Polymer physics, the quantum harmonic oscillator, and the fabric of space-time

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Abstract- The density of probability associated to the random walk describing a polymer chain is took as the squared wave function of a harmonic oscillator of mass M. The insertion of this wave function in the Schroedinger equation reveals a structure of energy levels, where the quantum of energy depends on the radius of gyration of the polymer. Besides this, we compare this function with another derivate from the Bekenstein-Hawking entropy. This comparison suggests that to each lattice edge having the size of the Planck's length, in a vision of a discrete space-time, we could attach a continuum chain of infinite length. We call this picture the fabric of space-time.

1– Introduction

The simplest type of polymer is represented by a linear chain of monomers [1]. As was pointed out by Raposo et al [1], in the lattice model of polymers, a dilute macromolecule in a solvent is described by a random self-avoiding walk (SARW).

The distribution function for **r**, the end to end distance of an ideal polymer chain, according to de Gennes [2] is given by

$$P(r) = C \exp[- 3r^2/(2Na^2)], \tag{1}$$

where $r^2 \equiv \mathbf{r}.\mathbf{r}$, N and a are the number and size of monomers composing the chain, and C is the normalization constant.

In this very short note, we propose to interpret P(r) as a probability density tied to the wave function Ψ of a particle of mass M, which motion is described by a quantum harmonic oscillator. Then we write

$$P(r) \equiv \Psi^*\Psi = \Psi^2. \tag{2}$$

2 – The wave equation satisfied by Ψ

In the one- dimensional (1-d) case, equations (1) and (2) lead to the wave function $\Psi(x)$, namely

$$\Psi(x) = \sqrt{C} \exp[- x^2/(4Na^2)]. \tag{3}$$

Next let us consider the 1-d Schroedinger equation

$$[-\hbar^2/(2M)](d^2\Psi/dx^2) + V(x)\Psi = \varepsilon_0\Psi. \tag{4}$$

Calculating the second derivative of Ψ and inserting it in eq. (4) we get

$$\{[(-\tfrac{1}{2})\hbar^2\, x^2]/[4M(Na^2)^2] + [\hbar^2/(4MNa^2)] + V(x)\}\Psi = \varepsilon_0\Psi. \tag{5}$$

Making the identifications between dependent and independent of x terms, we have

$$V(x) = \tfrac{1}{2}\{\hbar^2/[M(2Na^2)^2]\}\, x^2 = \tfrac{1}{2}\, k\, x^2, \tag{6}$$

$$\varepsilon_0 = \tfrac{1}{2}\hbar^2/[M(2Na^2)] = \tfrac{1}{2}\hbar\omega, \tag{7}$$

$$\omega = (k/M)^{1/2}. \tag{8}$$

In polymer physics [1,2], by taking a chain embedding in a four- dimensional space-time, we can write

$$R^2 = 2\, R_g^{\,2} = 2Na^2. \tag{9}$$

In (9), R_g stands for the radius of gyration of the polymer chain. Besides this we define

$$R = \hbar/(Mc). \tag{10}$$

Inserting (10) into (7) we get

$$\varepsilon_0 = \tfrac{1}{2}M\, c^2. \tag{11}$$

Taking in account the excited states, we get the structure of energy levels

$$\varepsilon_n = (n + \tfrac{1}{2})\, Mc^2,\ n=0,1,2,\ldots \tag{12}$$

3 – Fabric of space-time

In a paper dealing with the origin of proton mass [3], we defined a wave function associated to the curved space-time given by

$$\Psi_C(r) = \sqrt{C}\, \exp[-\,\pi r^2/(L_{Pl}{}^2)]. \tag{13}$$

Making the identification between (3) and (13),being L_{Pl} the Planck's length, we obtain

$$L_{Pl}{}^2 = (2\pi/3)\, Na^2. \tag{14}$$

Result given by eq. (14) can be interpreted as follows: the Planck's length is the edge of the square unit cell which decors the surface horizon of radius r (please see reference [3]). Meanwhile the right side of (14) is proportional to the square of the radius of gyration of a polymer of length L and unit length a, namely

$$L = Na, \qquad \text{and} \qquad R_g^{\,2} = Na^2. \tag{15}$$

L_{Pl} is the end to end distance of a polymer of length L and unit length a. We can take the simultaneous limit of L $\to\infty$, and a $\to 0$, but maintaining the end to end distance L_{Pl} finite. In this way, a continuous infinite string (polymer), have its two ends tied by a finite edge. This feature can be understood as operating a fabric of space-time, where a discrete space emerges from a continuous object.

In a paper dealing with the cosmological constant problem [4], the universe world line has been thought as a lattice model of a polymer (SARW) [1]. There, the universe world line was represented by a linear chain of monomers, which size was set up equal to the Planck's length. It is possible to think of a continuum polymer chain (L= Na $\to\infty$, a $\to 0$, but with Na^2 finite), as being connected to the two extremities of each monomer. To be more specific we write

$$N a^2 = L_{Pl}^2. \tag{16}$$

A photon traveling between two neighboring points of the lattice does it with a finite velocity c and with a finite duration equal to the Planck's time τ_{Pl}. On the other hand this photon knows from beforehand the locus of the point to be reached. We propose that this information is sent to this photon with an infinite speed in order to transpose the chain of infinite length in a finite time. We call "fabric of space-time" to these alternatives continuum paths.

As a means to better clarify the role of the fabric of space-time on this polymer model of the universe, we must stress that in the paper [4]: "Cosmological constant and polymer physics", the universe world line is represented by a finite chain containing $\mathcal{N}$ monomers. Indeed from reference [4], we have

$$R_\Lambda = \mathcal{N}^{1/2} L_{Pl}. \tag{17}$$

We observe that there [4], the radius of gyration, or the end-to-end distance was associated to the cosmological constant and $\mathcal{N}=10^{61}$is a great number, although finite. The picture we trace out in connection with the fabric of the space-time concept is that we have $\mathcal{N}$ infinite strings attached to the extremities of the "monomers", which compose this segmented and finite chain representing the universe world line.

References

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