A NOTE ON SMARANDACHE BL-ALGEBRAS

CELESTIN LELE AND JEAN B. NGANOU

Abstract. Using some new characterizations of ideals in BL-algebras, we revisit the paper of A. Borumand, and al.[1] recently published in this Journal. Using the concept of MV-center of a BL-algebra, we give a very simple characterization of Smarandache BL-algebra. We also restate some of the results and provide much simpler proofs. Among other things, we notice that Theorem 3.17 and Theorem 3.18 of [1] are not true and they affect a good portion of the paper. Since Definition 3.19, Examples 3.20, 3.21, Theorem 3.22, Remark 3.23 and Remark 3.24 are based on a wrong Theorem, they are completely irrelevant.

Key words: BL-algebra, BL-ideal, BL-filter, MV-center, Samarandache BL-algebra.

1. Introduction

The Study of Smarandache Algebraic Structures was initiated by Raul Padilla [7] following a paper written by Florentin Smarandache entitled "Special Algebraic Structures". A Smarandache Structure on a set A means a weak structure W such that there exits a proper subset B of A which is embedded with a strong structure S. Since then, the subject has been pursued by a growing number of researchers. In his research, Padilla treated the Smarandache Algebraic Structures mainly with associative binary operations. In [15], a systematic treatment of the basic non-associative groupoids was given by W. B. Vasantha Kandasamy who studied the concept of Smarandache groupoids, subgroupoids, ideals of groupoids, semi-normal subgroupoids, Smarandache Bol groupoids, strong Bol groupoids and obtained many interesting results about them. Smarandache groupoids exhibit simultaneously the properties of semigroups and of groupoids. The study of Smarandache semirings, Smarandache ring and Smarandanche semifields have been developed by many authors [8], [9]. Intensive study of Smarandache BCI-algebras has been given in [4]. Recently, A. Borumand and al. [1] defined the notion of Smarandache BL-algebra and studied some of its main properties. We recall that BL-algebras were invented by Hájek [3] in order to study the basic logic framework of fuzzy set theory. The study of BL-algebras has experienced a tremendous growth over the recent years from both a logical and algebraic standpoints [5], [6], [10], [14].

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This note has two main goals that both focus mainly on the paper by A. Borumand and al. recently published in this journal. On one hand, we use some new characterizations of ideals in BL-algebras to offer a simpler presentation of some of the results and their proofs. On the other hand, it is established that some of the main results of the paper are false, for instance Theorem 3.22 where it is asserted that the given relation is a congruence is false. Subsequently, the quotient algebra constructed and used is no longer valid.
We begin by presenting several characterizations of BL-ideals in BL- algebras that open the gate to a more accessible treatment and use of BL-ideals. Using the concept of MV-center of a BL-algebra, we give a very simple characterization of Smarandache BL-algebras. We proceed to present complete justifications as to why the results mentioned above are false.

2. Preliminaries and Notations

Definition 2.1. A *t-norm* is a binary operation T on $[0,1]$, that is commutative, associative, monotone, and has 1 as an identity element. T is a continuous t-norm if it is a t-norm and is a continuous mapping of $[0,1]^2$ into $[0,1]$.

Example 2.2. The following are important examples of continuous t-norms:

(i) Lukasiewicz t-norm : $T(x,y) = \max(0, x+y-1)$.
(ii) Gödel t-norm : $T(x,y) = \min(x,y)$.
(iii) Product t-norm : $T(x,y) = x.y$.

Note that the dual notion of t-norm is a t-conorm : A t-conorm is a binary operation T over $[0,1]$, that is commutative, associative, monotone, and has 0 as an identity element.

Lemma 2.3. *Let T be a continuous t-norm. Then there is a unique operation $x \to y$ satisfying, for all $x,y,z \in [0,1]$, the condition $T(x,z) \leqslant y$ iff $z \leqslant (x \to y)$, namely $x \to y = \max\{z/T(x,z) \leqslant y\}$.*

Definition 2.4. The operation $x \to y$ from Lemma 2.3 is called the residium of the t-norm.

The following operations are residual of the three t-norms above :

(i) Lukasiewicz implication : $x \to y = \begin{cases} 1 & , \ if \ \ x \leqslant y \\ \min(1-x+y;1) & , \ otherwise. \end{cases}$
(ii) Gödel implication : $x \to y = \begin{cases} 1 & , if \ \ x \leqslant y \\ y & , \ otherwise. \end{cases}$
(iii) Product implication : $x \to y = \begin{cases} 1 & , if \ \ x \leqslant y \\ y/x & , \ otherwise. \end{cases}$

Definition 2.5. A *BL-algebra* is a nonempty set L with four binary operations $\wedge, \vee, \otimes, \to$, and two constants $0, 1$ satisfying:

BL-1 $(L, \wedge, \vee, 0, 1)$ is a bounded lattice;
BL-2 $(L, \otimes, 1)$ is a commutative monoid;
BL-3 $x \otimes y \leq z$ iff $x \leq y \to z$. (Residuation);
BL-4 $x \wedge y = x \otimes (x \to y)$ (Divisibility);
BL-5 $(x \to y) \vee (y \to x) = 1$ (Prelinearity).

The main examples of BL-algebras are from the unit interval $[0, 1]$ endowed with the structure induced by continuous t-norms. Every BL-algebra has the complementation operation defined by $\bar{x} = x \to 0$.
A BL-algebra satisfying the double negation is called an MV-algebra, that is $\bar{\bar{x}} = x$.
The following is the most comprehensive list of properties of BL-algebras.

Proposition 2.6. [1],[3],[11], [12]. *For any BL-algebra $(L, \wedge, \vee, \otimes, \to, 0, 1)$, the following properties hold for every $x, y, z \in L$:*

1. $x \leq y$ *iff* $x \to y = 1$;
2. $x \to (y \to z) = (x \otimes y) \to z$;
3. $x \otimes y \leq x \wedge y$;
4. $(x \to y) \otimes (y \to z) \leq x \to z$;
5. $x \vee y = ((x \to y) \to y) \wedge ((y \to x) \to x)$;
6. $x \to (y \to z) = y \to (x \to z)$;
7. $(x \vee y) \to z = (x \to z) \wedge (y \to z)$;
8. $x \to y \leq (y \to z) \to (x \to z)$;
9. $y \to x \leq (z \to y) \to (z \to x)$;
10. *If* $x \leq y$, *then* $y \to z \leq x \to z$ *and* $z \to x \leq z \to y$;
11. *If* $x \vee \bar{x} = 1$, *then* $x \wedge \bar{x} = 0$;
12. $y \leq (y \to x) \to x$;
13. $x \leq y \to (x \otimes y)$;
14. $x \otimes (x \to y) \leq y$;
15. $1 \to x = x; x \to x = 1; x \to 1 = 1; x \leq y \to x, x \leq \bar{\bar{x}}, \bar{\bar{\bar{x}}} = \bar{x}$;
16. $\bar{0} = 1$ *and* $\bar{1} = 0$;
17. $x \otimes \bar{x} = 0$, $x \otimes y = 0$ *iff* $x \leq \bar{y}$;
18. $x \leq y$ *implies* $x \otimes z \leq y \otimes z$;
19. $x \leq y$ *implies* $z \to x \leq z \to y, y \to z \leq x \to z, \bar{y} \leq \bar{x}$;
20. $\overline{(x \otimes y)} = x \to \bar{y}$;
21. $(x \to y) \to (z \to z) = (x \wedge y) \to z, \overline{(x \wedge y)} = \bar{x} \vee \bar{y}, \overline{(x \vee y)} = \bar{x} \wedge \bar{y}$;
22. $\overline{\overline{(x \to y)}} = \bar{\bar{x}} \to \bar{\bar{y}}, \overline{\overline{(x \wedge y)}} = \bar{\bar{x}} \wedge \bar{\bar{y}}, \overline{\overline{(x \vee y)}} = \bar{\bar{x}} \vee \bar{\bar{y}}$;
23. $x \to y \leq (x \otimes z) \to (y \otimes z)$;
24. $x \vee y = 1$ *implies* $x \otimes y = x \wedge y$;
25. $x \otimes (y \vee z) = (x \otimes y) \vee (x \otimes y), x \otimes (y \wedge z) = (x \otimes y) \wedge (x \otimes y)$;
26. $(x \wedge y) \to x = (y \to x) \vee (z \to x)$;
27. $x \to (y \wedge z) = (x \to y) \wedge (x \to y)$;
28. $\bar{\bar{x}} \otimes \bar{\bar{y}} = \overline{\overline{x \otimes y}}$.

We would like to point out that some of theses properties are redundant. For instance, 20 can be obtained from 2 by setting $z = 0$, but we prefer to list all these to make their uses obvious.

A subset F of a BL-algebra $(L, \wedge, \vee, \otimes, \rightarrow, 0, 1)$ is called a *BL- filter* if it satisfies:

F1 : $1 \in F$;

F2 : For every $x, y \in F$, $x \otimes y \in F$; and

F3 : For every $x, y \in L$, if $x \leq y$ and $x \in F$, then $y \in F$.

It is clear from $F3$ and $x \leq \bar{\bar{x}}$, that $x \in F$ implies $\bar{\bar{x}} \in F$.
A deductive system of a BL-algebra L is a subset F containing 1 such that for all $x, y \in L$;

$$x \rightarrow y \in F \quad \text{and} \quad x \in F \quad \text{imply} \quad y \in F.$$

It is known that in a BL-algebra, BL- filters and deductive systems coincide [12].
In the literature, as for example in [2], MV-algebras are also defined as algebras $(M, \oplus, ^*, 0)$ satisfying:

MV-1 $(M, \oplus, 0)$ is an Abelian monoid;

MV-2 $(x^*)^* = x$;

MV-3 $0^* \oplus x = 0^*$;

MV-4 $(x^* \oplus y)^* \oplus y = (y^* \oplus x)^* \oplus x$

The two definitions of MV-algebras are equivalent through the following transfer.
Given a BL-algebra $(L, \wedge, \vee, \otimes, \rightarrow, 0, 1)$ satisfying the double negation, define $\oplus$ and * by: $x^* = \bar{x}$ and $x \oplus y = \bar{x} \rightarrow y$.
Then $(L, \oplus, 0)$ satisfies MV-1 through MV-4.
Conversely, given an algebra $(M, \oplus, 0)$ satisfying MV-1 through MV-4, define the operations $\wedge, \vee, \otimes, \rightarrow$ by:
$x \otimes y = (x^* \oplus y^*)^*$; $x \rightarrow y = x^* \oplus y$; $x \wedge y = x \otimes (y \oplus x^*)$; $x \vee y = x \oplus (y \otimes x^*)$; $x^* = \bar{x}$ and $1 = \bar{0}$ where $\bar{x} = x \rightarrow 0$.
Then $(M, \wedge, \vee, \otimes, \rightarrow, 0, 1)$ is a BL-algebra satisfying the double negation.
For any BL-algebra L, the subset $MV(L) = \{\bar{x}, x \in L\}$ is the largest MV-sub algebra of L and is called the MV-center of L [13].
The addition in the MV-center is defined by $\bar{x} \oplus \bar{y} = \overline{x \otimes y}$ for any $\bar{x}, \bar{y} \in MV(L)$. A detailed treatment of the MV-center is found in [13]. For every $x, y \in L$, we adopt the following notation, $x \oslash y := \bar{x} \rightarrow y$. We can observe that if the BL-algebra is an MV-algebra, the operation $\oslash$ and $\oplus$ are the same. In particular, in the MV-center of L, $\oslash$ coincides with $\oplus$.

Lemma 2.7. *In every BL-algebra L, the operation $\oslash$ is associative. That is for every $x, y, z \in L$, $(x \oslash y) \oslash z = x \oslash (y \oslash z)$.*

Proof. Let $x, y, z \in L$, then $x \oslash (y \oslash z) = x \oslash (\bar{y} \rightarrow z) = \bar{x} \rightarrow (\bar{y} \rightarrow z) = (\bar{x} \otimes \bar{y}) \rightarrow z = \overline{\overline{(\bar{x} \otimes \bar{y})}} \rightarrow z = (\overline{\bar{x} \rightarrow \bar{\bar{y}}}) \rightarrow z = (\overline{\bar{x} \rightarrow y}) \rightarrow z = (x \oslash y) \oslash z$ □

Remark 2.8. If L is a BL-algebra that is not an MV-algebra, then there exists an element $x \in L$ such that $\bar{\bar{x}} \neq x$. Hence $x \oslash 0 \neq 0 \oslash x$ and we conclude that the operation $\oslash$ is not commutative in general.

In this work, we will make use of the above definitions and notations without further notice and from the context, it should be clear to the reader which of the definitions is being used.

3. BL-Ideals

Apart from their logical interests, BL-algebras and MV-algebras also have many important algebraic properties. In this section, despite the lack of suitable algebraic addition in BL-algebras, we analyze the notion of BL-ideal in general BL-algebras which coincides with the notion of ideal in MV-algebras.

Definition 3.1. Let $(L, \wedge, \vee, \otimes, \to, 0, 1)$ be a BL-algebra and I a non empty subset of L. We say that I is an *ideal or BL-ideal* of L if it satisfies:

I1 : For every $x, y \in I$, $x \oslash y \in I$; and
I2 : For every $x, y \in L$, if $x \leq y$ and $y \in I$, then $x \in I$.

It is easy to see that for any BL-ideal I, $0 \in I$ and for every $x \in L$, $x \in I$ if and only if $\bar{\bar{x}} \in I$. It is also clear that the intersection of any family of BL-ideals of a BL-algebra L is again a BL-ideal of L.
The following result is a characterization of BL-ideal.

Theorem 3.2. *A non empty set I of a BL-algebra L is a BL-ideal*

$$\text{iff for every } x, y \in L, \quad \bar{x} \otimes y \in I \quad \text{and} \quad x \in I \quad \text{imply} \quad y \in I.$$

Proof. Assume that I is a BL-ideal and let $x, y \in L$ such that $x, \bar{x} \otimes y \in I$.
Since $x, \bar{x} \otimes y \in I$, we have $x \oslash (\bar{x} \otimes y) \in I$. We also have $y \leq x \oslash (\bar{x} \otimes y) \in I$, from which we conclude that $y \in I$.
Conversely, assume that for every $x, y \in L$, $x, \bar{x} \otimes y \in I$ implies $y \in I$. To show that I is an ideal, let $x, y \in L$ such that $x \leq y$ and $y \in I$.
Since $x \leq y$, we have $\bar{y} \leq \bar{x}$ and $\bar{y} \otimes x \leq \bar{x} \otimes x = 0 \in I$. We obtain $y, \bar{y} \otimes x \in I$ and apply the hypothesis to conclude that $x \in I$.
In addition, let $x, y \in I$. We observe that $\bar{x} \otimes (x \oslash y) = \bar{x} \wedge y \leq y \in I$. So, $x, \bar{x} \otimes (x \oslash y) \in I$ and from the hypothesis, it follows that $x \oslash y \in I$. □

Remark 3.3. Let I be a BL-ideal, then for every $x, y \in I$, we have $x \vee y \in I$ and $x \wedge y \in I$.
To see this, suppose that $x, y \in I$, then since $\bar{x} \otimes y \leq y \in I$, we obtain $\bar{x} \otimes y \in I$. But $\bar{x} \otimes (x \vee y) = \bar{x} \otimes y \in I$, which implies by Theorem 3.2 that $x \vee y \in I$. It is clear from $I1$ that $x \wedge y \in I$.

Remark 3.4. If a BL-algebra is an MV-algebra, $x \oslash y = x \oplus y$. Hence the concept of BL-ideal coincides with the well known notion of ideal in MV-algebras.

The following result is another characterization of BL-ideals.

Theorem 3.5. *A subset I of a BL-algebra L is a BL-ideal if and only if the following conditions hold:*

J1: $0 \in I$*; and*
J2: *For every $x, y \in L$, if $x \in I$ and $\overline{(\bar{x} \to \bar{y})} \in I$, then $y \in I$.*

Proof. Assume that I is a BL-ideal. It is clear that $0 \in I$.
Let $x, y \in L$ such that $x \in I$ and $\overline{(\bar{x} \to \bar{y})} \in I$. We must prove that $y \in I$. First, we observe that $\bar{x} \otimes \bar{\bar{y}} = \overline{\overline{\bar{x} \otimes \bar{\bar{y}}}} = \overline{(\bar{x} \to \bar{y})} \in I$. We have $x, \bar{x} \otimes \bar{\bar{y}} \in I$ and we apply Theorem 3.2 and obtain $\bar{\bar{y}} \in I$, from which it follows that $y \in I$.
Conversely, assume that J1 and J2 hold.
Setting $y = \bar{\bar{x}}$ in J2, we obtain that $x \in I$ implies $\bar{\bar{x}} \in I$. To show that I is an ideal, by Theorem 3.2, let $x, y \in L$ such that $x, \bar{x} \otimes y \in I$. Since $\bar{x} \otimes y \in I$, we have $\overline{\overline{(\bar{x} \otimes y)}} \in I$. But since $\overline{(\bar{x} \to \bar{y})} = \overline{\overline{\bar{x} \otimes \bar{\bar{y}}}} = \overline{\overline{(\bar{x} \otimes y)}}$, then $\overline{(\bar{x} \to \bar{y})} \in I$. Now, we apply J2 and obtain that $y \in I$.

□

Remark 3.6. The above Theorem enables us to see that our definition of BL-ideal coincide with the definition given in[1],[16]. It is worth noting that the definition of BL-ideal given in [1],[16] is hard to use and the authors did not mention that it is an extension of the definition of the Ideal in MV-algebras.

Example 3.7. Let $X = \{0, a, b, c, d, e, f, 1\}$ be such that $0 < a < b < c < 1$, $0 < d < e < f < 1$, $a < e$ and $b < f$. Define $\otimes$ and $\to$ as follows:

$\otimes$	0	a	b	c	d	e	f	1
0	0	0	0	0	0	0	0	0
a	0	a	a	a	0	a	a	a
b	0	a	a	b	0	a	a	b
c	0	a	b	c	0	a	b	c
d	0	0	0	0	d	d	d	d
e	0	a	a	a	d	a	a	e
f	0	a	a	b	d	a	e	f
1	0	a	b	c	d	e	f	1

$\to$	0	a	b	c	d	e	f	1
0	1	1	1	1	1	1	1	1
a	d	1	1	1	d	1	1	1
b	d	f	1	1	d	f	1	1
c	d	e	f	1	d	e	f	1
d	c	c	c	c	1	1	1	1
e	0	c	c	c	d	1	1	1
f	0	b	c	c	d	f	1	1
1	0	a	b	c	d	e	f	1

Then $(X, \wedge, \vee, \otimes, \to, 0, 1)$ is a BL-algebra which is not MV-algebra. $I = \{0, d\}$ and $J = \{0, a, b, c\}$ are proper BL-ideals of X. $F = \{c, 1\}$ and $G = \{d, e, f, 1\}$ are proper BL-filters of X. One should observe that unlike in MV-algebras, filters and ideals are not dual under complementation. For instance, $\bar{J} = \{\bar{x}, x \in J\} = \{d, 1\}$ is not a filter and $\bar{G} = \{\bar{x}, x \in G\} = \{0, c\}$ is not a BL-ideal. It is also easy to see that a BL-ideal is a lattice ideal. But a lattice ideal is not always a BL-ideal since $A = \{0; a\}$ is a lattice ideal which is not a BL-ideal.

Example 3.8. Let $(L, \wedge, \vee, \otimes, \rightarrow, 0, 1)$ be a BL-algebra and X a nonempty set. The set of functions from $X \rightarrow L$, L^X has a natural structure of BL-algebra with the operations defined pointwise. Fix any element $x_0 \in X$ and consider
$I = \{f \in L^X, f(x_0) = 0\}$. Routine computations prove that I is an ideal of L^X.

4. Smarandache BL-algebra and Smarandache ideals

In this section $(L, \wedge, \vee, \otimes, \rightarrow, 0, 1)$ is a BL-algebra.

Definition 4.1. [1] A Smarandache BL-algebra is defined to be a BL-algebra L in which there exists a proper subset Q of L such that:

S1 : $0, 1 \in Q$ and $|Q| > 2$; and
S2 : Q is an MV-algebra under the operations of L.

Remark 4.2. We observe that if L is a BL-algebra that is not an MV-algebra, the above definition simply means that $|MV(L)| > 2$. This is due to the fact that $MV(L)$ is the largest MV-subalgebra of L [13].

Definition 4.3. [1] A non empty subset I of a BL-algebra L is called Smarandache ideal of L related to Q (or briefly Q-Smarandache ideal of L) if it satifies:

C1 : For every $x, y \in L$, if $x \in I$, $y \in Q$ and $y \leq x$, then $y \in I$; and
C2 : For every $x, y \in I$, $x \oplus y \in I$.

Remark 4.4. It should be noted that even though $x \oplus y$ continues to make sense when $x, y \in L$, this addition is no longer commutative in L. Keeping the same notation as the authors of [1] did in C2 above led to mistakes in some of the proofs in [1]. This is why the authors inadvertently concluded on the proofs of some results that turned out to be false. To avoid this risk of confusion, [C2] may be rephrased as : For every $x, y \in I$, $x \oslash y \in I$.

Remark 4.5. [1] If I is Q-Smarandache ideal for every MV-subalgebra Q of L, we simply say that I is a Smarandache ideal of L.

Theorem 4.6. [1] *If I is a BL-ideal of a BL-algebra L, then I is a Smarandache ideal of L.*

Proof. A long proof of this theorem was given in [1]. But the result follows directly from Definition 3.1, Definition 4.3 and Remark 4.5. □

The following example prove that the converse of this theorem is not true.

Example 4.7. [1] Let $L = \{0, a, b, c, d, e, f, 1\}$ Define $\otimes$ and $\rightarrow$ as follows:

$\otimes$	0	a	b	c	d	1
0	0	0	0	0	0	0
a	0	b	d	d	0	a
b	0	b	b	0	0	b
c	0	d	0	c	d	c
d	0	0	0	d	0	d
1	0	a	b	c	d	1

$\to$	0	a	b	c	d	1
0	1	1	1	1	1	1
a	d	1	a	c	c	1
b	c	1	1	c	c	1
c	b	a	b	1	a	1
d	a	1	a	1	1	1
1	0	a	b	c	d	1

Then $(L, \wedge, \vee, \otimes, \to, 0, 1)$ is a BL-algebra which is an MV-algebra. $Q = \{0, b, c, 1\}$ is the only proper MV-algebra containing in L. $I = \{0, b, c, 1\}$ is a Q-Smarandache ideal (in fact a Smaradache ideal) of a BL-algebra L. One should observe that I is not a BL-ideal since $c, \overline{(\bar{c} \to \bar{d})} \in I$, but $d \notin I$.

Lemma 4.8. *Let S be any non empty set of a BL-algebra L, the following statements are equivalent:*

i : *For every $x, y \in L$, $\bar{x} \otimes \bar{\bar{y}} \in S$ implies $\bar{x} \otimes y \in S$.*
ii : *For every $x \in L$, $\bar{\bar{x}} \in S$ implies $x \in S$.*

Proof. *i* $\Rightarrow$ *ii*: By setting setting $x = 0$ in *i*.
ii $\Rightarrow$ *i*: Let $\bar{x} \otimes \bar{\bar{y}} \in S$, we must show that $\bar{x} \otimes y \in S$. We observe that $\bar{x} \otimes \bar{\bar{y}} = \bar{\bar{\bar{x}}} \otimes \bar{\bar{y}} = \overline{\overline{\bar{x} \otimes y}} \in S$. We apply the hypothesis and obtain that $\bar{x} \otimes y \in S$. □

The following result that was stated and proved in [1] is clearly false.

Theorem 4.9. [1] *If I is a Smarandache ideal of a BL-algebra L such that for every $x, y \in L$, $\bar{x} \otimes \bar{\bar{y}} \in I$ implies $\bar{x} \otimes y \in I$, then I is an ideal of L.*

Note that in the proof of this Theorem, the authors used the fact that y is an element of every MV-algebra containing in L which is not part of the assumption. As in Example 4.7, $I = \{0, b, c, 1\}$ is a Q-Smarandache ideal (in fact a Smaradache ideal) of the BL-algebra L and for every $x \in L$, $\bar{\bar{x}} \in I$ implies $x \in I$. One should observe that I is not a BL-ideal of L.

Theorem 4.10. [1] *Define the relation $\sim_Q$ on a Q-Smarandache BL-algebra L by: For every $x, y \in L$, $x \sim_Q y$ if and only if $x \to y \in Q$ and $y \to x \in Q$. It is claimed in* [1] *that $\sim_Q$ is a congruence on L which is not true.*

To see this, we consider the following example [1, Ex. 3.21].

Example 4.11. Let $L = \{0, a, b, c, d, e, f, g, 1\}$. Define $\otimes$ and $\to$ as follows:

⊗	0	a	b	c	d	e	f	g	1
0	0	0	0	0	0	0	0	0	0
a	0	0	a	0	0	a	0	0	a
b	0	a	b	0	a	b	0	a	b
c	0	0	0	0	0	0	c	c	c
d	0	0	a	0	0	a	c	c	d
e	0	a	b	0	a	b	c	d	e
f	0	0	0	c	c	c	f	f	f
g	0	0	a	c	c	d	f	f	g
1	0	a	b	c	d	e	f	g	1

→	0	a	b	c	d	e	f	g	1
0	1	1	1	1	1	1	1	1	1
a	g	1	1	g	1	1	g	1	1
b	f	g	1	f	g	1	f	g	1
c	e	e	e	1	1	1	1	1	1
d	d	e	e	g	1	1	g	1	1
e	c	d	e	f	g	1	f	g	1
f	b	b	b	e	e	e	1	1	1
g	a	b	b	d	e	e	g	1	1
1	0	a	b	c	d	e	f	g	1

Then $(L, \wedge, \vee, \otimes, \rightarrow, 0, 1)$ is a BL-algebra. $Q = \{0, d, 1\}$ is an MV-algebra which is properly contained in L. So, L is a Q-Smarandache BL-algebra. For every $x, y \in L$, let $x \sim_Q y$ if and only if $x \rightarrow y \in Q$ and $y \rightarrow x \in I$. $\sim_Q$ is not a congruence on L. For example, $a \sim_Q e$ and $0 \sim_Q 1$ but $a \rightarrow 0 \nsim_Q e \rightarrow 1$. Therefore, $\sim_Q$ is not compatible with $\rightarrow$.

Remark 4.12. We would like to stress on the fact that the quotient algebra that the authors attempted to construct is actually hopeless, at least in the direction taken by the authors. In fact, it is not hard to see that with the relation $\sim_Q$ defined as it was, $[0] = [1] = Q$ and since the implication on the quotient was the projection of the implication of L, the quotient L/Q (if it made sense) would always be trivial. This point seems to have evaded completely the authors.

5. Conclusion

The concept of BL-ideal in BL-algebras enables us to revisit the paper [1] and also settle many important points that were still missing in BL-algebras. For future work, we could use this new concept of BL-ideal and the many characterizations given here to study important properties of BL-algebras and related structures with fuzzy applications.

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Department of Mathematics, University of Oregon, Eugene, OR 97403
E-mail address: `celestin@uoregon.edu`

Department of Mathematics, University of Oregon, Eugene, OR 97403
E-mail address: `nganou@uoregon.edu`