Four conjectures about three subsets of pairs of twin primes

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Abstract. In this paper are stated four conjectures about three subsets of pairs of twin primes, i.e. the pairs of the form (p^2 + q – 1, p^2 + q + 1), where p and q are primes (not necessarily distinct), the pairs of the form (p + q – 1, p + q + 1), where p, q and q + 2 are all three primes and the pairs of the form (p^2 + q – 1, p^2 + q + 1), where p, q and q + 2 are all three primes.

Conjecture 1:
Any pair of twin primes (a, b) greater than (29, 31) can be written as (a = p^2 + q – 1, b = p^2 + q + 1), where p and q are primes (not necessarily distinct).

Verifying the conjecture:
(for the first 5 such pairs of twin primes)

For (a, b) = (29, 31), we have:
a = 5^2 + 5 – 1 = 29; b = 5^2 + 5 + 1 = 31.
For (a, b) = (41, 43), we have:
a = 5^2 + 17 – 1 = 41; b = 5^2 + 17 + 1 = 43.
For (a, b) = (59, 61), we have:
a = 7^2 + 11 – 1 = 59; b = 7^2 + 11 + 1 = 61.
For (a, b) = (71, 73), we have:
a = 5^2 + 47 – 1 = 71; b = 2^2 + 47 + 1 = 73;
a = 7^2 + 23 – 1 = 71; b = 7^2 + 23 + 1 = 73.
For (a, b) = (101, 103), we have:
a = 7^2 + 53 – 1 = 101; b = 7^2 + 53 + 1 = 103.

Note:
The conjecture can also be formulated in the following way: For any pair of twin primes (t, t + 2), where t is greater than or equal to 29, there exist two primes a, b such that (t + 1) – a^2 = b.

Verifying the conjecture:
(for the following 5 pairs of twin primes)

: 108 – 25 = 83; 108 – 49 = 59;
: 138 – 25 = 113; 138 – 49 = 89; 138 – 121 = 17;
: 150 – 49 = 101; 150 – 121 = 29;
: 180 – 49 = 131; 180 – 121 = 59; 180 – 169 = 11;
: 192 – 25 = 167; 192 – 121 = 71; 192 – 169 = 23.

Note:
The conjecture is also checked for the next twenty pairs of twin primes, with the lesser term equal to: 197, 227, 239, 269, 281, 311, 347, 419, 431, 462, 521, 569, 599, 617, 641, 659, 809, 821, 827, 857.

Conjecture 2:
There exist an infinity of pairs of primes of the form (p^2 + q – 1, p^2 + q + 1), where p and q are primes (not necessarily distinct).

Note:
This conjecture is equivalent with the Conjecture about the infinity of twin primes if the Conjecture 1 from above is true, in such case being representative for the entire set of the pairs of twin primes or may state something different (but still implying the infinity of the pairs of twin primes) if the Conjecture 1 is not true, in such case being representative for an infinite subset of the set of the pairs of twin primes. The two conjectures below also implies the infinity of the pairs of twin primes.

Conjecture 3:
There exist an infinity of pairs of primes (p, q), where p + 2 and q + 2 are also primes, such that p – q + 1 = t, where t is also prime.

Examples:
: 227 – 197 + 1 = 31, so (p, q, t) = (227, 197, 31);
: 239 – 227 + 1 = 13, so (p, q, t) = (239, 227, 13).

Note:
The conjecture can also be formulated in the following way: There exist an infinity of pairs of primes (p, p + 2), such that p = t + q – 1, where t is prime and the numbers q and q + 2 are also primes.

Conjecture 4:
There exist an infinity of pairs of primes (p, q), where p + 2 and q + 2 are also primes, such that p – q + 1 = t^2, where t is also prime.

Examples:
: 569 – 521 + 1 = 7^2, so (p, q, t) = (569, 521, 7);
: 851 – 857 + 1 = 5^2, so (p, q, t) = (851, 857, 5).

Note:
The conjecture can also be formulated in the following way: There exist an infinity of pairs of primes (p, p + 2), such that p = t^2 + q – 1, where t is prime and the numbers q and q + 2 are also primes.