Estimation of 'Drainable' Storage – a Geomorphological Approach

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Abstract

Storage of water within a drainage basin is often estimated indirectly by analyzing the recession flow curves as it cannot be directly estimated with the aid of available technologies. However, two major problems with recession analysis are: i) late recession flows, particularly for large basins, are usually not observed ii) and early recession flows indicate that initial storage is infinite, which is not realistic. We address this issue by using the recently proposed geomorphological recession flow model (GRFM), which suggests that storage-discharge relationship for a recession event is exponential for the early recession phase and power-law for the late recession phase, being distinguished from one another by a sharp transition. Then we obtain a simple expression for the 'drainable' storage within a basin in terms of early recession curve characteristics and basin geomorphology. The predicted storage matches well with the observed storage ($R^2 = 0.96$), indicating the possibility of reliably estimating storage in river basins for various practical purposes.

Keywords: drainable storage, discharge, complete recession curve, active drainage network, GRFM.

1. Introduction

Terrestrial water storage is the key entity that determines flows in river channels, climate and the fresh water ecosystem [e.g. 1, 18, 8]. It is not possible to directly measure storage, say in a drainage basin, with the use of available technology. For example, GRACE (Gravity Recovery and Climate Experiment) satellites can measure storage fluctuation, but cannot measure absolute storage [e.g. 18]. Hydrologists generally use conceptual models to estimate storage, particularly by analyzing recession flows or streamflows during no-rain periods [e.g., 6, 7, 23, 12, 3, 21, 13, 24], because during these periods discharge (observable entity) in river channels is controlled only by storage in the basin. If a basin is allowed to fully drain, initial 'drainable' storage (the part of storage that transforms into streamflows, henceforth simply called as storage) will be equal to total flow volume. A major problem in regard to the estimation of initial storage, however, is that most of the times we do not observe a 'complete' recession curve (a recession curve lasting till discharge approaches zero), because inter-storm time gaps are usually shorter than recession timescales (time periods for which recession events last), particularly in case of large basins. Thus, in practice streamflows during early phases of recession periods only are considered in an analysis.

For most natural basins early recession flows indicate that initial storage is infinite for any finite initial discharge, which is unrealistic. We try to solve this problem by using geomorphological recession flow model (GRFM) proposed by Biswal and Marani [3], which suggests that early recession flows are characteristically different from late recession flows: storage-discharge relationship is exponential for early recession periods, whereas it exhibits a power law relationship for late recession periods. We then follow suitable analytical methods to obtain a simple expression for the initial storage within a basin in terms of the properties of early recession flows and the channel network morphology. Using observed daily streamflow data from 27 USGS basins we then compare predicted initial storage matches with observed initial storage.

2. The Problem of a Single Storage-Discharge Relationship

A radical change in recession analysis was introduced by Brutsaert and Nieber [6] who expressed $-dQ/dt$ as a function of Q (Q being discharge observed at the basin outlet at time t), thus eliminating the necessity of defining a reference time. They found that $-dQ/dt$-Q relationship for recession periods generally show a power law relationship of the type:

$$-\frac{dQ}{dt} = kQ^{\alpha} \tag{1}$$

For recession periods, the classical mass balance equation takes the form:

$$\frac{dS}{dt} = -Q \tag{2}$$

as inflow into the system is zero in such cases (S is storage within the basin at time t). It should be noted that evaporation and other components are missing in eq. (2); however, this does not affect our analysis as we are interested only to estimate the amount of storage that turns into streamflows. Combining eq. (1) and (2) one can obtain the relationship between storage and discharge in integral form as

$$\int_{Q_0}^{Q} \frac{dQ}{Q^{\alpha-1}} = k \int_{S_0}^{S} dS \tag{3}$$

where Q_0 and S_0 respectively are discharge and storage at time $t = 0$. Biswal and Marani [3] analyzed early recession flows and found that α for a basin remains fairly constant across recession events but k varies greatly from one event to another, implying that recession flow curves must be analyzed individually. Individual analysis of early recession flow curves reveals that for most natural basins the value of α is close to 2 [3, 21, 4, 13, 22], in which case eq. (3) gives:

$$\ln Q_0 - \ln Q = k \cdot (S_0 - S) \tag{4}$$

or discharge is an exponential function of storage: $Q = Q_0 \cdot e^{-k(S_0-S)}$ [see also 10]. If a basin drains completely, both S and Q will be equal to zero. Using these values in eq. (4) one can find that S_0 is infinite, which is unrealistic. This implies that $-dQ/dt$ vs. Q curve cannot follow a single power law relationship (as described by eq. (1)) with $\alpha = 2$ throughout a recession event. In the next section we address this issue by using GRFM.

3. GRFM and Storage Estimation

Many details of the hydrological processes occurring in a basin can be found to be encoded in the morphology of the drainage network. Over the past few decades much research has been carried out to identify the signatures of the channel network morphology in the hydrological response produced by it, particularly in regard to surface flows [e.g. 16, 17, 14, 15]. More recently, Biswal and Marani [3] proposed a modelling framework (GRFM) which suggests that the gradual shrinkage of the active drainage network or the ADN (the part of the drainage network that is actively draining at a particular time) [e.g., 9, 25] controls recession flows in the basin. GRFM connects recession flow properties with the channel network morphology by assuming that the flow generation per unit ADN length, q, remains constant during a recession event. Thus at any point of time Q can be expressed as: $Q = q \cdot G(t)$, where $G(t)$ is the total length of the ADN at time t. Furthermore, they assumed that the speed at which the ADN heads move in downstream direction, c ($c = dl/dt$ or $l = c \cdot t$, where l is the distance of a ADN head from its farthest source or channel head at time t), remains constant in space and time. That means, discharge can also be expressed as a function of l:

$$Q = q \cdot G(l) \tag{5}$$

$G(l)$ being the geomorphic recession curve for the basin. Figure 1a shows $G(l)$ vs. l curve for Arroyo basin (106.71 sq km, California) obtained by using 30 m resolution USGS digital elevation model and imposing a flow accumulation threshold of 100 pixels. The expression for $-dQ/dt$ can then be obtained as:

$$-\frac{dQ}{dt} = -q \cdot \frac{dl}{dt} \cdot \frac{dG(l)}{dl} = q \cdot c \cdot N(l) \tag{6}$$

where $N(l)$ is the number of ADN heads at distance l or time t. Using eq. (5) and (6), the expression for the geomorphic counterpart of $-dQ/dt$ vs. Q curve (eq. 1)) can be obtained as

$$N(l) = \rho \cdot G(l)^{\alpha} \tag{7}$$

where $\rho = kq^{\alpha-1}/c$.

The $N(l)$ vs. $G(l)$ curve of a basin typically displays two scaling regimes, AB and BC, easily distinguishable from one another ([3], also see Figure 2a). The regime AB corresponds to early recession flows, and for most basins the geomorphic α for this phase is also nearly equal to 2 (i.e. both geomorphic α and observed α are nearly equal to 2 for the regime AB), suggesting that the model is able to capture key details of a recession flow curve. Defining the geomorphic storage $V(l)$ as: $-d\{V(l)\}/dl = G(l)$, the expression for the geomorphic storage-discharge relationship for $\alpha = 2$ can be obtained by using eq. (7):

$$\frac{dG(l)}{G(l)} = \rho \cdot dV(l) \tag{8}$$

Similar to the derivation of eq. (4), integration of eq. (8) from $\{G(0), G(l)\}$ to $\{V(0), V(l)\}$ yields

$$\ln G(l) - \ln G(0) = \rho \cdot \{V(l) - V(0)\} \tag{9}$$

i.e. the geomorphic storage-discharge relationship for part AB is exponential: $G(l) = G(0) \cdot e^{\rho\{V(l)-V(0)\}}$. Figure (3a) shows $V(l)$ vs. $G(l)$ curve for Arroyo basin displaying exponential relationship for its AB portion. The transition point B is very noticeable. Note that eq. (4) can be easily retrieved from eq. (9) using the relationships $Q = q \cdot G(l)$ and $S = -\int Qdt = -q \cdot \int G(l) \cdot dt/dl \cdot dl = q/cV(l)$. $N(l)$ is always equal to 1 for the phase BC as only the mainstream of the channel network contributes, which also means that $\alpha = 0$ for this phase (see Figure 2a). $G(l)$ for this phase is thus $L - l$, where L is the length of the mainstream of the channel network, and $V(l) = 1/2(L-l)^2$. That means,

$$V(l) = \frac{1}{2}G(l)^2 \tag{10}$$

Figure (4a) separately shows BC portion of the $V(l)$ vs. $G(l)$ curve for Arroyo basin. Using the expressions for S $\{S = q/c \cdot V(l)\}$ and Q $\{Q = q \cdot G(l)\}$ it can be found that the storage-discharge relationship of BC portions of real recession curves also follow a power law relationship with exponent equal to 2: $S \propto Q^2$.

If the power law scaling transition (i.e. α changes from 2 to 0) takes place at the length $l = l^*$, and the corresponding $G(l)$ is $G(l^*)$, using eq. (9) one can find that for $\alpha = 2$, $\ln G(0) - \ln G(l^*) = \rho \cdot \{V(0) - V(l^*)\}$. Eq. (7) suggests that $N(0) = \rho G(0)^2$, where $N(0)$ is the number of channel heads in the drainage network or $N(l)$ at $l = 0$, and $N(l^*) = 1 = \rho G(l^*)^2$ as N(l) is always 1 for the BC phase. Now noting that $V(l^*) = 1/2 \cdot (L - l^*)^2 = 1/2 \cdot G(l^*)^2 = 1/(2\rho)$, eq. (9) can be used to obtain:

$$V(0) = \frac{1}{2\rho}\{1 + \ln N(0)\} \tag{11}$$

According to Shreve [20], $N(0)$ (number of channel heads in the network) is proportional to basin area (A): $N(0) = \Psi \cdot A$, where Ψ is the constant of proportionality and A is the basin area. Thus,

$$V(0) = \frac{1}{2\rho}\left(1 + \ln A + \psi\right) \tag{12}$$

where $\psi = \ln \Psi$. Recalling that initial storage $S_0 = q/c \cdot V(0)$ and $k = c\rho/q$ for $\alpha = 2$, the expression for S_0 can be obtained as

$$S_0 = \frac{1}{2k}\left(1 + \ln A + \psi\right) \tag{13}$$

In the next section we analyze real recession curves from a number of basins and evaluate the predictability of eq. (13).

4. Analysis of Observed Recession Flow Curves

In this study, we use daily average streamflow data and carefully identify 'complete' recession curves for 27 USGS basins that are relatively unaffected by human activities (see Table 1 of the online supporting material). Theoretically, both Q and $-dQ/dt$ should continuously decrease over time during a recession period; however, almost always, this criteria is not satisfied due to errors (numerical errors, measurement errors, etc.), particularly associated with very low streamflows (i.e. with the BC parts of recession curves). Here we visually select relatively smooth looking recession curves starting from their respective peaks and lasting till discharge approaches zero (for e.g., see Figure 1a'). Q and $-dQ/dt$ are computed by following Brutsaert and Nieber [6] as: $Q = (Q_t + Q_{t+\Delta t})/2$ and $-dQ/dt = (Q_t - Q_{t+\Delta t})/\Delta t$ (here Δt is 1 day). For convenience we denote Q_z as the discharge during a recession event in the z-the after the recession peak. $-dQ/dt$ generally increases from $z = 0$ to $z = 1$, possibly because discharge during this period is likely to be significantly controlled by storm flows, and then it keeps on decreasing [3]. Thus, we consider that $t = 0$ at $z = 1$. A recession curve is then considered if atleast 3 data points starting from $t = 0$ (i.e. they belong to the AB part, see Table 1) show robust $-dQ/dt$-Q power law relationship ($R^2 > 0.7$) with $\alpha = 2 \pm 0.25$ (see, for e.g., Figure 2a'). Note that BC phases of observational $-dQ/dt$-Q curves cannot be produced, even for the complete recession curves, as late recession flows are very much dominated by observational and other errors, and $-dQ/dt$ is particularly sensitive to such errors. In total, we select 121 complete recession curves from the 27 basins for our analysis (Table 1). S_z, storage in the basin in the z-th day after the peak, can be computed as:

$$S_z = \Delta t \cdot \sum_{i=z}^{Z} Q_i \tag{14}$$

where Z is the number of days for which the recession event lasts or the timescale of the recession curve.

The observed recession curves display storage-discharge patterns very similar to those of the geomorphic recession curves. The AB parts of observed recession curves display exponential S-Q relationship as predicted by eq. (9) (see Figure 3a'). The BC parts exhibit power law Q-S relationship with exponent nearly equal to 2 (see Figure 4a'), though not in all cases because of high degree of errors associated with this part. It should be noted here that the discontinuation of exponential S-Q relationship was also reported in some past studies [e.g. 11, 2]. We then obtain observed S for $t = 0$ (S_0^o) for each recession curve following eq. (14). We follow least square regression method and compute k for each recession curve by fixing α of its AB part at 2 to predict the initial storage. Note that for modelling of initial storage (S_0^m) using eq. (13) the value of ψ needs to be determined from recession flow data as we do not have information on the values of $N(l)$ and $G(l)$ for $t = 0$. We compute the value of ψ for each recession curve of Arroyo basin by putting S_0^o in eq.(13). The reason for selecting Arroyo basin is that it has maximum number of recession curves (17 in total) and therefore it is expected to give a better representative value of ψ. The mean value ψ for the basin is found to be nearly equal to 1, which gives the expression for S_0:

$$S_0 = \frac{1}{k}\left(1 + \frac{1}{2}\ln A\right) \tag{15}$$

We thereafter use eq. (15) to compute S_0^m for all the selected recession curves from all the basins including Arroyo by using eq. (13). Figure 5 shows the plot between S_0^m and S_0^o for the selected recession curves (including those from Arroyo basin) with correlation $R^2 = 0.96$. Good R^2 correlation indicates the ψ values is universal.

The main motivation behind this study was to obtain a simple analytical expression for initial storage, and for this reason particularly, the strong relationship between S_0^m and S_0^o is quite remarkable. Our results also indicate that eq. (15) is a universal relationship, although this aspect needs to be rigorously tested. Potential implications of the observations in this study, therefore, include better management of fresh water resources and ecosystems and better hydrological inputs for climate models. The little amount scatter and the observation that the trend line does not have a slope exactly equal to 1 in Figure05 (obtained slope is 1.06) are possibly because of GRFM's assumption that q and c remain constant during a recession event. It should be also noted that the present study uses relatively small and homogeneous basins (drainage area ranging from 2.85 sq km to 595.70 sq km) as it is not possible for us to obtain complete recession curves for large basins (say the Mississippi river basin). Thus, further investigation is require to analyze storage-discharge relationships of large river basins that can even witness spatial variation in climate and geology [e.g. 19]. Future studies may introduce meaningful modifications to GRFM [e.g. 5, 13] to model storage more accurately, particularly for large basins.

5. Summary and Conclusions

The state of the art technologies do not enable us to estimate water stored within a drainage basin. A viable option for this purpose is the analysis of recession flow curves. In natural basins we usually observe early recession flows. Late recession flows are hardly observed because no-rain periods most of the times are shorter than recession timescales, particularly in case of larger basins. Early recession flows across basins are typically characterized by a power law relationship: $-dQ/dt = kQ^{\alpha}$ with α being nearly equal to 2, i.e. $-dQ/dt = kQ^2$. If we assume this relationship to continue throughout a recession period (i.e. till discharge approaches zero), storage will be infinite for any finite initial discharge, which is unrealistic. We addressed this storage estimation problem here using geomorphological recession flow model (GRFM).

GRFM suggests that a $-dQ/dt$ vs. Q curve exhibits two distinct scaling regimes: AB, which corresponds to early recession flows, and BC, which corresponds to late recession flows. While the regime AB gives $\alpha = 2$, α for the regime BC is 0 according to the model. Thus storage-discharge relationship is exponential for the regime AB and power law for the regime BC with exponent equal to 2. Using data from 27 basins we found that the observed recession curves, like the modelled (geomorphic) recession curves, display exponential discharge-storage relationship for AB parts and power law relationships for BC parts. We then followed suitable analytical methods and obtained a simple expression for the initial basin storage, S_0, as a function of k and basin geomorphology: $S_0 = 1/(2k)\,(1 + \ln A + \psi)$, ψ being equal to 1. We observed that the modelled initial storage, S_0^m, matches well with the observed initial storage, S_0^o ($R^2 = 0.96$). Results here are indicative of the possibility that GRFM can be used to reliably model 'drainable' storage in basins to answer many practical and scientific questions related to water resources.

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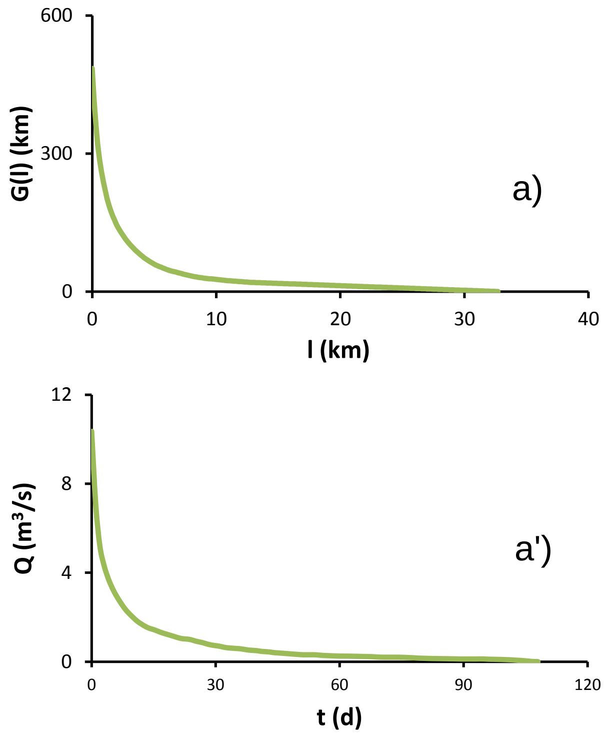

Figure 1: a) The $G(l)$ vs. l curve (or the geomorphic recession curve) for Arroyo basin (106.71 sq km). The channel network for the basin was obtained by imposing a flow accumulation threshold equal to 100 pixels. a') A sample observed recession curve (Q vs. t curve, from 3/20/1973 to 7/8/1973) from the basin, which looks similar to the geomorphiic recession curve of the basin.

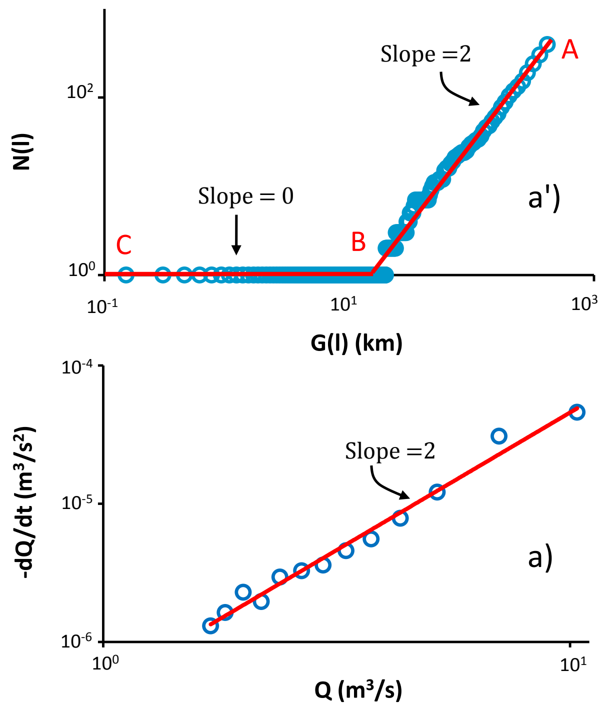

Figure 2: a) $N(l)$ vs. $G(l)$ curve for Arroyo basin (106.71 sq km), which displays two distinct scaling regimes: AB that corresponds to early recession flows ($\alpha = 2$) and BC that corresponds to late recession flows ($\alpha = 0$). The channel network for the basin was obtained by imposing a flow accumulation threshold equal to 100 pixels. a') AB part of an observed recession curve (lasting from 3/20/1973 to 7/8/1973) from the basin displaying $-dQ/dt$ vs. Q power law relationship with power law exponent nearly equal to 2. Note that BC parts of observed recession curves are generally dominated by significant errors. (Red lines indicate slopes in the log-log planes.)

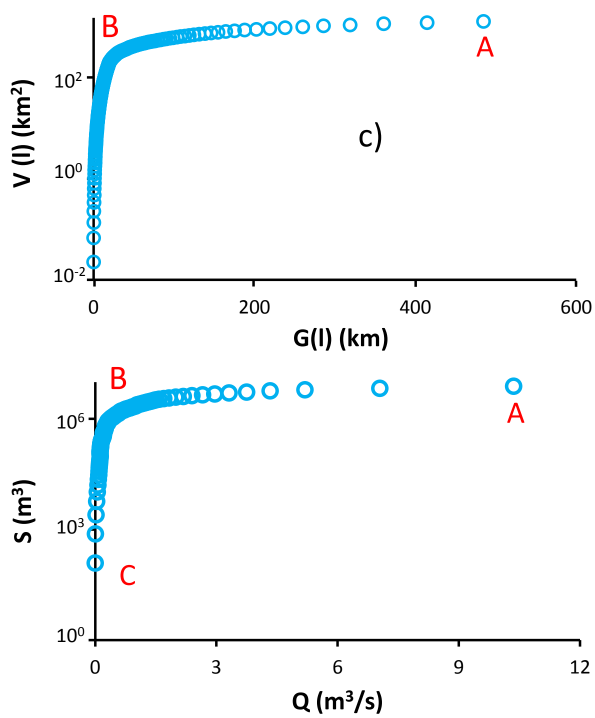

Figure 3: a) The $V(l)$ vs. $G(l)$ curve or the geomorphic storage-discharge curve of Arroyo basin (106.71 sq km) and a') a selected observed recession curve from the basin (from 3/20/1973 to 7/8/1973) displaying two distinct scaling relationships in semi logarithmic planes: exponential relationship for the regime AB and power law relationship for the regime BC (not clearly visible here).

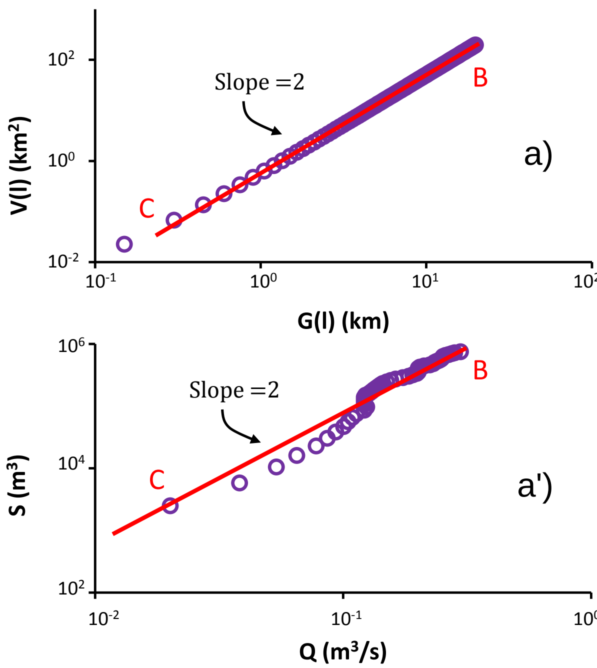

Figure 4: a) The BC portion of the $V(l)$ vs. $G(l)$ curve of Arroyo basin (106.71 sq km) in log-log plot displaying a power law relationship with exponent equal to 2 (indicated by red line). a') The BC portion of a selected observed recession curve (from 3/20/1973 to 7/8/1973) also displaying a power law relastionship with exponent close to 2.

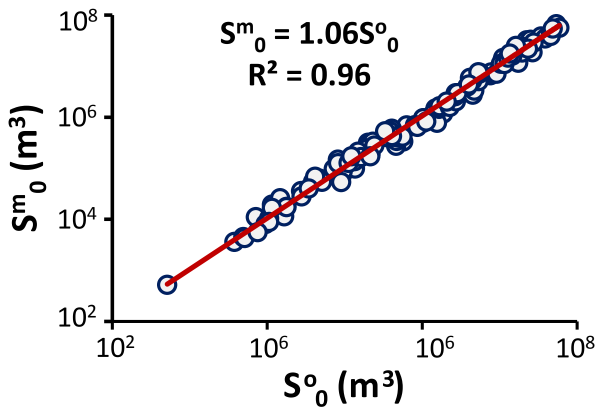

Figure 5: The plot between modelled initial storage (S_0^m) vs. observed initial storage (S_0^o) for the 121 recession curves selected in this study. Good correlation ($R^2 = 0.96$) indicates that the predictions are reliable.