MORE SMARANDACHE CONJECTURES ON PRIMES' SUMMATION
(GENERALIZATIONS OF GOLDBACH AND POLIGNAC CONJECTURES)

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&1. ODD NUMBERS.

A) Any odd integer n can be expressed as a combination of three primes as follows:

1) As a sum of two primes minus another prime (k=3, s=1):
n = p+q-r, where p, q, r are all prime numbers.
Do not include the trivial solution: p = p+q-q when p is prime.

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For example:  1 = 3+5-7    = 5+7-11  = 7+11-17  = 11+13-23 = ... ;
              3 = 5+5-7    = 7+19-23 = 17+23-37 = ... ;
              5 = 3+13-11  = ... ;
              7 = 11+13-17 = ... ;
              9 = 5+7-3    = ... ;
             11 = 7+17-13  = ... .
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a) Is this conjecture equivalent with Goldbach's Conjecture (any odd integer >=9 is the sum of three primes)?
b) Is the conjecture true when all three prime numbers are different?
c) In how many ways can each odd integer be expressed as above?

2) As a prime minus another prime and minus again another prime (k=3, s=2):
n = p-q-r, where p, q, r are all prime numbers.

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For example:  1 = 13-5-7 = 17-5-11 = 19-5-13 = ... ;
              3 = 13-3-7 = 23-7-13 = ... ;
              5 = 13-3-5 = ... ;
              7 = 17-3-7 = ... ;
              9 = 17-3-5 = ... ;
             11 = 19-3-5 = ... .
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a) Is this conjecture equivalent with Goldbach's Conjecture (any odd integer >=9 is the sum of three primes)?
b) Is the conjecture true when all three prime numbers are different?
c) In how many ways can each odd integer be expressed as above?

B) Any odd integer n can be expressed as a combination of five primes as follows:

3) n = p+q+r+t-u, where p, q, r, t, u are all prime numbers, and t <> (different from) u. [k=5, s=1]

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For example:  1 = 3+3+3+5-13   = 3+5+5+17-29 = ... ;
              3 = 3+5+11+13-29 = ... ;
              5 = 3+7+11+13-29 = ... ;
              7 = 5+7+11+13-29 = ... ;
              9 = 7+7+11+13-29 = ... ;
             11 = 5+7+11+17-29 = ... .
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a) Is the conjecture true when all five prime numbers are different?
b) In how many ways can each odd integer be expressed as above?

4) n = p+q+r-t-u, where p, q, r, t, u are all prime numbers, and t, u <> p, q, r. [k=5, s=2]

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For example:  1 = 3+7+17-13-13  = 3+7+23-13-19 = ... ;
              3 = 5+7+17-13-13  = ... ;
              5 = 7+7+17-13-13  = ... ;
              7 = 5+11+17-13-13 = ... ;
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9 = 7+11+17-13-13 = ... ;
11 = 7+11+19-13-13 = ...
a) Is the conjecture true when all five prime numbers are different?
b) In how many ways can each odd integer be expressed as above?

5) n = p+q-r-t-u, where p, q, r, t, u are all prime numbers, and r, t, u <> p, q. [k=5, s=3]
For example: 1 = 11+13-3-3-17 = ... ;
3 = 13+13-3-3-17 = ... ;
5 = 3+29-5-5-17 = ... ;
7 = 3+31-5-5-17 = ... ;
9 = 3+37-7-7-17 = ... ;
11 = 5+37-7-7-17 = ...
a) Is the conjecture true when all five prime numbers are different?
b) In how many ways can each odd integer be expressed as above?

6) n = p-q-r-t-u, where p, q, r, t, u are all prime numbers, and q, r, t, u <> p. [k=5, s=4]
For example: 1 = 13-3-3-3-3 = ... ;
3 = 17-3-3-3-5 = ... ;
5 = 19-3-3-3-5 = ... ;
7 = 23-3-3-5-5 = ... ;
9 = 29-3-5-5-7 = ... ;
11 = 31-3-5-5-7 = ...
a) Is the conjecture true when all five prime numbers are different?
b) In how many ways can each odd integer be expressed as above?

Etc.

&2. EVEN NUMBERS.

A) Any even integer n can be expressed as a combination of two primes as follows:

1) n = p - q, where p, q are both primes [k=2, s=1].
For example: 2 = 7 - 5 = 13 - 11 = ... ;
4 = 11 - 7 = ... ;
6 = 13 - 7 = ... ;
8 = 13 - 5 = ...
a) In how many ways can each odd integer be expressed as above?

B) Any even integer n can be expressed as a combination of four primes as follows:

2) n = p + q + r - t, where all p, q, r, t are primes [k=4, s=1].
For example: 2 = 3 + 3 + 3 - 7 = 3 + 5 + 5 - 11 = ... ;
4 = 3 + 3 + 5 - 7 = ... ;
6 = 3 + 5 + 5 - 7 = ... ;
8 = 11 + 5 + 5 - 13 = ...
a) Is the conjecture true when all four prime numbers are different?
b) In how many ways can each odd integer be expressed as above?

3) n = p + q - r - t, where all p, q, r, t are primes [k=4, s=2].
For example: 2 = 11 + 11 - 3 - 17 = 11 + 11 - 13 - 7 = ... ;
4 = 11 + 13 - 3 - 17 = ... ;
6 = 13 + 13 - 3 - 17 = ... ;
8 = 11 + 17 - 7 - 13 = ...

a) Is the conjecture true when all four prime numbers are different?
b) In how many ways can each odd integer be expressed as above?

4) n = p - q - r - t, where all p, q, r, t are primes [k=4, s=3].

For example: 2 = 11 - 3 - 3 - 3 = 13 - 3 - 3 - 5 = ... ;
4 = 13 - 3 - 3 - 3 = ... ;
6 = 17 - 3 - 3 - 5 = ... ;
8 = 23 - 3 - 5 - 7 = ...

a) Is the conjecture true when all four prime numbers are different?
b) In how many ways can each odd integer be expressed as above?

Etc.

GENERAL CONJECTURE:
Let k >= 3, and 1 <= s < k, be integers. Then:

i) If k is odd, any odd integer can be expressed as a sum of k-s primes (first set) minus a sum of s primes (second set) [such that the primes of the first set is different from the primes of the second set].

a) Is the conjecture true when all k prime numbers are different?
b) In how many ways can each odd integer be expressed as above?

ii) If k is even, any even integer can be expressed as a sum of k-s primes (first set) minus a sum of s primes (second set) [such that the primes of the first set is different from the primes of the second set].

a) Is the conjecture true when all k prime numbers are different?
b) In how many ways can each even integer be expressed as above?

References:

[1] Smarandache, Florentin, "Collected Papers", Vol. II, Kishinev University Press, Kishinev, article <Prime Conjecture>, p. 190, 1997.
[2] Smarandache, Florentin, "Conjectures on Primes' Summation", Arizona State University, Special Collections, Hayden Library, Tempe, AZ, 1979.