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The zeros of Riemann's Function And Its Fundamental Role In Quantum Mechanics

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Abstract

This paper presents a proof of the fundamental connection between the zeros of the Riemann function and quantum mechanics. Two results that unify gravity and electromagnetism, by exact calculation, both the elementary electric charge and mass of the electron. These two results depend directly on the sum of the imaginary parts of the zeros of the Riemann function, exactly following the Hilbert-Polya conjecture. This summatory, is the exponential sums of all the negative values of the imaginary part of all zeros of the Riemann function. The main consequences are: scales of the Planck length gravity becomes repulsive, through the interaction of the gravitinos. Special relativity is a special case of a generalization, in which the geometry of a wormhole (hyperbolic geometry) implies that the energy of the tachyon states is zero, only if the velocity at the outer surface of the wormhole is infinite, or what is the same: an observer at rest can not distinguish an infinite speed of zero velocity, both are equivalent. There is not mere speculation; since only under this assumption the mass of the electron as a function of the non-trivial zeros of the Riemann zeta function is calculated. Time ceases to exist, takes the value zero. These wormholes would explain the quantum entanglement, as well as resolve the paradox of information loss in black holes. Fundamental constants used in this calculation are: elementary charge, gravitational Newton constant, Planck mass, mass of the electron, and fine structure constant for zero momentum.

keywords: Riemann hypothesis; Möbius function; Mertens function; zeros zeta Riemann and quantum mechanics; electric charge; Planck mass

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1 Introduction

In 1859 Riemann formulated his hypothesis, in his doctoral thesis: "On the prime numbers less than a given magnitude." As this hypothesis was not essential for his doctoral thesis, did not try to give a proof of it. But he knew Riemann zeros nontrivial are distributed around the line $s = \frac{1}{2} + it$, and knew also that all nontrivial zeros should be in the range $0 \leq \Re(s) \leq 1$

Riemann found, a few zeros that were on the critical line with real part $1/2$ and suggested that all non-trivial zeros have real part $1/2$; and this is the Riemann hypothesis.

In 1896, Hadamard and de la Vallée-Poussin independently proved that any zero could be on the line $\Re(s) = 1$

United with other properties of the nontrivial zeros, demonstrated by Riemann, this showed that the non-trivial zeros should be on the critical band $0 < \Re(s) < 1$

In 1914, Hardy showed that there is an infinite number of zeros $\Re(s) = 1/2$, however there was still the possibility that an infinite number of zeros were in critical band $\Re(s) \neq 1/2$

2 The Function $\zeta(s)$ of Riemann.

The Riemann zeta function is defined, for $\Re(s) > 1$

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} \quad (2.1)$$

The zeta function has a singularity at $s = 1$, which reduces divergent harmonic series. Being s , the complex number defined as $s = \sigma + it$

The previous series is a prototype of Dirichlet series which converges absolutely an analytic function for s such that $\sigma > 1$ and diverges for all other values of s

Riemann showed that the function defined by the series in the half-plane of convergence can be continued analytically to all complex values $s \neq 1$. for $s = 1$ is the harmonic series, which diverges to $+\infty$

Hence the Riemann zeta function is a meromorphic function in the whole complex s-plane, which is holomorphic everywhere except for a simple pole at $s = 1$ with residue 1.

2.1 The analytic extension of the function $\zeta(s)$ of Riemann.

The Riemann zeta function satisfies, the so-called Riemann functional equation.

$$\zeta(s) = 2^s \pi^{s-1} \sin\left(\frac{\pi s}{2}\right) \Gamma(1-s) \zeta(1-s) \quad (2.2)$$

Where $\Gamma(s)$ is the gamma function, which is an equality of meromorphic functions valid in the whole complex plane.

This equation relates the values of the Riemann zeta function at points s and $1-s$. Functional equation (due to the properties of the sine function) implies that $\zeta(s)$ has a simple zero at each negative integer $s = -2n$. These are called the trivial zeros $\zeta(s)$. For s with positive integer number, the product $\sin(\pi s/2)\Gamma(1-s)$ is regular and the functional equation relates to the values of the Riemann zeta function at negative integers even and odd positive integers.

The functional equation was established by Riemann in his 1859 paper, "On the number of primes less than a given magnitude" and is used to construct the analytic continuation first. An equivalent relationship was conjectured by Euler over a hundred years before, in 1749, for the Dirichlet eta function (alternating zeta function, $\zeta^*(s)$).

$$\eta(s) = \zeta^*(s)$$

$$\eta(s) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^s} = \frac{1}{1^s} - \frac{1}{2^s} + \frac{1}{3^s} - \frac{1}{4^s} + \dots \quad (2.3)$$

This Dirichlet series is the alternating sum, corresponding to the expansion of the Dirichlet series of the Riemann zeta function, $\zeta(s)$; and for this reason the Dirichlet eta function is also known as the zeta alternate function, also symbolized $\zeta^*(s)$. The following simple relationship holds:

$$\zeta^*(s) = (1 - 2^{1-s})\zeta(s) \quad (2.4)$$

The eta function zeros include all the zeros of the zeta function: the infinity of negative integers (real simple zeros equidistant), an infinity of zeros along the critical line, none of which are known to be multiple and over 40% of which have been shown to be simple, and zeros in the critical strip hypothetical but not on the critical line, if they do, should be placed at the vertices of rectangles symmetrical about the x-axis and the critical line and whose multiplicity is unknown. Furthermore, the factor $(1 - 2^{1-s})$ adds an infinity of simple complex zeros, located at equidistant points on the line $\Re(s) = 1$ at $s_n = 1 + 2n\pi i / \ln(2)$ where n is a nonzero integer.

Hardy gave a simple proof of the functional equation for eta function, which is:

$$\frac{\eta(1-s)}{\eta(s)} = -\frac{(2^s - 1)}{\pi^s (2^{s-1} - 1)} \Gamma(s) \cos(\pi s/2) \quad (2.5)$$

3 A brief historical overview of the RH and quantum mechanics.

The original Kaluza-Klein theory, had the huge success; able to unify the equations of electromagnetism and the GR. A theory compactified fifth dimension allowed this union. This theory was dismissed by the discrepancy; unable to obtain the real and exact value of the electron mass.

Subsequently, emerged superstring theory, which, despite its successful and rich mathematical apparatus, have been unable to calculate, for example, the elementary charge, the electron mass, etc.

On the other hand, it began to investigate the Riemann hypothesis and its possible connection to quantum mechanics.

In a development that has given substantive force to this approach to the Riemann hypothesis through functional analysis, Alain Connes has formulated a trace formula that is actually equivalent to the Riemann hypothesis. This has therefore strengthened the analogy with the Selberg trace formula to the point where it gives precise statements. He gives a geometric interpretation of the explicit formula of number theory as a trace formula on noncommutative geometry of Adele classes.

In mathematics, the Hilbert–Pólya conjecture is a possible approach to the Riemann hypothesis, by means of spectral theory. A possible connection of Hilbert–Pólya operator with quantum mechanics was given by Pólya. The Hilbert–Pólya conjecture operator is of the form $1/2 + iH$ where H is the Hamiltonian of a particle of mass m that is moving under the influence of a potential $V(x)$. The Riemann conjecture is equivalent to the assertion that the Hamiltonian is Hermitian, or equivalently that V is real.

Michael Berry and Jon Keating have speculated that the Hamiltonian H is actually some quantization of the classical Hamiltonian xp , where p is the canonical momentum associated with x .

The square root function; appears recurrently in all results involving the truth of the Riemann hypothesis. This same function, plays a key role in the main functional forms of quantum mechanics.

This recurring algebraic form, is the function: $\sqrt{\quad}$ and all equivalences of the Riemann hypothesis involve estimates of the type $O(x^{1/2})$, as a fundamental element.

4 The connections: the Riemann hypothesis with physics.

The square root function is, for example, in the following fundamental identities:

1. Probability density: $(\rho_t(x))^{1/2} = \psi(x, t)$
2. Matter-antimatter asymmetry (baryon physical density) as a function dependent on the sum of the Möbius function, by their equivalents: the R-parity or operator of supersymmetry $(-1)^F; (-1)^B$; where F and B are respectively the number of fermions and bosons.
3. One of the dualities of string theory: $f(x) = \sqrt{x}; 2df(x) = \frac{1}{f(x)}$
4. Probability in a given volume: $P(v) = \int_v |\psi(x, t_0)|^2 dx$
5. The total power: $E_t = (m^2c^4 + p^2c^2)^{1/2}$

6. The Planck mass, Planck time and Planck length: $m_{pk} = (\frac{\hbar c}{G_N})^{1/2}$
7. The symmetry of the vacuum with respect to the electric charge, taking into account that the origin of the electric charge is due to the unification of gravity with the electromagnetic field: $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} m_{pk} \sqrt{G_N}}{\pm e \cdot n^s} = \left(\frac{m_{pk} \sqrt{G_N}}{\pm e} \right) \cdot \left(\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^s} \right) = 0$; $s = \frac{1}{2} + it_n$ Where $Im(s) = t_n$, corresponds to a zero of $\zeta(s)$.

This last case: the symmetry of the vacuum from electrical charges, is which will allow the calculation of the elementary electric charge, as a direct function of the imaginary parts of the zeros of the Riemann function. Also this same function of the zeros of Riemann's function, yields the mass of the electron, in an equation where the electromagnetic field and the gravitational feature functional equality.

4.1 The five basic solutions of the equation of energy-momentum, as factorisation of imaginary energies.

In this section we show five basic solutions of the equation of energy-momentum. These five solutions imply the existence of states of energy, mass and momentum, imaginary. If these states are interpreted as coexisting, then you get three consequences: 1) the existence of imaginary mass means faster than the speed of light, these components being imaginary, virtual states. There is no contradiction with quantum mechanics and special relativity, since the same relativity equation for the four spatial coordinates, using the imaginary light speed, cti . 2) As a result of the four positive energy solutions, and as a vector sum, you get on one hand the minimum limit of the uncertainty principle. And for real particles, it seems that while the particle is not observed, measured, the particle is found in two states at once. 3) The extension of these four states of positive energy to seven dimensions, to calculate the fine structure constant to zero momentum.

Also virtual particles, which violate the uncertainty principle, involve above the speed of light.

Even in the early universe, the process of inflation, implies above the speed of light. Is established that the vacuum, in the absence of energy, takes over the speed of light. What is more, due to the zero energy of the tachyons, the energy disappears, time is canceled and the speed is infinite: All this because of the zeros of the Riemann zeta function. Exists only in this state, a pure hyperbolic space, which if taken as wormholes, defined by a hyperboloid of one sheet, then allows perfectly explain entanglement states, and as this infinite speed of these wormholes; pure space, zero energy, zero time. Allows instant transmission, change of state of the other particle interlocking. No causality violation exists; since no energy transmission, so that there is no signal sending. It is only the space that interconnects the two particles, with infinite speed (not distinguishable of zero velocity, relative to an observer at rest), which transmits the state change, so instant

$$E^2 = m^2 c^4 + p^2 c^2 = \begin{array}{|l} (imc^2 + pc)(-imc^2 + pc) = E_1^2 \\ (imc^2 - pc)(-imc^2 - pc) = E_2^2 \\ (mc^2 + ipc)(mc^2 - ipc) = E_3^2 \\ (-mc^2 + ipc)(-mc^2 - ipc) = E_4^2 \\ i(imc^2 + pc)(mc^2 + ipc) = -E_5^2 \end{array} \quad (1)$$

$$\begin{pmatrix} imc^2 - pc & -imc^2 + pc \\ imc^2 + pc & -imc^2 - pc \end{pmatrix} = 0$$

$$\begin{pmatrix} mc^2 + ipc & mc^2 - ipc \\ -mc^2 + ipc & mc^2 - ipc \end{pmatrix} = 0$$

$$E_T = \sqrt{E_1^2 + E_2^2 + E_3^2 + E_4^2} = 2E \quad (2)$$

In the solutions of table one (1), it can be seen that the square of the energy, can be factored in five different ways, with each solution, the product of the sum of two energies: one component of the energy-momentum and another component owing to mass-energy equivalence.

Each factor of the five solutions, is the sum of an imaginary component and a real. Later this table will extend to seven dimensions by octonions. Table (1) is equivalent to obtain by the rotations of the spin. And just five spins are sufficient to obtain the four solutions of positive energy, and one of negative energy. The vector sum of the positive energy, expressed in equation (2) directly involve the uncertainty principle; minimum value of uncertainty.

That is:

$$E_T = \sqrt{E_1^2 + E_2^2 + E_3^2 + E_4^2} = 2E = \min(2\Delta E) = \frac{\hbar}{\Delta t}$$

4.1.1 The five solutions of the energy-momentum equation, derived by rotations of the spins.

These five solutions; rotating the value of energy by the spines; must obey the Pauli exclusion principle. Thus, it appears that each spin appears four times each in the four solutions of positive energy.

$$E_1^2 = (imc^2 \cdot \exp 2\pi i \cdot 1 + pc \cdot \exp 2\pi i \cdot 1)(imc^2 \cdot \exp 2\pi i \cdot \frac{1}{2} + pc \cdot \exp 2\pi i \cdot 1) \quad E_2^2 = (imc^2 \cdot \exp 2\pi i \cdot 1 + pc \cdot \exp 2\pi i \cdot \frac{1}{2})(imc^2 \cdot \exp 2\pi i \cdot \frac{1}{2} + pc \cdot \exp 2\pi i \cdot \frac{3}{2})$$

$$E_3^2 = (imc^2 \cdot \exp 2\pi i \cdot 2 + pc \cdot \exp 2\pi i \cdot 2)(imc^2 \cdot \exp 2\pi i \cdot 2 + pc \cdot \exp 2\pi i \cdot \frac{3}{2})$$

$$E_4^2 = (imc^2 \cdot \exp 2\pi i \cdot \frac{3}{2} + pc \cdot \exp 2\pi i \cdot 2)(imc^2 \cdot \exp 2\pi i \cdot \frac{1}{2} + pc \cdot \exp 2\pi i \cdot \frac{3}{2})$$

$$-E_5^2 = i(imc^2 + pc)(mc^2 + ipc) = i^2 m^2 c^4 + i^2 p^2 c^2$$

$$-E_5^2 = i(imc^2 \cdot \exp 2\pi i \cdot 0 + pc \cdot \exp 2\pi i \cdot 0)(imc^2 \cdot \exp 2\pi i \cdot 0 + pc \cdot \exp 2\pi i \cdot 0)$$

As one can observe, with the five known spins is sufficient to obtain the five basic solutions.

Obviously there are other combinations of rotations of spin, which give equivalent solutions.

Now, a twist, with a negative value of spin is also a solution. Therefore:

$$E_1^2 = (imc^2 \cdot \exp 2\pi i \cdot -1 + pc \cdot \exp 2\pi i \cdot -1)(imc^2 \cdot \exp 2\pi i \cdot \frac{-1}{2} + pc \cdot \exp 2\pi i \cdot -1) \quad E_2^2 = (imc^2 \cdot \exp 2\pi i \cdot -1 + pc \cdot \exp 2\pi i \cdot \frac{-1}{2})(imc^2 \cdot \exp 2\pi i \cdot \frac{-1}{2} + pc \cdot \exp 2\pi i \cdot \frac{-3}{2})$$

$$E_3^2 = (imc^2 \cdot \exp 2\pi i \cdot -2 + pc \cdot \exp 2\pi i \cdot -2)(imc^2 \cdot \exp 2\pi i \cdot -2 + pc \cdot \exp 2\pi i \cdot \frac{-3}{2})$$

$$E_4^2 = (imc^2 \cdot \exp 2\pi i \cdot \frac{-3}{2} + pc \cdot \exp 2\pi i \cdot -2)(imc^2 \cdot \exp 2\pi i \cdot \frac{-1}{2} + pc \cdot \exp 2\pi i \cdot \frac{-3}{2})$$

$$-E_5^2 = i(imc^2 + pc)(mc^2 + ipc) = i^2 m^2 c^4 + i^2 p^2 c^2$$

$$-E_5^2 = i(imc^2 \cdot \exp 2\pi i \cdot -0 + pc \cdot \exp 2\pi i \cdot -0)(imc^2 \cdot \exp 2\pi i \cdot -0 + pc \cdot \exp 2\pi i \cdot -0)$$

The negative energy solution, admitting only integer values of spin, and since bosons can occupy the same quantum state, for this negative energy solution, the spin could have taken the values 1 or 2. For consistency (completeness) to the negative energy is assigned the spin zero.

4.1.2 The real particles and five solutions of energy-momentum.

The five solutions that have been dealt, are solutions to the vacuum energy momentum, so naturally it has obtained the minimum value of the uncertainty principle, which corresponds to the vacuum fluctuations. For the energy momentum equation, to a real particle, it is necessary to have three spines integers, rotating energy in the case of the negative square of the energy. in this way obtained:

$$E_{T(r)} = \sqrt{E_1^2 + E_2^2 + E_3^2 + E_4^2 - 3E_5^2} = E_0$$

Fulfilled, that the ratio of the squares of the negative and positive energy, is precisely the square of the modulus of the spin 1/2 (electron, leptons):

$$\left[\frac{-3E_5^2}{4E_{1,\dots,4}^2} \right] = \left(\frac{1}{2} + 1\right)\frac{1}{2}; \quad \sqrt{\left[\frac{-E_5^2}{4E_{1,\dots,4}^2} \right]} = \frac{1}{2} = \min(\Delta x \cdot \Delta p/\hbar) = \sin^2(2\pi/8) = \cos^2(\text{spin } 1) = s = 1/2$$

4.1.3 Impossibility of the existence of spin, higher two.

If there is a spin greater than two, then immediately there would be a additional solution of positive energy. If this were so, then, the minimum value 1/2, the uncertainty principle would not be met, which is not possible. And therefore the maximum value for the spin can not be greater than the spin 2.

Proof. Case a) $s = n/2; n > 2$

□

$$\begin{aligned} E_1^2 &= (imc^2 \cdot \exp 2\pi i \cdot 1 + pc \cdot \exp 2\pi i \cdot 1)(imc^2 \cdot \exp 2\pi i \cdot \frac{1}{2} + pc \cdot \exp 2\pi i \cdot 1) \\ E_2^2 &= (imc^2 \cdot \exp 2\pi i \cdot 1 + pc \cdot \exp 2\pi i \cdot \frac{1}{2})(imc^2 \cdot \exp 2\pi i \cdot \frac{1}{2} + pc \cdot \exp 2\pi i \cdot \frac{3}{2}) \\ E_3^2 &= (imc^2 \cdot \exp 2\pi i \cdot 2 + pc \cdot \exp 2\pi i \cdot 2)(imc^2 \cdot \exp 2\pi i \cdot 2 + pc \cdot \exp 2\pi i \cdot \frac{3}{2}) \\ E_4^2 &= (imc^2 \cdot \exp 2\pi i \cdot \frac{3}{2} + pc \cdot \exp 2\pi i \cdot 2)(imc^2 \cdot \exp 2\pi i \cdot \frac{1}{2} + pc \cdot \exp 2\pi i \cdot \frac{3}{2}) \\ E_5^2 &= (imc^2 \cdot \exp 2\pi i \cdot \frac{n}{2} + pc \cdot \exp 2\pi i \cdot 2)(imc^2 \cdot \exp 2\pi i \cdot \frac{1}{2} + pc \cdot \exp 2\pi i \cdot \frac{n}{2}) \\ E_T &= \sqrt{E_1^2 + E_2^2 + E_3^2 + E_4^2 + E_5^2} = \sqrt{5}E > \min(2\Delta E) = \frac{\hbar}{\Delta t} \end{aligned}$$

Proof. Case b) $s = n; n > 2$

□

$$\begin{aligned} E_1^2 &= (imc^2 \cdot \exp 2\pi i \cdot 1 + pc \cdot \exp 2\pi i \cdot 1)(imc^2 \cdot \exp 2\pi i \cdot \frac{1}{2} + pc \cdot \exp 2\pi i \cdot 1) \\ E_2^2 &= (imc^2 \cdot \exp 2\pi i \cdot 1 + pc \cdot \exp 2\pi i \cdot \frac{1}{2})(imc^2 \cdot \exp 2\pi i \cdot \frac{1}{2} + pc \cdot \exp 2\pi i \cdot \frac{3}{2}) \\ E_3^2 &= (imc^2 \cdot \exp 2\pi i \cdot 2 + pc \cdot \exp 2\pi i \cdot 2)(imc^2 \cdot \exp 2\pi i \cdot 2 + pc \cdot \exp 2\pi i \cdot \frac{3}{2}) \\ E_4^2 &= (imc^2 \cdot \exp 2\pi i \cdot \frac{3}{2} + pc \cdot \exp 2\pi i \cdot 2)(imc^2 \cdot \exp 2\pi i \cdot \frac{1}{2} + pc \cdot \exp 2\pi i \cdot \frac{3}{2}) \end{aligned}$$

$$E_5^2 = (imc^2 \cdot \exp 2\pi i \cdot n + pc \cdot \exp 2\pi i \cdot n)(imc^2 \cdot \exp 2\pi i \cdot n + pc \cdot \exp 2\pi i \cdot \frac{3}{2})$$

$$E_T = \sqrt{E_1^2 + E_2^2 + E_3^2 + E_4^2 + E_5^2} = \sqrt{5}E > \min(2\Delta E) = \frac{\hbar}{\Delta t}$$

4.1.4 The electron mass I.

As demonstrated in previous work ("Quantum Information and Cosmology: the Connections", pag 3-4), the scaling law plays an essential role, since on the one hand seems to represent an amount of entropy and information intrinsic to different vacuum states. This allowed us to obtain the value of the baryon density, if the vacuum of space-time-energy is considered, as a group E8 Lattice.

"The group E8 with dimension 248 complex, real dimension 496. The E8 root system is a rank 8 root system containing 240 root vectors spanning R8. 240 vectors are equivalent to the number of hyperspheres, in eight dimensions, which can touch a equivalent hypersphere without any intersections. This number, 240, forming a lattice, is the maximum quantity of particles of the vacuum, in its state of minimal energy, as we demonstrate in the essay of last year. In this paragraph, we obtain the same result by the five microstates derived from the five solutions of the equation (3). Similarly, 240 is representable by a seven-dimensional vector, with octonions, as follows:

Fibonacci numbers dividers 240: 1, 2, 3, 5, 8, $\sum_{F_n/240} F_n^2 = 103 = \lfloor 2 \cdot \ln(m_{pk}/m_e) \rfloor$; 137 = 11^2 + 6^2 = $\lfloor \alpha^{-1} \rfloor$, $x_7 = 1e_1 + 2e_2 + 3e_3 + 5e_4 + 8e_5 + 11e_6 + 6e_7$, $\|x_7\| = 240$

The Kissing number of lattice 8d, $k(8d) = 240$; It has the following main property: $k(8) = \sum_{d=2, d \neq 5}^{d=7} k(d) = k(2) + k(3) + k(4) + \dots$

$$\dots + k(6) + k(7) = 6 + 12 + 24 + 72 + 126$$

Another fundamental property of this summation is: each $k(d)$ (Kissing number), is the product of n two-dimensional planes, so that $k(d) = nk(2d)$, ie:

$$k(8) = 240 = 1 \cdot k(2) + 2 \cdot k(2) + 4 \cdot k(2) + 12 \cdot k(2) + 21 \cdot k(2) \quad (4), \quad 1 + 2 + 4 + 12 + 21 = 40 = k(5d)$$

Equality given by the expression (4) is the basis of the strong holographic principle. By this principle, the fundamental properties of space-time-mass are holograms of two-dimensional surfaces with six spheres that touch each other at center, the seventh. That is, the compactifying, circular, of the seven dimensions, is a plane holographed seven circles. The proof that this principle is correct comes from the calculation of the value of the Higgs vacuum and the Higgs boson mass. Each circle has a curvature: $1/r$

The scaling law, which is essential, is immediately obtained as the sum of all circular curvatures between two intervals.

$$\sum_{r_2}^{r_1} \frac{dr}{r} = \int_{r_1}^{r_2} \frac{dr}{r} = \ln(r_1/r_2) \quad (5)$$

This scaling law is deductible, of the uncertainty principle, considering that the vacuum is the disintegration of photon pairs (same speed c) to any particle-antiparticle pairs. $\Delta m_f(\text{photon}), \Delta r_1 \Delta m_{f1} c \geq \hbar/2$; $\Delta r_2 \Delta m_{f2} c \geq \hbar/2$. Taking the minimum possible value of uncertainty: $\min(\Delta r_1 \Delta m_{f1} c) = \min(\Delta r_2 \Delta m_{f2} c) = \hbar/2$, $2\Delta r_1 \Delta m_{f1} = 2\Delta r_2 \Delta m_{f2} = \hbar/c$. Changing the photon by the particle-antiparticle pair: $2\Delta r_1 \Delta m_1 = 2\Delta r_2 \Delta m_2 \rightarrow 2(\Delta r_1 / \Delta r_2) = 2(\Delta m_2 / \Delta m_1)$ $r_1 \geq r_2, m_2 \leq$

$$m_1 \text{ .Applying equation (5): } 2 \sum_{r_2}^{r_1} \frac{d\Delta r}{\Delta r} = 2 \int_{r_2}^{r_1} \frac{d\Delta r}{\Delta r} = 2 \ln(r_1/r_2) = -2 \sum_{m_1}^{m_2} \frac{d\Delta m}{\Delta m} = -2 \int_{m_1}^{m_2} \frac{d\Delta m}{\Delta m} = -2 \ln(m_2/m_1)$$

Considering the vacuum as the state of lowest mass with electric charge (electrons) stable (lifetime of the particle, infinity) and its field of bosons (photons) then we obtain $k(8d)$, with a slight asymmetry due to the baryon density, as the sum of electron-positron pairs and the inverse of the fine structure (zero momentum) constant, as the number of photons. Scaling for the mass and radius are the Planck mass and the Planck length. $2 \ln(m_{Pk}/m_e) + \alpha^{-1} = 103.0556830739 + 137.035999174 = 240.091682247$, $(240.091682247 - 240)/2 = 0.0458411235 \cong \Omega_b$

As can be seen these logarithms are amounts of information, because:"

$$[2 \ln(m_{Pk}/m_e) + \alpha^{-1}] = 240 = \|x_7\|$$

As previously mentioned, in the four solutions of positive energy and considering additional solutions by the negative sign of the spin, then it is possible to obtain the electron mass, if the areas defined by the spins is applied, in the context of the loop quantum theory of gravity. Since the only parameters that are intrinsic; five solutions to the energy-momentum, are the number of solutions and spines. The equation for the electron mass is semi-empirical, but we have no doubt of its validity, both for consistency, as for the accuracy of the calculation.

$m_\tau = \text{tau mass}$; $m_\mu = \text{muon mass}$; $m_e = \text{electron mass}$; $m_{Pk} = \text{Planck mass}$

$$\alpha = \text{fine structure constant} - \text{zero momentum} = \frac{1}{137.035999174}$$

$$\ln(m_{Pk}/m_e) = [8 \sum_s \sqrt{(s+1)s} + (\pi \cdot \ln(m_\tau/m_\mu))/\ln(\alpha)] \cdot (1 + \frac{\alpha^2}{4}) = 51.5278415653 \quad (3)$$

$$8 \sum_s \sqrt{(s+1)s} = 8(\sqrt{(1+1)1} + \sqrt{(2+1)2} + \dots$$

$$+ \dots \sqrt{(\frac{1}{2}+1)\frac{1}{2}} + \sqrt{(\frac{3}{2}+1)\frac{3}{2}} + \sqrt{(0+1)0}) = 53.32976306$$

$$(\pi \cdot \ln(m_\tau/m_\mu))/\ln(\alpha) = (\pi \cdot 2.822369798 / -4.920243659)$$

$$(1 + \frac{\alpha^2}{4}) = 1 + 3.328209653 \cdot 10^{-6}$$

$$\frac{m_\tau}{m_e} = 3477.151012 ; \frac{m_\mu}{m_e} = 206.768284 ; \frac{m_e}{m_e} = 1 ; a_e = 1.15965218076 \cdot 10^{-3}$$

$a_e = \text{Anomalous magnetic moment of the electron}$

$$\ln(m_{Pk}/m_e) = [8 \sum_s \sqrt{(s+1)s} + \ln \left\{ 2 \cdot \left(\frac{m_\tau}{m_e} + \frac{m_\mu}{m_e} + \frac{m_e}{m_e} - \alpha^{-1} \right) \right\} / \ln(\alpha)] \cdot (1 + \frac{4}{(a_e^{-1}+1)^2}) \quad (4)$$

4.1.5 Extension of the four positive energy solutions to seven dimensions by octonions.

The four solutions of positive energy, can be interpreted perfectly as equivalent to four vectors in four dimensions. As will be shown in later sections, there are literally seven compactified dimensions, three extended and one temporal.

The extension of the solutions in the table (1) to seven dimensions by octonions, regardless of the unit value e_0 (imaginary energies only virtual) and counting the negative signs of the spins, gives us a total of 112 spin-dependent solutions. **Adding a factor of two, because of the disintegration of virtual photon pairs.** And it is precisely this amount, the 112 roots with integer entries obtained from:

$$(\pm 1, \pm 1, 0, 0, 0, 0, 0, 0)$$

By taking an arbitrary combination of signs and an arbitrary permutation of coordinates, and 128 roots with half-integer entries obtained from:

$$(\pm \frac{1}{2}, \pm \frac{1}{2})$$

The 112 roots with integer entries form a D8 root system. The E8 root system also contains a copy of A8 (which has 72 roots) as well as E6 and E7.

This is also equivalent to the total number of factors, two for each solution, to make the extension to seven dimensions. Next, the entire table is given.

These, 112 solutions form 14 groups of four by two four-dimensional solutions. And 14, that is precisely the dimension of the Lie group G2. G2 is the name of three Simple Lie groups (a complex form, compact real form and real split form), Their Lie algebras g_2 , as well as some algebraic groups. They are the smallest of the five exceptional Lie groups simply. G2 has rank 2 and dimension 14. It has two basic representations, With dimension 7 and 14.

In later sections, the link between the group G2 and zeros of the Riemann zeta function is displayed.

$(\pm \exp 2\pi i \cdot s)(e_1 mc^2 + pc)(-e_1 mc^2 + pc) = E_{e_1,1}^2$
$(\pm \exp 2\pi i \cdot s)(e_1 mc^2 - pc)(-e_1 mc^2 - pc) = E_{e_1,2}^2$
$(\pm \exp 2\pi i \cdot s)(mc^2 + e_1 pc)(mc^2 - e_1 pc) = E_{e_1,3}^2$
$(\pm \exp 2\pi i \cdot s)(-mc^2 + e_1 pc)(-mc^2 - e_1 pc) = E_{e_1,4}^2$
.....

Eights factors x \pm sings rotations by spins =16

$(\pm \exp 2\pi i \cdot s)(e_7 mc^2 + pc)(-e_7 mc^2 + pc) = E_{e_7,1}^2$
$(\pm \exp 2\pi i \cdot s)(e_7 mc^2 - pc)(-e_7 mc^2 - pc) = E_{e_7,2}^2$
$(\pm \exp 2\pi i \cdot s)(mc^2 + e_7 pc)(mc^2 - e_7 pc) = E_{e_7,3}^2$
$(\pm \exp 2\pi i \cdot s)(-mc^2 + e_7 pc)(-mc^2 - e_7 pc) = E_{e_7,4}^2$

$$16 \cdot 7 = 112 = 240 - 2^7$$

Negative energy: 14 solutions. 14 negative energy solutions, and counting the three odd spins: 42. Total sum of positive and negative energies: 112- 42 = 70

$$112E_+^2 + ||14E_-^2|| = \dim(Lattice\ 7d) = 126$$

And the number 70 meets three important conditions: 1) This is the only integer number whose square is the sum of consecutive squares of the first 24 integers. $24 \Rightarrow SU(5) \Rightarrow 4d!$ Lattice 24d. 2) $7!+1=71^2$. 3) The value of the energy of the vacuum, applying the entropic change of scale, is quite closely:

$$E_{vacuum} \approx (m_{Pk}c^2 \cdot \exp -70)/2 = 2.426 \cdot 10^{-3} eV$$

$$(\pm \exp 2\pi i \cdot s)(e_1, \dots, e_7) \cdot ((e_1, \dots, e_7)mc^2 + pc) \cdot (mc^2 + (e_1, \dots, e_7)pc) = -E_{(e_1, \dots, e_7)5}^2$$

4.1.6 The interaction matrix of the energies, the spins and the electric charges.

There is an equivalence or isomorphism between the five solutions of the energy-momentum, the five spines and five electric charges, not only for being the same number, otherwise identical also for interaction matrix. A matrix, that correspond to vacuum, the sum of all matrix elements must be zero. In this way the three matrix; with 25 interactions, are:

E	$-E_1^2$	$-E_2^2$	$-E_3^2$	$-E_4^2$	$+E_5^2$
$+E_1^2$	0	0	0	0	+2E
$+E_2^2$	0	0	0	0	+2E
$+E_3^2$	0	0	0	0	+2E
$+E_4^2$	0	0	0	0	+2E
$-E_5^2$	-2E	-2E	-2E	-2E	0

s	0	1/2	1	3/2	2
0	0	1/2	1	3/2	2
-1/2	-1/2	0	1/2	1	3/2
-1	-1	-1/2	0	1/2	1
-3/2	-3/2	-1	-1/2	0	1/2
-2	-2	-3/2	-1	-1/2	0

q	-1	-1/3	+2/3	4/3	1/3
1	0	+2/3	+5/3	+7/3	+4/3
1/3	-2/3	0	+3/3	+5/3	+2/3
-2/3	-5/3	-3/3	0	+2/3	-1/3
-4/3	-7/3	-5/3	-2/3	0	-3/3
-1/3	-4/3	-2/3	+1/3	+3/3	0

The 112 solutions, extending to seven dimensions by octonions, by adding the 25 matrix interactions of electric charges, spins or energies, you get precisely the integer part of the inverse of the fine structure constant, that is: $112 + 25 = 137 = 2^7 + 2^3 + 2^0 = \lfloor \alpha^{-1} \rfloor$

As you can see, 137 is the sum of all possible states of polarization of the photon, seven, three and zero dimension (time). This is equivalent to the sum of states, seven, three and zero qubits.

Taking into account the sum of the three matrixs of 25 elements of interaction (energies, electric charges and spins), then you need to add three more states for the 240 non-zero roots of the lattice in R8, which represents the vacuum. But in this case, breaking the vacuum is produced by the group E6, discounting the eight fundamental states.

$$3 \cdot 27 + 3 \cdot 27 + 3 \cdot 25 + 3 = 240$$

This new state added to complete the dimension of the lattice in R8, is twenty-six dimensions-states of the original superstring theory, in which tachyons appear.

$$3 \cdot 27 + 3 \cdot 27 + \{(3 \cdot 26) = \dim[E6]\} = 240 = \dim(Lattice R8)$$

The 11 dimensions are the result of the reduction of the 26 dimensions, for the group SU(4), or the sum of all possible projections of the spins, ie the vacuum goes to a lower energy state, and stable to

fluctuate by virtual particles (spin states, electric charge and energy).

These 11 dimensions are exactly the sum of the powers of the decomposition of 137, the vacuum, corresponding to photons, by powers of two, or qubits states.

$$2^7 + 2^3 + 2^0 = 137; 7d + 3d + 0d \text{ (time)}$$

The 26 dimensions are generated by a matrix of five elements, to which is added an imaginary state, which would be the time. $(5 + i)(5 - i) = 26d = 26 \text{ states}$

$$5d + 5d + (i \cdot -i)d = 10d + 1d_{time}$$

$$26d - \sum_s 2s + 1 = 11d = 26d - \dim[SU(4)]$$

Thus the ground state, the lowest energy vacuum, and stable, is the decomposition by photons and electrons, both particles with an average life without limits. Thus, it holds that the vacuum in its state of lowest energy, and stable, is broken down by the lattice group R8, in the sum (electron-positron pairs + photons): $240 = [2 \ln(m_{Pk}/m_e)] + [\alpha^{-1}] = 103 + 137$

Both 103 and 137 are surfaces of spheres in five and a seven dimensions, given by:

$$103 = 1^2 + 2^2 + 3^2 + 5^2 + 8^2; 103 = \sum_{F_n/240} F_n^2; \text{ where: } F_n \text{ The Fibonacci numbers divisors of 240}$$

$$137 = 1^2 + 2^2 + 3^2 + 5^2 + 8^2 + 11^2 + 4^2; 11^2 = SU(11) + U(1); 4^2 = SU(4) + U(1)$$

Therefore the length, due to the electrons coupling photons is:

$$\sqrt{103/137} \approx \cos(2\pi/12) = \cos 30^\circ$$

The proper length, the coupling of the twenty-six states and one hundred and twelve momentum energy solutions, is:

$$\sqrt{26/112} \approx \sin_W^{eff}(M_Z)(\overline{m_s}) = 0.2321428571$$

The mass of the electron as a function of the length of the spins, less length, vacuum electron coupling photons, is:

$$\ln\left(\frac{m_{Pk}}{m_e}\right) = \frac{[8 \sum \sqrt{(s+1)s} - (\sqrt{103/137})/(\sqrt{26/112})]}{[1 + \alpha \cdot (2\pi \cdot 26)^{-1}]} = \dots = 51.5278419284783 \text{ (5)}$$

Other expressions equivalent to (5), are:

$$\ln\left(\frac{m_{Pk}}{m_e}\right) \approx \frac{[8 \sum \sqrt{(s+1)s} + (\sqrt{240 \cdot \alpha - 1} + 8)/\ln \alpha]}{[1 + 11/(2\pi \cdot m_{Vh}/m_e)]^{-1}} = 51.5278417770775 \text{ (6)}$$

Where m_{Vh} , is mass equivalent vacuum Higgs = 246.221202 GeV

The value of the coupling, given by: $\sqrt{103/137} \sim \cos \theta_W$; Although an accurate approximation is given by: $\frac{\sqrt{103/137}}{(120 \cdot \alpha)^{1/8}} = \cos \theta_W = 0.8815866541 = \frac{M_W}{M_Z} \implies (91.1876 \text{ GeV} = M_Z) \cdot 0.8815866541 = 80.389 \text{ GeV}$

$$\frac{s \sum \sqrt{(s+1)s} + \left[8 + \cos\left(\frac{2\pi}{\ln\left(\frac{M_W + M_Z}{2 \cdot m_e}\right)}\right)\right]}{[1 + (\frac{2 \cdot M_W}{m_e})^{-1}]^{-1}} \Big/ \ln \alpha = \dots = \ln\left(\frac{m_{Pk}}{m_e}\right) = 51.5278413077842 \text{ (7)}$$

4.1.7 The breakdown of the group E8, and its relation to $\left\lfloor \ln \frac{m_{Pk}}{m_e} \right\rfloor = 103$;and $\lfloor \ln(\alpha^{-1}) \rfloor$

One way to incorporate the standard model of particle physics into heterotic string theory is the symmetry breaking of E8 to its maximal subalgebra $SU(3) \times E6$. Complex dimension $E6 = 78$.

The 248-dimensional adjoint representation of E8, when similarly restricted, transforms under $E6 \times SU(3)$ as: $(8,1) + (1,78) + (3,27) + (3,\overline{27})$.

The breakdown of the group-lattice E8 is given by the twenty-five states of the matrix, to spin, electric charge or energy; with the numbers 103 and 137 and with the following equalities:

$$\left\lfloor \ln \frac{m_{Pk}}{m_e} \right\rfloor = 103 - 25 = 78 ; (3, 27) + (3, \overline{27}) = 162 = 137 + 25 \quad (8)$$

$$(78/162) = 0.481481481 \approx \sin \hat{\theta}_w(M_Z)(\overline{m_s}) \quad (9) , 78 + 162 = 240 = \dim(\text{lattice } 8d)$$

$$8 \sum_s \sqrt{(s+1)s} - \frac{\sqrt{2 \cdot \ln(\frac{m_{Pk}}{m_e}) \cdot \alpha}}{(78/162)} \approx \exp(1/\rho(8d))$$

$$\rho(8d) = \frac{\pi^4}{384} = \text{packing density lattice } 8d$$

$$8 \sum_s \sqrt{(s+1)s} - \frac{\sqrt{2 \cdot \ln(\frac{m_{Pk}}{m_e}) \cdot \alpha}}{(78/162)} - \dots$$

$$\dots - \left(\frac{2 \cdot \ln(\frac{m_{Pk}}{m_e}) \cdot \alpha^{-1}}{10} \sqrt{2 \cdot \ln(\frac{m_{Pk}}{m_e}) \cdot \alpha} \right)^{-1} = \ln\left(\frac{m_{Pk}}{m_e}\right) = 51.52784145 \quad (10)$$

4.1.8 The instability of the vacuum and the five solutions of the equation energy momentum.

As showed, in a previous section, the stability of the vacuum is defined by the value of the fifth solution of the energy equation impulse, since on the one hand, the value of the total energy of real particles is obtained, and in secondly, the minimum value of uncertainty for virtual particles is obtained; or, equivalently, the value of the zero point energy. $2E_0 = \hbar\omega$

For real particles, it is necessary to have the three states of the three integers spines, including zero. In this way we obtain:

$$-E_{5,1}^2 = i(imc^2 \cdot \exp 2\pi i \cdot 0 + pc \cdot \exp 2\pi i \cdot 0)(imc^2 \cdot \exp 2\pi i \cdot 0 + pc \cdot \exp 2\pi i \cdot 0)$$

$$-E_{5,2}^2 = i(imc^2 \cdot \exp 2\pi i \cdot 1 + pc \cdot \exp 2\pi i \cdot 1)(imc^2 \cdot \exp 2\pi i \cdot 1 + pc \cdot \exp 2\pi i \cdot 1)$$

$$-E_{5,3}^2 = i(imc^2 \cdot \exp 2\pi i \cdot 1 + pc \cdot \exp 2\pi i \cdot 1)(imc^2 \cdot \exp 2\pi i \cdot 1 + pc \cdot \exp 2\pi i \cdot 1)$$

$$E_T = \sqrt{E_1^2 + E_2^2 + E_3^2 + E_4^2 - E_{5,1}^2 - E_{5,2}^2 - E_{5,3}^2} = E_0 = m^2c^4 + p^2c^2$$

For virtual particles, the four states of positive energy is only necessary.

$$E_T = \sqrt{E_1^2 + E_2^2 + E_3^2 + E_4^2} = 2E_0 = \min\left(\frac{\hbar}{\Delta t} = 2\Delta E\right) = \hbar\omega$$

The stability of the vacuum, is directly dependent on the fifth solution of the energy momentum equation, not take imaginary values for the square of the energy, for which the factorization of the same, is multiplied by the imaginary unit.

Now, there is another solution, equally admissible: the square of the energy is imaginary, that is:

$$iE_5^2 = (imc^2 + pc)(mc^2 + ipc) = i(m^2c^4 + p^2c^2) \quad (11)$$

Counting the three states of integer spin (including zero) rotating energies, there is a complex vector (**seven-dimensional vector**) given by the four non-imaginary solutions and three imaginary energies, ie:

$$E_1^2 + E_2^2 + E_3^2 + E_4^2 + iE_{5,1}^2 + iE_{5,2}^2 + iE_{5,0}^2$$

So the square of the total energy is given, by the previous module complex vector:

$$E_T^2 = \sqrt{(E_1^2 + E_2^2 + E_3^2 + E_4^2)^2 + (E_{5,1}^2 + E_{5,2}^2 + E_{5,0}^2)^2} = \sqrt{16E^4 + 9E^4} = 5E^2$$

Thus, the value of the energy, is: $E_T = \sqrt{5}E$ (12)

Now this energy value is greater than zero point energy and the minimum value of uncertainty. Therefore, this value of the vacuum is unstable and has to come down to at least the value of the zero point energy or its equivalent, the minimum of uncertainty. This stabilization of the vacuum, having reduced its value, is the originator of the masses of the electron (the lowest possible energy level, with life of the particle at $t = \text{infinity}$, and electric charge), muon and tau, as demonstrated . And in theory the energy of any elementary particle.

4.1.9 Stabilization of the vacuum: electrically charged lepton masses. First application of the Riemann zeta function.

Conditions that must meet the dimensionless value of the energy, to stabilize the vacuum:

1. The energy function must be zero (conservation of energy)
2. The derivative of the energy has to be $\sqrt{5}$
3. The derivative of the energy must include the minimum value of zero point energy. $2E_1 = \hbar\omega$
4. This value of the energy can not be greater than 2, to meet the minimum value of the uncertainty principle.
5. The value of the instability of the vacuum energy, has to be a subtraction of the energy value of zero point energy. And it has to be an integer quantized value (subtraction).

Therefore, we have: 1) $f(E_1) = 0$; 2) $df(E_1) = \sqrt{5}$; 3) $df(E_1) = 2E_1 - y$

$$\frac{\sqrt{5}+y}{2} = E_1$$

The minimum value of this quantized energy substracion, is one. As has finally equation:

$$\frac{\sqrt{5}+1}{2} = E_1; df(E_1) = 2E_1 - 1; \int df(E_1) = \int (2E_1 - 1)dE_1 = E_1^2 - E_1 + c = 0; c = 1$$

$$E_1^2 - E_1 - 1 = 0 \quad (13)$$

As you can see, this dimensionless value, is the golden ratio. We will use the notation φ_E , to refer to this value of dimensionless energy.

Now, equation (13) has another solution with a lower negative value, given by: $-\frac{1}{\varphi_E}$. Therefore, this value is the minimum of the energy given by equation (13). For this value of the energy is positive, the equation (13) should become:

$$E_2^2 + E_2 - 1 = 0; \frac{df(E_2)+1}{2} = E_2 = \frac{\sqrt{5}-1}{2} = \frac{1}{\varphi_E} \quad (14)$$

And here an interesting result appears: The inverse of the value of the energy value is another solution of the equation (13). This result bears a strong resemblance to one of the dualities of superstring theory: duality S.

4.1.9.1 Derivation of minimum energy value, applying the Riemann zeta function.

Be the Riemann zeta function, with $s = \frac{1}{2} + it_n$. The alternating zeta function is used, and being $s = \frac{1}{2} + it_n$ a nontrivial zero for said alternating function: $\zeta(s) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^s \cdot (1-2^{s-1})}$. Being t_n , the real value of the imaginary part of n nontrivial zero.

For a complex number $\frac{1}{2} + it$; the following exclusive (only s with real part 1/2) properties hold:
a) Properties of commutation.

$$\frac{dx^s}{s} = \frac{1}{x^{\bar{s}}}, \frac{dx^{\bar{s}}}{\bar{s}} = \frac{1}{x^s} \implies x^{\bar{s}} dx^s + dx^{\bar{s}} x^s = 1 \quad ; \quad dx^s x^{\bar{s}} - x^s dx^{\bar{s}} = 2it; \quad x^s dx^{\bar{s}} - dx^s x^{\bar{s}} = -2it$$

b) The inverse function equal to the derivative of the function with conjugate exponent s, divided by the same complex exponent: $\frac{dx^{\bar{s}}}{\bar{s}} = \frac{1}{x^s}, \frac{dx^s}{s} = \frac{1}{x^{\bar{s}}}$

The condition of conservation of energy, or zero energy function, is expressed as: $E \cdot \zeta(s) = 0$

The derivation of equation (14) is, with $E = \sqrt{5}$:

$$(E^{\bar{s}} dE^s - it_n)E - (E^s dE^{\bar{s}} + it_n) = \frac{df(E_2)+1}{2} = E_2 = \frac{\sqrt{5}-1}{2} \quad (15)$$

However, for this energy is consistent, equivalent to the quantum vacuum zero-point energy: $E_0 = \frac{\hbar\omega}{2} = \frac{1}{2 \cdot \varphi_E} = \frac{E_2}{2}$. Noting that $\frac{1}{2 \cdot \varphi_E} = \cos(2\pi/5)$; you can reach the conclusion, that the stabilization energy of the vacuum is given by the coupling of the trigonometric functions of the angle $2\pi/5$

Considering an oscillation of the energy given by: $\Delta E = \frac{E_2}{2} + \frac{df(E_2)+1}{E_2}$

And representing the coupling of energy for the sine and cosine of the angle $2\pi/5$, the two solutions to the square of the energies are obtained, in de Sitter space (hyperbolic), where the logarithmic scaling law, would in fact areas of hyperbolic sectors of the de Sitter space. For this reason and consistent with Table 1 of the energies and the hyperbolic space; finally have the masses of the muon and tau leptons:

$$\ln(m_\mu/m_e) = (\cos(2\pi/5) + 2)^2 - \alpha^2 \cdot \cos(2\pi/\varphi^2) \quad (16); \quad \alpha = 1/137.035999174; \quad \varphi = \frac{\sqrt{5}+1}{2}$$

$$\ln(m_\mu/m_e) = (\cos(2\pi/5)+2)^2 - \alpha^2 \cdot \cos(2\pi/\varphi^2) = 5.33159874620381; \quad \frac{m_\mu}{m_e} = \exp(5.33159874620381) = 206.7682800511$$

$$\ln(m_\tau/m_e) = (\sin(2\pi/5) + 2)^2 - [8 \sum_s \sqrt{(s+1)s} - \ln(m_{Pk}/m_e)]^{-1} + \frac{\sin^2(2\pi/5) \cdot \alpha}{2\pi \cdot \ln(m_\mu/m_e)} = \dots$$

$$\dots = 8.15396846567741 \quad (17)$$

$$\frac{m_\tau}{m_e} = 3477.15067443364$$

$$\ln(m_{Pk}/m_e) = \left(\frac{2}{\varphi}\right)^2 + 2 \cdot 5^2 - 1 / \left[\left(\sqrt{\frac{1}{2} \sqrt{\frac{\pi^2}{3} - 2}} \right) \cdot \pi \cdot \alpha^{-2} \right] = 51.5278415513752 \quad (18)$$

$2 \cdot 5^2 \implies$ **two matrix spins and electric charges**

A very interesting and exact relations: 1) Entropic uncertainty principle only for space $\frac{\ln(\sqrt{2\pi})+1/2}{\varphi \cdot \ln \varphi} + 1 = 2.82238298354115 \approx \ln\left(\frac{m_\tau}{m_\mu}\right)$

2) Higgs boson mass: $\ln(m_h/m_e) = (\varphi^8 - \varphi^7 + \varphi^6 - \varphi^5 + \varphi^4 - \varphi^3 + \varphi^2 - \varphi) - 8 \cdot 2 = 12.4164078649988 \implies m_h = 126.123905 \text{ GeV}$ (Higgs boson mass)

3) Ratio proton/electron mass : $\ln^2(m_{Pk}/m_t) - \ln(m_{Pk}/m_\mu) + \sqrt[240]{\frac{m_{Vh}}{m_e} + 248 \cdot \pi^2 - [2 \cdot \ln(m_{Pk}/m_e) + \pi]} \dots = \frac{m_p}{m_e} = 1836.15267245$

4) Electron mass: $\ln(m_{Pk}/m_e) = \varphi^8 + \ln[(5 \cdot \ln \pi + 4)^2] + 1 / [m_Z/m_e - m_\tau/m_e - m_\mu/m_e - m_e/m_e] = 51.52784155$

4.1.9.2 The value of the Higgs vacuum. $\Delta E_{00} = \frac{E_2}{2} + \frac{df(E_2)+1}{E_2}$; $\Delta E_{01} = \frac{df(E_1)}{E_2} = \frac{E_T = \sqrt{5}E}{E_2} = (E_2 + 3) = (E_1^2 + 1) = (E_1 + i)(E_1 - i)$

$(E_2 + 3)^2 = (E_1^2 + 1)^2 = \varphi^4 + 2\varphi^2 + 1$; $d(\varphi^4 + 2\varphi^2 + 1) - 11 = 4\varphi^3 + 4\varphi - 11 = \ln(m_h/m_e)$; $SU(5) = SU(3) \times SU(2) = 8 \cdot 3$; $8 + 3 = 11$

$(E_2 + 3)^2 - \frac{(M_W/M_Z)^2 = (\cos \theta_W)^2}{240 - \dim(E_6)} = \ln(m_{Vh}/m_e)$ (19); $M_W =$ **boson W mass**; $M_Z =$ **boson Z mass**

4.1.9.3 Fine structure constant at zero momentum. $\alpha = \{(\varphi/[\zeta(2)+1] + \frac{163}{6})^{-1} + 137\}^{-1} = (137.0359991741)^{-1}$ (20); $\zeta(2) =$ **Sums of the curvatures of all quantized spherical radius, with integer values. Sum from 1 to ∞**

1^2	$= 1$
1^2	$= 1$
$1^2 + 2^2$	$= 5$
$1^2 + 2^2 + 3^2$	$= 14$
$1^2 + 2^2 + 3^2 + 5^2$	$= 39$
$1^2 + 2^2 + 3^2 + 5^2 + 8^2$	$= 103$
S_T	$= 163$

Arithmetic mean: $163/6$; $\{1, 1, 2, 3, 5, 8\} =$ The first five Fibonacci numbers; dividers lattice R8, with dimension: 240

4.1.10 The isomorphism between the Higgs potential and the five solutions equation energy momentum.

From equation (12), a potential energy is derived, given by: $E_T^4 = 25E^4$. Extended imaginary values only; If quaternions are used and counting the double rotation of the spins, a total of positive energy are obtained: $2 \cdot 12 \cdot E^2 = \dim[SU(5)]E^2 = 7 \cdot 4 \cdot 2E^2 - 2^5$

$2^5 =$ maximum number of supercharges, equivalent to all possible states (0, 1) of five solutions of the energy equation momentum.

$7 \cdot 4 \cdot 2E^2 =$ Number of states of positive energy, generated by octonions (only extended imaginary values) and the two rotations of the spins.

The symmetry of the vacuum requires that the number of fermions and bosons is the same, except the Higgs boson, the particle of dark matter and the graviton, ie: $12E_B^2 + 12E_F^2$. Precisely in equation (14) is the key, if the holographic principle is admitted (reduction of the seven dimensions, for the volume of a torus, with dimension six, radius).

Be a Higgs potential, given by the Mexican hat function: $|\phi|^4 + |\phi|^2 \iff E^4 + E^2 = (E^2 + iE)(E^2 - iE)$; $(E^2 + iE) \iff (E^2 + E)$

By integration, you can get this potential energy isomorphic to the Higgs potential, or Mexican hat; obtained by:

$$\int \int 12E^2 dE + \int \int 2dE = E^4 + E^2. \text{ Fulfilling with the holographic principle, because: } (\int \int 12E^2 dE) \cdot (\int \int 2dE) = E^6$$

Then, it will show that this isomorphism is strictly accurate. Getting the value of the Higgs vacuum, as the ratio between the mass of the electron, by: 1) $(24^4 + 24^2)(\sqrt{6} - 1) + 2 \ln(m_{Pk}/m_e) = 481843.8708$; 2) $(25^4 + 25^2)(1 + \sin_{eff}^2 \hat{\theta}_W) = (25^4 + 25^2)(1 + \ln^2 \varphi) = 481849.7362 = \frac{m_{Vh}}{m_e}$

$$3) [(26^4 + 2 \cdot 26^2) / \sin(2\pi/5)] - \frac{26 \cdot \ln 26}{\ln \ln 26} = 481842.8592; 2 \cdot 26 + 26 = 78 = \dim(E6)$$

$$4) ([2\varphi^3 - 3] \cdot \varphi)^6 + (\ln \ln \varphi)^{-12} = 481842.9943$$

4.1.10.0.1 The isomorphism $|\phi|^4 + |\phi|^2 \iff E^4 + E^2$. Calculation of the elementary electric charge. As will be shown in the final section, the initial formulation of Kaluza-Klein, adding a fifth dimension, compactified, which allowed him to derive the equations of electromagnetism and the RG theory of Einstein, lacked the partition function of the imaginary parts of the zeros of the Riemann zeta function. Since the electron is stable vacuum scale, with a minimum of energy and electric charge, the exact isomorphism between the Higgs potential (Mexican hat function) and the derivation of an equivalent potential energy, given in the previous section, we yields the elementary electric charge, exactly as the initial Kaluza-Klein theory.

Being the SU(7) group, a direct function of the dimensions, eight and four (double holography; eight dimensions, to four) or subtraction of the table-array octonions and quaternions, ie: $\dim[SU(7)] = 8^2 - 4^2 \implies SU(5) = \dim[SU(7)]/2$. The potential energy, isomorphic to the Higgs potential, is derived by the double holography, by double integration of basic potential, and isomorphic to $12E^2 + 2dE$. Also double holography four dimensions, to two dimensions, such as lattice. Being 12; twelve fermions-bosons of the standard model. And the term 2dE; differential oscillation of the vacuum, equivalent to a minimum of uncertainty, or zero point energy of the vacuum.

$$\int \int 12E^2 dE + \int \int 2dE = E^4 + E^2 \iff |\phi|^4 + |\phi|^2$$

Take as potential, as a function of the logarithm Planck-mass electron mass, ratio. $V(E/c^2) = \ln^4(m_{Pk}/m_e) + \ln^2(m_{Pk}/m_e)$

Standard deviation of the electric charges.

If the vacuum is symmetric with respect to the electric charges, in the sense that there is the same probability, that arise from quantum fluctuations of virtual particles, for different electric charges, then you need to take into account a standard deviation of quantized electric charges, if and only if the physical entity space-time - energy, is actually a physical manifestation of a unitary entity, in which the spins, electric charges, etc. are different characteristics interrelated single physical entity, character, that assume, purely geometric and topological. Emphasis has to be done, we are dealing with a vacuum of virtual particles, and therefore unobservable. This means that if the deviation of the electric charges of the virtual particles, due to the unification, or unitarity; occurs, then you probably will not be able to observe and therefore measured. But this deviation yea shall appear in the calculation of the elementary electric charge, using the result of Kaluza- Klein and isomorphic Higgs potential, which has been shown before.

Now, this deviation must meet the requirement of invariance of the closed circular elementary electric charge circuit; dependent rotations of the spin given by: $\exp(2\pi is) = 1 \rightarrow s = 0, 1, 2$; $\exp(2\pi is) = -1 \rightarrow s = \frac{1}{2}, \frac{3}{2}$

Thus, with the hypothetical electric charges $4/3$ and $1/3$, it has a standard deviation for the electric charges (continuous transformation of virtual particles of all different electric charges). The square of the standard deviation with the arithmetic mean value equal to zero, is the vector sum of the square of the electric charges, ie:

$$\sigma^2(q, \mu(q) = 0) = \sum_q q^2 = \frac{31}{9}$$

Applying the invariance, which has been formulated, we have: $\sigma^2(q, \mu(q) = 0) - \sigma_0^2(q) = 1 \implies \sigma_0^2(q) = \sigma^2(q, \mu(q) = 0) - 1 = \frac{31}{9} - 1 = \frac{22}{9}$

But this deviation must be modified to take into account, both the three color charges of the strong interaction, as well as virtual particle-antiparticle pairs, which introduce a multiplicative factor of two. Therefore, the final deviation is expressed as:

$$\sigma(q) = 2\sqrt{\frac{(\sigma_0^2(q)/5)}{3}} = 0.8073734276 ; \sigma(q) \approx \ln^{-1}[\sigma^2(q, \mu(q) = 0)]$$

$$\sigma(q) = \ln^{-1}[\sigma^2(q, \mu(q) = 0)] - [(\ln(m_\tau/m_e) \cdot \ln(m_{Pk}/m_e))]^{-1} + [(m_\tau \cdot m_\mu/m_e^2) \cdot \exp([\frac{12\pi}{27}]^4/8)]^{-1}$$

The direct relationship between the spins and the electric charges, with the manifestation of the principle of unitarity, it is clear, if we consider the sum of all the electric charges to particles-antiparticles, just as probabilities, dependent on the cosine of the spin 2 (graviton) and the electron spin $1/2$. And these two boson-fermion particles are precisely those that are involved in the unification of gravity and electromagnetism, to calculate the elementary electric charge.

When these probabilities, unit (1, -1), also can be treated as two states, and therefore as an entropy. Since the entropic uncertainty principle or Hirschman uncertainty, is equal to: $\ln \pi + 1$. The final entropy, adding the contribution of the electric charges and the coupling of the five spines, we have: $5(\ln \pi + 1 - 2) = H(p, x, q, s)$. p = momentum, x = position, q = electric charges, s = spin.

$$H(q) = 2$$

$$\sum_q q = \frac{-1}{3} + \frac{1}{3} + \frac{2}{3} + \frac{4}{3} - 1 = 1 = -\cos^2(s=1/2) + \cos^2(s=1/2) + \cos^2(s=2) + 2\cos^2(s=2) - 1 = \exp_{s=0,1,2}(2\pi i s)$$

$$\sum_{-q} -q = \frac{1}{3} - \frac{1}{3} - \frac{2}{3} - \frac{4}{3} + 1 = -1 = \cos^2(s=1/2) - \cos^2(s=1/2) - \cos^2(s=2) - 2\cos^2(s=2) + 1 = \exp_{s=\frac{1}{2},\frac{3}{2}}(2\pi i s)$$

Finally, the elementary electric charge, taking into account the potential isomorphic to the Higgs potential, the deviation of the electric charges, and the entropy, is expressed as:

$$[\ln^4(m_{Pk}/m_e) + \ln^2(m_{Pk}/m_e)]/5 \cdot (\ln \pi - 1) = m_{Pk}/\sqrt{(\pm e \cdot \sigma(q))^2/16\pi G_N} \implies \pm e = \dots$$

$$\dots = \pm \sqrt{\frac{m_{Pk}^2 \cdot 16\pi \cdot G_N \cdot 5^2 (\ln \pi - 1)^2}{[\ln^4(m_{Pk}/m_e) + \ln^2(m_{Pk}/m_e)]^2 \cdot \sigma^2(q)}} = 1.602159586 \cdot 10^{-19} C$$

The very small difference between the theoretical value and the experimental; should surely to the value of constant of gravity, known mostly inaccurate. For the calculation, we have adopted a value of the constant of gravitation equal to: $G_N = 6.67428 \cdot 10^{-11} N \cdot m^2/Kg^2$

The expression of the electric charge, also obeys the following empirical amazing identities:

$$1) (m_W/m_e) \cdot 5^2 \cdot 2 = \frac{m_{Pk}}{\sqrt{\pm e^2/G_N \cdot 16\pi}} ; 2) (44 + \frac{4\pi}{137}) \frac{m_Z}{m_e} = \frac{m_{Pk}}{\sqrt{\pm e^2/G_N \cdot 16\pi}} ; 3) (\zeta(2) + 4)^2 \frac{m_h}{m_e} = \frac{m_{Pk}}{\sqrt{\pm e^2/G_N \cdot 16\pi}}$$

$$4) \left(\frac{m_\tau \cdot m_\mu}{m_e^2} \right) \cdot \ln \left(\frac{m_\tau \cdot m_\mu}{m_e^2} \right) \cdot \frac{\sqrt{26}}{2\pi} = \frac{m_{Pk}}{\sqrt{\pm e^2/G_N \cdot 16\pi}}$$

The entropy $H(p, x, q, s)$; as a contribution of the tau lepton, and muon. $H(q) = 2 = H\left(\ln\left[\int \frac{dm_1}{m_1} = \int \frac{m_2 dm_2}{2}\right] + C_1 + \int \frac{dm}{m} + C_2\right) - C_1 - C_2$; $m_2^2 = \ln(m_1) + C_1$; $m_2^2 = \varphi^2$

$$\exp(\varphi^2) - \frac{5^2}{112} + \frac{\alpha}{2\pi \cdot (2+\varphi^{-2})^4} = \ln\left(\frac{m_\mu \cdot m_\tau}{m_e^2}\right); \int \frac{dm}{m} + C_2 = \ln(e^2/\varphi^2) - (\cos^4 \theta_W \cdot 4 \ln^2(m_{Pk}/m_e))^{-1}$$

$$\int \frac{dm}{m} = \ln((e^2/\varphi^2))$$

$$(e^2/\varphi^2) - (\cos^4 \theta_W \cdot 4 \ln^2(m_{Pk}/m_e))^{-1} = \ln(m_\tau/m_\mu)$$

$$\exp(\varphi^2) - \exp[-\sqrt{\exp(\sigma(q))}] + [(e^2/\varphi^2) \cdot 4\pi \ln(m_{Pk}/m_e) - \cos \theta_{s=2}]^{-1} = \ln\left(\frac{m_\mu \cdot m_\tau}{m_e^2}\right) ; H(p) = \ln \varphi^2 + \ln(e^2/\varphi^2) = 2$$

4.1.11 Masses of quarks and neutrinos.

This section will show, so semi-empirical, and applying the principle of conservation of energy, equivalent to the function of the total energy is zero, it is possible to calculate the masses of the quarks and the average mass of neutrinos, taking into account the oscillatory aspect of the neutrinos. The semi-empirical relationships and very notorious, are:

1. Direct relationship of the trigonometric functions of the angle $2\pi/5$ with the angle $\theta_{c12} = 13.04^\circ$ of Cabibbo–Kobayashi–Maskawa matrix. $\tan \theta_{c12} = \sin^2 \hat{\theta}_W (M_Z)(\overline{m_s}) = \ln^2 \varphi = 0.2315648207 \Rightarrow \theta_{c12} = 13.0378^\circ$ (21)

$\sin(2\pi/5) + \frac{\tan\theta_{c12}}{10} = \cos\theta_{c12} = 0.9742129984$ (22). We think the factor 10 represents the sum of bosons with zero rest mass, ie: $8g + 1\gamma + 1G = 10$

- The group E6, and the dimension of the lattice group R8, 240. $(240\alpha + 2)/(240 - \dim(E6)) = \frac{\tan\theta_{c12}}{10} = 0.0231565717$
- The E6 group and the group SU(9). $\dim(E6)/SU(9) = \cos\theta_{c12}/\cos\theta_{c23} = \cos 13.0378^\circ / (\cos 2.38^\circ \pm 0.06^\circ) = 0.975$ The relation of the breakdown of the lattice dimension R8, with the group E6, which is based on $240 - 78 = 2 \cdot 9^2$; being the matrix 9×9 , a hypothetical general matrix, the interaction between the matrix of neutrino oscillation or leptons (with electric charge), and the matrix of change of flavor of quarks.
- $\cos\theta_{c12} - \frac{\tan\theta_{c12}}{10} = d(\frac{\ln(\cos\theta_{c12})}{10} + \sin\theta_{c12})$; $\ln(\cos\theta_{c12}) \approx -\frac{1}{6 \sum_{q=1}^6 \ln(m_q/m_e)}$, $q = \text{quark}$ (23)

With today's more precise values of the quark masses, seems to be a similar function to that shown for the charged leptons, which uses the sine of the angle $(2\pi/5)$, and that calculates the sum of the logarithms of the quarks with equal isopin and electric charge.

4.1.11.0.2 Quarks masses. $m_u = 2.16 \text{ MeV}$; $m_d = 4.6 \text{ MeV}$; $m_s = 93.5 \text{ MeV}$; $m_c = 1.275 \text{ GeV}$; $m_b = 4.18 \text{ GeV}$; $m_t = 173.1 \text{ GeV}$, $m_e = 0.510998928 \text{ MeV}$

Sum logarithms quark mass ratios u, c, t, relative to the mass of the electron. Isopin = 1/2. Factor 3, because the three color charges.

$$\sum_{q_1=u,c,t} \ln(m_{q_1}/m_e) = 21.99659778 \approx 2 \cdot (\frac{3}{\sin^2(2\pi/5)})^2 = 22.00124224$$

Sum logarithms quark mass ratios d, s, b, relative to the mass of the electron. Isopin = -1/2

$$\sum_{q_2=d,s,b} \ln(m_{q_2}/m_e) = 16.41624763 \approx 2 \cdot (3 \cdot \sin(2\pi/5))^2 + (\ln(m_\tau/m_e) - 2)^{-1}/9 = 16.41626384$$

$$\sum_{q_1=u,c,t} \ln(m_{q_1}/m_e) + \sum_{q_2=d,s,b} \ln(m_{q_2}/m_e) = \sum_{q=1}^6 \ln(m_q/m_e) = 38.41284541 \approx 2 \cdot ([\frac{2}{\varphi} + \sin\theta_{c12}] \cdot 3)^2 - \Omega_b = 38.41232157$$

$$\Omega_b = \text{baryon density} = (\alpha^{-1} + 2 \cdot \ln(m_{Pk}/m_e) - 240)/2$$

$$\sum_{q=1}^6 \ln(m_q/m_e) \approx 12\pi + [3 \cdot \sin^2(2\pi/5)] - 2 = 38.41263733$$
; $\exp(-1/\sum_{q=1}^6 \ln(m_q/m_e)) = 0.9743029778 \approx \cos\theta_{c12}$

$$[2 \cdot (\frac{3}{\sin^2(2\pi/5)})^2] / [2 \cdot (3 \cdot \sin(2\pi/5))^2] = \sin^6(2\pi/5)$$
; $18^2 \cdot 2 = 2 \cdot [240 - \dim(E6)]$

$$2 \cdot [240 - \dim(E6)] \cdot \sin^6(2\pi/5) = 240 + \ln^{-1}(\alpha/2)$$
; $(2/5)^2 \cdot \sin^6(2\pi/5) = 0.1184016994 = \alpha_s(M_Z)$

The masses of the quarks obey a dependent function trigonometric functions Cabiboo angle θ_{c12} , which can be found in our previous work: "Simple Formulas that Generates the Quarks Masses"

4.1.11.1 Neutrino masses and dark matter candidate Whereas the oscillation of the three neutrinos, one could think of an average mass of them. If we group the particles of the standard model (maximum energy equal to the value of the vacuum Higgs) for bosons and fermions, considering that the total energy, represented by logarithms (particle electron mass ratio) must be zero, then it must meet for fermions:

$$\sum_{q=1}^6 \ln(m_q/m_e) + \sum_{l=1}^6 (m_l/m_e) = 0 \quad (24)$$

$$\sum_{l=1}^6 (m_l/m_e) = \sum_{l_1=\tau,\mu,e} \ln(m_{l_1}/m_e) + \sum_{l_2=\nu_e,\nu_\mu,\nu_\tau} \ln(m_{l_2}/m_e) = 3 \cdot \ln(\overline{m_\nu}/m_e)$$

Arithmetic mean neutrino mass: $\overline{m_\nu}$; Thus, equation (24) becomes: $-\sum_{q=1}^6 \ln(m_q/m_e) - \sum_{l_1=e,\mu,\tau} \ln(m_{l_1}/m_e) = 3 \cdot \ln(\overline{m_\nu}/m_e)$ (25)

So the average neutrino mass, is: $[-\sum_{q=1}^6 \ln(m_q/m_e) - \sum_{l_1=e,\mu,\tau} \ln(m_{l_1}/m_e)]/3 = (-51.89841274/3)$

$$\sum_{q=1}^6 \ln(m_q/m_e) + \sum_{l_1=e,\mu,\tau} \ln(m_{l_1}/m_e) \approx \ln(m_{Pk}/m_e)(1 + \exp(-\frac{\pi^2}{2})) = 51.89842376$$

$$\overline{m_\nu} = m_e \cdot \exp(-51.89841274/3) = 510998.928 \text{ eV} \cdot 3.068564412 \cdot 10^{-8} = 1.568033125 \cdot 10^{-2} \text{ eV}$$

$$|\Delta m_{32}^2| = 2.43 \cdot 10^{-3} \text{ eV}^2 ; \Delta m_{21}^2 = 7.54 \cdot 10^{-5} \text{ eV}^2$$

$$(3 \cdot \overline{m_\nu} / \cos^2 \theta_{c12})^2 = |\Delta m_{32}^2| = (3 \cdot 1.568033125 \cdot 10^{-2} \text{ eV} / \cos^2 \theta_{c12})^2$$

$$\Delta m_{21}^2 = (1 - \ln 2) \cdot (1.568033125 \cdot 10^{-2} \text{ eV})^2 ; (1.568033125 \cdot 10^{-2} \text{ eV} / \cos_W \text{ eff}(M_z)(\overline{m_s}))^2 \approx \Delta m_{21}^2 - |\Delta m_{32}^2|$$

The sum of the logarithms for bosons has to be (using equation (25)), then the sum equal to the fermions. Thus, we have (bosons with zero rest mass does not contribute):

$$\sum_B \ln(m_B/m_e) = \sum_{q=1}^6 \ln(m_q/m_e) + \sum_{l_1=e,\mu,\tau} \ln(m_{l_1}/m_e)$$

$$\sum_B \ln(m_B/m_e) = \ln(m_{w_+}/m_e) + \ln(m_{w_-}/m_e) + \ln(m_Z/m_e) + \ln(m_h/m_e) = 48.44081971$$

As you can observe, this sum is not equal to: $\sum_{q=1}^6 \ln(m_q/m_e) + \sum_{l_1=e,\mu,\tau} \ln(m_{l_1}/m_e) = 51.89841274$

This mismatch, may be possibly due to lack of the sum of bosons, another boson, which we believe is the dark matter candidate. With this hypothesis, it would have to the boson mass of this dark matter, would be:

$$\sum_{q=1}^6 \ln(m_q/m_e) + \sum_{l_1=e,\mu,\tau} \ln(m_{l_1}/m_e) - \sum_B \ln(m_B/m_e) = \ln(m_D/m_e) = 3.457593039 \Rightarrow$$

$$m_D = m_e \cdot \exp(3.457593039)$$

$$m_D = (0.510998928 \text{ MeV}) \cdot 31.7404861 = 16.21935437 \text{ MeV} \approx 16 \text{ KeV}$$

It is also possible that the mass of the dark matter obeys as a ratio of the value m_D , among the five solutions of energy-momentum, so it could be that: $m_{D2} = (0.510998928 \text{ MeV}) \cdot 31.7404861/5 = 3.243870887 \text{ KeV}$

The logarithm of: $\ln(m_D/m_e) = 3.45759303$; it has the following empirical properties: 1) $\ln(m_D/m_e) \approx (2\pi) \cdot \sqrt[4]{4\pi\alpha}$

2) Entropic uncertainty and the partition function of the 14 bosons, eight gluons, one photon, two W, one Z, one Higgs boson and, one boson to dark matter:

$$(\ln \pi + 1)/\sqrt{\ln(m_D/m_e)} - 1 = \exp(14)/[m_{Vh}/m_e]$$

$$3) 10/\ln(m_D/m_e) = \Omega_m = 0.3150550363$$

4.2 The zeros of the Riemann zeta function: derivation of elementary electric charge, and mass of the electron.

4.2.1 The relativistic invariance of the elementary electric charge.

As demonstrated, in this last section, the relativistic invariance of the elementary electric charge, is based on that solely depends on the canonical partition function of the imaginary parts of the zeros of the Riemann zeta function. And since the imaginary parts of the zeros of the zeta function, are pure and constant numbers; immediately relativistic invariance of electric charge is derived. Being the Planck mass other relativistic invariant, since there can be no higher mass to the Planck mass, this invariance is guaranteed.

4.2.2 Partition function (statistical mechanics).

Be considered, the coupling of the electromagnetic field to gravity, as represented by a bath of virtual particles, whose thermodynamic state is in equilibrium and there is no exchange of matter. Being a thermal bath whose temperature is constant, invariant, then its energy is infinite (in principle, ideally). Thermodynamic temperature canonical ensemble system can vary, but the number of particles is constant, invariant. That this theoretical approach, is exactly according to the values of the elementary electric charge, and mass of the electron, suggests that space-time-energy to last the unification scale, would behave like black holes, or even, as we shall see later, with wormholes with throat open. These wormholes, following a hyperbolic de Sitter space can explain the quantum entanglement, and the call action at a distance, or non-locality of quantum mechanics.

This partition function of the canonical ensemble, as is well known, is:

$$Z = \sum_s \exp -\beta E_s ; \text{ where the "inverse temperature", } \beta, \text{ is conventionally defined as } \beta \equiv \frac{1}{k_B T} ;$$

with k_B denoting Boltzmann's constant. Where E_s , is the the energy.

Will use for the dimensionless factor; βE_s , the change by the imaginary parts of the nontrivial zeros of the Riemann function $\zeta(s)$

This change is justified, for the simple reason that the vacuum is neutral with respect to the electric charges, ie the value of the electric charge of the vacuum is zero. These zeros can be expressed by the Riemann function, applying the Kaluza-Klein formulation for electric charges; dependent Planck mass, and as we will show by the partition function of canonical ensemble. Thus the zeros of the vacuum to the electric charge is expressed as: $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} m_{Pk}}{n^s \cdot \sqrt{\pm e^2/16\pi \cdot G_N}} = 0 ; s = \frac{1}{2} + it_k ; \zeta(s) = 0$ (26)

Equation (26), and therefore the behavior of the electric field strength with distance, depends on the value of s , because of (26) is obtained, using the conjugate of s : $\frac{(-1)^{n-1} m_{Pk}}{n^s \cdot n^{\bar{s}-\frac{1}{2}} \cdot (\pm e^2/16\pi \cdot G_N)} = \frac{m_{Pk}}{\pm \sqrt{(\pm e)^2/16\pi \cdot G_N}}$

Therefore, by using the canonical partition ensemble, making the substitution of the imaginary parts of the nontrivial zeros of the Riemann function, and taking into account the deviation of the electric charge, the equation is obtained relating the gravity with electric charge and the nontrivial zeros of the Riemann function. The calculation of the partition function has been performed with wolfram math program, version 9. For this calculation we used the first 2000 nontrivial zeros, value more than enough for the accuracy required. Although using the first six zeros, would also be sufficient. The code of this calculation is as follows:

$$\frac{1}{\sum_{n=1}^{2000} e^{-N[\Im(\rho_n), 15]}} = \frac{1}{\sum_{n=1}^{2000} \exp-(N[Im[ZetaZero[n]], 15])} = 1374617.45454188 ; \text{ Given that for values greater than 2000; } \exp-Im(\rho_n) \approx 0 ; \text{ You can write the equality as (by changing rho to s) as: } \left(\sum_{s_n}^{\infty} \exp-Im(s_n) \right)^{-1} \approx 1374617.45454188 \quad (27)$$

Finally, the equation that unifies the gravitational and electromagnetic field, by elementary electric charge, is: $m_{Pk} = \left(\sum_{s_n}^{\infty} \exp-Im(s_n) \right)^{-1} \cdot \sqrt{(\pm e \cdot \sigma(q))^2/G_N} = 2.176529059 \cdot 10^{-8} \text{ Kg} \quad (28) ;$ The value obtained for the Planck mass is in excellent agreement. The very slight difference, surely is that the constant of gravitation has a very high uncertainty about the other universal constants. Thus, making a speculative exercise, we can give a value for the gravitational constant:

$$G_N = \left[\left(\sum_{s_n}^{\infty} \exp-Im(s_n) \right)^{-2} \cdot (\pm e \cdot \sigma(q))^2 \right] / m_{Pk}^2 = 6.674841516 \cdot 10^{-11} \text{ N} \cdot \text{m}^2/\text{Kg}^2 \quad (29)$$

4.2.3 Derivation of the partition function of canonical ensemble by the special and unique properties of the Riemann zeta function, for complex values s , with real part $1/2$.

The function x^r , to a value of $1/2$, in the set of real numbers, is the only one that has the property, for which its derivative is $1/2$ the inverse of this function, that is: $d(x^{1/2}) = \frac{1}{2 \cdot x^{1/2}}$ This function has the same property, for complex values of the exponent, such that: $r = s = \frac{1}{2} + it ; dx^{\bar{s}}/\bar{s} = 1/x^s ; dx^s/s = 1/x^{\bar{s}} \quad (31)$

4.2.3.1 Commutation properties From equation (31), the following four identities are derived: **1)** $x^s dx^{\bar{s}} = \bar{s} ; 2)$ $x^{\bar{s}} dx^s = s ; 3)$ $\frac{dx^{\bar{s}}}{\bar{s}} - \frac{1}{x^s} = 0 ; 4)$ $\frac{dx^s}{s} - \frac{1}{x^{\bar{s}}} = 0$

Of the identities (1) and (2) are derived, by commutation of the conjugates of the exponents, s, \bar{s} ; the following identities:

$$\mathbf{1) } x^s dx^{\bar{s}} + x^{\bar{s}} dx^s = \mathbf{1} \quad \mathbf{2) } x^s dx^{\bar{s}} - x^{\bar{s}} dx^s = -2it \quad \mathbf{3) } x^{\bar{s}} dx^s - x^s dx^{\bar{s}} = 2it \quad (32)$$

From the identities (31) and (32) immediately derives the following corollary:

Corollary 4.1. *Only for complex values, s , with real part $1/2$, the three commutation properties, expressed in differential equations are satisfied.*

Conditions that must meet the equation derived from the commutators (32), and the identities (31)

1. Must include the invariance of the sum of the quantized electric charges. This sum is equivalent to the difference between the standard deviation of the electric charges with zero arithmetic mean, and the standard deviation, which has been developed previously, that is: $\sigma^2(q, \mu(q) = 0) = \sum_q q^2 = \frac{31}{9}$ (33) ; $\sigma^2(q, \mu(q) = 0) - \sigma_0^2(q) = 1 = \sum_q q$ (34)
2. The neutrality of the vacuum, in relation to the electric charges, or zero value of the electric charges of the vacuum, is the sum of infinite "oscillators", whose function is the Riemann zeta function applied to the ratio of Planck mass and the mass derived, from elementary electric charge and gravitational constant; fulfilling the equation obtained by Kaluza-Klein, to unify electromagnetism and gravity, adding a fifth dimension compactified on a circle.
$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1} m_{pk} \sqrt{G_N}}{\pm e \cdot n^s} = \left(\frac{m_{pk} \sqrt{G_N}}{\pm e} \right) \cdot \left(\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^s} \right) = 0$$
 (35)
3. The complex value s , can only be with real part $1/2$, since only for $s = 1/2 + it$, it is possible to derive from the commutators, both the invariance of the sum of the electric charges and the function of canonical ensemble, as will be demonstrated below.
4. The value of the energy is the lowest possible, with integer values.

With these four conditions, we have: 1) $E^s dx^{\bar{s}} + E^{\bar{s}} dx^s = 1 = \sigma^2(q, \mu(q) = 0) - \sigma_0^2(q) = \sum_q q$; $E =$ energy

$$2) E^s dE^{\bar{s}} = \bar{s}; E^{\bar{s}} dE^s = s; 3) [(E^s dE^{\bar{s}})E - E/2]/Ei = -t_n$$

$$4) \sum_{n=1}^{\infty} \frac{(-1)^{n-1} m_{pk} \sqrt{G_N}}{\pm e \cdot n^s} = 0 = \sum_{E=1}^{\infty} \frac{dE^{\bar{s}}}{\bar{s}} - \frac{1}{E^s} = \sum_{E=1}^{\infty} \frac{dE^s}{s} - \frac{1}{E^{\bar{s}}}$$

4.2.3.1.1 Derivation of partition function of canonical ensemble: ratio, elementary electric charge and kalulaza-Klein equation. The introduction of a fifth coordinate; allowed obtaining Theodor Kaluza, the quantization of electric charge; unifying Maxwell's equations (electromagnetism) and the RG Albert Einstein's equations. The derivation of a much higher mass, that the mass of electron, and other problems of the theory, led to dismiss it as a realistic theory, according to experimental physical data.

As we will demonstrate shortly, this theory lacked renormalization by canonical partition function of statistical mechanics (thermodynamics), derived from the imaginary parts of the zeros of the Riemann function $\zeta(s) = 0$; $s = \frac{1}{2} + it_n$

In the framework of the theory we are developing in this work, this fifth dimension corresponds to the three isomorphisms: five electric charges, five spines, five solutions of the energy equation momentum.

As a beginning assumption, assume that a thermodynamically large system is in thermal contact with the environment, with a temperature T , and both the volume of the system and the number of constituent particles are fixed. This kind of system is called a canonical ensemble. Let us label with $s = 1, 2, 3, \dots$ the exact states (microstates) that the system can occupy, and denote the total energy of the system when it is in microstate s as E_s . Generally, these microstates can be regarded as analogous to discrete quantum states of the system.

$$Z = \sum_s \exp\left(-\frac{E_s}{k_B T}\right)$$

The equation for the elementary electric charge, according to the initial theory of Kaluza (see bibliography) is: $q_n = m_n \cdot \sqrt{16\pi G_N}$

From equations (31), (32) and (34) with the conditions imposed, the following development is obtained, leading to accurate calculation of the elementary electric charge, as a partition function of the imaginary parts of the nontrivial zeros Riemann's function $\zeta(s)$. Partition function exactly equivalent to the canonical partition function of statistical mechanics (thermodynamics).

a) $E_0/c^2 = m_0$ b) $dm^s/s \cdot m^s = (1/m^s) \cdot (1/m^{\bar{s}}) = 1/m_0$, $d\bar{m}^{\bar{s}}/\bar{s} \cdot m^{\bar{s}} = (1/m^s) \cdot (1/m^{\bar{s}}) = 1/m_0$
c) $m_0(dm^s/m^s) = s$; $m_0(d\bar{m}^{\bar{s}}/m^{\bar{s}}) = \bar{s}$; $m_0 \equiv \sigma^2(q, \mu(q) = 0) - \sigma_0^2(q) = \sum_q q = 1$; d)
 $(dm^s/im^s) - 1/2i = -t_n$

f) We make the change (dm^s/im^s) , by (dm_1/m_1) ; g) $-(dm_1/m_1) + 1/2i = -t_n$; $(d\bar{m}^{\bar{s}}/im^{\bar{s}}) - 1/2i = (dm_2/m_2) - 1/2i = -t_n$

h) $-(dm_1/m_1) + 1/2i + (dm_2/m_2) - 1/2i = -t_n - t_n$; $-(dm_1/m_1) + (dm_2/m_2) = -t_n - t_n$
 $-(dm_1/m_1) + (dm_2/m_2) = -t_n - t_n \rightarrow (dm_2/m_2) = (d - m_3/m_3) = -(dm_3/m_3) = -(dm_1/m_1)$

Having two elementary electric charges with signs $-$, $+$, because: $\pm q_n = m_n \cdot \pm \sqrt{16\pi G_N}$; Then, the following two differential equations for the real value of the imaginary part of the nontrivial zeros Riemann's function is obtained:

i1) $-(dm_1/m_1) = -t_n$; i2) $-(dm_3/m_3) = -t_n$; j) $\int_{m_5}^{m_4} -(dm_1/m_1) = -t_n$; $m_4 < m_5$; $\ln(m_4/m_5) = -t_n$
 $\int_{m_5}^{m_4} -(dm_3/m_3) = -t_n$; $m_4 < m_5$; $\ln(m_4/m_5) = -t_n$; $(m_4/m_5) = \exp(-t_n)$

Finally, making the infinite sum nontrivial zeros Riemann's function (with the above approach; 7.2.2, with the first 2000 zeros), the two solutions (negative electric charge and positive), these are obtained, taking into account the standard deviation of the electric charge $\sigma(q) = 0.8073734276$:

$$(35) \sum_n \frac{m_{n-}}{m_0} = \sum_n \exp(-t_n) = \sqrt{(-e \cdot \sigma(q))^2 \cdot G_N / m_{Pk}}; m_{Pk} / \sqrt{(-e \cdot \sigma(q))^2 \cdot G_N} = \left[\sum_n \exp(-t_n) \right]^{-1}$$

$$(36) \sum_n \frac{m_{n+}}{m_0} = \sum_n \exp(-t_n) = \sqrt{(e \cdot \sigma(q))^2 \cdot G_N / m_{Pk}}; m_{Pk} / \sqrt{(e \cdot \sigma(q))^2 \cdot G_N} = \left[\sum_n \exp(-t_n) \right]^{-1}$$

Performing the calculation with a value of the gravitational constant, the conjectured by equation (29), it has the value of the electric charge, with excellent accuracy:

$$\left[\sum_n^{\infty} \exp(-t_n) \right]^{-1} \approx 1374617.45454188 = m_{Pk} / \sqrt{(e \cdot \sigma(q))^2 \cdot G_N} \rightarrow \dots$$

$$\dots \rightarrow \pm e = \sqrt{m_{Pk}^2 \cdot G_N} / \left(1374617.45454188 \cdot \sigma(q) \right)^2$$

$$\pm e = \sqrt{m_{Pk}^2 \cdot G_N} / \left(1374617.45454188 \cdot \sigma(q) \right)^2 = 1.602176565 \cdot 10^{-19} \text{ C} \quad (37)$$

However, if taken as a two solutions, also valid: $\begin{cases} -(dm_1/m_1) + 1/2i = -t_n \\ -(dm_3/m_3) - 1/2i = -t_n \end{cases}$ It has the following empirical equations:

$$1) (m_{Pk}/\sqrt{8\pi})/\sqrt{(e \cdot \sigma(q))^2 \cdot 16 \cdot G_N} = \left(\pi \cdot \left[3 + \left(\frac{2}{\varphi^3} \right)^2 \right] + \frac{2 \cdot \sin^2(2\pi/l_\gamma) - 1}{(m_\tau/m_e + m_\mu/m_e + m_e/m_e)} \right) \cdot 2 \cdot \left[\sum_n^{\infty} \exp(-t_n) \right]^{-1} \cdot [\exp(1/2i) + \exp(-1/2i)]$$

$\left(\frac{2}{\varphi^3} \right)^2 = \sin^2 \theta_W = 1 - \frac{M_W^2}{M_Z^2}$; Probability particle in a box potential: $l_\gamma = \sqrt{\alpha^{-1}/4\pi}$; $P(2, l_\gamma) = 2 \cdot \sin^2(2\pi/l_\gamma)/l_\gamma$

$$2) (m_{Pk}/\sqrt{8\pi})/\sqrt{(e \cdot \sigma(q))^2 \cdot 16 \cdot G_N} = \left\{ \left(\prod_q \right)^{-1} + \left[\frac{1}{(m_\tau/m_e + m_\mu/m_e + m_e/m_e)} \right] + \exp-(11 + \frac{137}{112}) \right\} \cdot 2 \cdot \left[\sum_n^{\infty} \exp(-t_n) \right]^{-1} \cdot [\exp(1/2i) + \exp(-1/2i)]$$

$\left(\prod_q \right)^{-1} = \left(\frac{-1}{3} \cdot \frac{1}{3} \cdot \frac{2}{3} \cdot \frac{4}{3} \cdot -1 \right)^{-1} = \frac{240 - \text{Dim}(E6)}{4^2}$; $[\exp(1/2i) + \exp(-1/2i)] = 1.755165123780746 \simeq 1 + \frac{2\pi}{\ln 2 \cdot 12}$ Bekenstein bound, in informational terms: $I \leq (2\pi k_B \cdot R \cdot E) / \hbar c \cdot \ln 2$

A similar empirical identity:

$$2 \cdot \left[\sum_n^{\infty} \exp(-t_n) \right]^{-1} \cdot [\exp(1/2) + \exp(-1/2)] \cdot \left(\sqrt{248 + \sin \theta_W \cdot \sigma(q)} / 2 \right) = \dots$$

$$\dots = (m_{Pk}/\sqrt{8\pi})/\sqrt{(e \cdot \sigma(q))^2 \cdot 16 \cdot G_N}; \sin \theta_W = \sqrt{1 - \frac{M_W^2}{M_Z^2}}$$

4.2.3.2 Physical interpretation of the non-trivial zeros Riemann's function. Being the vacuum oscillations of virtual particle - antiparticle pairs, and since they do not obey the equation of energy - momentum, quantum mechanics allows these particles may have a higher speed of light. Because of this, the equation of energy - momentum can be factored in energies with imaginary components. Applying strictly, literally, the non-trivial zeros Riemann's function, you can get a zero energy if the speed is infinite, and thus the energy of the tachyon states is zero. But this implies that special relativity has to be generalized so that, on one hand, for real particles, the speed limit is the speed of light, while for particles in tachyonic state, its speed is infinite. Simply put: you can get this generalization of SR, if the space-time, on the last scale of his quantization; consists of wormholes

, which are the deformation of a torus, by breaking through his larger circle. This deformation, the torus becomes a hyperboloid of one sheet, ie: in a wormhole with the throat open.

In this way, a space of De Sitter according to the RG is obtained. These wormholes would be the stabilization of the attractive forces of electric charges, and repulsive gravitational forces. It is not merely a theory, since only under this interpretation is possible, as we shall see in the next section, obtaining the mass of the electron, as a function of the elementary electric charge, the gravitational constant and the Planck mass. Thus, gravity must return a repulsive force, at scales of the Planck length.

For an observer at rest; infinite speed is equivalent to a zero velocity, since a point moving at infinite speed is completely equivalent to a fixed point.

For this generalization of the SR, it is necessary to establish two equations, infinitesimal change or infinitesimal measure distances, in a hyperboloid. This is expressed by the measurement of the derivative of arcsin(x), the circle which is within the boundary of the light cone, and that belongs to the great circle of the open throat of the wormhole. This equation determines the SR, for a speed limit c. For exceeding the speed of light, the distance belongs to the derivative of arcosh(x), of the outer surface of the wormhole, outside the cone of light. Thus we have:

$$\frac{d}{dx} \operatorname{arccos} h(x) = \frac{1}{\sqrt{x^2-1}} ; x^2 = \frac{v^2}{c^2} ; v > c ; \frac{d}{dx} \operatorname{arcsin}(x) = \frac{1}{\sqrt{1-x^2}} ; x^2 = \frac{v^2}{c^2} ; v < c \quad (38)$$

The equivalence of the zeros of the Riemann function, imply that time is canceled (becomes zero), the energy is zero, and only pure space left. The non-locality of quantum mechanics when the phenomena of entanglement manifest, would be the result of the connection of particles interlaced by these wormholes. An instantaneous change (infinite speed) of the topology of wormholes, would be the consequence of the change of an observable of one of the two ends of the particles interconnected by wormholes. Likewise, this same non-locality of these wormholes, the last fabric of space-time, would allow to resolve the paradox of information loss from black holes. Thus the singularity of black holes would not exist; since spacetime would be quantized with a dynamic deformed torus-wormhole type. Would exist a minimum length.

$$v = \infty ; t' = \frac{t_0}{\sqrt{(\frac{v}{c})^2-1}} = 0 = t_0 \cdot \zeta(s) ; E' = \frac{E_0}{\sqrt{(\frac{v}{c})^2-1}} = 0 = E \cdot \zeta(s) ; l' = l_0 \cdot \sqrt{(\frac{v}{c})^2-1} = \infty = l_0/\zeta(s)$$

An infinite length, which is just a result of walking on an infinite speed zero time, a finite distance, infinitely. For example by rotating or also by the existence of ideal hyperbolic triangles, whose perimeter is infinite and whose area is π

There are results that seem to confirm the existence of these ideal hyperbolic triangles. A triangle of this type would be the equivalent of a minimum Feymann diagram (three particles). One of these results with an accuracy of the order of QED, is electron magnetic moment anomaly. An ideal triangle, its inscribed circle, form a triangle with sides: $l_{\Delta} = 4 \cdot \ln \varphi ; \varphi = (1 + \sqrt{5})/2$

$$a_e = 1.15965218076 \cdot 10^{-3} ; a_e = [2\pi\alpha^{-1} + 3 \cdot 4 \cdot \ln \varphi - (\frac{1}{\alpha \cdot 112} - 1)^{-1} + \frac{1}{248 + \ln(\ln(m_{Pk}/m_e))}]^{-1} = 1.15965218075 \cdot 10^{-3}$$

$$\ln(\ln(m_{Pk}/m_e)) \approx \frac{384}{\pi^4} = \rho^{-1}(l8d) \quad (\text{density lattice packings of hyperspheres in eight dimensions})$$

$$\alpha^{-1} = 137.035999174$$

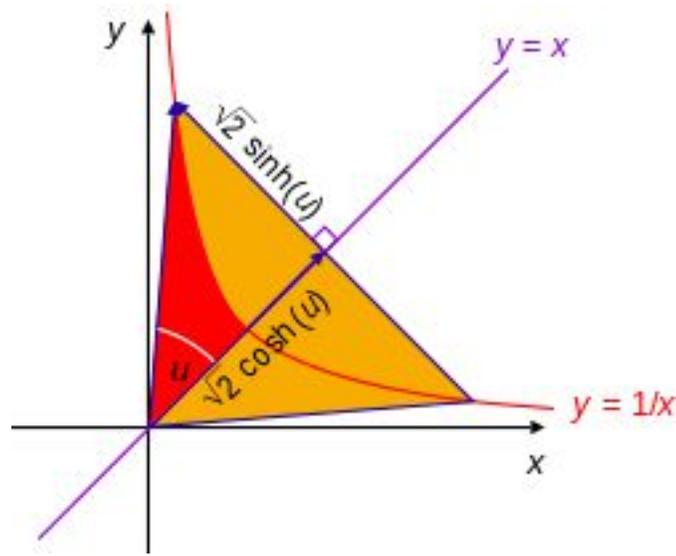
There are also results that confirm this hyperbolic space. It is noteworthy to highlight that in a hyperbolic space of any dimension, a holographic principle is fulfilled, that is: Any hyperbolic triangle, the three points are in the same plane, ie form a surface.

$$l_\gamma = \sqrt{\alpha^{-1}/4\pi} ; 6 \cdot l_\gamma^2 + \ln^2 \varphi - \ln \left\{ \left[\sum_n^\infty \exp(-t_n) \right]^{-1} \right\} + \frac{1}{(240 \cdot \alpha - 1) \cdot 112 \cdot 128} = \ln(m_{Pk}/m_e) = 51.5278415721$$

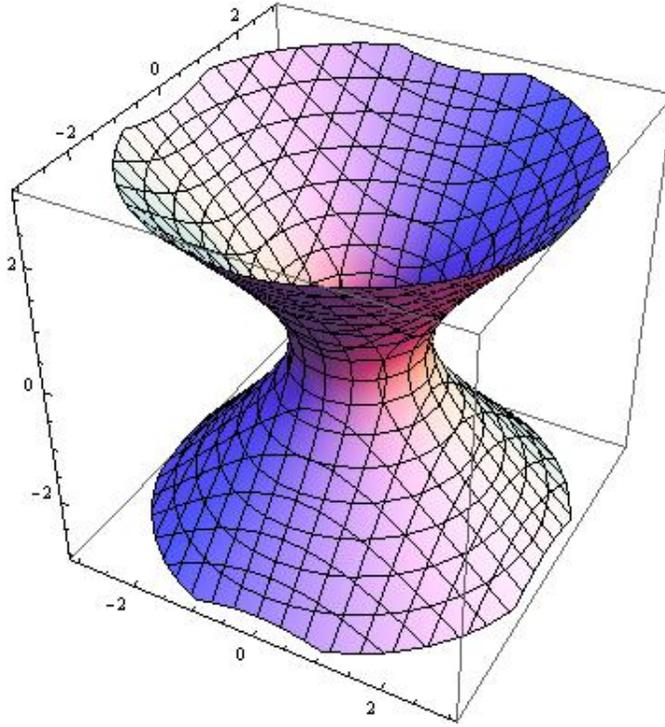
$R_8 \rightarrow 112$ roots with integer ; $(\pm 1, \pm 1, 0, 0, 0, 0, 0, 0)$; 128 roots with half - integer

$$\left(\pm \frac{1}{2}, \pm \frac{1}{2} \right)$$

Similarly, the logarithmic scaling law, would be areas of sectors of hyperbolic triangles. When in standard position, a hyperbolic sector determines a hyperbolic triangle. And its inverse; coordinates of a hyperbolic space, ie: $S_{hs\Delta} = \ln(m_1/m_2)$; $x' = \sinh(S_{hs\Delta}) + \cosh(S_{hs\Delta})$



Hyperbolic sector



wormhole

4.2.4 The mass of the electron.

Being the electron, the mass of the vacuum lower, with electric charge and completely stable (infinite lifetime), and on the other hand, the Planck mass is the maximum possible, if indeed, the non-trivial zeros of the Riemann function represent stabilizing a deformed torus; become a wormhole, with gravitational and electromagnetic, fully matched forces, then you can set requirements to be the equation, which equals each of the zeros Riemann's function.

Conditions These conditions would be: a) the sum of the electromagnetic and gravitational part must be zero. b) In the equation the term breaking torus must appear. c) The curvature of space-time, according to general relativity, it must be possible derive directly.

d) The equation must contain the partition function zeros Riemann's.

The equation $2\pi^2 \cdot (\pm e) \cdot \left[\sum_n^{\infty} \exp(-t_n) \right] - 2 \cdot \sqrt{m_{Pk} \cdot m_e \cdot G_N} = 0 ; 2\pi^2 = \text{volume torus} \quad (39)$

$$\sqrt{m_{Pk} \cdot m_e} = m_o ; \int_0^{2\pi} m \cdot dm = 2\pi^2 ; e \cdot \exp(2\pi i n) = 1 \cdot e ; e \cdot \exp(2\pi i n/2) = -1 \cdot e ; n \in \{N\}$$

$$m_e = \pi^4 (\pm e)^2 \left[\sum_n^{\infty} \exp(-t_n) \right]^2 / m_{Pk} \cdot G_N = 9.10938291 \cdot 10^{-31} \text{Kg} ; G_N = 6.674841516 \cdot 10^{-11} \text{N} \cdot \text{m}^2 / \text{Kg}^2 ; m_{Pk} = \sqrt{\frac{\hbar c}{6.674841516 \cdot 10^{-11}}}$$

In the above equation the volume of a torus appears in three dimensions. Can also derive the angle of curvature of general relativity, because:

$$4\pi^4(\pm e)^2 \cdot \left[\sum_n^\infty \exp(-t_n) \right]^2 / m_e \cdot c^2 \cdot l_{Pk} = \frac{4 \cdot m_{Pk} \cdot G_N}{c^2 \cdot l_{Pk}} = \theta = 4 \quad (40)$$

The gravitino mass. Equation (39) requires that the gravitational force, at scales of the Planck length, is repulsive. The only possible candidate is the gravitino, which occurs naturally in both theories of supersymmetry, supergravity and string theory. Therefore, from equation (39) can be derived for the gravitino mass, taking into account the spin 3/2, the mass:

$$m_{3/2} = \sqrt{m_{Pk} \cdot m_e \cdot (s+1)_{s=3/2}} \quad (41)$$

Equation (41) is more than pure speculation; since the gravitino field is conventionally written as four-vector index. With the mass of the gravitino, according to equation (41) a mass ratio of four grade (four-vector index) is obtained. In the numerator, the four potency of gravitino mass. And in the denominator the product of the Planck mass, electron mass and equivalent mass Higgs vacuum. The result is the mass of unification, in GUT theory.

$$(m_{3/2})^4 \cdot 4^3 / (m_{Pk} \cdot m_e \cdot m_{Vh}) = m_{GUT} = 4.065067121 \cdot 10^{-11} K_g$$

$$\ln\left(\frac{m_{GUT}}{m_Z}\right) \approx \frac{10\pi}{28} \cdot [\alpha^{-1}(U(1)) - \alpha^{-1}(SU(2))]; \quad \alpha^{-1}(U(1)) \approx 59.2; \quad \alpha^{-1}(SU(2)) \approx 29.6$$

Observe, that in equation (40), the volume factor appears eight dimensions. Also, in this equation the entropy of a black hole is obtained by multiplying by π . This last operation; implies a volume factor in ten dimensions.

$$\left(4\pi^4(\pm e)^2 \cdot \left[\sum_n^\infty \exp(-t_n) \right]^2 / m_e \cdot c^2 \cdot l_{Pk} \right) \cdot \pi = 4\pi; \quad 4\pi^4/4 \cdot \dim[SU(5)] = V_{8d} = \frac{\pi^4}{24}; \quad \dim(l8) = 240; \quad \frac{4\pi^4 \cdot \pi}{2 \cdot 240} = V_{10d} = \frac{\pi^5}{120}$$

The curvature angle $\frac{4 \cdot m_{Pk} \cdot G_N}{c^2 \cdot l_{Pk}} = \theta = 4 \text{ rad}$; has the following outstanding properties:

1. $\tan(4)/5 = \sin^2 \hat{\theta}_W(M_Z)(\overline{m_s}) = 0.2315642564 \approx \ln^2 \varphi = 0.2315648207$
2. $\sin_W^{eff}(M_Z)(\overline{m_s}) / \cos(\theta_{c23}/2) = \tan(4) - \sqrt{4-\pi} = 0.2313185316$; $\theta_{c23} = 2.38^\circ \pm 0.06^\circ$ (Cabibbo-Kobayashi-Maskawa matrix)
3. $7 \cdot \tan 4 - 8 = \sin^2(2\theta_{13}) = 0.104748976 \rightarrow \theta_{13} \approx 9.44188^\circ$; (Neutrino oscillation, mixing angles)
4. $\cos(2\pi/\varphi^2) / \sin 4 = 0.9743214142 \approx \cos \theta_{c12}$
5. $\sin^2 4 / -\cos 4 \approx \cos \theta_W^{eff}$; $\sin^2 4 / -\cos 4 = 0.8762420356$
6. $1 - (\sin^2 4 / -\cos 4)^2 \approx \exp(\zeta(1/2)) = \exp(-1.4603545088095) = 0.2321539595$

4.2.5 The value of the energy of the vacuum.

The value of the vacuum energy, with positive density, is simply the logarithm of the partition function obtained from the non-trivial zeros of Riemann's function, multiplied by five possible energy solutions, by factoring. Thus we have:

$$\ln(m_{Pk}/m_v) = 5 \cdot \ln\left(\left[\sum_n^{\infty} \exp(-t_n)\right]^{-1}\right); \ln(m_{Pk}/m_v) = 5 \cdot 14.13368604$$

$$\frac{m_{Pk}}{m_v} = \exp(5 \cdot 14.13368604) \quad (42)$$

$$\frac{m_{Pk}c^2}{\exp(5 \cdot 14.13368604) \cdot (\pm e)} = 2.487423271 \cdot 10^{-3} eV$$

4.3 The expansion of the Universe.

Equation (39) establishes a zero, equivalent to one of the nontrivial zeros Riemann's function. Since there are infinitely many zeros, necessarily, there is a summation of infinite zeros, given by equation (39). Our interpretation of this sum is to be continually created, and infinitely quantum units of space-time, ie: an expansion of space comes.

The Hubble constant; would really double the frequency of the energy of the vacuum. As we have shown in one of our works ("Quantum Information and Cosmology: the Connections"), in the phase of inflation of the universe, the equation that determines the inflation factor is:

$$H_0 = t_{Pk}^{-1} \cdot \sqrt{\alpha^{-1}/4\pi} \cdot \exp(\exp(\pi^2/2)) = (4.3378224104 \cdot 10^{17})^{-1} s^{-1}$$

$$H_0 \approx t_{Pk}^{-1} \cdot \exp(5 \cdot 2 \cdot \ln([\sum_n^{\infty} \exp(-t_n)]))/3 = (1/4.329164509 \cdot 10^{17} s)$$

4.4 The reality of the existence of eight extra dimensions.

The reality of the existence of eight extra dimensions, apart from the three spatial dimensions not compacted, is demonstrated by the existence of an equation that is a function of all the essential constants of quantum physics, and determines a radius, by a space eight dimensions. Two radii are obtained corresponding to the value of the Higgs vacuum and the Higgs boson mass, the latter just two years after its experimental confirmation.

$$\left(\frac{(k_e \cdot \pm e)^2 \cdot (G_N) \cdot (\hbar c) \cdot r_e^5}{m_e \cdot c^6}\right)^{1/8} = l_{H_0}; k_e = 8.9875517873681764 \times 10^9 N \cdot m^2 / C^2 (m/F); r_e = \frac{\hbar}{2\pi \cdot m_e c}$$

$$\frac{\hbar}{2\pi \cdot l_{H_0} \cdot c \cdot \sqrt[4]{2}} \Big/ \left(1 + \frac{1}{(12\pi)^2}\right) = m_{Vh} \text{ (equivalent mass vacuum Higgs)}$$

$$\left(\frac{(k_e \cdot \pm e)^2 \cdot (G_N) \cdot (\hbar c) \cdot r_e^5 \cdot 24}{m_e \cdot c^6 \cdot \pi^4}\right)^{1/8} = l_{H_1}; \frac{\hbar}{2\pi \cdot l_{H_1} \cdot c} \cdot \left(\frac{1}{2} \sqrt{\frac{\pi^2}{3} - 2}\right) = m_h \text{ (Higgs boson mass)}$$

$$\text{Particle in a box (lattice): } \Delta x \Delta p / (\hbar/2) = \left(\frac{1}{2} \sqrt{\frac{n^2 \cdot \pi^2}{3} - 2}\right); \min n = 1$$

Conclusion Humbly, we think that this work demonstrates the extraordinary role of the non-trivial zeros of the Riemann zeta function in quantum mechanics.

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