APPROACH TO SOLVE P VS NP BY USING BIJECTION REDUCTION

KOBAYASHI, KOJI

1. Abstract

This article describes about that P is not NP by using bijection reduction between each problems. If injective reduction of each directions between CNFSAT and HornSAT exist, bijection between CNFSAT and HornSAT also exist. If P is NP, this bijection is polynomial time. But HornSAT description is polynomial complex and CNFSAT description is exponential complex. It means that there is no bijection in polynomial time. Therefore P is not NP.

2. Preparation

In this article, we will use words and theorems of References [1, 2, 3] in this paper. About problem and turing machine types, we use description as follows;

Definition 1. We will use the term "$A \preceq_M B$" that injection reduction from A to B that compute complexity class M exist, "$A \sim_M B$" that bijection reduction between A and B that compute complexity class M exist.

Define concrete problem as follows;

Definition 2. We will use the term "$HornSAT$" as a HornCNF Satisfiability problem set. To simplify, $p \in HornSAT$ description is arranged by HornCNF partially ordered set structure. We will use the term "$CNFSAT$" as a CNF Satisfiability problem set. To simplify, $p \in CNFSAT$ description is arranged by CNF clauses variables set.

Define problems cardinals within finite.

Definition 3. We will use the term "Problem cardinals" and "$|P|_n$" as a cardinals of problem that input length is n.

3. P is not NP

Prove $P \neq NP$ by using cardinals difference between $HornSAT$ and $CNFSAT$. All $A \in P$ have injection reduction to $HornSAT$ and all $B \in NP$ have injection reduction to $CNFSAT$. Therefore polynomial time reduction as bijection between $HornSAT$ and $CNFSAT$ exists if $P = NP$. But $HornSAT$ description is polynomial complex and $CNFSAT$ description is exponential complex. It means that there is no polynomail time reduction between $HornSAT$ and $CNFSAT$. Therefore $P \neq NP$.

Theorem 4. *Logarithm space reduction as injection from $A \in P$ to $HornSAT$ exist that output size bigger than input size. And polynomial time reduction as*

injection from $B \in NP$ to $CNFSAT$ exist that output size bigger than input size. That is,
$\forall A \in P\,(A \preceq_L HornSAT)$
$\forall B \in NP\,(B \preceq_P CNFSAT)$

Proof. This is trivial because some Turing Machine can compute output that include input structure. For example, output include input that will not affect $HornSAT$ and $CNFSAT$ clauses. Therefore, output become unique and bigger than input. □

Theorem 5. *If $P = NP$, there exists polynomial time bijection reduction between $HornSAT$ and $CNFSAT$. That is,*
$(P = NP) \rightarrow HornSAT \sim_p CNFSAT$

Proof. From $P = NP$, we can define injection reduction
$f : CNFSAT \rightarrow HornSAT$, $g : HornSAT \rightarrow CNFSAT$
and bijection $h : CNFSAT \rightarrow HornSAT$ is

$$h(x) = \begin{cases} f(x) & if\ \left(f^{-1} \circ g^{-1}\right)^k (x)\ exist\, and\, g^{-1} \circ \left(f^{-1} \circ g^{-1}\right)^k (x)\ not\, exist \\ g^{-1}(x) & \text{others} \end{cases}$$

from The Cantor-Bernstein-Schroeder theorem[4].

Mentioned above 4, pDTM can compute h because f^{-1}, g^{-1} reduce output size and $f^{-1} \circ g^{-1}$ can repeat atmost $O(n^c)$ times. Therefore, this theorem was shown. □

Theorem 6. $|HornSAT|_n = O(n^c)$, $|CNFSAT|_n = O(c^n)$

Proof. This is trivial by constraint of clauses description. HornSAT clauses have atmost one positive literal in each clauses. Therefore we can arrange HornSAT clauses by positive literal and matrix of negative literals existences. And this matrix have meaning Triangular matrix because each clauses imply positive literals by usnig unit resolution. Therefore $|HornSAT|_n = O(n^c)$.

But CNFSAT clauses have no limit like HornSAT. We can build CNFSAT as direct product of clauses that made same variables set. Therefore $|CNFSAT|_n = O(c^n)$. □

Theorem 7. $P \neq NP$

Proof. We prove it using reduction to absurdity. We assume that $P = NP$. Mentioned above 5, $HornSAT \sim_p CNFSAT$.

But mentioned above 6, $|HornSAT|_n = O(n^c)$ and $|CNFSAT|_n = O(c^n)$. Therefore bijection require $O(c^n)$ size $HornSAT$ to map to $CNFSAT$. Therefore pDTM cannot compute this bijection and contradict $P = NP$.

Therefore, this theorem was shown than reduction to absurdity. □

References

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