The Concept of the Effective Mass Tensor in GR

Newton's Bucket Argument

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Abstract: *In the papers [1, 2] we presented the concept of the effective mass tensor (EMT) in General Relativity (GR). According to this concept under the influence of the gravitational field the bare mass tensor* $m_{\mu\nu}^{bare}$ *becomes the EMT* $m_{\mu\nu}$*. The concept of the EMT is a new physical interpretation of GR, where the curvature of space-time has been replaced by the EMT. In this paper we consider again the concept of the EMT in the GR but in the aspect of Newton's bucket argument.*

keywords: *the effective mass tensor, the equation of motion, Newton's bucket argument.*

I. Introduction

In the papers [1, 2] we presented the concept of *the effective mass tensor* (EMT) in General Relativity (GR). According to this concept under the influence of the gravitational field *the bare mass tensor* $m_{\mu\nu}^{bare}$ [2] becomes the EMT $m_{\mu\nu}$. The concept of the EMT is a new physical interpretation of GR, where the curvature of space-time has been replaced by the EMT. In this paper we again consider the concept of the EMT in the but in the aspect of Newton's bucket argument.

As we know from the papers [1, 2] the metric tensor $g_{\mu\nu}$ we can express by the EMT $m_{\mu\nu}$

$$g_{\mu\nu} = \frac{m_{\mu\nu}}{m} \tag{1}$$

where: m is *the bare mass* of the body, the space-time components μ, ν = 0, 1, 2, 3.

Therefore the metric

$$ds^2(g_{\mu\nu}) = ds^2(m_{\mu\nu}) \tag{2}$$

where: $ds^2(g_{\mu\nu}) = g_{\mu\nu}dx^\mu dx^\nu$ and $ds^2(m_{\mu\nu}) = \frac{m_{\mu\nu}}{m}dx^\mu dx^\nu$.

Let us consider the Lagrangian function for the free body in the curved space, which is moving with the small velocity $\frac{dx^{\mu}}{d\tau}$ ($\frac{dx^{\mu}}{d\tau} << c$), where c is the speed of the light, τ is the proper time.

$$L = \frac{1}{2} m \cdot g_{\mu\nu} \frac{dx^{\mu}}{d\tau} \frac{dx^{\nu}}{d\tau} \tag{3}$$

If in the eq. (3) we replace the metric tensor $g_{\mu\nu}$ with the EMT $m_{\mu\nu}$ (see eq. (1)) then we have

$$L = \frac{1}{2} m_{\mu\nu} \frac{dx^{\mu}}{d\tau} \frac{dx^{\nu}}{d\tau} \tag{4}$$

The equation of motion for the Lagrangian function (4) have the form

$$\frac{d^2 x^{\beta}}{d\tau^2} + \Gamma^{*\beta}_{\mu\nu} \frac{dx^{\mu}}{d\tau} \frac{dx^{\nu}}{d\tau} = 0 \tag{5}$$

where the term $\Gamma^{*\beta}_{\mu\nu}$ we will call *the modified Christoffel symbols of the second kind* and

$$\Gamma^{*\beta}_{\mu\nu} = \frac{1}{2} m^{\beta\alpha} \left(\frac{\partial m_{\alpha\mu}}{\partial x^{\nu}} + \frac{\partial m_{\alpha\nu}}{\partial x^{\mu}} - \frac{\partial m_{\mu\nu}}{\partial x^{\alpha}} \right) \tag{6}$$

In the weak gravitational field we can decompose of the EMT of the body to the simple form: $m_{\mu\nu} = m^{bare}_{\mu\nu} + m^{*}_{\mu\nu}$, where: $m^{bare}_{\mu\nu} = m \cdot \eta_{\mu\nu} = \mathrm{diag}(-m,+m,+m,+m)$ we will call *the bare mass tensor*, $\eta_{\mu\nu}$ is the Minkowski tensor, $m^{*}_{\mu\nu} = m \cdot \left| h_{\mu\nu} \right| << 1$ is a small EMT "perturbation" [1]. Note that in the absence of the gravitational field the EMT becomes the bare mass tensor $m_{\mu\nu} \rightarrow m^{bare}_{\mu\nu}$.

The modified Christoffel symbols (6) (with accuracy to first order) have the form

$$\Gamma^{*i}_{00} = \frac{1}{2m} \delta^{ij} \partial_j m^{*}_{00} \tag{7}$$

(components i and j are the Roman indices to denote spatial components: i, j = 1, 2, 3) and similarly

$$\Gamma^{*i}_{0j} = -\frac{\delta^{ik}}{2m} \left(\partial_j m^{*}_{0k} - \partial_k m^{*}_{0j} \right) \tag{8}$$

Now the equation of the motion (5) have the form

$$\frac{d^2 x^i}{dt^2} = -\delta^{ij} \partial_j \left(\frac{c^2 m^{*}_{00}}{2m} \right) + \frac{\delta^{ik} c}{m} \left(\partial_j m^{*}_{0k} - \partial_k m^{*}_{0j} \right) \frac{dx^j}{dt} \tag{9}$$

where we omitted the term $\partial_0 m^{*}_{\mu\nu}$. The second right term in the eq. (9) is velocity-dependent and is associated with the rotation and *the Coriolis acceleration*.

II. The equation of motion in the rotating reference system

Let us consider two coordinating systems: nonrotating system *K* with coordinates (*T*, *X*, *Y*, *Z*) and rotating system $K^{'}$ with the small angular velocity ω with coordinates (*t*, *x*, *y*, *z*), $\boldsymbol{\omega} = (0,0,\omega)$ respect to the all bodies in the Universe [3]. In the *K* system line element has the form

$$c^2 d\tau^2 = c^2 dT^2 - dX^2 - dY^2 - dZ^2 \tag{10}$$

Let us define new coordinates (*t*, *x* , *y*, *z*) by

$$\begin{aligned} T &\equiv t \\ X &\equiv x\cos\omega t - y\sin\omega t \\ Y &\equiv x\sin\omega t + y\cos\omega t \\ Z &\equiv z \end{aligned} \tag{11}$$

The coordinate system *K′* (*t*, *x*, *y*, *z*) is rotating relative to the coordinate system *K*(*T*, *X* ,*Y*, *Z*) with the angular velocity ω. In the $K^{'}$ system line element has the form

$$c^2 d\tau^2 = \left(c^2 - \omega^2 r^2\right)dt^2 + 2\omega y\,dx\,dt - 2\omega x\,dy\,dt - dx^2 - dy^2 - dz^2 \tag{12}$$

where: $r^2 = x^2 + y^2$ is a radial distance from the origin of the rotating system to the observer. In the slowly rotating coordinating system with the angular velocity ω [3] a small EMT "perturbation" $m^{*}_{\mu\nu}$ we can express in the matrix form

$$m^{*}_{\mu\nu} = m\begin{bmatrix} -\dfrac{\omega^2 r^2}{c^2} & \dfrac{\omega y}{c} & -\dfrac{\omega x}{c} & 0 \\ \dfrac{\omega y}{c} & 0 & 0 & 0 \\ -\dfrac{\omega x}{c} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{13}$$

The EMT we can express by the equation

$$m_{\mu\nu} = m\begin{bmatrix} 1-\dfrac{\omega^2 r^2}{c^2} & \dfrac{\omega y}{c} & -\dfrac{\omega x}{c} & 0 \\ \dfrac{\omega y}{c} & -1 & 0 & 0 \\ -\dfrac{\omega x}{c} & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \tag{14}$$

When the body is not rotates ($\omega = 0$) then

$$m_{\mu\nu} = m^{bare}_{\mu\nu} \tag{15}$$

If we use an approximation

$$m_{00}^{*} = -m\frac{\omega^2 r^2}{c^2} \tag{16}$$

in eq. (9) and $\left(\partial_j m_{0k}^{*} - \partial_k m_{0j}^{*}\right)$ terms then the equation of motion of the body in a slowly rotating system [3] is

$$\begin{aligned} \frac{d^2x}{dt^2} &= \omega^2 x + 2\omega\frac{dy}{dt} \\ \frac{d^2y}{dt^2} &= \omega^2 y - 2\omega\frac{dx}{dt} \\ \frac{d^2z}{dt^2} &= 0 \end{aligned} \tag{17}$$

These equation is identical with the classical Newtonian equation in the slowly rotating system.

III. The rotating liquid mirror with mercury instead the bucket water

Let us consider the rotating liquid mirror (LM) with mercury, instead the bucket with water [4]. According to the Berkeley and Mach point of view the rotation (only) with respect to the *fixed star sphere* (FSS, all bodies in the Universe) gives curvature of the surface of the mercury. If we were to take the LM of the mercury, with the utmost care, to the Earth's pole, we would find that the surface of the mercury assumes a paraboloidal shape, even when the LM is at the rest relative to the Earth (the Earth with LM is rotating relative to the FSS).

According to the concept of the EMT when the LM with the mercury **rotates** respect to the FSS then the 00 component of a small EMT "perturbation" m_{00}^{*} is expressed by the formula

$$m_{00}^{*} = -m\frac{\omega^2 r^2}{c^2} \tag{18}$$

and then equation of motion (9) have the form

$$\begin{aligned} \frac{d^2x}{dt^2} &= \omega^2 x \\ \frac{d^2y}{dt^2} &= \omega^2 y \\ \frac{d^2z}{dt^2} &= 0 \end{aligned} \tag{19}$$

where we omitted the Coriolis acceleration. The surface of mercury has the paraboloid shape and depends on the radius of mirror [4].

But if LM with the mercury **not rotates** respect to the FSS then the EMT of the mercury is expressed by the formula

$$m_{\mu\nu} = m_{\mu\nu}^{bare} \tag{20}$$

and then equation of motion (9) have the form

$$\frac{d^2x}{dt^2}=0$$
$$\frac{d^2y}{dt^2}=0 \qquad (21)$$
$$\frac{d^2z}{dt^2}=0$$

and surface of the mercury would be flat. The experiment with LM with the mercury could confirm (or not) the Berkeley and Mach point of view and also the concept of the EMT.

IV. Clock in the rotating reference frame

Clock rests in the system $K^{'}$ has the coordinates $x = const, y = const, z = const$. Therefore

$$c^2 d\tau^2 = \left(c^2 - \omega^2 r^2\right) dt^2 \qquad (22)$$

and finally

$$d\tau = \left(1 - \frac{\omega^2 r^2}{c^2}\right)^{1/2} dt \qquad (23)$$

This means that the clock resting in the rotating system $K^{'}$ runs slower than the clock in system associated with the inertial system K, because effective mass is different in the rotating system $K^{'}$ than in the inertial system K (see eq. (14)).

V. Conclusion

In this paper we considered the concept of the EMT in the aspect of Newton's bucket argument. According to this concept in the slowly rotating system with respect to the FSS the bare mass tensor becomes the EMT and is expressed by the eq. (14). Rotation with respect to FSS will change the effective mass of mercury, its surface and changes in the running of the clock.

The experiment with LM with the mercury could confirm (or not) the Berkeley and Mach point of view and also the concept of the EMT.

References

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