A pure geometric approach to derive quantum gravity from general relativity

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Abstract

In this paper scalar-tensor gravity is derived from the Schwarzschild solution of General Relativity. The solution is also extended to a maximal and complete manifold. A well-defined relationship between the scalar product of metric and the scalar field is revealed. A theoretical quantum test particle is constructed on the basis of Compton wavelength and General Relativity. It is also demonstrated how the rest mass of such particle depends on the background geometry of the space, which explains the correlation between the scalar field and the curvature.

Introduction

This paper describes a step-by-step transformation from general relativity to a certain kind of geometric quantum gravity. The basic concept was born in 1989 but it was first introduced only in 2000 [1]). At that time only the basic idea was outlined without extensive computations or detailed explanation of the whole approach. This approach is significantly different from the usual one.

This paper is a qualitative analysis based on well known textbook computations. The main purpose was rather to introduce and explain how this kind of quantum gravity related to general relativity, than to provide a general mathematical solution of this problem. Most of the results were obtained by using pure textbook computations. For the case of simplicity only the static, spherically symmetric case is covered here. Additional limitation is that albeit this approach might raise several new quantitative questions, most of them are not answered here.

The content of this paper is organized into three main chapters.

In the first chapter ("Geometry and topology of black holes") we analyze Schwarzschild metric, as the simplest solution of General Relativity. First we discuss the topology and the geometrical extensions of the solution, then we describe two steps transforming the solution into a form, which we consider being appropriate for creating quantum gravity.

In the second chapter ("Relationship between (static) geometry and gravity") we reveal that there is a well defined relationship between the static curvature of space – in this case the Ricci scalar – and the scalar gravitational potential.

In the third chapter ("Quantum test particle") as a final step we introduce a special object – a theoretical quantum particle – which is constructed on the basis of General Relativity. We demonstrate how the mass of such particle depends on the curvature of the space –explaining at least qualitatively the correspondence between the gravitation potential and background curvature.

Finally we try to sum up and draw a conclusion of the paper as well as to make some recommendations, wherever it seems to be reasonable.

Geometry and topology of black holes

There are several solutions of General Relativity. However the first and most frequently used one is the static and spherically symmetric – so called Schwarzschild solution. To keep things simple and to be able to focus on the main purpose of this paper we will also start over this special solution.

Standard coordinates

Schwarzschild solution of General Relativity uses standard coordinate system. In this special case these coordinates are often called Schwarzschild coordinates as well. The form of the metric is shown in equation (1) [3].

(1) $$ds^2 = A(r)dt^2 + B(r)dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

We can state that this is a usual approach describing space-time. The main property of this coordinate system is that the circumference of each centrally symmetric sphere can be expressed the same as in case of flat space. (See equation (2)) In general the distances are preserved on the surface of the centrally symmetric spheres.

(2) $$l = 2r\pi$$

At the same time radial distances can be expressed from the coordinates only by using the appropriate metric coefficient. Consequently Schwarzschild coordinate system – as a projection – does not preserve the shape of objects. (Which means that in case of a spherical object $\Delta x_{radial} < \Delta x_{\Theta} = \Delta x_{\varphi}$) We will come back to this statement later, when we compare coordinate systems, but first we will inspect the geometry of the Schwarzschild solution.

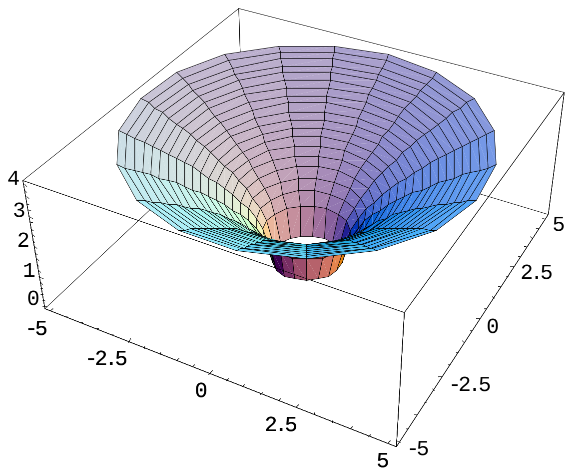
Geometry of the Schwarzschild black hole

The geometry of the Schwarzschild black hole is usually [4] explained by a section of the black hole at t=const., φ=Π/2 plane. The Gaussian curvature of this section is

(3) $$R_{Gauss} = -\frac{r_g}{2r^3} ,$$

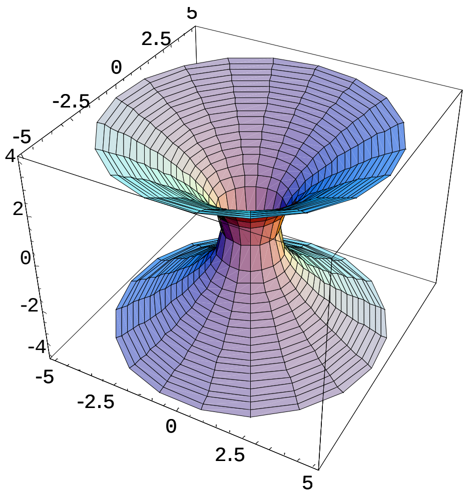
where ‘r_g‘ is the Schwarzschild radius and ‘r‘ is the radial coordinate. The equivalent surface of revolution – which has the same curvature – can be expressed as

(4) $$z = \sqrt{r_g(r - r_g)}$$



1. Geometry of the Schwarzschild black hole – equivalent surface of revolution

It can be seen both from equation (4) and figure 1 that there is a split at the event horizon. This split is caused by a coordinate singularity. There was always a natural intention to get rid of this singularity and continue the model beyond this point. The natural resolution of this question is so called “Einstein-Rosen Bridge” [5], which is constructed by merging two Schwarzschild black holes at the event horizon. (See figure 2. below) Let me remark, that this is a space-like extension, as we used t=const. surfaces.



2. The Einstein-Rosen Bridge

Isotropic coordinates

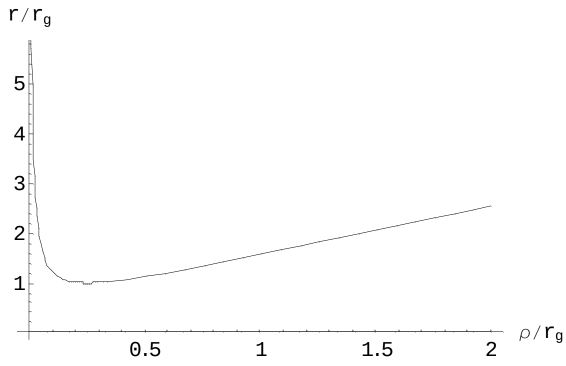
Schwarzschild solution can be transformed to isotropic – so called or conform-Euclidean – coordinates. In this case the form of the metric is

$$ds^2 = A'(\rho)dt^2 + B'(\rho)(d\rho^2 + \rho^2 d\theta^2 + \rho^2 \sin^2 \theta d\varphi^2) \qquad (5)$$

where ρ is the isotropic radial coordinate.

In isotropic coordinate the distances are not equal to coordinate distances in any direction; however the shape of the object is preserved. (In case of a spherical object $\Delta x_{radial}=\Delta x_{\Theta}=\Delta x_{\varphi}$) Change of coordinates from standard to isotropic form of metric (6) is well known from textbooks [3][4]. Here we provided the diagram as well (Figure 3), which helps us interpret the result of the transformation.

(6) $$r = \rho(1+\frac{r_g}{4\rho})^2$$



3. Standard radial coordinate depending on the isotropic radial coordinate

From the diagram it is obvious, that the standard radius can be defined as a function of the isotropic one, while the inverse of this function gives ambiguous results. Question arises: which coordinate system should we consider being the fundamental one? Well known solutions and coordinate systems are using or built on standard coordinates (Schwazschild, Eddington-Finkelstein, Kruskal-Szekeres, Penrose, Kerr [5]) but due to some major advantages we prefer using the isotropic one.

The main advantage of the isotropic approach is that the event horizon the boundary of the solution is naturally extended beyond the coordinate singularity. Additionally this extended manifold is found to be maximal, as the geodesics either can be extended to the infinity or terminates at the origin. Moreover the manifold is geodesically complete, as the coordinate origin behaves like infinity: the affine parameter along those geodesics, which are terminating there, can also be extended to infinite values. (The concept of Einstein-Rosen Bridge is also confirmed by this approach.) [5]

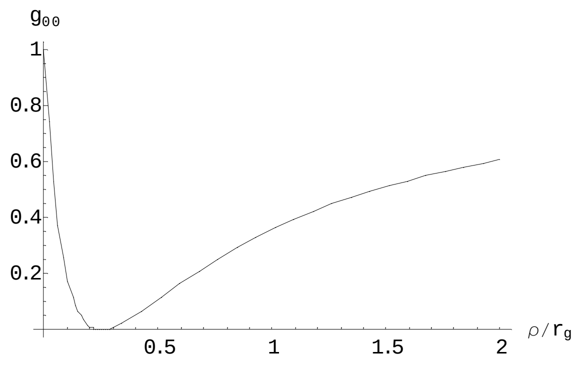
It is also obvious from the figure that the domain of standard coordinate system is restricted to the $r_g < r < \infty$ region. Any speculation regarding to the region "inside" the event horizon ($r < r_g$) is out of its domain therefore such result is physically meaningless. (Using isotropic coordinate system one can avoid this mistake and will find that Killing vector never gets space-like.)

Properties of isotropic metric

There are some additional interesting properties of the isotropic form of Schwarzschild solution, which can be read out from the metric. The metric itself can be found in textbooks:

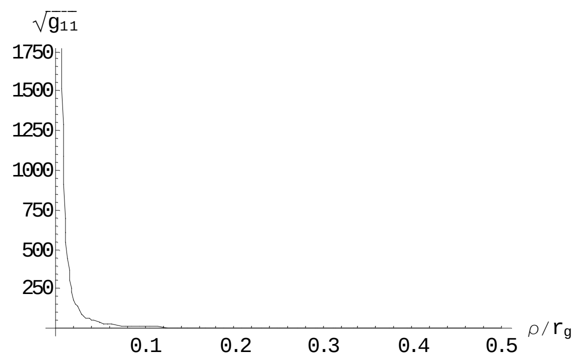
(7) $$ds^2 = \left(\frac{1-\frac{r_g}{4\rho}}{1+\frac{r_g}{4\rho}}\right)^2 dt^2 + \left(1+\frac{r_g}{4\rho}\right)^4 (d\rho^2 + \rho^2 d\theta^2 + \rho^2 \sin^2\theta d\varphi^2)$$

First we will have a look at g_{00}. (See Figure 4) At $\rho = r_g/4$ there is a minimum of g_{00}, which equals to zero. At the origin the value of g_{00} is one: the same value as it has at the infinity.



4. g_{00} of the isotropic metric of Schwarzschild black-hole

Then, if we have a look at g_{11}, we can see that its value becomes infinity at the origin. (See Figure 5) Moreover if we integrate the path along a geodesic we found that the path is infinite as well – as we mentioned previously.



5. $g_{11}^{1/2}$ of the isotropic metric of Schwarzschild black-hole

Therefore we can state that both g_{00} and g_{11} indicates that the coordinate origin behaves like infinity, which also confirms that this coordinate system reveal an extended Schwarzschild solution: both sides of the Einstein-Rosen Bridge. According to this approach we can interpret isotropic form of the solution so that there is another universe inside a Schwarzschild black hole. This picture might remind us of the multiversum theory of Lee Smolin [9][10].

Relationship between (static) geometry and gravity

Non-linearity and synchronization

General relativity is a non linear theory. Usually this is the first and main reason mentioned if someone tries to explain, why general relativity has no successful quantum theory. What does non-linearity mean? What is the cause of it? Is it possible to eliminate this problem? To answer these questions first we examine the mathematical representations we use.

In general we represent physical quantities with numbers, or vectors. Usually the union of two quantities is represented by the sum of the values assigned to the mentioned two quantities. Such representation is called linear representations. If the representation is non-linear, then for the addition first we have to transform the quantities to a linear scale, then the result has to be transformed back to the original, non-linear representation. For creating a quantum theory we prefer (and need) linear representation of quantities; therefore our first step is to eliminate this nonlinearity of general relativity. [12]

As we stated in earlier articles [2][6] the main problem is the nonlinearity of the mass and energy scale, which causes an ambiguity in the length scale as well. The root cause of the problem has been addressed by physicists in the early 60s, like Dicke [7], who wrote, that “(The statement that) a hydrogen atom on Sirius has the same diameter as one on Earth … is either a definition or else is meaningless.” The same holds for the rest mass of particles, which is in the context of General Relativity independent of the location “by definition”.

In case of a static space-time such nonlinearity can be eliminated by synchronization. [2][6] As a result of the synchronization we decompose the 4D space-time into the product of a 3D space by the real line, which might be referred in this special static case (if a preferred frame is bound to the Schwarzschild black-hole) as universal time or at least a special-relativistic time.. With this separation the mathematical complexity is reduced as well, therefore we can easily examine further properties of the solution. The real consequence is that energy can be expressed as

(8) $$\varepsilon_0 = -c\delta S / \delta x_0$$

(9) $$\varepsilon_0 = \frac{m_0 c^2 \sqrt{g_{00}}}{\sqrt{1-\frac{v^2}{c^2}}},$$

which is also well known from textbooks [3]

Assuming free fall this amount – the sum energy – remains unchanged during the movement of object or particle. If we have a closer look to this equation we will conclude that this is not pure tensor gravity any more, but scalar-tensor gravity, where the rest mass of particle is location dependent.[6] More precisely

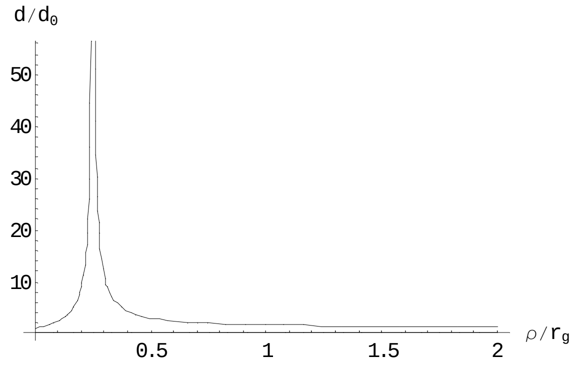
(10) $$m_r = m_0 \sqrt{g_{00}}$$

Naturally the equation of motion differs from the usual, as there is a new member [8][6] caused by the scalar field.

(11) $$(d/ds)(mu_i) - \frac{1}{2} m g_{jk,i} u^j u^k - m_{,i} = 0$$

This new, third member represents a real force, which is actually the Newtonian gravitational attraction.

After applying the aforementioned conform transformation the size of the particle is also not constant; it is inverse proportional to the rest mass [11]. At the event horizon, where g_{00} as well as m_r is zero the size of particle becomes infinite. (See the Compton-wavelength later.) We have to remark that infinite particle size indicates at least two things. One is that particle disperses at such place; the second is that in such circumstances the dispersed particle will not be point-like, therefore the gravitational field cannot be considered locally homogenous – violating the main assumption behind Einsteinian Equivalence Principle (EEP). It also means that the Einstein Vacuum Field Equation, and so General Relativity is not applicable there.

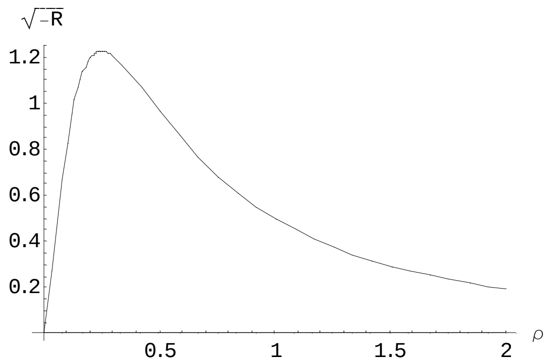


6. Locally measured relative size of particle

Curvature of 3D space

After we synchronized the space-time in this preferred frame with the applied conform-transformation, we have a 3D curved space and an universal time. The curvature of this 3D space can be expressed by the Ricci scalar, which is

(12) $$R = -2^{16}\frac{3r^4}{2(1+4r)^8}$$



7. Ricci scalar of the 3D Schwarzschild space

The diagram of curvature (Ricci scalar) confirms our assumption: that the space at the coordinate origin behaves the same as at the infinity: scalar curvature is zero at that point too. The curvature only has a single maximum and it is located at the event horizon.

Additionally the diagram of the curvature of 3D space and g_{00} before the synchronization seems to correlate; therefore we tried to find the exact correlation, which is found to be:

(13) $$g_{00} = 1 - \sqrt[4]{\frac{2}{3}(-R)}$$

Even if this expression is a heuristic one and might require at least some refinement in the current form, we concluded and proved that gravitational potential is related to (and can be expressed from) the geometry of space. This correspondence explicitly cancels the judgment of Brans and Dicke, who stated that scalar product of the metric is inappropriate to be a gravitational scalar field [8].

Let us remark, that this correspondence is not trivial at all, because in this extended domain ($0 < \rho < \infty$) neither g_{00} nor Ricci are bijective functions, consequently their inverse is not a function.

The last thing that we will analyze in this paper is: why and how the curvature affects the rest-mass of particles.

Quantum test particle

In this chapter we will introduce a theoretical quantum test particle [1], which is not only theoretical, but also instable construction. Still it is suitable for demonstrating how the curvature of space can affect the rest-mass of a particle.

The idea of quantum particle

There are two characteristic sizes or lengths that can be assigned to an elementary object. The first one is the Schwarzschild radius, which is

(14) $$r_s = \frac{2Gm}{c^2}$$

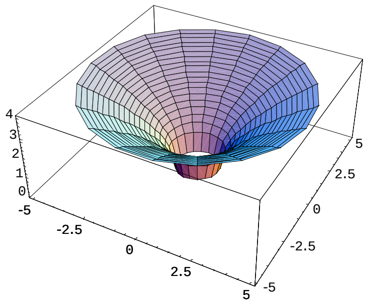
The second one is the Compton wavelength. If one would like to create standing wave from a radiation of Compton-wavelength as closed loop, the radius of this formation (hereafter **quantum radius - r_Q**) also depends on the number of full waves: a quantum number (hereafter "k"). The expression for the mentioned quantum radius is

(15) $$r_Q = \frac{k}{2\pi}\lambda_{Compton} = \frac{hk}{2\pi mc}$$

How can these two things be merged?

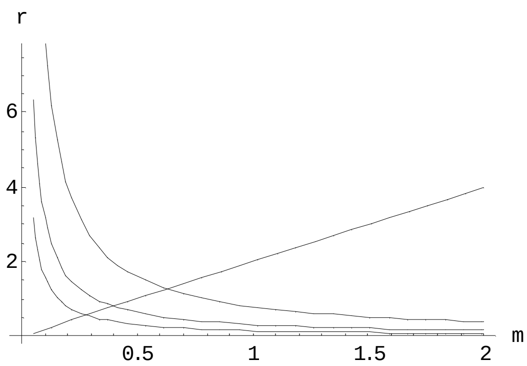
We can imagine a black hole. It is well known, that there is a certain sphere at $3/2\ r_s$, [3][4] which is suitable for creation of electromagnetic standing waves. We will call it **geometric radius (r_g)**. Of course such wave has its own energy that increases the rest mass of the actual black hole, but usually this increase is so small, that it can be neglected. Now let us imagine an extreme situation, when the black hole is so small that this increase is significant, in even more extreme situation the whole rest mass (energy) arises from this spherical standing wave. We call this latter formation a "theoretical quantum particle" [1].

The geometry of such formation is easy to imagine in two dimensions (see the figure below), but we need to extend it to 3 dimensions; so the standing wave must be spherical. We have to admit that the whole formation is not realistic, because even if the energy distribution is spherical, it is instable: any deviation causes the wave either to fall into the hole or disperse. Yet we will use this theoretical construction, because it is suitable for studying qualitatively how such kind of geometrical constructions behave in curved space.



8. Theoretical quantum-particle in 2D

There is quantum a solution for the particle size when the geometric and the quantum radiuses are equal ($r_g=r_Q$). Of course there are different solutions for each quantum number, which is shown on the following diagram.



9. Particle size (h=G=c=1) for different quantum numbers (k=1,2,5)

The exact solution for each quantum number also defines its rest mass. The mass of such theoretical quantum particle can be expressed as

$$m_k = \sqrt{k}\sqrt{\frac{hc}{8\pi G}} \quad (16)$$

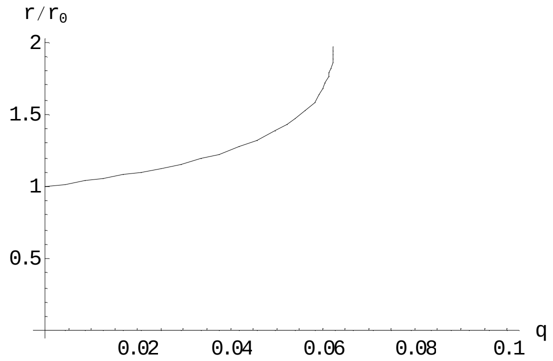
From this equation it seems that – in case h, c and G are real constants – rest mass is a definite and constant value for each quantum number.

The effect of background curvature

In the previous subchapter a geometric quantum particle has been introduced. Here we examine how the size and the rest mass of such particle are affected by the curvature of the background space. For the case of simplicity when we refer to its value, we use uniform background curvature in all directions and use the relative inverse radius of the corresponding 3-sphere ($q = r_S/R$).

First we examine how the geometric radius of a "particle" with constant mass depends on the background curvature. This dependency can be seen on equation 17 and figure 10.

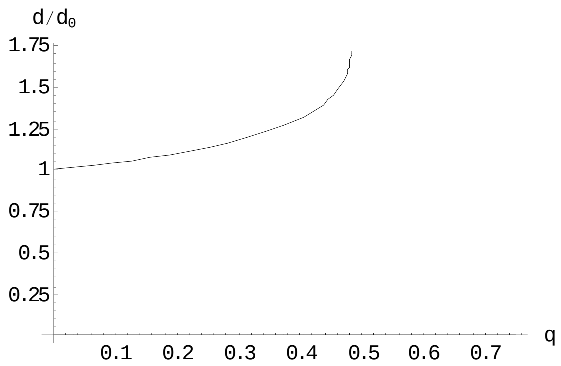
(17) $$r_g = \frac{3}{2} r_s \frac{1+\sqrt{1-16q}}{2q}$$



10. Relative change of the geometric radius depending on the background curvature

From the diagram we can read out not only that the size of the particle increases with the background curvature, but it seems that the size gets infinite at a certain, critical curvature level.

Going one step further we can recall that particle size is a quantum radius, which is related to the Compton wavelength, therefore if the size increases, then the rest mass and the corresponding Schwarzschild radius have to decrease – somewhat compensating the original change. (See Figure 11.)



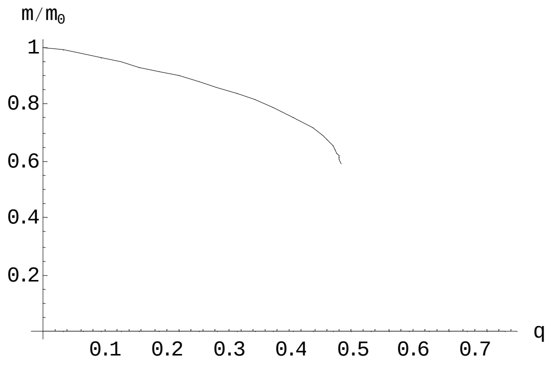
11. Relative change of the particle size – depending on the background curvature

On figure 11 we can see that the overall change (increase) of the particle size is smaller than it was seen in case of constant particle mass; however the particle size still gets infinite at a critical background curvature.

As we mentioned the mass of our theoretical quantum particle also changes with the background curvature. We expressed this variable rest mass with equation (18) (substituting G=1, h=1, c=1, k=1)

(18) $$m = \frac{\pi}{2\,6^{1/3}\left(-9\pi^4 q + \sqrt{3}\sqrt{-2\pi^9 + 27\pi^8 q^2}\right)^{1/3}} + \frac{\left(-9\pi^4 q + \sqrt{3}\sqrt{-2\pi^9 + 27\pi^8 q^2}\right)^{1/3}}{2\,6^{2/3}\pi^2}$$

This dependency is illustrated on the following diagram.



12. "Gravitational" red-shift – caused by the background curvature

We concluded that the rest mass of such particle decreases with increased curvature. Let us also remind that this relative decrease of mass can be detected as "gravitational" red-shift.

This correspondence means that in our model the curvature of background space can represent the scalar field which is actually the basis of our scalar-tensor gravity. The results have also confirmed that this geometric construction is a good candidate for being a test particle and illustrating the behavior of such quantum gravitational theory.

Conclusion

In this paper we described three steps starting from the Schwarzschild solution of general relativity to a certain kind of geometric quantum gravity. First we transformed the solution to synchronized conform-Euclidean coordinates then we revealed the correlation of the static geometry and the gravitational potential, finally we described a geometrical quantum test-particle, which explains this correlation. In general we used computations and transformations which are well known from textbooks.

We concluded that the quantum gravity introduced in this paper is able to derive gravitation attraction from the geometrical deformation of space without any force-carrying particle (e.g. graviton), or mysterious field. The paper might also induce a fundamental review of general relativity. There are topologic and cosmological conclusions as well, because in this paper we confirmed the multiverse theory and gave a new basis of explaining the evolution of universes.

There are many additional things to do to refine this theory. We mention only some of them, like refinement of the correspondence between gravitational mass and metric, which is essential; and enhancing the particle model and the correlation between the scalar-products of metric, and the gravitational-potential – which is also a big challenge. We have to mention the non-symmetric and non static case as well as the multi-object problem.

I would like to thank a couple of physicists who helped me with advice and guidance to labor and polish this theory. Thanks for their openness and patience reading and reviewing my ideas and theory, when it was not even semi-finished.

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