The description of the physical field with spin 1/3.

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Abstract

viXra.org 1402.0167 gives general description of raising vector space W in the power M/L. The result is the new vector space V. In this paper we take W - the 8 – dimensional generalization of our 4 – dimensional vector space. Then we raise W in the power 1/3. The result is the 2 – dimensional vector space V. The metric and algebraic tensors for V are the same as in viXra.org 1402.0176.

After that we take some vector from V and use it for construction of Lagrangian. And for simplicity we restrict us by only first 4 dimensions of W. Then, from the principle of minimal action, we get the equations for our vector. And we derive that vector from these equations.

Then we define the tensor and vector of energy – momentum for this Lagrangian. And also we find the density of spin tensor and (with the help of the algebraic tensor) the density of spin vector. The numerical cofactor in them is 1/3. So we consider that spin of this vector is 1/3. It coincides with the power of W for V (vector space we took our vector from).

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1) The generalization of our 4-dimensional space W_1 to the 8-dimensional space W.

Let us take $V = W^{1/3}$ as the vector space $V_1 = W_1^{1/2}$. Then $\vec{n}_\alpha$ - basis of V, and $\vec{e}_\mu$ - basis of W.

$\alpha = 1, 2 \qquad \mu = 1,2,3,4,5,6,7,8$

So, as $W = V \otimes V \otimes V$, then we have :

$$\vec{e}_\mu = e_\mu{}^{\alpha\beta\gamma} \cdot \vec{n}_\alpha \otimes \vec{n}_\beta \otimes \vec{n}_\gamma \qquad (1.0)$$

and choose the cobasics (definition of cobasic see in [1]) so :

$$\begin{aligned}
\vec{e}_1 &= 1 \cdot \vec{n}_1 \otimes \vec{n}_1 \otimes \vec{n}_1 \\
\vec{e}_2 &= i_1 \cdot \vec{n}_2 \otimes \vec{n}_1 \otimes \vec{n}_1 \\
\vec{e}_3 &= i_2 \cdot \vec{n}_1 \otimes \vec{n}_2 \otimes \vec{n}_1 \\
\vec{e}_4 &= i_1 \cdot i_2 \cdot \vec{n}_2 \otimes \vec{n}_2 \otimes \vec{n}_1 \qquad (1.1) \\
\vec{e}_5 &= i_3 \cdot \vec{n}_1 \otimes \vec{n}_1 \otimes \vec{n}_2 \\
\vec{e}_6 &= i_1 \cdot i_3 \cdot \vec{n}_2 \otimes \vec{n}_1 \otimes \vec{n}_2 \\
\vec{e}_7 &= i_2 \cdot i_3 \cdot \vec{n}_1 \otimes \vec{n}_2 \otimes \vec{n}_2 \\
\vec{e}_8 &= i_1 \cdot i_2 \cdot i_3 \cdot \vec{n}_2 \otimes \vec{n}_2 \otimes \vec{n}_2
\end{aligned}$$

Here the hypercomplex numbers i_1, i_2, i_3 behave so :

$i_\alpha \cdot i_\alpha = -1 \quad (1.2) \qquad i_\alpha \cdot i_\beta = -i_\beta \cdot i_\alpha \ (\alpha \neq \beta) \quad (1.3) \qquad i \cdot i = -1 \ (1.4) \qquad i \cdot i_\alpha = i_\alpha \cdot i \ (1.5)$

We have for V also as for V_1 the same metric and algebraic tensors :

$$(\vec{n}_\alpha,\vec{n}_\beta)=q_{\alpha\beta} \quad (1.6) \qquad [\vec{n}_\alpha\times\vec{n}_\beta]=\vec{n}_\gamma\cdot f^{\gamma}{}_{\alpha\beta} \quad (1.7)$$

$$(\vec{n}_1,\vec{n}_1)=1 \quad (1.8) \qquad (\vec{n}_1,\vec{n}_2)=0 \quad (1.9)$$
$$(\vec{n}_2,\vec{n}_1)=0 \quad (1.10) \qquad (\vec{n}_2,\vec{n}_2)=1 \quad (1.11)$$

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$$[\vec{n}_1\times\vec{n}_1]=\vec{n}_1 \quad (1.12) \qquad [\vec{n}_1\times\vec{n}_2]=\vec{n}_2 \quad (1.13)$$
$$[\vec{n}_2\times\vec{n}_1]=\vec{n}_2 \quad (1.14) \qquad [\vec{n}_2\times\vec{n}_2]=\vec{n}_1 \quad (1.15)$$

For metric tensors in W and in V we have :

$$g_{\mu\nu}=e_\mu{}^{\alpha\beta\gamma}\cdot e_\nu{}^{\delta ns}\cdot(\vec{n}_\alpha,\vec{n}_\delta)\cdot(\vec{n}_\beta,\vec{n}_n)\cdot(\vec{n}_\gamma,\vec{n}_s)=e_\mu{}^{\alpha\beta\gamma}\cdot e_\nu{}^{\delta ns}\cdot q_{\alpha\delta}\cdot q_{\beta n}\cdot q_{\gamma s} \quad (1.16)$$

If $\beta=\gamma=\alpha$ and $n=s=\delta$ then we derive :

$$q_{\alpha\delta}=(e^{\nu}{}_{\delta\delta\delta}\cdot e^{\mu}{}_{\alpha\alpha\alpha}\cdot g_{\mu\nu})^{\frac{1}{3}} \quad (1.17)$$

And for algebraic tensor in W - $F^{\sigma}{}_{\mu\nu}$ - we have :

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$$[\vec{e}_\mu\times\vec{e}_\nu]=\vec{e}_\sigma\cdot F^{\sigma}{}_{\mu\nu} \quad (1.18)$$

$$[(e_\mu{}^{\alpha\beta\gamma}\cdot\vec{n}_\alpha\otimes\vec{n}_\beta\otimes\vec{n}_\gamma)\times(e_\nu{}^{\delta ns}\cdot\vec{n}_\delta\otimes\vec{n}_n\otimes\vec{n}_s)]=e_\sigma{}^{pql}\cdot\vec{n}_p\otimes\vec{n}_q\otimes\vec{n}_l\cdot F^{\sigma}{}_{\mu\nu} \quad (1.19)$$
$$e_\mu{}^{\alpha\beta\gamma}\cdot e_\nu{}^{\delta ns}\cdot[\vec{n}_\alpha\times\vec{n}_\delta]\otimes[\vec{n}_\beta\times\vec{n}_n]\otimes[\vec{n}_\gamma\times\vec{n}_s]=$$
$$=e_\mu{}^{\alpha\beta\gamma}\cdot e_\nu{}^{\delta ns}\cdot(\vec{n}_p\cdot f^{p}{}_{\alpha\delta})\otimes(\vec{n}_q\cdot f^{q}{}_{\beta n})\otimes(\vec{n}_l\cdot f^{l}{}_{\gamma s})=$$
$$=e_\mu{}^{\alpha\beta\gamma}\cdot e_\nu{}^{\delta ns}\cdot f^{p}{}_{\alpha\delta}\cdot f^{q}{}_{\beta n}\cdot f^{l}{}_{\gamma s}\cdot\vec{n}_p\otimes\vec{n}_q\otimes\vec{n}_l \quad (1.20)$$

From (1.19) and (1.20) we have :

$$e_\mu{}^{\alpha\beta\gamma}\cdot e_\nu{}^{\delta ns}\cdot f^{p}{}_{\alpha\delta}\cdot f^{q}{}_{\beta n}\cdot f^{l}{}_{\gamma s}=e_\sigma{}^{pql}\cdot F^{\sigma}{}_{\mu\nu} \quad (1.21)$$
$$F^{\sigma}{}_{\mu\nu}=e^{\sigma}{}_{pql}\cdot e_\mu{}^{\alpha\beta\gamma}\cdot e_\nu{}^{\delta ns}\cdot f^{p}{}_{\alpha\delta}\cdot f^{q}{}_{\beta n}\cdot f^{l}{}_{\gamma s} \quad (1.22)$$

If we take $\sigma,\mu,\nu=1,2,3,4$, then we get the same algebraic tensor as for W_1. If we take in (1.16) $\mu,\nu=1,2,3,4$, then we get the same metric tensor as for W_1. So we see, that subspace W_2 ($\vec{e}_\mu$, $\mu=1,2,3,4$) of the space W ($\vec{e}_\mu$, $\mu=1,2,3,4,5,6,7,8$) coincide with the space W_1 in all significant features. Then we can say, that W is 8-dimensional generalization of W_1.

2) Lagrangian for the field with spin 1/3.

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Let us take the field $\varphi_\alpha(x_\mu)$ from V ($V=W^{\frac{1}{3}}$). And restrict μ so : $\mu=1, 2, 3, 4$. And now we construct the Lagrangian :

$$L=i\cdot e_\mu{}^{\alpha\beta\gamma}\cdot\varphi_\alpha\cdot\varphi_\beta\cdot\partial^\mu\varphi_\gamma \quad (2.1)$$

And for simplicity we will consider that

$$\frac{\partial\varphi_\gamma}{\partial x_\mu}=\partial^\mu\varphi_\gamma=0 \quad (2.2) \qquad \mu=5, 6, 7, 8 \quad (2.3)$$

3) Equations for φ_m.

The principle of minimal action gives us the equations for φ_m:

$$\frac{\partial L}{\partial \varphi_m} = 0 \quad (3.1) \qquad m = 1,\ 2 \quad (3.2)$$

$$i \cdot [e_\mu{}^{m\beta\gamma} \cdot \varphi_\beta \cdot \partial^\mu \varphi_\gamma + e_\mu{}^{\alpha m\gamma} \cdot \varphi_\alpha \cdot \partial^\mu \varphi_\gamma - e_\mu{}^{\alpha\beta m} \cdot \partial^\mu (\varphi_\alpha \cdot \varphi_\beta)] = 0 \quad (3.3))$$

Let us try $\varphi_\delta(x_\nu) = f_\delta \cdot e^{-i \cdot k^\nu \cdot x_\nu}$ (3.4)

$$f_\delta = a_\delta + i_1 \cdot b_\delta + i_2 \cdot c_\delta + i_1 \cdot i_2 \cdot d_\delta \quad (3.5)$$

m=1

$$e_\mu{}^{1\beta\gamma} \cdot \varphi_\beta \cdot \partial^\mu \varphi_\gamma + e_\mu{}^{\alpha 1\gamma} \cdot \varphi_\alpha \cdot \partial^\mu \varphi_\gamma - e_\mu{}^{\alpha\beta 1} \cdot \partial^\mu (\varphi_\alpha \cdot \varphi_\beta) = 0 \quad (3.6)$$

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$$e_1{}^{111} \cdot f_1 \cdot k^1 \cdot f_1 + e_3{}^{121} \cdot f_1 \cdot k^3 \cdot f_1 - 2 \cdot e_1{}^{111} \cdot k^1 \cdot f_1 \cdot f_1 - 2 \cdot e_2{}^{211} \cdot k^2 \cdot f_2 \cdot f_1 -$$
$$- 2 \cdot e_3{}^{121} \cdot k^3 \cdot f_1 \cdot f_2 - 2 \cdot e_4{}^{221} \cdot k^4 \cdot f_2 \cdot f_2 = 0 \quad (3.7)$$

$$k^3 \cdot i_2 \cdot f_1 \cdot f_1 - k^1 \cdot f_1 \cdot f_1 - 2 \cdot k^2 \cdot i_1 \cdot f_2 \cdot f_1 - 2 \cdot k^3 \cdot i_2 \cdot f_1 \cdot f_2 - 2 \cdot i_1 \cdot i_2 \cdot k^4 \cdot f_2 \cdot f_2 = 0 \quad (3.8)$$

m=2

$$e_\mu{}^{2\beta\gamma} \cdot \varphi_\beta \cdot \partial^\mu \varphi_\gamma + e_\mu{}^{\alpha 2\gamma} \cdot \varphi_\alpha \cdot \partial^\mu \varphi_\gamma - e_\mu{}^{\alpha\beta 2} \cdot \partial^\mu (\varphi_\alpha \cdot \varphi_\beta) = 0 \quad (3.9)$$

From (2.2), (2.3), (3.4), (3.9) we derive :

$$i_2 \cdot (k^2 + k^3) \cdot f_1 \cdot f_1 + 2 \cdot i_1 \cdot i_2 \cdot k^4 \cdot f_2 \cdot f_1 = 0 \quad (3.10)$$

As quaternion algebra is the algebra with division, then

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$$i_2 \cdot (k^2 + k^3) \cdot f_1 = -2 \cdot i_1 \cdot i_2 \cdot k^4 \cdot f_2 \quad (3.11)$$

If we now multiply (3.11) on i_2 from the left, then we get

$$(k^2 + k^3) \cdot f_1 = 2 \cdot i_1 \cdot k^4 \cdot f_2 \quad (3.12)$$

As k^α $(\alpha = 1, 2, 3, 4)$ are the real numbers, then we can write :

$$f_1 = 2 \cdot i_1 \cdot f_2 \cdot \frac{k^4}{(k^2 + k^3)} \quad (3.13)$$

Let us designate $\alpha = \dfrac{k^4}{(k^2 + k^3)}$ (3.14)

Then $f_1 = 2 \cdot \alpha \cdot i_1 \cdot f_2$ (3.15)

From (3.8) and (3.15) we can get :

$$(k^3 \cdot \alpha^2 \cdot i_2 - k^1 \cdot \alpha^2 - k^2 \cdot \alpha) \cdot 4 \cdot i_1 \cdot f_2 \cdot i_1 \cdot f_2 = (-2 \cdot \alpha \cdot k^3 + k^4) \cdot 2 \cdot i_1 \cdot i_2 \cdot f_2 \cdot f_2 \quad (3.16)$$

As our algebra is the one with division, then :

$$(k^3 \cdot \alpha^2 \cdot i_2 - k^1 \cdot \alpha^2 - k^2 \cdot \alpha) \cdot 4 \cdot i_1 \cdot f_2 \cdot i_1 = (-2 \cdot \alpha \cdot k^3 + k^4) \cdot 2 \cdot i_1 \cdot i_2 \cdot f_2 \quad (3.17)$$

Let us designate : $\beta = k^1 \cdot \alpha^2 + k^2 \cdot \alpha$ (3.18)

$\gamma = k^4 - 2 \cdot \alpha \cdot k^3$ (3.19)

From (3.5), $\delta = 2$, (3.17), (3.18), (3.19) we result those 4 equations :

$1: \quad -2\cdot\alpha^2\cdot k^3\cdot c_2 + 2\cdot\beta\cdot a_2 = -\gamma\cdot d_2$ (3.20)

$i_1: \ 2\cdot\alpha^2\cdot k^3\cdot d_2 + 2\cdot\beta\cdot b_2 = -\gamma\cdot c_2$ (3.21)

$i_2: \ -2\cdot\alpha^2\cdot k^3\cdot a_2 - 2\cdot\beta\cdot c_2 = \gamma\cdot b_2$ (3.22)

$i_1\cdot i_2: \ 2\cdot\alpha^2\cdot k^3\cdot b_2 - 2\cdot\beta\cdot d_2 = \gamma\cdot a_2$ (3.23) To the Content

From (3.20) and (3.21) we have :

$$c_2 = \frac{(4\cdot\beta\cdot\alpha^2\cdot k^3\cdot a_2 - 2\cdot\gamma\cdot\beta\cdot b_2)}{(\gamma^2 + (2\cdot\alpha^2\cdot k^3)^2)} \qquad (3.24)$$

From (3.22) and (3.24) we get : $A\cdot a_2 = B\cdot b_2$ (3.25)

Here $A = -2\cdot\alpha^2\cdot k^3 - \dfrac{8\cdot\beta^2\cdot\alpha^2\cdot k^3}{[\gamma^2 + (2\cdot\alpha^2\cdot k^3)^2]}$ (3.26)

$$B = \gamma - \frac{4\cdot\gamma\cdot\beta^2}{[\gamma^2 + (2\cdot\alpha^2\cdot k^3)^2]} \qquad (3.27)$$

From (3.23), (3.20), (3.24) we have :

$$C\cdot a_2 = D\cdot b_2 \qquad (3.28)$$

Here $C = 4\cdot\dfrac{\beta^2}{\gamma} - \dfrac{16\cdot\beta^2\cdot(\alpha^2\cdot k^3)^2}{[\gamma^2 + (2\cdot\alpha^2\cdot k^3)^2]} - \gamma$ (3.29)

$$D = -2\cdot\alpha^2\cdot k^3 + \frac{8\cdot\beta^2\cdot\alpha^2\cdot k^3}{[\gamma^2 + (2\cdot\alpha^2\cdot k^3)^2]} \qquad (3.30)$$

From (3.25), (3.28) we get one equation for k^1, k^2, k^3, k^4: $A\cdot D = B\cdot C$ (3.31)

And now we derive the solution for φ_α: a_2 - any real number, and it fully defines f_2 *and* f_1.

4) Tensor and vector of energy-momentum for this Lagrangian.

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From [2] (page 19 {36} equation over (2.9)) we take the definition of the tensor of energy-momentum :

$$T_l^{\ k} = \frac{\partial L}{\partial\left(\dfrac{\partial\varphi_n}{\partial x^k}\right)}\cdot\frac{\partial\varphi_n}{\partial x^l} - L\cdot\delta_l^{\ k} \qquad (4.1)$$

$$T^l{}_k = \frac{\partial L}{\partial\left(\dfrac{\partial\varphi_n}{\partial x_k}\right)}\cdot\frac{\partial\varphi_n}{\partial x_l} - L\cdot\delta^l{}_k = i\cdot e_k{}^{\alpha\beta n}\cdot\varphi_\alpha\cdot\varphi_\beta\cdot(\partial^l\varphi_n) - L\cdot\delta^l{}_k =$$

$$= e_k{}^{\alpha\beta n}\cdot\varphi_\alpha\cdot\varphi_\beta\cdot k^l\varphi_n - L\cdot\delta^l{}_k \qquad (4.2)$$

$$P_k = \int T^1{}_k\cdot d^8x \qquad (4.3)$$

P_k is the vector of energy-momentum. From (2.1), (2.2), (2.3), (4.2) we have :

$$T^1{}_k = k^1\cdot e_k{}^{abn}\cdot j_a\cdot j_b\cdot j_n - d^1{}_k\cdot(i\cdot e_m{}^{abn}\cdot j_a\cdot j_b\cdot\partial^m j_n) \qquad (4.4)$$

$$T^1{}_1 = k^1\cdot e_1^{111}\cdot j_1\cdot j_1\cdot j_1 - (k^1\cdot e_1^{111}\cdot j_1\cdot j_1\cdot j_1 + k^2\cdot e_2^{211}\cdot j_2\cdot j_1\cdot j_1 +$$

$$+k^3 \cdot e_3{}^{121} \cdot j_1 \cdot j_2 \cdot j_1 + k^4 \cdot e_4{}^{221} \cdot j_2 \cdot j_2 \cdot j_1) =$$
$$= -(i_1 \cdot k^2 \cdot j_2 \cdot j_1 \cdot j_1 + i_2 \cdot k^3 \cdot j_1 \cdot j_2 \cdot j_1 + i_1 \cdot i_2 \cdot k^4 \cdot j_2 \cdot j_2 \cdot j_1) \qquad (4.5)$$

$$T^1{}_2 = k^1 \cdot e_2{}^{211} \cdot \varphi_2 \cdot \varphi_1 \cdot \varphi_1 = k^1 \cdot i_1 \cdot \varphi_2 \cdot \varphi_1 \cdot \varphi_1 \qquad (4.6)$$

$$T^1{}_3 = k^1 \cdot e_3{}^{121} \cdot \varphi_1 \cdot \varphi_2 \cdot \varphi_1 = k^1 \cdot i_2 \cdot \varphi_1 \cdot \varphi_2 \cdot \varphi_1 \qquad (4.7)$$

$$T^1{}_4 = k^1 \cdot e_4{}^{221} \cdot \varphi_2 \cdot \varphi_2 \cdot \varphi_1 = k^1 \cdot i_1 \cdot i_2 \cdot \varphi_2 \cdot \varphi_2 \cdot \varphi_1 \qquad (4.8)$$

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5) Spin tensor.

From [2] (2.15) we take the expression for spin tensor :

$$S^k{}_{lm} = -\frac{\partial L}{\partial(\partial u_i / \partial x^k)} \cdot u_j(x) \cdot A^j{}_{i,lm} \qquad (5.1)$$

And from [2] page 20 {37} we take the formula, lower then formula (2.11) :

$$\delta u_i = \sum_{j,k<l} A^j{}_{ikl} \cdot u_j(x) \cdot \delta\omega^{kl} \qquad (5.2)$$

So we have : $S^\lambda{}_{mn} = -\frac{\partial L}{\partial(\partial_\lambda \varphi_\beta)} \cdot \frac{\partial \varphi_\beta}{\partial \omega^{mn}}$ (5.3)

Let us take a vector $\vec{A} = \vec{e}_\mu \cdot x^\mu$ from W and then rotate the system of coordinates in the plane, given by two vectors $\vec{e}_\mu$ and $\vec{e}_\nu$ on the angle $\delta\omega^\mu{}_\nu$. The whole vector $\vec{A}$ remains the same, only it's projections x^μ on the new basic vectors would change. So we can write :

$$\frac{\partial}{\partial \omega^{mn}}(\vec{e}_\mu \cdot x^\mu) = 0 \qquad (5.4)$$

$$\frac{\partial \vec{e}_\mu}{\partial \omega^{mn}} \cdot x^\mu + \vec{e}_\mu \cdot \frac{\partial x^\mu}{\partial \omega^{mn}} = 0 \qquad (5.5)$$

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Here

$$\frac{\partial x^\mu}{\partial \omega^{mn}} = x^\nu \cdot A_\nu{}^\mu{}_{mn} \qquad (5.6) \qquad\qquad \frac{\partial \vec{e}_\mu}{\partial \omega^{mn}} = \vec{e}_\nu \cdot B^\nu{}_{\mu mn} \qquad (5.7)$$

From (5.5), (5.6), (5.7) we have : $A_\nu{}^\mu{}_{mn} = -B^\mu{}_{\nu mn}$ (5.8)

In uncurved W projections x^μ change so (from [1] page 20 {37} over the (2.11)) :

$$\delta x^\mu = x^\nu \cdot \delta\omega^\mu{}_\nu = x^\nu \cdot g_{\nu\lambda} \cdot \delta\omega^{\mu\lambda} = x^\nu \cdot g_{\nu n} \cdot \delta^\mu{}_m \cdot \delta\omega^{mn} = x^\nu \cdot A_\nu{}^\mu{}_{mn} \cdot \delta\omega^{mn} \qquad (5.9)$$

So $A_\nu{}^\mu{}_{mn} = g_{\nu n} \cdot \delta^\mu{}_m$ (5.10)

Further, we can use the connection of W with V :

$$\vec{A} = \vec{e}_\mu \cdot x^\mu = (\vec{n}_\alpha \cdot y^\alpha) \otimes (\vec{n}_\beta \cdot y^\beta) \otimes (\vec{n}_\gamma \cdot y^\gamma) \qquad (5.11)$$

If vector $\vec{A}$ stay unchanged, then $\vec{n}_\beta \cdot y^\beta$ also stay unchanged, and we can write :

$$\frac{\partial}{\partial \omega^{mn}} \cdot (\vec{n}_\beta \cdot \varphi^\beta) = \frac{\partial \vec{n}_\beta}{\partial \omega^{mn}} \cdot \varphi^\beta + \vec{n}_\beta \cdot \frac{\partial \varphi^\beta}{\partial \omega^{mn}} = 0 \qquad (5.12)$$

$$\frac{\partial \varphi^{\beta}}{\partial \omega^{mn}} = \varphi^{s} \cdot C_{s}{}^{\beta}{}_{mn} \qquad (5.13) \qquad\qquad \frac{\partial \vec{n}_{\beta}}{\partial \omega^{mn}} = \vec{n}_{s} \cdot D^{s}{}_{\beta mn} \quad (5.14)$$

From (5.12), (5.13), (5.14) we get : $C_{s}{}^{\beta}{}_{mn} = -D^{\beta}{}_{smn}$ (5.15)

For symmetric metric tensor in V_2 $q_{\alpha\beta}(y)$ we have the same formula for Christoffel symbols as for W_1 :

$$\frac{\partial \vec{n}_{\beta}}{\partial y^{\delta}} = \vec{n}_{s} \cdot \Gamma^{s}{}_{\beta\delta} \quad (5.16) \qquad\qquad \Gamma^{s}{}_{\beta\delta} = \tfrac{1}{2} \cdot q^{sr} \cdot \left(\frac{\partial q_{r\beta}}{\partial y^{\delta}} + \frac{\partial q_{\delta r}}{\partial y^{\beta}} - \frac{\partial q_{\beta\delta}}{\partial y^{r}} \right) \qquad (5.17)$$

Now we can get $D^{s}{}_{\beta mn}$: **To the Content**

$$\frac{\partial \vec{n}_{\beta}}{\partial \omega^{mn}} = \frac{\partial \vec{n}_{\beta}}{\partial y^{\delta}} \cdot \frac{\partial y^{\delta}}{\partial \omega^{mn}} = \vec{n}_{s} \cdot \tfrac{1}{2} \cdot q^{sr} \cdot \left(\frac{\partial q_{r\beta}}{\partial y^{\delta}} + \frac{\partial q_{\delta r}}{\partial y^{\beta}} - \frac{\partial q_{\beta\delta}}{\partial y^{r}} \right) \cdot \frac{\partial y^{\delta}}{\partial \omega^{mn}} =$$

$$= \vec{n}_{s} \cdot \tfrac{1}{2} \cdot q^{sr} \cdot \left(\frac{\partial q_{r\beta}}{\partial \omega^{mn}} + \frac{\partial q_{\delta r}}{\partial \omega^{mn}} \cdot \delta^{\delta}{}_{\beta} - \frac{\partial q_{\beta\delta}}{\partial \omega^{mn}} \cdot \delta^{\delta}{}_{r} \right) = \vec{n}_{s} \cdot \tfrac{1}{2} \cdot q^{sr} \cdot \left(\frac{\partial q_{r\beta}}{\partial \omega^{mn}} + \frac{\partial q_{\beta r}}{\partial \omega^{mn}} - \frac{\partial q_{\beta r}}{\partial \omega^{mn}} \right) =$$

$$= \vec{n}_{s} \cdot \tfrac{1}{2} \cdot q^{sr} \cdot \left(\frac{\partial q_{r\beta}}{\partial \omega^{mn}} \right) \qquad (5.18)$$

From (5.14), (5.18) we have : $D^{s}{}_{\beta mn} = \tfrac{1}{2} \cdot q^{sr} \cdot \dfrac{\partial q_{r\beta}}{\partial \omega^{mn}}$ (5.19)

Now we'll get one useful formula for uncurved W using (5.7), (5.8), (5.9) :

$$\frac{\partial g_{\mu\nu}}{\partial \omega^{mn}} = \frac{\partial (\vec{e}_{\mu}, \vec{e}_{\nu})}{\partial \omega^{mn}} = \left(\frac{\partial \vec{e}_{\mu}}{\partial \omega^{mn}}, \vec{e}_{\nu} \right) + \left(\vec{e}_{\mu}, \frac{\partial \vec{e}_{\nu}}{\partial \omega^{mn}} \right) = (\vec{e}_{s}, \vec{e}_{\nu}) \cdot B^{s}{}_{\mu mn} + (\vec{e}_{\mu}, \vec{e}_{s}) \cdot B^{s}{}_{\nu mn} =$$

$$= -g_{s\nu} \cdot A_{\mu}{}^{s}{}_{mn} - g_{\mu s} \cdot A_{\nu}{}^{s}{}_{mn} = -g_{s\nu} \cdot g_{\mu n} \cdot \delta^{s}{}_{m} - g_{\mu s} \cdot g_{\nu n} \cdot \delta^{s}{}_{m} = -(g_{m\nu} \cdot g_{\mu n} + g_{\mu m} \cdot g_{\nu n})$$

So for uncurved W we have : $\dfrac{\partial g_{\mu\nu}}{\partial \omega^{mn}} = -(g_{m\nu} \cdot g_{\mu n} + g_{\mu m} \cdot g_{\nu n})$ (5.20)

From (2.1), (5.13), (5.14) we have :

$$S_{\lambda mn} = -i \cdot e_{\lambda}{}^{\alpha\beta\gamma} \cdot \varphi_{\alpha} \cdot \varphi_{\beta} \cdot \frac{\partial \varphi_{\gamma}}{\partial \omega^{mn}} = -i \cdot e_{\lambda}{}^{\alpha\beta\gamma} \cdot \varphi_{\alpha} \cdot \varphi_{\beta} \cdot \varphi_{s} \cdot D^{s}{}_{\gamma mn} \qquad (5.21)$$

From (5.19), (1.5), (1.6), (1.7) we have :

$$S_{1mn} = -i \cdot e_{1}{}^{111} \cdot \varphi_{1} \cdot \varphi_{1} \cdot \varphi_{s} \cdot \tfrac{1}{2} \cdot q^{sp} \cdot \frac{\partial q_{p1}}{\partial \omega^{mn}} = -i \cdot \tfrac{1}{2} \cdot \varphi_{1} \cdot \varphi_{1} \cdot \left(\varphi_{1} \cdot q^{11} \cdot \frac{\partial q_{11}}{\partial \omega^{mn}} + \varphi_{2} \cdot q^{22} \cdot \frac{\partial q_{21}}{\partial \omega^{mn}} \right) \quad (5.22)$$

$q^{11} = q^{22} = 1 \qquad q^{12} = q^{21} = 0 \qquad (5.23)$ **To the Content**

From (1.17), (5.20), (5.23) we have :

$$\frac{\partial q_{11}}{\partial \omega^{mn}} = \tfrac{1}{3} \cdot \frac{1}{(q_{11})^{2}} \cdot \frac{\partial g_{\mu\nu}}{\partial \omega^{mn}} \cdot e^{\mu}{}_{111} \cdot e^{\nu}{}_{111} = -\tfrac{1}{3} \cdot (g_{m\nu} \cdot g_{\mu n} + g_{\mu m} \cdot g_{\nu n}) \cdot e^{\mu 111} \cdot e^{\nu 111} =$$

$$= -\tfrac{1}{3} \cdot (e_{n}{}^{111} \cdot e_{m}{}^{111} + e_{m}{}^{111} \cdot e_{n}{}^{111}) \qquad (5.24)$$

From (1.16) we have :

$$q_{\alpha\delta} \cdot q_{\beta r} \cdot q_{\gamma s} = g_{\mu\nu} \cdot e^{\mu}{}_{\alpha\beta\gamma} \cdot e^{\nu}{}_{\delta rs} \qquad (5.25)$$

$$q_{21} \cdot q_{11} \cdot q_{11} = g_{\mu\nu} \cdot e^{\mu}{}_{211} \cdot e^{\nu}{}_{111} \qquad (5.26)$$

$$\frac{\partial q_{21}}{\partial w^{mn}} = \frac{1}{(q_{11})^2} \cdot \frac{\partial g_{mn}}{\partial w^{mn}} \cdot e^{m\,211} \cdot e^{n\,111} = -(g_{mn} \cdot g_{mn} + g_{mm} \cdot g_{nn}) \cdot e^{m\,211} \cdot e^{n\,111} =$$
$$= -(e_n^{\ 211} \cdot e_m^{\ 111} + e_m^{\ 211} \cdot e_n^{\ 111}) \qquad (5.27)$$

From (5.22), (5.23), (5.24), (5.27) we get :
$$S_{1mn} = -i \cdot \tfrac{1}{2} \cdot j_1 \cdot j_1 \cdot [j_1 \cdot (-\tfrac{1}{3}) \cdot (e_n^{\ 111} \cdot e_m^{\ 111} + e_m^{\ 111} \cdot e_n^{\ 111}) + j_2 \cdot (-(e_n^{\ 211} \cdot e_m^{\ 111} + e_m^{\ 211} \cdot e_n^{\ 111}))] \ (5.28)$$

As usual, $\delta\omega^{mn} = -\delta\omega^{nm} \quad (n \neq m)$ (5.29) and $\delta\omega^{mn} = \delta\omega^{nm} \quad (n = m)$ (5.30)
So :
$$S_{1mn} \cdot \delta\omega^{mn} = i \cdot \tfrac{1}{2} \cdot \varphi_1 \cdot \varphi_1 \cdot [\varphi_1 \cdot \tfrac{1}{3} \cdot 2 \cdot e_n^{\ 111} \cdot e_m^{\ 111}] \cdot \delta\omega^{mn} \qquad (5.31)$$

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And we have eventually the density of spin tensor :
$$S^1{}_{mn} = g^{11} \cdot S_{1mn} = \tfrac{1}{3} \cdot i \cdot \varphi_1 \cdot \varphi_1 \cdot \varphi_1 \cdot e_n^{\ 111} \cdot e_m^{\ 111} \quad (n = m = 1) \qquad (5.32)$$
For density of spin vector we have (1.0), (1.1), (1.22) :
$$S^p = g^{pk} \cdot F_k^{\ mn} \cdot S_{1mn} = g^{pk} \cdot F_k^{\ 11} \cdot S_{111} \qquad (5.33)$$
For W_2 we have {[3] – the definition of algebraic tensor} :
$$S^1 = g^{11} \cdot F_1^{\ 11} \cdot S_{111} = S_{111} = \tfrac{1}{3} \cdot i \cdot \varphi_1 \cdot \varphi_1 \cdot \varphi_1 \qquad (5.34)$$
$$S^v = 0 \quad (v = 2, 3, 4) \qquad (5.35)$$

From (5.34), (5.35) we see that spin of φ_β equals 1/3.

Literature

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Input finished 21/10-2011

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Abstract – 6/11 – 2011

The correction of mistakes and introducing the citation numbers of viXra – 1/03-2014