The cubic equation and 137.036

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A special case of the cubic equation, distinguished by having an unusually economical solution, is shown to relate to both the fine structure constant inverse (approximately 137.036) and the sines squared of the quark and lepton mixing angles.

I. INTRODUCTION

Earlier, the author showed how a *slightly asymmetric equation* [1] (that is, an equation whose left- and right-hand sides are very similar) produced the experimental value of the fine structure constant inverse from physics (approximately 137.036) [2, 3], which elsewhere was tied to the sines squared of the quark and lepton mixing angles [4, 5]. Here, a related equation, similarly inspired by the fine structure constant inverse but distinguished by being a cubic equation with an unusually economical solution, is linked to these same physical constants.

II. SPECIAL CASE OF THE CUBIC EQUATION

Let

$$K = \left(\frac{m+x}{n}\right)^3 + (m+x)^2 \quad , \tag{2.1}$$

where x is a variable, and K, m, and n are positive constants such that

$$m = \frac{n^3}{3} \quad . \tag{2.2}$$

III. ITS FIRST SOLUTION

Then, by letting

$$L = \left(\frac{m}{n}\right)^3 + (m)^2 \tag{3.1}$$

$$v = \sqrt[3]{\frac{K \pm \sqrt{K\,(K-L)}}{L/2}} - 1 \tag{3.2}$$

$$w = v - 1 + \frac{1}{v} \tag{3.3}$$

and choosing K and n so that $K \geq L$, we have

$$x = m\,(w-1) \quad , \tag{3.4}$$

the more complex of the two solutions to be given for Eq. (2.1). It follows that

$$m + x = mw \quad , \tag{3.5}$$

so that

$$K = \left(\frac{mw}{n}\right)^3 + (mw)^2 \quad , \tag{3.6}$$

which highlights just how closely K and L interrelate.

IV. ITS MINIMAL CASE

The smallest integer solution to Eq. (2.2)

$$m = 9 \quad \text{and} \quad n = 3$$

is notable, simply because it is minimal.

V. A SPECIAL SOLUTION

And, for the special solution $x = 1$, this m and n cause Eq. (2.1) to give

$$\begin{aligned} K &= \left(\frac{9+1}{3}\right)^3 + (9+1)^2 \\ &= 137.\overline{037} \quad . \end{aligned} \tag{5.1}$$

Here, the constant K is close enough — within one thousandth of one per cent — to the physicists' 137.036 [2, 3] to suggest that looking for a connection might turn up interesting mathematics.

VI. FINE STRUCTURE CONSTANT INVERSE

In fact it does [6]. Let

$$K = 137.036 \qquad m = 9 \qquad n = 3 \quad ,$$

so that

$$\begin{aligned} L &= (m/n)^3 + (m)^2 \\ &= (9/3)^3 + 9^2 \\ &= 3^3 + 3^4 \\ &= 108 \\ v &\approx 1.393\,479\,170\,916^{\pm 1} \\ w &\approx 1.111\,107\,407\,399 \end{aligned}$$

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and

$$\begin{aligned} x &= m\,(w-1) \\ &\approx 9 \times (1.111\,107\,407\,399 - 1) \\ &\approx 9 \times 0.111\,107\,407\,399 \\ &\approx 0.999\,966\,666\,591 \\ &\approx 1 - 1/29\,999.932\,142\,743\,338 \quad . \end{aligned}$$

Then, the above values for x, m, and n cause Eq. (2.1) to give

$$\begin{aligned} 137.036 = &\left[\left(\frac{9+1}{3}\right) - \frac{1}{3 \times 29\,999.932\ldots}\right]^3 \\ &+ \left[(9+1) - \frac{1}{29\,999.932\ldots}\right]^2 \quad , \end{aligned} \tag{6.1}$$

which, in turn, branches into two important offshoots

$$29\,999.932\ldots \approx 3 \times (9+1)^4 \tag{6.2}$$

and

$$137.036 = 999.999/3^3 + 99.999 \quad . \tag{6.3}$$

Observe that 29 999.932 is an unexpectedly *round* number, whose remarkable connections to $3 \times (9+1)^4$ and $999.999/3^3 + 99.999$ have already been explored concretely in [6] and abstractly in [1]. The above round number and these intriguing offshoots identify 137.036 as a noteworthy number *independent of its role in physics*, while simultaneously pointing to Eq. (2.1) as an equation worthy of greater attention.

VII. QUARK AND LEPTON MIXING ANGLES

By the same reasoning the following four values, *which can be reproduced from the sines squared of the quark and lepton angles* (as shown in [5])

$$\begin{array}{cc} \dfrac{10}{3} & \dfrac{1}{3 \times 29\,999.932\ldots} \\ 10 & \dfrac{1}{29\,999.932\ldots} \end{array} \quad ,$$

are likewise of some purely mathematical interest independent of *their* role in physics. And, by the same reasoning, they likewise suggest that Eq. (2.1) is an equation worthy of greater attention.

VIII. ITS SECOND SOLUTION

The most obvious way to further explore Eq. (2.1) is to restate it in the form of the general cubic equation and then solve it using the general cubic's classical solution. The classical solution to the general cubic equation

$$ax^3 + bx^2 + cx + d = 0 \tag{8.1}$$

is

$$x = \sqrt[3]{\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} + \sqrt[3]{\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} - r \quad , \tag{8.2}$$

where

$$\left.\begin{aligned} p &= \frac{c}{a} - \frac{b^2}{3a^3} \\ q &= -\frac{2b^3}{27a^3} + \frac{bc}{3a^2} - \frac{d}{a} \\ r &= \frac{b}{3a} \quad . \end{aligned}\right\} \tag{8.3}$$

Though complicated, tellingly, these complexities largely vanish when a, b, c, and d derive from Eq. (2.1). So, when Eq. (2.1) is expanded into the general cubic equation we get these coefficients

$$\left.\begin{aligned} a &= 1 & b &= 6m \\ c &= 9m^2 & d &= m(4m^2 - 3K) \end{aligned}\right\} \tag{8.4}$$

in terms of m and K. Substituting the coefficients of Eq. (8.4) into Eq. (8.3) allows *simplifying* Eq. (8.2) to get

$$x = \sqrt[3]{t + \sqrt{t^2 - m^6}} + \sqrt[3]{t - \sqrt{t^2 - m^6}} - 2m \quad , \tag{8.5}$$

the second solution to Eq. (2.1), a solution that is notably economical given the compactness of

$$\begin{aligned} t &= m^3 - \frac{d}{2} \\ &= m\left(1.5K - m^2\right) \quad . \end{aligned} \tag{8.6}$$

IX. SOLUTION ECONOMY

The complexity of the above solution can be objectively assessed by comparing it against the exceptionally-simple classical solution to the *depressed cubic*, a natural off-the-shelf benchmark; the depressed cubic is merely the general cubic equation without its quadratic term (i.e., $b = 0$). Assuming the coefficient of the depressed cubic's leading term equals one (i.e., $a = 1$), we have

$$x^3 + cx + d = 0 \tag{9.1}$$

for the depressed cubic. Substituting this equation's coefficients into Eq. (8.3) gives

$$\left.\begin{aligned} p &= c \\ q &= -d \\ r &= 0 \end{aligned}\right\} \tag{9.2}$$

so that Eq. (8.2) gives this compact solution to Eq. (9.1)

$$x = \sqrt[3]{\frac{-d}{2} + \sqrt{\frac{d^2}{4} + \frac{c^3}{27}}} + \sqrt[3]{\frac{-d}{2} - \sqrt{\frac{d^2}{4} + \frac{c^3}{27}}} \quad . \tag{9.3}$$

This solution, along with the solution to the general cubic, date back to Cardano's renaissance masterwork *Ars Magna*.

X. PHYSICALLY INSPIRED RESULTS?

The above solution to the depressed cubic is about as complex as the second solution to Eq. (2.1): that is, Eqs. (8.5) and (8.6). This shows that Eq. (2.1), like the depressed cubic, is elementary enough to be of interest to mathematicians. But does it also deserve to engage the interest of physicists?

To help shed light on this, consider that the second solution to Eq. (2.1) emerged from straightforward application of classical mathematics *with 137.036 providing a helpful hint about the existence of such a compact solution.* (More specifically, it was the proximity of 137.036 to $137.\overline{037}$ that suggested that Eq. (2.1) might be mathematically interesting. In fact, it *is* mathematically interesting, particularly because of its unusually economical solution, a solution rivaling in compactness that of the celebrated depressed cubic.)

But why should a *physical constant* such as 137.036 aid in the discovery of *purely mathematical results*? Unless, of course, these results are not purely mathematical, but actually share with the celebrated fine structure constant physical significance.

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[6] J. S. Markovitch, "Some mathematics inspired by 137.036" (2013) `http://www.vixra.org/abs/1312.0016`.