Electrodynamics in The Model of 4D Matter

I. The electromagnetic fields

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Abstract

Hydrodynamical model of four-dimensional medium similar to the ideal fluid is presented in the first four sections. As the medium is supposed to be incondensable and located in a limited volume, the expansion of the Euler equation to four dimensions are applied to describe its behavior. Electromagnetic field is derived as the consequence of the velocity field of the medium. The amalogue of the Helmholtz equation for vorticity of the velocity field and the analogue of the Maxewll's equations are obtained. The former let to treat the 4D electromagnetic field as the frozen field into the medium. The latter has a problem with the sign at the displacement term.

I. Introduction

It is the common to say that now the contemporary physics consists from many different areas, slightly interconnected or almost disconnected from each other. Despite the essential successes made in every area one can hardly name this state in physics by other word then that of a crisis. This state that has been widely debated for a long time in our opinion is bound up with the scarce "ground" on which trials to understand the whole set of phenomena of nature has been undertaken. Most of the scientists are endeavoring to "jam" all physical events into three-dimensional space. It is justifiable while the objects of classical physics are considered. However, such handling failed when applied to micro and macro objects, to particles and to galaxies. That is why multidimensional spaces and united space and time are used in the string theory of the fundamental particles and in the theory of relativity. That is why the problem of the creation of the Grand Unification Theory and Theory of Everything is arisen and one direction to solve it is the model proposed here.

We assume that there is only one extra spatial dimension having the same properties as the usual three dimensions needs to be introduced. Of course, it is not a novel idea. There is the 5D theory of Kaluza [1] as the extension of the general theory of relativity and there are many other theories used the same approach, see as the example [2], but as one can see below our handling differs from it in some essential moments. One of them consists in the meaning of the time. We think of it as a simple parameter which is not bound up with the space anyhow. In this point we diverge from the theory of relativity.

However, space is not the thing we intend to study. Our aim is to try to understand what matter, the substance, is and what kinds of motion it may have. We are using space and time only for the description of the substance and its movements in this model. Of course, no one does know what matter is in essence. Therefore to describe matter in this model of the reality by the simplest means, the simplest space among all others is chosen, the four- dimensional Euclidean space with positively defined metric. In the attempt to clarify it our consideration comes here from only one hypothesis – there is additional space for matter. In addition to it we assume that such 4D matter does not fill all space. Our task is to see what can come from these conditions.

We are to introduce the some terms that will be employed through the whole paper because the introduction of the additional dimension demands it. As we suppose that all matter is consist of a 4D homogeneous medium and think of it as a four-dimensional closed manifold with a border, we may call the latter as the world. Obviously, it has three dimension. The particles which the medium consists of are supposed to be very tiny and their structure is not considered here. So matter may be treated as a continuous medium beginning from some scale. While the whole manifold is supposed to be situated in the four-dimensional Euclidean space, the world has the induced metric that is non-Euclidean in general. If this manifold is supposed to be compact the metric is Riemanian one.

As we will see at the next section the boundary of the 4D medium is able to propagate the light and other electromagnetic waves. Some model of photon that is compatible with this conclusion was presented in paper [3] and will be given in details in the next paper. That is why only the world is observable and therefore it is named so. In that sense the world as the border hypersurface of 4D medium can be considered as the classical luminiferous ether which so long and unsuccessfully is being sought out. It really fills all “space” keeping in mind that this space generated by the border of the 4D medium.

The elementary particles such as electrons, protons and so on are thought of as not the point-like objects but as 4D vortices in the medium. In some rough approximation they can be treated as the one-dimensional threads, strings or, more precisely, vortex lines. The place of the particle in the visible world is determined by the vent-hole of the vortex, by the position where it is risen out on the border of the medium. The aggregations of the vortexes form the compound particles, atoms, molecules and so on up to all material objects of the world.

Due to in the global scale the world exerts the surface tension the manifold is to have a form close to 4D-sphere. Also we may name the entire manifold as the universe to distinguish it from the world. Of course, we can't exclude the possibility of the existence of the other external universes, supposedly spherical in shape and distributed from each other on some distances in the infinite space. But we never can see them or adopt any information from them because the light can't spread among them through the empty space of 4D vacuum. Here we will make an attempt to study the behavior of the medium in one of the universes. If the radius of the universe is big (as in the case of our Universe), at the rather small range the world may be described as 3D Euclidean space too. However the world is not flat in general even if the perturbations in the universe such as waves, particles, stars and galaxies are not taken into account. Therefore there are various scales where the picture proposed can apply on, from micro-world to macro-world.

The behavior of the two adjacent vortices was studied [4] and the law of the their interaction close to the Newton's Universal law of gravitation was obtained as the effect of the curvature of the world. Here the internal behavior of the medium is in our consideration.

II. The Basic Equations

In general we can describe the world as a border hypersurface of the 4D region by the equation

$$f(x,t)=0 \tag{2.1}$$

where $x = \{x_1, x_2, x_3, x_4\}$ are the coordinates of the space points. The medium supposed to be situated inside of the border hypersurface. As it was told the time t is an ordinary parameter independent from the space. The dimensionality of the hypersurface is three and the knowledge the function f is a final task in our interpretation because the positions and the behavior of all objects in the world can be found from the form of the function f. It is supposed so because any object in the universe is “made” of from the 4D medium with its visual part on the common 3D border.

While the world is not flat it is supposed to undergo the pressure due to (hyper-) surface tension lineally depending from curvature of the border hypersurface:

$$\mathrm{p}=\sigma\,\mathrm{K} \tag{2.2}$$

where σ is a coefficient of the surface tension and K is the mean curvature of the hypersurface. Here we refer to p as a pressure density, the force effecting on the unit of the hypersurface. In the case of four dimensions p and σ has the dimensionalities $[M^1 L^{-2} T^{-2}]$ and $[M^1 L^{-1} T^{-2}]$, respectively. If under this pressure the universe takes the form of a sphere of radius R, the pressure reacts on the border with direction of its gradient inwards the medium to the center of sphere and its value can be rather small for big radius

$$\mathrm{p}_0=\frac{3\sigma}{\mathrm{R}} \tag{2.3}$$

Nevertheless it might be big enough to hold the entire universe together. It is possible if σ has a big value for 4D medium. In more complicated cases the mean curvature must be considered as the function of form of the hypersurface f.

The state of the smooth spherical world when there is no any movement of the medium being able to disturb its surface we may call 3D vacuum. But in general it is not so and the any piece of the medium cannot be always at rest moving with their velocity $u(x,t)=\frac{\partial x}{\partial t}\equiv\partial_t x\equiv\dot{x}$ in such a manner that their average density (in rather small region of the space) is not changed. This means that the medium can be treated as incondensable and the divergence of u vanishes

$$\partial\cdot\mathrm{u}=0 \tag{2.4}$$

where symbol ∂ stands for partial derivatives with respect to four coordinates x. As a result, the total flow through the closed hypersurface f, or the whole world, is vanished too and the 4D volume of the medium is constant. This property makes the medium to be alike to the ideal fluid with no viscosity.

Therefore we can take the equation of the motion for the small piece of the medium in the form similar to Euler equation for ideal fluid or fluid without viscosity extended to four dimensions:

$$\dot{\mathrm{u}}+(\mathrm{u}\cdot\partial)\mathrm{u}+\frac{1}{\rho_4}\partial\mathrm{p}=0 \tag{2.5}$$

where ρ_4 is the density of the medium with dimensionality $[M^1 L^{-4} T^0]$. Because the latter is supposed to be constant in all its volume, the pressure is endowed with the sense of potential.

It is easy to consider the static case when the velocity field is constant for all universe. It means the whole universe is moving with that velocity. As it ensues from Eq.(2.5), pressure p will be constant inside the medium. This statement is known as the Pascal's law. For the spherical universe the constant pressure is determined by external value in Eq.(2.3). So the last member in Eq.(2.5) is significant only on the curved hypersurface where the gradient of p doesn't vanish.

In some approximation the hypersurface of the universe for the rather small area can be considered as the tangent space of the simplest case of the flat hypersurface when Eq. (2.1) looks like that

$$f=x_4=0 \tag{2.6}$$

In this case the pressure is vanished and the medium alleged to be occupied the half space, e.g. where $x_4>0$. It is the “usual” three-dimensional Euclidean space we are associated with the World. For our Universe where we all live the World really seems for us as a flat space but it is not flat for whole Universe in the model considered.

We can choose local system of coordinates so that the first three axes will belong to the world and the fourth axis x_4 will be normal to it. Then the Eq.(2.5) dissevers to as follows:

$$\dot{\mathbf{u}}+(\mathbf{u}\cdot\nabla)\mathbf{u}+\mathrm{u}_4\partial_4\mathbf{u}+1/\rho_4\nabla\mathrm{p}=0 \tag{2.7}$$

$$\dot{u}_4+\mathbf{u}\cdot\nabla u_4+u_4\partial_4 u_4+1/\rho_4\partial_4 p=0 \qquad (2.8)$$

where ∇ is the usual partial derivative with respect to three coordinates $\boldsymbol{r}=\{x_{1,}x_{2,}x_3\}$. We took the same notation for velocity u in Eqs. (2.7 and 2.8) as in Eq. (2.5) because the various form of derivative (∇ instead of ∂) is using so they can't mix up. In addition to it 3D vectors are emphasized. Thus $\boldsymbol{u}$ is the velocity field that is complanar to the hypersurface at the point $\boldsymbol{r}$. Eq. (2.5) then will be wrote as

$$\nabla\cdot\boldsymbol{u}+\partial_4 u_4=0 \qquad (2.9)$$

The hypersurface f is supposed to be free. It means that there are not any external constraints and that the equation

$$\dot{f}+u_f\cdot\partial f=0 \qquad (2.10)$$

is true under condition (2.1). The lower index f stands for the value which must be given on the border. If the form of hypersurface can be presented in the form of graph $x_4=x_4(\boldsymbol{r},t)$, where $\boldsymbol{r}=\{x_{1,}x_{2,}x_3\}$, Eq.(2.10) will look like

$$u_4+\mathbf{u_f}\cdot\nabla x_4=0 \qquad (2.11)$$

Hence to know the behavior of hypersurface f one needs to know the border velocity field u_f which is satisfied Eq.(2.5) under condition (2.4).

III. The Field Invariants

There are additional relations that could be useful while solving the task. For the purpose to find one of them we multiply Eq.(2.5) represented by components

$$\dot{u}_i+u_k\partial_k u_i+1/\rho_4\partial_i p=0 \qquad (3.1)$$

to u_i and summarize. Hereafter the latin indices takes values from 1 to 4 and the usual rule of summation for repeating indices is presumed. Then it is easy to get the following equation

$$\dot{w}+u\cdot\partial w=0 \qquad (3.2)$$

where w is the constant in Bernoulli like equation and can be treated as the energy density of the medium unit of 4D volume with kinetic and potential parts being multiplied on the constant density ρ_4 :

$$w=\frac{1}{2}u^2+p/\rho_4 \qquad (3.3)$$

The last term is supposed to be independent from the time explicitly.

We may determine 3D energy density referred to the small 3D volume of the border hypersurface as follows

$$W=\frac{dU}{dV}=\rho_4\int d\,x_4 w \qquad (3.4)$$

where U is the energy, V is 3D volume given on the border and integration is taken over x_4 axis from the border to some value L where the velocity field is vanished. So we can express the last equation as $W=\rho_3 w_m$, where $\rho_3=L\rho_4$ is three-dimensional density and $w_m=\frac{1}{L}\int dx_4 w$ is w averaged over fourth dimension.

The law of energy density conservation (3.2) also can be put down in a short form as

$$d_t w=0 \qquad (3.5)$$

if use the notation d_t instead of $\partial_t+(u\cdot\partial)$ as the full or the material derivative in 4D space. It means that the energy density w keeps constant value not in the fixed point of the space but along the streamline belonging to so-called Bernoulli surface (it means 3D hypersurface) as it will be shown below. Knowing the energy density, one can get the Euler equation (2.5) from the Lagrangian density $L=1/2u^2-p/\rho_4$, keeping in mind that u is patently time and space dependent.

The form of the streamline can be determined from the equations

$$\frac{dx_1}{u_1}=\frac{dx_2}{u_2}=\frac{dx_3}{u_3}=\frac{dx_4}{u_4} \tag{3.6}$$

As in 3D fluids, it coincides with the trajectory of the selected element of the medium unless the velocity field is time dependent. Because of $\partial_k f=\partial_i f\,\partial_k x_i=\frac{u_i}{u_k}\partial_i f$ due to Eqs.(3.6), the set of the conjugate streamlines can form the Bernoulli surface where the value of energy density is the same and which also satisfied Eq.(2.10). It means that the Bernulli surface can be treated as the free hypersurfaces in the bulk of the 4D medium determined by the Eq.(2.1) and the border hypersurface, the world, is also the Bernulli surface.

But the external surface is differed from the internal surfaces in one essential aspect. The pressure acting on the former is not in equilibrium as on the latter. The medium points of the inner streamline has the same values of the pressure from the both sides of the Bernoulli surface but with opposite signs. That is why the existence of the pressure outside of the border cannot be implied for the incondensable medium.

The sum of the energy (3.4) on the all conjugate streamlines localized in some 4D volume forms the full energy U of the elementary particle or other material object being in this volume as the projecture on the border, on the world. In the next paper we will give the detailed model of such construction.

The property of the additivity of the energy means that the streamlines belonging to different Bernulli surfaces can mix up under their juxtaposition. Really, if two Bernoulli surfaces f_1 and f_2 satisfy equations $d_t f_1=0$ and $d_t f_2=0$, the common Bernoulli surface $f=f_1+f_2$ will also satisfy Eq.(2.10) $d_t f=0$. That is why this property of the superposition of the hypersurfaces is used in [4] to derive the law of gravitation. There it was sufficient to consider Eq.(3.1) without the second term, the convective derivative.

From Eq.(2.5) the law of “momentum” density conservation, also along the streamline, can be represented in a similar way if the last member $-\partial p$, the “force” density, or acceleration of the medium particle caused by the curvature of the border hypersurface, is vanished

$$d_t u=0 \tag{3.7}$$

Of course, this is the reflection of the fact that the velocity field belongs to the streamline. By other words, the streamline forms by the velocity field.

By multiplying Eq.(3.1) on x_j, changing indexes i and j and making subtraction we can easily get the conservation law of “impulse-momentum” density

$$d_t m=0 \tag{3.8}$$

where m stands for the tensor of impulse-momentum density of the small piece of the 4D medium

$$m_{ij}=x_i u_j-x_j u_i \tag{3.9}$$

The relation (3.8) will be true only if discard the last member in Eq.(3.1). Otherwise the right part will be equal to the torque density $x_i\partial_j p-x_j\partial_i p$. Obviously, it may not vanish only on the curved border and under violation of the symmetry of the pressure distribution in the 2D plane constructed from the directions of the axes marked by i and j.

IV. The Electromagnetic Field

Let us again consider Eq.(2.5) and transform it as follows

$$\dot{u}+F\,u+\partial w=0 \tag{4.1}$$

where F is antisymmetric tensor of rank two composed from the partial derivatives of u:

$$F_{ij}=\partial_i u_j-\partial_j u_i \tag{4.2}$$

It is easy to check that the gradient of F vanishes as it follows from definition (4.2):

$$(dF)_{ijk}=\partial_i F_{jk}-\partial_j F_{ik}+\partial_k F_{ij}=0 \tag{4.3}$$

We may denote $F_{4\alpha}$ as e_α and $F_{\alpha\beta}$ as h_γ where Greek indices accept their values cyclically from 1 to 3. Then the sets $\mathbf{e}=\{e_\alpha\}$ and $\mathbf{h}=\{h_\alpha\}$ will form the 3D-vectors. The vector $\mathbf{h}=\nabla\times\mathbf{u}$ is the usual vorticity of the field **u**, so the tensor F is its 4D extension. The vector $\boldsymbol{e}=\partial_4\boldsymbol{u}-\nabla u_4$ can be also considered as the vorticity involving the fourth dimension. Thus the record of Eq.(4.1) can be considered as the 4D extension of the Euler equation in the Gromeka-Lamb form.

Here it is needed to note that the orientation of the coordinate axes may be chosen fully arbitrary and may not be compatible with the border hypersurface. This means that the vectors $\boldsymbol{e}$ and $\boldsymbol{h}$ are interchangeable. But it is not so in vicinity of the border if we want to have usual three-dimensional axes marked as $\mathbf{r}=\{x_1, x_2, x_3\}$. Therefore we can choose orientation of the axis x_4 to be normal to the border which suppose hereafter to be flat as in Eq. (2.6).

Using vectors **e** and **h** in Eq.(4.3), one can get two equations

$$\nabla\times\mathbf{e}-\partial_4\mathbf{h}=0 \tag{4.4}$$

$$\nabla\cdot\mathbf{h}=0 \tag{4.5}$$

Calculating the divergence of F with respect to the one of the indices we get the following expression

$$\partial_k F_{ik}=4\pi\,J_i \tag{4.6}$$

where it is used the denotation given with the help of (2.4)

$$4\pi\,J=-\partial^2 u$$

(4.7)

The Eq.(4.6) resolves into two with the help of vectors **e** and **h**

$$\nabla\cdot\mathbf{e}=4\pi\,\rho \tag{4.8}$$

$$\nabla\times\mathbf{h}-\partial_4\mathbf{e}=4\pi\,\mathbf{j}/c \tag{4.9}$$

where c is some constant with dimensionality of velocity, **j** and ρ are components of 4D vector $J=(\mathbf{j}/c,\rho)$ or explicitly

$$4\pi\,\rho=-\partial^2 u_4=-(\nabla^2 u_4+\partial_4^2 u_4) \tag{4.10}$$

$$4\pi\,\mathbf{j}/c=-\partial^2\mathbf{u}=-(\nabla^2\mathbf{u}+\partial_4^2\mathbf{u}) \tag{4.11}$$

Taking into account Eq.(2.4) one can get from Eq.(4.7)

$$\partial\cdot J=\nabla\cdot\mathbf{j}/c+\partial_4\rho=0 \tag{4.12}$$

It is easy to notice the similarity of Eqs. (4.4, 4.5) and (4.8, 4.9) with those of Maxwell for electromagnetic fields **E** and **H** and Eq.(4.12) with equation of continuity. The components of 4D velocity field of medium would play the role of electromagnetic potentials **A** and ϕ . Then the quantities ρ and j ought be considered as ones having relation to the charge density and current density, respectively.

In addition, the Eq.(4.1) being expressed through vectors $\mathbf{e}$ and $\mathbf{h}$ will be taken the similar form as in the Lorentz equation for the charge particle

$$\dot{\mathbf{u}} - \mathbf{u}\times\mathbf{h}+u_4\mathbf{e}+\nabla w = 0 \qquad (4.13)$$

and as the change of its kinetic energy [?]

$$\dot{u}_4 + \mathbf{u}\cdot\mathbf{e}+\partial_4 w = 0 \qquad (4.14)$$

There is no explicit place for the charge in these two equations. But as it follows from the Eq.(4.10) the sign of the charge can be associated with the direction of the velocity field along the fourth dimension that is presented in this equation. Last equations also contain additional terms with the force density ∂w which can not be implied, however, if the motion along the streamline is considered where w takes constant value.

The similarity with the Maxwell's equations will be closer if we could change the spatial derivative ∂_4 in Eqs.(4.4, 4.9) by the temporal one -1/c ∂_t, where c is the constant speed of light. This could be done if Eq.(2.5) would simplify in such a way that it takes a form

$$\dot{u}+ c\,\partial_4 u=0 \qquad (4.15)$$

Then the similar relations would be true for every vector's derivative from u and its combinations and therefore the changing demanded above seems to be quite legitimate under assumption

$$(\mathbf{u}\cdot\nabla)u+u_4\partial_4 u=c\,\partial_4 u \qquad (4.16)$$

The more stronger condition

$$u_4=c \qquad (4.17)$$

gives $\partial_4 u_4=0$ that leads in view of (2.9) to an analogue of the Coulomb gauge $\mathrm{div}\,\boldsymbol{A}=0$

$$\nabla\cdot\boldsymbol{u}=0 \qquad (4.18)$$

However, then Eq.(4.15) for the fourth component of velocity field becomes the edentity and the condition (4.16) would mean that the convective derivative $(\boldsymbol{u}\cdot\nabla)u$ is neglected. If these conditions will be adopted to be valid, there should be no sense in the introduction of the fourth component of the velocity field and the intrinsic field F_{ij} should be postulated as in the classical electrodynamics but not be introduced from the equation of motion as it was made above. Therefore it would erode all purport of the theory. And of course, the fourth component of the velocity field u_4 must vanish at the border and doesn't take there the enormous value of the speed of light. So such assumption can be used only far away from the border.

It is left only one formal difference between Maxwell equations and those were been obtaining here. The sign at the second member in Eq.(4.9), the analogue of the Maxwell's displacement current, don't correspond to that usually figured. The problem lies on the indeterminacy of the orientation of the embedded 3D-hypersurface into the 4D-manifold. Indeed in 3D space the vector **h** is an axial vector, or pseudo-vector, and its direction in 4D space can be freely referred to the 4th axis too because the latter is normal to any vector in 3D space. Therefore the direction of **h** may be chose arbitrary in the 2D plane composed by its direction in 3D space and the 4th axis and so it may be by the arbitrary way inverted in 3D space. But when we map the **h** vector onto the border 3D space, onto the world, its orientation can not be arbitrary and must be taken in concordance with the orientation of the border.

But even then there is a main distinction between the electromagnetic potentials and the velocity field u. The former are defined in 3D space, in the flat world, and the latter is in 4D space of the universe. Therefore they will never be fully in comparison. We may call the fields **e** and **h** the intrinsic electrical and magnetic fields to distinguish its from “extrinsic” fields **E** and **H**, respectively, that is using in the classical physics.

We can calculate time derivative of F by the straightforward way substituting $\dot{u}$ from Eq. (3.1) into the determination of F (4.2). Then we get

$$\begin{aligned}&\dot{F}_{ij}+\partial_i((u_k\partial_k)u_j)-\partial_j((u_k\partial_k)u_i)=\\&=\dot{F}_{ij}+u_k\partial_k F_{ij}+\partial_i u_k\partial_k u_j-\partial_j u_k\partial_k u_i\end{aligned} \tag{4.20}$$

After adding $\pm\partial_k u_i \partial_k u_j$ and making some arrangements, the next relation that express the extension of the Helmholtz theorem for 4D vorticity appears

$$\dot{\mathsf{F}}+\mathsf{L}_{\mathsf{u}}\mathsf{F}=0 \tag{4.21}$$

where L_u is the Lie derivative with respect of field u. This relation shows that the intrinsic electromagnetic field is “frozen” into the velocity field. It is carrying along by the field u. In the case of extrinsic field it is not so because the integration along the fourth dimension and the Lie derivative are not commuted. This can be shown if Eq.(4.33) will be presented with the help of vectors **e** and **h**

$$\begin{aligned}&\dot{\mathbf{e}}+(\mathbf{u}\cdot\nabla)\mathbf{e}+\mathsf{u}_4\partial_4\mathbf{e}+(\mathbf{e}\cdot\nabla)\mathbf{u}+\mathbf{e}\partial_4\mathsf{u}_4+\mathbf{h}\times\nabla\mathsf{u}_4=0\\&\dot{\mathbf{h}}+(\mathbf{u}\cdot\nabla)\mathbf{h}+\mathsf{u}_4\partial_4\mathbf{h}-(\mathbf{h}\cdot\nabla)\mathbf{u}-\mathbf{h}\partial_4\mathsf{u}_4+\mathbf{e}\times\nabla\mathsf{u}_4=0\end{aligned} \tag{4.22}$$

We see that there are additional terms along with the Lie derivatives in 3D space $L_{\boldsymbol{u}}\boldsymbol{e}=(\boldsymbol{u}\cdot\nabla)\boldsymbol{e}-(\boldsymbol{e}\cdot\nabla)\boldsymbol{u}$ and $L_{\boldsymbol{u}}\boldsymbol{h}=(\boldsymbol{u}\cdot\nabla)\boldsymbol{h}-(\boldsymbol{h}\cdot\nabla)\boldsymbol{u}$

The two first terms in two last equations may be included in the full time derivatives with respect to some 3D-hypersurface parallel to the border $\mathsf{D}_t=\partial_t+(\mathbf{u}\cdot\nabla)$. It means that the time derivatives of the field will be measured with respect to the points of the medium belonging to this hypersurface but not with respect to the point of space. The third terms can be changed with the help of Eqs.(4.4, 4.9) to be closer to the form of the Maxwell's equations. But the three last terms in Eqs.(4.22) are redundant from the point of view of the classical electrodynamics. They are vanishing if the condition (4.17) is implied. Then these pair of equations together with Eqs.(4.5, 4.8, 4.12) will be an analogue of the set of main equations in electrodynamics:

$$\begin{aligned}&\frac{1}{c}\mathsf{D}_t\mathbf{e}+\nabla\times\mathbf{h}=\frac{4\pi}{c}\mathbf{j}\\&\frac{1}{c}\mathsf{D}_t\mathbf{h}+\nabla\times\mathbf{e}=0\end{aligned} \tag{4.23}$$

Again the signs in the first equation is not corresponded to the right signs of the Ampere equation.

V. The Maxwell's Equations

The only procedure we can made with the intrinsic fields is to conform its orientation with the orientation of the border. Then axis x_4 will be directed along the normal to the border outwards the medium and, in order to get the "usual" electromagnetic fields from the intrinsic ones, it is sufficient to take the mean value of intrinsic fields along the fourth dimension given with the square root of 4D density ρ_4 . By that we will bring into accord the dimensionalities of the extrinsic fields. While calculating the mean value the integration must be taken along the fourth coordinate from the some value L , where the fields **e** and **h** (and velocity field, respectively) vanish, and up to border hypersurface f given by Eq.(2.1):

$$\mathbf{E} = \frac{\sqrt{\rho_4}}{L}\int dx_4 \mathbf{e} = \frac{\sqrt{\rho_4}}{L}\left(\mathbf{u_f} - \nabla \int dx_4 u_4\right)$$
$$\mathbf{H} = \frac{\sqrt{\rho_4}}{L}\int dx_4 \mathbf{h} = \frac{\sqrt{\rho_4}}{L}\nabla\times\int dx_4 \mathbf{u} \qquad (5.1)$$

Here $\boldsymbol{u_f}$ is the value of the velocity $\boldsymbol{u}$ at the border. Hereafter the extrinsic fields are the mean 4D vorticity multiplied on the coefficient $\sqrt{\rho_4}$. We see that these determinations are nohow dependent from the intrinsic fields. Then we can determine the electromagnetic potentials as follows

$$\mathbf{A} = \frac{\sqrt{\rho_4}}{L}\int dx_4 \mathbf{u}$$
$$\varphi = \frac{\sqrt{\rho_4}}{L}\int dx_4 u_4 \qquad (5.2)$$

As we can see they are the mean 4D velocity with accuracy of the factor $\sqrt{\rho_4}$. So with its help the electrical and magnetic fields will look like in the classical theory

$$\mathbf{E} = -1/c\dot{\mathbf{A}} - \nabla\varphi$$
$$\mathbf{H} = \nabla\times\mathbf{A} \qquad (5.3)$$

To meet the determination of $\boldsymbol{E}$ given in Eq.(5.1) with the classical one in Eq.(5.3) we are compelled to admit that

$$\sqrt{\rho_4}\boldsymbol{u_f}/L = -1/c\,\dot{\boldsymbol{A}} \qquad (5.4)$$

Thus the sense of the vector potential $\boldsymbol{A}$ becomes obvious. Integrating this equation we give

$$\boldsymbol{A} = -\sqrt{\rho_4}c\,\boldsymbol{s}/L + \boldsymbol{A_0} \qquad (5.5)$$

where $s = \boldsymbol{u_f}t$ is the path gone by the particle of the medium along the border for the time t and $\boldsymbol{A_0}$ is some additive constant. It means that the expression $c\,\boldsymbol{s}/L$ is in the close relation with the mean velocity $\boldsymbol{u}$ as it follows from Eq.(5.2).

From the Eqs. (5.3) we easily get the first pair of Maxwell's equations

$$\nabla\times\boldsymbol{E} + \frac{1}{c}\dot{\boldsymbol{H}} = 0$$
$$\nabla\cdot\boldsymbol{H} = 0 \qquad (5.6)$$

They are compatible with Eqs.(4.4) and (4.5) when those will be integrated along x_4 -axis taking into account that the border value of the magnetic field $\boldsymbol{h_f} = \nabla\times\boldsymbol{u_f}$ with the coefficient $\sqrt{\rho_4}/L$ is equal to $-1/c\,\dot{\boldsymbol{H}}$ as the rotor of $\sqrt{\rho_4}\dot{\boldsymbol{u}}_f$ in accordance with Eq.(5.4).

Giving the divergence of the first equation in (5.1) or integrating (4.8) we get the next Maxwell's equation

$$\nabla\cdot\boldsymbol{E} = 4\pi\rho_{cl} \qquad (5.7)$$

where ρ_{cl} stands for the classical charge density which is referred to the three-dimensional case

$$4\pi\rho_{cl} = \frac{4\pi\sqrt{\rho_4}}{L}\int dx_4\rho = -\nabla^2\varphi - \frac{\sqrt{\rho_4}}{L}(\partial_4 u_4)_f \quad , \qquad (5.8)$$

where the value of $\partial_4 u_4$ is given on the border in the last term and it is vanished under Coulomb gauge. It is the Poinson equation for ϕ . Calculating the difference of $\nabla \times \boldsymbol{H}$ and $1/c\dot{\boldsymbol{E}}$ using definitions (5.1), the last Maxwell's equation will look like the following

$$\nabla \times \boldsymbol{H} - 1/c\,\dot{\boldsymbol{E}} = 4\pi\, \boldsymbol{j}_{cl}/c \quad , \tag{5.9}$$

where $\boldsymbol{j}_{cl}$ stands for the definition of the classical current density

$$\frac{4\pi}{c}\boldsymbol{j}_{cl} = \nabla \times \nabla \times \boldsymbol{A} - \frac{\sqrt{\rho_4}}{Lc}\dot{\boldsymbol{u}}_f + \frac{1}{c}\nabla\dot{\varphi} \quad . \tag{5.10}$$

Such determination of the current density meets the problem with the proper sign at the second term in Eq.(5.9), the Maxwell's displacement current, and leads to the proper form of the equation of continuity as in the classical electrodynamics

$$\nabla \cdot j_{cl} + \dot{\rho}_{cl} = 0 \tag{5.11}$$

It is the consequence of the equation of the incompressibility (2.9) because it was used in the form $\nabla \cdot \boldsymbol{u}_f + (\partial_4 u_4)_f = 0$.

By the integrating of (2.9) we can get the next relation

$$\nabla \cdot \boldsymbol{A} + \frac{\sqrt{\rho_4}}{L} u_4{}^f = 0 \tag{5.12}$$

So we can elucidate the notion of the Coulomb and Lorentz gauges. The former means that there is no fourth component of the velocity at the border and all the medium at the border is moving along the border. The latter means that $\frac{\sqrt{\rho_4}}{L} u_4{}^f = \frac{1}{c}\dot{\varphi} = \frac{\sqrt{\rho_4}}{c}\dot{u}_4{}^m$. After the integration over time we get

$$u_4{}^m = \frac{c}{L}\int u_4{}^f\, dt \tag{5.13}$$

If we set the range of integration equals to T= L/c, to time needed for light to go the distance L, we get

$$u_4{}^m = \overline{u_4{}^f} \tag{5.14}$$

where the line over symbol means the mean value for the time T. So the Lorentz gauge is such gauge when the mean value of the fourth component of velocity field on the range L is equals to the mean velocity of the border value of this component averaged time T.

The condition $u_4{}^f = 0$ for Coulomb gauge contradicts with Eq.(4.17). It can be explained as so that in the classical electrodynamics there is no any borders where the speed of light would be reaching zero value. There the infinite space is considered.

The Maxwell's equation (5.9) can obtain by integrating Eq.(4.9) as well. A few manipulations with the border value of the intrinsic electrical field $\boldsymbol{e}_f$ must be done, again leading to Eq.(5.4) to prove that it is equal to $-1/c\,\dot{\boldsymbol{E}}$ with the coefficient $\sqrt{\rho_4}$.

Thus we get the Maxwell's equations from the intrinsic velocity field u making only one assumption (5.4) concerning the meaning of the vector potential $\boldsymbol{A}$. This assumption stems from the restricted equation (4.15) and therefore the Maxwell's equations ought not be considered as the final equations for the behavior of the electrical and magnetic fields in this theory.

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