Let $\varphi = \varphi(s)$ be a pseudo null curve of constant breadth. Then;*
i) There is a relation among curvature functions as

$$\frac{\sigma}{\tau} = \frac{s+c}{2}. \tag{10}$$

ii) There are no spherical pseudo null curve of constant breadth in Minkowski space-time.
iii) Position vector of a pseudo null curve of constant breadth can be expressed

$$\varphi^* = \varphi + (s+c)N + 2B_2. \tag{11}$$

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