There are Infinitely Many Sets of N-Odd Primes and Pairs of Consecutive Odd Primes

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Abstract

Let us consider positive odd numbers which share a prime factor>1 as a kind, then the positive directional half line of the number axis consists of infinite many equivalent line segments on same permutation of χ kinds' odd points plus odd points amongst the χ kinds' odd points, where $\chi \geq 1$. We will prove together that there are infinitely many sets of n-odd prime numbers and pairs of consecutive odd prime numbers by the mathematical induction with aid of such equivalent line segments and odd points thereof, in this article.

Keywords

Sets of n-odd prime numbers, Pairs of consecutive odd prime numbers, Mathematical induction, Odd points, Positive directional half line of the number axis, RLSS $_{№1\sim№\chi}$, Sets of •μ(•s)+b(◦s)•, Pairs of •υ(◦s)•, The coexisting theorem, №1 RLS $_{№1\sim№\chi}$, Set of ♠μ(♠s)+b(◦s)♠, Pair of ♠υ(◦s)♠.

Basic Concepts

Suppose n >1, and $\kappa_1<\kappa_2<...<\kappa_{n-1}$ are n-1 natural numbers, and J_χ, $J_\chi+\kappa_1$, $J_\chi+\kappa_2$, $J_\chi+\kappa_3$, ... $J_\chi+\kappa_{n-1}$ are all odd prime numbers, then we call $(J_\chi,J_\chi+\kappa_1,J_\chi+\kappa_2,J_\chi+\kappa_3,...J_\chi+\kappa_{n-1})$ a set of n-odd prime numbers. Thereupon we conjecture that for any positive odd prime number J_p, if a number of residue's classes which n integers 0, κ_1, ... κ_{n-1} divide respectively by modulus J_p is less than J_p, then there are infinitely many sets of n-odd prime numbers which differ orderly by κ_1, $\kappa_2-\kappa_1$, $\kappa_3-\kappa_2$,...and $\kappa_{n-1}-\kappa_{n-2}$. We term the conjecture as n-odd prime numbers' conjecture. For example, when $n \geq 2$, and $\kappa_1=2$, it contains twin prime numbers' conjecture. In addition, it contains 3-odd prime numbers' conjecture when $n \geq 3$, $\kappa_1=2$ and $\kappa_2=6$. And so on and so forth…

Evidently, if modulus $J_p \geq J_\chi+\kappa_{n-1}$, then each odd prime number of such a set of odd numbers belongs in a residue class, thus number n of n-odd prime numbers is less than J_p. If modulus $J_p \leq J_\chi$, then number n of n-odd prime numbers may be greater than J_p. For example, a set of 16-odd prime numbers (13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73) for modulus J_4 (i.e. 11), it has 16 odd prime numbers of 10 residue's classes because 17≡61(mod 11), 19≡41(mod 11), 23≡67(mod 11), 29≡73 (mod 11), 31≡53(mod 11), and 37≡59(mod 11) plus 13, 43, 47, and 71.

In addition, there is such a conjecture, namely if there is a pair of consecutive odd prime numbers which differ by 2k, then there are surely infinitely many pairs of consecutive odd prime numbers which differ by 2k, where k is a natural number. This conjecture needs still us to prove it. When k=1, it is the very twin prime numbers' conjecture evidently.

Everyone knows, each and every odd point at positive directional half line of the number axis expresses a positive odd number. Also infinite many a distance between two consecutive odd points at the positive directional half line equal one another.

Let us use the symbol "•" to denote an odd point, whether • is in a formulation or it is at the initial positive directional half line of the number axis. Moreover the positive directional half line is marked merely with symbols of odd points. Please, see following first illustration.

First Illustration

We use also symbol "•s" to denote at least two odd points in formulations. Then, the number axis's positive directional half line which begins with odd point 3 is called the half line for short thereinafter.

We consider smallest positive odd prime number 3 as №1 odd prime number, and consider positive odd prime number J_χ as №χ odd prime number, where $\chi \geq 1$, then odd prime number 3 is written as J_1 as well.

And then, we consider positive odd numbers which share prime factor J_χ as №χ kind of odd numbers. If an odd number contains α one another's-different prime factors, then the odd number belongs in α kinds of odd numbers concurrently, where $\alpha \geq 1$.

There is an only odd prime number J_χ within №χ kind's odd numbers. Excepting J_χ, we term others as №χ kind of odd composite numbers.

If one • is defined as an odd composite point, then we must change symbol "◦" for its symbol "•". And use symbol "◦s" to denote at least two definite odd composite points in formulations.

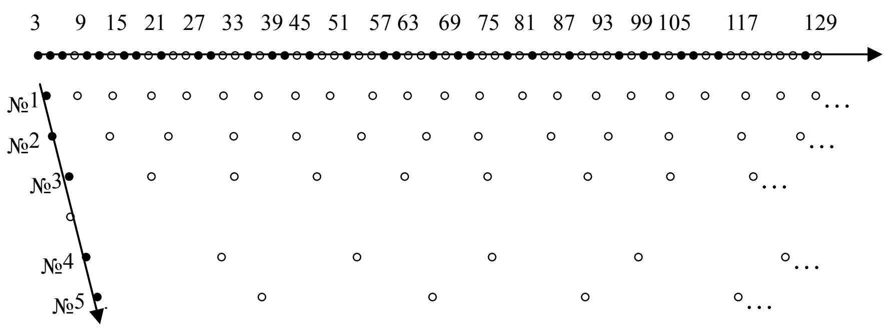
In course of the proof, we shall change ◦s for •s at places of $\sum$№χ $[\chi \geq 1]$

kind's odd composite points according as χ is from small to large.

Since №χ kind's odd numbers are infinitely many a product which multiplies every odd number by J_χ, so there is a №χ kind's odd point within consecutive J_χ odd points at the half line.

Therefore any one another's permutation of χ kind's odd points plus odd points amongst the χ kind's odd points assumes always infinite many recurrences on same pattern at the half line, irrespective of their prime/composite attribute.

We analyze seriatim №χ kind of odd points at the half line according to χ =1, 2, 3 … in one by one, and range them as second illustration.



Second Illustration

We consider one another's equivalent shortest line segments at the half line in accordance with same permutation of χ kinds' odd points plus odd points amongst the χ kinds' odd points as recurring segments of the χ kinds' odd points.

We use character "$RLS_{№1\sim№\chi}$" to express a recurring segment of $\sum$№χ [$\chi \geq 1$] kind of odd points, and use character "$RLSS_{№1\sim№\chi}$" to express the plural. If one • is affirmed as an odd prime point, then this • is rewritten as one ♠ at the half line and/or in formulations, and symbol ♠s express at least two odd prime points in formulations. For example, the one another's permutation of certain kinds of odd points at №1 $RLS_{№1\sim№4}$, please, see following third illustration.

The permutation of odd prime & composite points at №1 $RLS_{№1\sim№4}$

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Third Illustration

Annotation: “♠” denotes an odd prime point; “○” denotes an odd composite point.

№1 $RLS_{№1}$ ends with odd point 7; №1 $RLS_{№1\sim№2}$ ends with odd point 31; №1 $RLS_{№1\sim№3}$ ends with odd point 211; №1 $RLS_{№1\sim№4}$ ends with odd point 2311.

Justly №1 $RLS_{№1\sim№\chi}$ begins with odd point 3. There are $\prod J_\chi$ odd points at each $RLS_{№1\sim№\chi}$, where $\chi \geq 1$, and $\prod J_\chi = J_1 * J_2 * \ldots * J_\chi$.
Undoubtedly one $RLS_{№1\sim№(\chi+1)}$ consists of consecutive $J_{\chi+1}$ $RLSS_{№1\sim№\chi}$, and they link one by one.

Since none of any kind's odd composite points coincides with odd point 1 on the left of №1 $RLS_{№1\sim№\chi}$, then none of any kind's odd composite points coincides with the odd point which closes on the left of №2 $RLS_{№1\sim№\chi}$ according to the definition of recurring segments of the χ kinds' odd points. The odd point which closes on the left of №2 $RLS_{№1\sim№\chi}$ is exactly the most right odd point of №1 $RLS_{№1\sim№\chi}$. Thus the most right odd point of №1 $RLS_{№1\sim№\chi}$ is an odd prime point always. Namely $2\prod J_\chi+1$ is an odd prime number always.

Number the ordinals of odd points at seriate each $RLS_{№1\sim№\chi+y}$ by consecutive natural numbers which begin with 1, namely from left to right each odd point at seriate each $RLS_{№1\sim№\chi+y}$ is marked with from small to great a natural number ≥ 1 in the proper order, where $y \geq 0$.
Then, there is one №(χ+y) kind's odd point within $J_{\chi+y}$ odd points which share an ordinal at $J_{\chi+y}$ $RLSS_{№1\sim№(\chi+y-1)}$ of a $RLS_{№1\sim№\chi+y}$.
Furthermore, there is one №(χ+y) kind's odd composite point within $J_{\chi+y}$ odd points which share an ordinal at $J_{\chi+y}$ $RLSS_{№1\sim№(\chi+y-1)}$ of seriate each

$RLS_{№1\sim№\chi+y}$ on the right of №1 $RLS_{№1\sim№\chi+y}$.

Odd prime points $J_1, J_2 \dots J_{\chi-1}$ and J_χ are at №1 $RLS_{№1\sim№\chi}$. Yet, there are χ odd composite points on ordinals of J_1 plus J_2 ...plus $J_{\chi-1}$ plus J_χ at seriate each $RLS_{№1\sim№\chi}$ on the right of №1 $RLS_{№1\sim№\chi}$. Thus №1 $RLS_{№1\sim№\chi}$ is a particular $RLS_{№1\sim№\chi}$ in contradistinction to each of others.

After change ◦s for •s at places of ∑№χ [$\chi \geq 1$] kind's odd composite points at the half line, if one • is separated from another • by μ •s plus b ◦s irrespective of their permutation, then express such a combinative form as a set of • μ(•s)+b(◦s) •, where $\mu \geq 0$, and $b \geq 0$.

If μ+2 •s of • μ(•s)+b(◦s) • are all defined as odd prime points, then the set of • μ(•s)+b(◦s) • is rewritten as a set of ♠ μ(♠s) +b(◦s) ♠. Further, if the set of ♠μ(♠s)+b(◦s) ♠ lies within consecutive J_χ odd points, and for odd prime number J_χ, a number of residue's classes which μ+2 odd prime numbers whereof μ+2 ♠s express divided respectively by modulus J_χ is less than J_χ, then, such a set of ♠ μ(♠s) +b(◦s) ♠ is the very a set of n-odd prime points, where $n=\mu+2$.

If two •s of • υ(◦s) • are defined as odd prime points, then the pair of •υ(◦s)• is rewritten as a pair of ♠ υ(◦s) ♠ , where $\upsilon \geq 0$.

When μ=0, a set of • μ(•s)+b(∘s) • is exactly a pair of •b(∘s)•, and a set of ♠ μ(♠s)+b(∘s) ♠ is exactly a pair of ♠ b(∘s) ♠ , where $b \geq 0$.

Let μ+ b=m, a set of • μ(•s)+b(∘s) • may be written as a set of • m(•s∘s) •, and a set of ♠ μ(•s)+b(∘s) ♠ may be written as a set of ♠ m(•s∘s) ♠ .

After change ∘s for •s at places of ∑№χ [$\chi \geq 1$] kind's odd composite points, $J_{\chi\text{-}h}$ at №1 $RLS_{№1\sim№\chi}$ is defined as an odd prime point, where $\chi > h \geq 0$, yet there are infinitely many •s on the right of J_χ at the half line, and every • is an undefined odd point on prime/composite attribute. Anyhow every prime factor of an odd number which each • at the right of J_χ expresses is greater than J_χ.

A set of • μ(•s)+b(∘s) • is negated according as any • of the set is defined as one ∘. Also a pair of • υ(∘s) • is negated according as either • of the pair is defined as one ∘. If a set of • μ(•s)+b(∘s) • can not always be negated, then it is precisely a set of ♠ μ(♠s)+b(∘s) ♠. Likewise, if a pair of • υ(∘s) • can not always be negated, then it is precisely a pair of ♠ υ(∘s) ♠.

From the definition for recurring segments of χ kinds' odd points, we can conclude that after change ∘s for •s at places of ∑№χ [$\chi \geq 1$] kind's odd composite points, if there is a set of ♠ μ(♠s)+b(∘s) ♠ within consecutive J_χ odd points on the right of J_χ at №1 $RLS_{№1\sim№\chi}$, then there is surely a set of

• $\mu(\bullet s)+b(\circ s)$ • on ordinals of the set of ♠ $\mu(♠s)+b(\circ s)$ ♠ at seriate each $RLS_{№1\sim№\chi}$ on the right of №1 $RLS_{№1\sim№\chi}$.

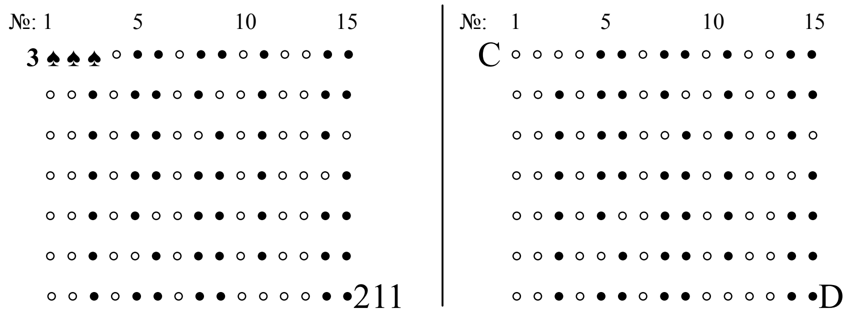
Without doubt, the converse proposition is tenable too. Namely after change ◦s for •s at places of ∑№χ [$\chi \geq 1$] kind's odd composite points, if there is a set of •$\mu(\bullet s)+b(\circ s)$ • within consecutive J_χ odd points at seriate each $RLS_{№1\sim№\chi}$ on the right of №1 $RLS_{№1\sim№\chi}$, and from left to right №k odd prime points of all sets of •$\mu(\bullet s)+b(\circ s)$ • share an ordinal, then there is surely a set of ♠ $\mu(♠s)+b(\circ s)$ ♠ on ordinals of any such set of • $\mu(\bullet s)+b(\circ s)$ •, at №1 $RLS_{№1\sim№\chi}$, where k = 1, 2, … μ+2.

Of course, every ♠ of the set of ♠ $\mu(♠s)+b(\circ s)$ ♠ and every prime factor of an odd number which each • of every such set of • $\mu(\bullet s)+b(\circ s)$ • expresses are greater than J_χ .

To be brief, after change ◦s for •s at places of ∑№χ [$\chi \geq 1$] kind's odd composite points, a set of ♠$\mu(♠s)+b(\circ s)$ ♠ on the right of J_χ at №1 $RLS_{№1\sim№\chi}$ and infinite many sets of •$\mu(\bullet s)+b(\circ s)$• on ordinals of the set of ♠$\mu(♠s)+b(\circ s)$♠ at seriate $RLSS_{№1\sim№\chi}$ on the right of №1 $RLS_{№1\sim№\chi}$ coexist at the half line.

We term the aforesaid conclusion as the coexisting theorem of a set of ♠$\mu(♠s)+b(\circ s)$♠ plus infinite many sets of •$\mu(\bullet s)+b(\circ s)$• at the half line, or term it as the coexisting theorem for short.

$J_{\chi+1}$ $RLSS_{№1\sim№\chi}$ of any $RLS_{№1\sim№(\chi+1)}$ may be folded at an illustration, one by one, so as to view conveniently, e.g. №1, №2 and №3 kinds' odd points at two $RLSS_{№1\sim№3}$ from the differentia, please, see following fourth illustration.



Fourth Illustration

After change ◦s for •s at places of №1 plus №2 plus №3 kinds' odd composite points, every ♠ denotes a definite odd prime point, and every • denotes an undefined odd point at prime/composite attribute, and every ◦ denotes a definite odd composite point, in the illustration. Line segment 3(211) is №1 $RLS_{№1\sim№3,}$ and line segment CD is any of seriate $RLSS_{№1\sim№3}$ on the right of №1 $RLS_{№1\sim№3}$.

The Proof

We will prove together that there are infinitely many sets of n-odd prime numbers and pairs of consecutive odd prime numbers by the mathematical induction with the aid of $RLSS_{№1\sim№\chi}$ and odd points thereof, thereinafter.

1. When χ=1, there is a set of ♠ ♠ alone on the right of J_1 at №1 $RLS_{№1}$, and the set of ♠ ♠ is a pair of ♠$\upsilon_{1(\circ s)}$♠ as well, i.e. twin odd prime points 5 and 7, where $\upsilon_1=0$.

When χ=2, there are ♠ ◦ ♠ ♠ ◦ ♠ ♠ ◦ ♠ ◦ ◦ ♠ ♠ on the right of J_2 at №1 $RLS_{№1\sim№2,}$ and these odd points contain several sets of ♠$\mu_{2(♠s)}+b_{2(\circ s)}$♠

within consecutive J_s odd points, including several pairs of ♠ $\upsilon_{2(°s)}$ ♠ within them, where $\mu_2 \leq 6$, $b_2 \leq 5$, $J_1 \leq J_s \leq J_5$, and $\upsilon_2 \leq 2$.

Evidently these pairs of ♠$\upsilon_{2(°s)}$♠ contain pairs of twin odd prime points.

When $\chi=3$, there are both sets of ♠ $\mu_{3(♠s)}+b_{3(°s)}$ ♠ within consecutive J_f odd points and pairs of ♠ $\upsilon_{3(°s)}$ ♠ on the right of J_3 at №1 $RLS_{№1\sim№3,}$ where $\mu_2 \leq \mu_3 \leq 41$, $b_2 \leq b_3 \leq 58$, $J_s \leq J_f \leq J_{27}=101$, and $\upsilon_2 \leq \upsilon_3=0$, 1, 2, 3, 4, 5 and 6.

Evidently these sets of ♠$\mu_{3(♠s)}+b_{3(°s)}$♠ embody certain sets of ♠$\mu_{2(♠s)}+b_{2(°s)}$♠, and these pairs of ♠ $\upsilon_{3(°s)}$ ♠ embody all pairs of ♠$\upsilon_{2(°s)}$♠.

For pairs of ♠$\upsilon_{3(°s)}$♠ on values of υ_3 at №1 $RLS_{№1\sim№3}$, we instance (11, 13), (13, 17), (23, 29), (89, 97), (139, 149), (199, 211) and (113, 127). Please, see preceding third illustration once again.

When $\chi=4$, there are both sets of ♠ $\mu_{4(♠s)}+b_{4(°s)}$ ♠ within consecutive J_a odd points and pairs of ♠ $\upsilon_{4(°s)}$ ♠ on the right of J_4 at №1 $RLS_{№1\sim№4\ ,}$ where $\mu_3 \leq \mu_4 \leq 337$, $b_3 \leq b_4 \leq 813$, $J_f \leq J_a \leq J_{189}=1151$, and $\upsilon_3 \leq \upsilon_4=0$, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 16.

Evidently these sets of ♠$\mu_{4(♠s)}+b_{4(°s)}$♠ embody certain sets of ♠$\mu_{3(♠s)}+b_{3(°s)}$♠, and these pairs of ♠ $\upsilon_{4(°s)}$ ♠ embody all pairs of ♠ $\upsilon_{3(°s)}$ ♠.

For pairs of ♠$\upsilon_{4(°s)}$♠ on values of υ_4 at №1 $RLS_{№1\sim№4}$, we instance (17, 19), (19, 23), (31, 37), (89, 97), (139, 149), (211, 223), (293, 307), (1831, 1847), (1259, 1277), (887, 907), (1669, 1693), (2179, 2203) and (1327, 1461). Please, see preceding third illustration once more.

2. When $\chi=\beta\geq4$, suppose that there are both sets of ♠ $\mu_{\beta(♠s)}+b_{\beta(\circ s)}$ ♠ within consecutive J_b odd points and pairs of ♠ $\upsilon_{\beta(\circ s)}$ ♠ on the right of J_β at №1 $RLS_{№1\sim№\beta}$, where $\mu_\beta\geq\mu_4$, $b_\beta\geq b_4$, $\upsilon_\beta\geq\upsilon_4$, $J_b\geq J_a$, and $J_\beta\geq J_4$. In addition, these sets of ♠ $\mu_{\beta(♠s)}+b_{\beta(\circ s)}$ ♠ embody any of sets of n-odd prime points on the right of J_1 at №1 $RLS_{№1\sim№\psi}$, and these pairs of ♠ $\upsilon_{\beta(\circ s)}$ ♠ embody any of pairs of consecutive odd prime points at №1 $RLS_{№1\sim№\psi}$, where $\psi<\beta$.

Let us suppose that any of sets of n-odd prime points on the right of J_1 at №1 $RLS_{№1\sim№\psi}$ is a set of ♠ $\mu_{p(♠s)}+b_{q(\circ s)}$ ♠; and any of pairs of consecutive odd prime points at №1 $RLS_{№1\sim№\psi}$ is a pair of ♠ $\upsilon_{\delta(\circ s)}$ ♠, where $\mu_p\geq\mu_4$, $b_q\geq b_4$, and $\upsilon_\delta\geq\upsilon_4$.

3. When $\chi=\eta>\beta$, prove that there are both sets of ♠ $\mu_{\eta(♠s)}+b_{\eta(\circ s)}$ ♠ within consecutive J_c odd points and pairs of ♠ $\upsilon_{\eta(\circ s)}$ ♠ on the right of J_η at №1 $RLS_{№1\sim№\eta}$, where $\mu_\eta\geq\mu_\beta$, $b_\eta\geq b_\beta$, $\upsilon_\eta\geq\upsilon_\beta$, $J_c\geq J_b$, and $J_\eta>J_\beta$. In addition, these sets of ♠ $\mu_{\eta(♠s)}+b_{\eta(\circ s)}$ ♠ must embody a set of ♠ $\mu_{p(♠s)}+b_{q(\circ s)}$ ♠ which needs us to prove, and these pairs of ♠ $\upsilon_{\eta(\circ s)}$ ♠ must embody a pair of ♠ $\upsilon_{\delta(\circ s)}$ ♠ which needs us to prove.

Proof . Since there is a set of ♠ $\mu_{p(♠s)}+b_{q(\circ s)}$ ♠ within consecutive J_b odd points on the right of J_β at №1 $RLS_{№1\sim№\beta}$, furthermore, we name the set of

$♠\mu_{p(♠s)}+b_{q(\circ s)}♠$ "$J_{\beta+d}\ \mu_{p(♠s)}+b_{q(\circ s)}\ J_g$", where $d\geq 1$ and $g=\beta+d+\mu_p+1$. Well then, let us first prove that there is a set of $♠\mu_{p(♠s)}+b_{q(\circ s)}♠$ on ordinals of $J_{\beta+d}\ \mu_{p(♠s)}+b_{q(\circ s)}\ J_g$ on the right of J_g at №1 $RLS_{№1\sim№g}$, hereinafter.

We know that every odd number >1 has a smallest prime factor except for 1 surely, yet the smallest prime factor of any odd prime number is exactly it itself.

If greatest one within respective smallest prime factors of b_q odd composite numbers whereof $b_{q(\circ s)}$ between $J_{\beta+d}$ and J_g express is written as J_φ, then the set of $J_{\beta+d}\ \mu_{p(♠s)}+b_{q(\circ s)}\ J_g$ is either at №1 $RLS_{№1\sim№\varphi}$ or out of №1 $RLS_{№1\sim№\varphi}$. If it is at №1 $RLS_{№1\sim№\varphi}$, then let $1\leq\chi_1\leq\varphi$. If it is out of №1 $RLS_{№1\sim№\varphi}$, then suppose that it is just at №1 $RLS_{№1\sim№\kappa}$, but it is not at №1 $RLS_{№1\sim№\kappa-1}$, then $\kappa>\varphi$, and let $1\leq\chi_2\leq\kappa$.

If the set of $J_{\beta+d}\ \mu_{p(♠s)}+b_{q(\circ s)}\ J_g$ is at №1 $RLS_{№1\sim№\varphi}$, then after change ◦s for •s at places of $\sum №\chi_1\ [1\leq\chi_1\leq\varphi]$ kind's odd composite points, there is a set of $•\mu_{p(•s)}+b_{q(\circ s)}•$ on ordinals of $J_{\beta+d}\ \mu_{p(♠s)}+b_{q(\circ s)}J_g$ at seriate each $RLS_{№1\sim№\varphi}$ on the right of №1 $RLS_{№1\sim№\varphi}$.

If the set of $J_{\beta+d}\ \mu_{p(♠s)}+b_{q(\circ s)}\ J_g$ is just barely at №1 $RLS_{№1\sim№\kappa}$, but it is out of №1 $RLS_{№1\sim№(\kappa-1)}$, then after change ◦s for •s at places of $\sum №\chi_2\ [1\leq\chi_2\leq\kappa]$ kind's odd composite points, there is a set of $•\mu_{p(•s)}+b_{q(\circ s)}•$ on ordinals of

$J_{\beta+d}\ \mu_{p(\spadesuit s)}+b_{q(\circ s)}\ J_g$ at seriate each $RLS_{\text{№}1\sim\text{№}\kappa}$ on the right of №1 $RLS_{\text{№}1\sim\text{№}\kappa}$.

Either there is $J_\varphi \geq \mu_p+bq+2$ or $J_\kappa \geq \mu_p+bq+2$, uniformly let it to equal J_ν. If J_φ or $J_\kappa<\mu_p+bq+2$, then suppose that J_ν is the smallest odd prime number which is not smaller than μ_p+bq+2.

Each set of $\bullet\mu_{p(\bullet s)}+b_{q(\circ s)}\ \bullet$ on ordinals of $J_{\beta+d}\ \mu_{p(\spadesuit s)}+b_{q(\circ s)}\ J_g$ considering aforementioned either case is rewritten as a set of $\underline{\bullet}\ \mu_{p(\underline{\bullet} s)}+b_{q(\circ s)}\ \underline{\bullet}$.

If some set of $\underline{\bullet}\ \mu_{p(\underline{\bullet} s)}+b_{q(\circ s)}\underline{\bullet}$ is defined as a set of $\spadesuit\ \mu_{p(\spadesuit s)}+b_{q(\circ s)}\ \spadesuit$, then the set of $\spadesuit\ \mu_{p(\spadesuit s)}+b_{q(\circ s)}\ \spadesuit$ is rewritten as a set of $\underline{\spadesuit}\ \mu_{p(\underline{\spadesuit} s)}+b_{q(\circ s)}\ \underline{\spadesuit}$.

Let $\nu+1\leq\omega\leq g$, since there is one №ω kind's odd point within consecutive J_ω odd points, and there is one №ω kind's odd point within J_ω odd points which share an ordinal at seriate J_ω $RLSS_{\text{№}1\sim\text{№}\omega\text{-}1}$, therefore there is a series of results as the following.

After successively change ◦s for •s at places of № (ν+1) kind's odd composite points, there are both $(J_{\nu+1}\text{-}\mu_p)$ sets of $\underline{\bullet}\ \mu_{p(\underline{\bullet} s)}+b_{q(\circ s)}\ \underline{\bullet}$ and μ_p sets of $\underline{\bullet}\ _{(\mu_p\text{-}1)(\underline{\bullet} s)}+_{(b_q+1)(\circ s)}\ \underline{\bullet}$ at seriate each $RLS_{\text{№}1\sim\text{№}(\nu+1)}$ on the right of №1 $RLS_{\text{№}1\sim\text{№}(\nu+1)}$. Of course, every prime factor of an odd number which each $\underline{\bullet}$ at here expresses is greater than $J_{\nu+1.}$

After successively change ◦s for •s at places of № (ν+2) kind's odd composite points, there are both $(J_{\nu+1}-\mu_p)(J_{\nu+2}-1)$ sets of $\underline{\bullet}_{\mu_{p(\underline{\bullet}s)}+b_{q(\circ s)}}\underline{\bullet}$ and $\mu_p(J_{\nu+2}-1)$ sets of $\underline{\bullet}_{(\mu_p-2)_{(\underline{\bullet}s)}+(b_q+2)_{(\circ s)}}\underline{\bullet}$ at seriate each $RLS_{№1\sim№(\nu+2)}$ on the right of №1 $RLS_{№1\sim№(\nu+2)}$. Of course, every prime factor of an odd number which each $\underline{\bullet}$ at here expresses is greater than $J_{\nu+2}$.

And so on and so forth…

Up to after successively change ◦s for •s at places of №g kind's odd composite points, there are both $(J_{\nu+1}-\mu_p)(J_{\nu+2}-1)(J_{\nu+3}-1)\ldots(J_g-1)$ sets of $\underline{\bullet}_{\mu_{p(\underline{\bullet}s)}+b_{q(\circ s)}}\underline{\bullet}$ and $\mu_p(J_{\nu+2}-1)(J_{\nu+3}-1)\ldots(J_g-1)$ pairs of $\underline{\bullet}_{(\mu_p+b_q)_{(\circ s)}}\underline{\bullet}$ at seriate each $RLS_{№1\sim№g}$ on the right of №1 $RLS_{№1\sim№g}$。Of course, every prime factor of an odd number which each $\underline{\bullet}$ at here expresses is greater than J_g。

Since the half line on the right of №1 $RLS_{№1\sim№g}$ has infinitely many $RLSS_{№1\sim№g}$, thus there are both infinitely many sets of $\underline{\bullet}_{\mu_{p(\underline{\bullet}s)}+b_{q(\circ s)}}\underline{\bullet}$ and infinitely many pairs of $\underline{\bullet}_{(\mu_p+b_q)_{(\circ s)}}\underline{\bullet}$ at the half line after successively change ◦s for •s at places of ∑№ω [ν+1≤ω≤g] kind's odd composite points. Concurrently, there are infinitely many sets of $\underline{\bullet}_{(\mu_p-1)_{(\underline{\bullet}s)}+(b_q+1)_{(\circ s)}}\underline{\bullet}$, infinitely many sets of $\underline{\bullet}_{(\mu_p-2)_{(\underline{\bullet}s)}+(b_q+2)_{(\circ s)}}\underline{\bullet}$, … and infinitely many sets of $\underline{\bullet}_{1(\underline{\bullet})\,(\mu_p+b_q-1)_{(\circ s)}}\underline{\bullet}$ at the half line. Of course, every prime factor of an odd number which each $\underline{\bullet}$ within aforementioned sundry sets expresses is greater than J_g.

Thus there are a set of $\spadesuit\, \mu_{p\,(\spadesuit s)+b_{q}(°s)}\, \spadesuit$, a set of $\spadesuit_{(\mu_{p}-1)(\spadesuit s)+(b_{q}+1)(°s)}\spadesuit$, … a set of $\spadesuit_{\,1(\spadesuit)\,(\mu_{p}+b_{q}-1)(°s)}\spadesuit$, and a pair of $\spadesuit_{(\mu_{p}+b_{q})(°s)}\spadesuit$ on the right of J_g at №1 $RLS_{№1\sim№g}$ according to aforesaid that coexisting theorem.

Thus it can seen, preceding results contain such a conclusion, namely there is a set of $\spadesuit\mu_{p(\spadesuit s)+b_{q}(°s)}\spadesuit$ on the right of J_g at №1 $RLS_{№1\sim№g}$. This is just the proposition which need us to prove.

In case a pair of $\spadesuit\, \upsilon_{\delta(°s)}\, \spadesuit$ which needs us to prove is embodied within the set of $\spadesuit\, \mu_{p\,(\spadesuit s)+b_{q}(°s)}\, \spadesuit$ plus the set of $\spadesuit_{\,(\mu_{p}-1)(\spadesuit s)+(b_{q}+1)(°s)}\spadesuit$ …plus the set of $\spadesuit_{1(\spadesuit)\,(\mu_{p}+b_{q}-1)(°s)}\spadesuit$ plus the pair of $\spadesuit_{\,(\mu_{p}+b_{q})(°s)}\spadesuit$ on the right of J_g at №1 $RLS_{№1\sim№g}$, the pair of $\spadesuit\, \upsilon_{\delta(°s)}\, \spadesuit$ is proven synchronously into the real too.

If a pair of $\spadesuit\, \upsilon_{\delta(°s)}\, \spadesuit$ which needs us to prove is not embodied within aforementioned sundry sets of n-odd prime points, then we can likewise apply the aforesaid way of doing according to the coexisting theorem to prove and get that there is a pair of $\spadesuit\upsilon_{\delta(°s)}\spadesuit$ or a set of $\spadesuit\mu_{p\,(\spadesuit s)+b_{q}(°s)}\spadesuit$ which embodies such a pair of $\spadesuit\upsilon_{\delta(°s)}\, \spadesuit$ on the right of J_g at №1 $RLS_{№1\sim№g}$, but values of g on two places are perhaps unlike.

Since the mathematical induction sets up a claim to $\chi=\eta>\beta$, whereas now has $\chi=g=\beta+d+\mu_p+1>\beta$, thus can replace g by η, therefore, we have proven that there is a set of ♠μ_p(♠s)+b_q(°s) ♠ and a pair of ♠υ_{δ}(°s) ♠ on the right of J_η at №1 $RLS_{№1\sim№\eta}$.

When vest further χ with a value which is greater than g, we likewise can continue to apply the aforesaid way of doing and the coexisting theorem to prove and get that there are another set of ♠μ_p(♠s)+b_q(°s)♠ and another pair of ♠υ_{δ}(°s) ♠ . And so on and so forth…

Though values of χ are not consecutive natural numbers under the prerequisite that it is proven there are sets of ♠μ_p(♠s)+b_q(°s) ♠ and pairs of ♠υ_{δ}(°s) ♠ by the aforesaid way of doing and the coexisting theorem, but, since there are infinitely many natural numbers at all events, so that there are infinitely many values of χ which accord with the claim. Therefore there are both infinitely many sets of ♠μ_p(♠s)+b_q(°s) ♠ and infinitely many pairs of ♠υ_{δ}(°s) ♠ .

Since a set of ♠μ_p(♠s)+b_q(°s) ♠ expresses a set of n-odd prime numbers, where n=μ_P+2, consequently there are infinitely many sets of n-odd prime numbers.

In addition, a pair of ♠υ_{δ}(°s) ♠ expresses a pair of consecutive odd prime numbers which differ by 2(υ_δ+1), consequently there are infinitely many pairs of consecutive odd prime numbers which differ by 2k, where k=υ_δ+1.

Taken one with another, we have proven that there are both infinitely many sets of n-odd prime numbers and infinitely many pairs of consecutive odd prime numbers which differ by 2k.