General Relativity as curvature of space

Peter H. Michalicka

February 20, 2014

Abstract

With the Planck 'constants' length, time, mass and acceleration will be shown, that a Quantum Gravity of the cosmos exists. This paper shows how Einstein's Field Equations in Friedmann Robertson Walker Metric solves the Planck Era context.

1 The Planck 'constants'

Planck length $\Delta x = \sqrt{\frac{Gh}{c^3}}$

Planck time $\Delta t = \sqrt{\frac{Gh}{c^5}}$

Planck mass $\Delta m = \sqrt{\frac{hc}{G}}$

Planck acceleration $\Delta a = \frac{c}{\Delta t} = \sqrt{\frac{c^7}{hG}}$

2 Modern Cosmology

Within modern cosmology the Einstein's Field Equations would be written with cosmological term Λ as follows (see [2] and [3]):

$$R_{ik} - \frac{1}{2} g_{ik} R - \Lambda g_{ik} = \frac{8\pi G}{c^4} T_{ik} \quad (2.0)$$

The solutions of the Field Equations in Friedmann-Roberson-Walker-Metric are:

FRW Equation (I)

$$\frac{\dot{R}^2}{R^2} = \frac{8\pi G\rho}{3} + \frac{\Lambda c^2}{3} - \frac{kc^2}{R^2} \quad (2.1)$$

FRW Equation (II)

$$\frac{\ddot{R}}{R} = \frac{\Lambda c^2}{3} - \frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) \quad (2.2)$$

Einstein abandoned the cosmological term Λ as his "greatest blunder" after Hubble's 1928 discovery that the distant galaxies are expanding away from each other. Within a Universe with ideal Quantum gas and without cosmological term Λ and geometry factor $k = 1$ the equation (2.1) and (2.2) will become:

FRW Equation (I)

$$\frac{\dot{R}^2}{R^2} = \frac{8\pi G\rho}{3} - \frac{c^2}{R^2} \quad (2.3)$$

and FRW Equation (II)

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) \quad (2.4)$$

With the relation $p = \frac{\rho c^2}{3}$ (Quantum gas) will change (2.4) as follows: FRW Equation (II)

$$\frac{\ddot{R}}{R} = -\frac{8\pi G\rho}{3} \quad (2.5)$$

Within Thermodynamics we assume $dE = TdS - pdV$ and an adiabatic process it holds $TdS = 0$. We become $d(\epsilon V) = d\epsilon V + \epsilon dV = -pdV$ and it follows:

With $d\epsilon = -(\epsilon + p)\frac{dV}{V}$ and the relation $p = \frac{\epsilon}{3}$ we assume:

$$\frac{d\epsilon}{\epsilon} = -\frac{4dV}{3V} \; or \; \epsilon \sim V^{-4/3} \sim R^{-4} \quad (2.6)$$

3 The Cosmic Background Radiation

Now we know the energy-density in the Planck-Era $\rho c^2 = \tilde{a}\Delta T^4 = \frac{3c^7}{8\pi hG}(\tilde{a} = Radiation\ constant = 7.5657e^{-16})$, If we assume $c = h = G = k_B = 1$ we get:

$$\frac{3}{8\pi} = \tilde{a}\Delta T^4 = \frac{8\pi^5}{15}T^4 \; or \; T^4 = \frac{45}{64\pi^6}$$

Now we get the Planck-Temperature $\Delta T = (\frac{45}{64\pi^6})^{1/4} = \frac{1}{6.08088337383} = 5.8404e^{31}\ K$

In Planck-Era the following 6 relations are valid:

$\Delta m\ \Delta x = \frac{h}{c}$ (3.1)

$\Delta m\ \Delta t = \frac{h}{c^2}$ (3.2)

$\frac{\Delta m}{\Delta a} = \frac{h}{c^3}$ (3.3)

$\frac{\Delta m}{\Delta x} = \frac{c^2}{G}$ (3.4)

$\frac{\Delta m}{\Delta t} = \frac{c^3}{G}$ (3.5)

$\Delta F = \Delta m\ \Delta a = \frac{c^4}{G}$ (3.6)

Now we could calculate the CBR (T_γ) as follows:

$$E_\gamma = \frac{hc}{x} = 6.08088337383 k T_\gamma\ with\ T_\gamma = 2.725K$$

We could calculate $m_\gamma = 2.5444e^{-39}kg$ and $x = \lambda_\gamma = 8.6828e^{-4}m$, also a $t_\gamma = \frac{\lambda_\gamma}{c} = 2.8963e^{-12}s$.

For the Bekenstein-Hawking-Radiation we receive:

$$m_{BH}c^2 = \frac{hc}{R}$$
$$E_{BH} = m_{BH}ax = \frac{h\nu}{c^2}\frac{m_{BH}c^3}{h}\frac{c}{\nu} = m_{BH}c^2$$

4 Gravitation as curvature of space

In macroscopic Scale the equation (3.1) til (3.6) will we rewritten as: (Entropy constant $\zeta = \frac{\Delta T}{T_\gamma} = 2.1432e^{31}$)

$M\ R = \zeta^4 \frac{h}{c}$ (4.1)

$M\ t = \zeta^4 \frac{h}{c^2}$ (4.2)

$\frac{M}{a} = \zeta^4 \frac{h}{c^3}$ (4.3)

$\frac{M}{R} = \frac{c^2}{2G}$ (4.4)

$\frac{M}{t} = \frac{c^3}{2G}$ (4.5)

$\Delta F = M\ a = \frac{c^4}{G}$ (4.6)

With $a = \frac{G\ M}{R^2} = \frac{M\ c^3}{\zeta^4 h}$ follows:

$$\frac{G}{R^2} = \frac{c^3}{\zeta^4 h} \quad (4.7)$$

Furthermore we receive from GR:

$$R = \zeta^2\ \Delta x => Radius\ of\ Universe\ R = 1.861e^{28} m$$
$$\frac{M}{R} = \frac{c^2}{2G} = \frac{\Delta m}{\Delta x} => Mass\ of\ Universe\ M = 1.253e^{55} kg$$
$$\frac{M}{t} = \frac{c^3}{2G} = \frac{\Delta m}{\Delta t} => Age\ of\ Universe\ t = 6.207e^{19} s$$

For $\dot{R}^2 = \frac{2GM}{R}$ is with (4.7): $\dot{R}^2 = 2MR\frac{c^3}{\zeta^4 h} = c^2$

The FRW Gleichung (I) (2.3) is as follows:

$$\frac{c^2}{R^2} = \frac{8\pi G\rho}{3} - \frac{c^2}{R^2}$$

or

$$\frac{1}{R^4} = \frac{4\pi\rho c^2}{3\zeta^4 hc}$$

We become the R^4 dependency of (2.6) as follows:

$$\frac{3\zeta^4 hc}{4\pi R^4} = \rho c^2 = \tilde{a} T_\gamma^4$$

We get also:

$$\frac{1}{R} = \frac{Mc}{\zeta^4 h}$$

5 References

1. A.Einstein, Sitz. Preuss. Akad. d. Wiss. Phys.-Math 142 (1917)
2. A.Friedman, Über die Krümmung des Raumes, Zeitschrift für Physik 10 (1): 377 – 386. (1922)
3. V.Sahni, The Case for a Positive Cosmological Λ-Term, astro-ph/9904398
4. S.M.Carroll, The Cosmological Constant, astro-ph/0004075
5. A.I.Arbab, Nonstandard Cosmology With Constant and Variable Gravitational and Variable Cosmological Constants and Bulk Viscosity, http://arxiv.org/pdf/gr-qc/0105027.pdf
6. M.S.Berman, Gen. Rel. Gravit. 23, 465(1991)