New Insight of Single Photon and Electron Interference

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Here we analyze the single photon and electron interference experiments by using an optical Mach-Zehnder and a 2-slits (2-pinholes) electron interferometer. According to Maxwell equations, the electromagnetic fields of the interference using the Mach-Zehnder Interferometers can be described by a localized optical wave in one path and non-localized vector and scalar potentials in the other path. In a quantum mechanical description of the interference, quantum-superposition state has been introduced. However, the single photon interference can also be calculated under the assumption that the states are expressed as the localized optical beam (a photon) and the non-localized potentials. Similarly, Maxwell equations can be divided into a localized electron beam and the non-localized potentials in case of the 2-slits (2-pinholes) single electron interferometer. From the analysis results, the non-localized potentials can be identified as an indefinite metric vector with zero probability amplitude and correspond to gauge fields introduced by gauge transformation of the second kind. The results show we can express quantum states without quantum-superposition state, which leads to an improved understanding of the uncertainty principle and resolution of paradox of reduction of the wave packet. The concept provides not only exactly the same calculation and experimental results using quantum-superposition state, but also can eliminate zero-point energy and cause spontaneous symmetry breaking without complexity. The results insist that Quantum theory is a kind of statistical physics.

INTRODUCTION

Basic concept of the quantum theory is the quantum-superposition states. Arbitrary states of a system can be described by pure states which are superposition of eigenstates of the system. Calculation results by the concept agree well with experiment. Without the concept, single photon or electron interference could not be explained. In addition to the interference, entangle states also could not be explained.

However the concept leads to the paradox of the reduction of the wave packet typified by "Schrödinger's cat" and "Einstein, Podolsky and Rosen (EPR)". [1, 2]

Although there were a lot of arguments about the paradoxes, recent paper related to the quantum interferences convince us of the validity of the concept. For example, the atom interference by using Bose-Einstein condensates (BECs) has been reported experimentally and theoretically. [3, 4] The coherence length of an electron or electron-electron interference by using the Aharonov-Bohm oscillations in an electronic MZI has been discussed theoretically. [5, 6] A plasmonic modulator utilizing an interference of coherent electron waves through the Aharonov-Bohm effect has been studied by the author. [7] The entangle states have been widely discussed experimentally and theoretically. [8–13] The photon interference by using nested MZIs and vibrate mirrors has been measured and analyzed. [14, 15] The double-slit electron diffraction has been experimentally demonstrated. [16] However, BECs, condensate and bosonization systems correspond to mixed states with or without coherence rather than pure states, and no paper can solve the paradoxes.

In this paper, we offer a new insight of the single photon and electron interference that can solve the paradoxes. According to the new insight, there is no concept of quantum-superposition and pure states whose probabilities are fundamental sense in nature. Only the concept of mixed states whose probabilities are statistical sense is valid in nature. The new insight gives us novel and important results, i,e., improved understanding of the uncertainty principle non-related to measurements, elimination of zero-point energy without artificial subtraction, derivation of spontaneous symmetry breaking without complexity and knowledge that Quantum theory is a kind of statistical physics.

In addition, new insight can conclude that the concept of entangle state is also not valid in nature though there have been reported the validity of the concept of entangle states. [8–13] We will discuss the entangle state by using the new insight in other letter. [17]

In section and , we show easy example of Gaussian photon beam or electron flow to explain that single photon or electron can be described by substantial (localized) photon or electron and non-localized potentials. In addition, more general description by using gauge invariance is offered. In section and , we show the calculation of the interferences by using states represent the substantial photon or electron and the non-localized potentials, which does not require quantum-superposition states. In section , we discuss the paradoxes related to quantum-superposition states, zero-point energy and spontaneous symmetry breaking. In section , we summarize the findings of this work.

The discussions in this paper are very simple to the same level as an introductory of quantum theory, because the quantum theory has a misunderstanding in such a fundamental concept and nature of nature will be simple.

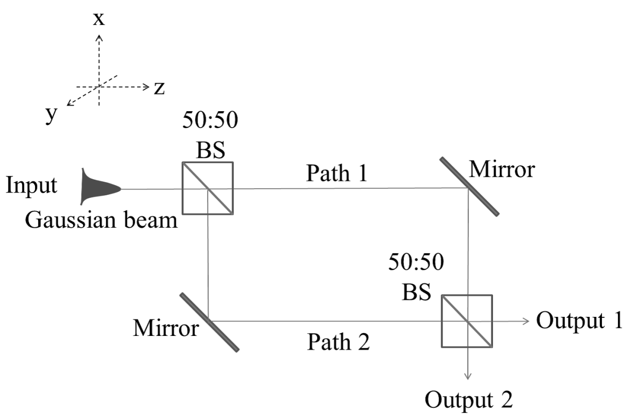

FIG. 1. Schematic view of MZI. BS:Beam Splitter.

CLASSICAL ELECTROMAGNETIC FIELD OF MZI - POTENTIALS AND PHOTON

Figure 1 shows schematic view of the Mach-Zehnder Interferometer (MZI) and coordinate system.

First we examine the input beam. Assume that an x-polarized optical beam propagates in z-direction with angular frequency ω and propagation constant β, the electric field $\mathbf{E}$ of the optical beam is well localized in the free space, e.g., the cross section profile of the electric field is expressed as Gaussian distribution.

Then, the electric field of the optical beam in the input can be expressed as follows.

$$\mathbf{E} = \mathbf{e}_x \cdot C_E \cdot \exp\left(-\frac{x^2+y^2}{w_0^2}\right) \cdot \cos(\omega t - \beta z) \quad (1)$$

Where, $\mathbf{e}_x$ is unit vector parallel to the x-axis. C_E is an arbitrary constant of which squared is proportional to the field intensity. w_0 is the radius of the optical beam. $\mathbf{E}$ and $\mathbf{B}$ are expressed by vector and scalar potentials as follows.

$$\begin{aligned} \mathbf{E} &= -\frac{\partial}{\partial t}\mathbf{A} - \nabla\phi \\ \mathbf{B} &= \nabla \times \mathbf{A} \end{aligned} \quad (2)$$

From (1) and (2), $\mathbf{A}$ is expressed by introducing a vector function $\mathbf{C}$ as follows.

$$\begin{aligned} \mathbf{A} &= -\frac{1}{\omega}\mathbf{e}_x \cdot C_E \cdot \exp\left(-\frac{x^2+y^2}{w_0^2}\right) \cdot \sin(\omega t - \beta z) + \mathbf{C} \\ \frac{\partial}{\partial t}\mathbf{C} &= -\nabla\phi \end{aligned} \quad (3)$$

By taking $\mathbf{C}$ as an irrotational vector function $\nabla \times \mathbf{C} = 0$ in order for $\mathbf{B}$ to localize in the space, for example, $\mathbf{C}$ and ϕ can be expressed by introducing an arbitrary scalar function λ as $\mathbf{C} = \nabla\lambda$ and $\nabla\left(\frac{\partial}{\partial t}\lambda + \phi\right) = 0$ respectively.

Then $\mathbf{B}$ is expressed as follows

$$\begin{aligned} \mathbf{B} &= \nabla \times \mathbf{A} \\ &= \frac{\beta}{\omega}\mathbf{e}_y \cdot C_E \cdot \exp\left(-\frac{x^2+y^2}{w_0^2}\right) \cdot \cos(\omega t - \beta z) \\ &\quad - \frac{2y}{\omega \cdot w_0^2}\mathbf{e}_z \cdot C_E \cdot \exp\left(-\frac{x^2+y^2}{w_0^2}\right) \cdot \sin(\omega t - \beta z) \end{aligned} \quad (4)$$

Therefore, $\mathbf{E}$ and $\mathbf{B}$ are localized in the free space in the input. In contrast, the vector and scaler potentials, which can not be observed alone, are not necessarily localized. The above localized form (1) is one example, other forms can be employed as will be described in the next section.

Note that, the Gaussian beam radius will be spatially expanded due to the free space propagation. However, the radius of the propagated beam $w(z)$ will be approximately 10.5mm when the beam with the initial radius $w_0 = 10$mm propagates $z = 100$m in free space. This value can be calculated by $w(z) = w_0\sqrt{1 + \left(\frac{\lambda z}{\pi w_0^2}\right)^2}$ when the wavelength $\lambda = 1\mu$m is applied. Then the spatially expansion of the beam will be negligible small when the paths of the MZI are less than several tens meters.

POTENTIALS AND ELECTRON

Figure 2 shows schematic view of a typical setup for the 2-slits (2-pinholes) single electron interference experiment. [16, 18]

An electron is launched from the electron source and propagates in right direction. According to the traditional explanation, the propagating electron passes through the both pinholes. However, we can obtain the interference pattern even if the electron passes through one of the two pinholes as described below.

The propagating electron can be identified as an electron beam whose space current density is $j = Nqv$, where N is the number of electron per unit volume, q is the electron charge and v is the electron velocity. When the radius of the electron beam is w_0, the current I can be expressed as $I = \pi w_0^2 j$. According to Biot-Savart Law, the propagation generates magnetic fields and potentials around the propagation path.

Assume that the electron propagates parallel to z-axis at a constant velocity. Then, the vector potentials around the propagation path are expressed as [18, 19]

$$\begin{aligned} A_x &= A_y = 0 \\ A_z &= \frac{I}{2\pi\varepsilon_0 c^2}\ln\frac{1}{r} \end{aligned} \quad (5)$$

where $r = \sqrt{x^2+y^2}$, ε_0 is the permittivity and c is the speed of light.

Therefore the vector potential clearly passes through not only the pinhole the electron passes through but also the opposite pinhole.

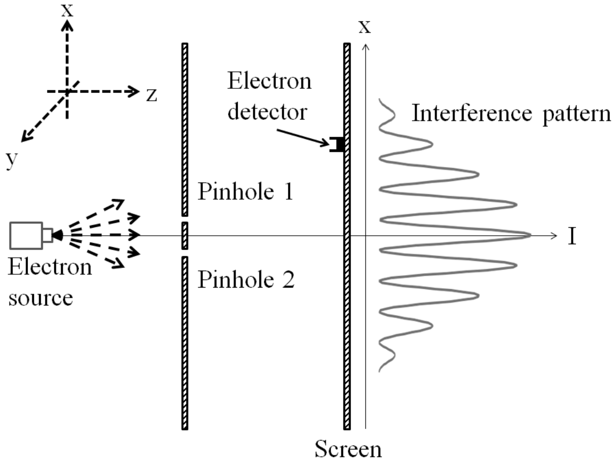

FIG. 2. Schematic view of a typical setup for the 2-slits (2-pinholes) single electron interference experiment.

However, we examine the following Maxwell equations to clarify the discussion.

$$\left(\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{A} - \nabla\left(\nabla\cdot\mathbf{A} + \frac{1}{c^2}\frac{\partial\phi}{\partial t}\right) = -\mu_0\mathbf{i}$$
$$\left(\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\phi + \frac{\partial}{\partial t}\left(\nabla\cdot\mathbf{A} + \frac{1}{c^2}\frac{\partial\phi}{\partial t}\right) = -\frac{\rho}{\varepsilon_0} \quad (6)$$

where μ_0 is the permeability and ρ is the electric charge density.

Here, we divide the potentials $\mathbf{A}$, ϕ into localized $\mathbf{A}_{\mathrm{l}}$, ϕ_{l} and non-localized $\mathbf{A}_{\mathrm{nl}}$, ϕ_{nl}. Then the equations (6) can be divided into following equations

$$\left(\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{A}_{\mathrm{l}} - \nabla\left(\nabla\cdot\mathbf{A}_{\mathrm{l}} + \frac{1}{c^2}\frac{\partial\phi_{\mathrm{l}}}{\partial t}\right) = -\mu_0\mathbf{i}$$
$$\left(\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\phi_{\mathrm{l}} + \frac{\partial}{\partial t}\left(\nabla\cdot\mathbf{A}_{\mathrm{l}} + \frac{1}{c^2}\frac{\partial\phi_{\mathrm{l}}}{\partial t}\right) = -\frac{\rho}{\varepsilon_0} \quad (7)$$

and

$$\left(\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{A}_{\mathrm{nl}} - \nabla\left(\nabla\cdot\mathbf{A}_{\mathrm{nl}} + \frac{1}{c^2}\frac{\partial\phi_{\mathrm{nl}}}{\partial t}\right) = 0$$
$$\left(\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\phi_{\mathrm{nl}} + \frac{\partial}{\partial t}\left(\nabla\cdot\mathbf{A}_{\mathrm{nl}} + \frac{1}{c^2}\frac{\partial\phi_{\mathrm{nl}}}{\partial t}\right) = 0 \quad (8)$$

If the electron beam can be expressed by localized waveform, just as an example, Gaussian with angular frequency ω and propagation constant β

$$\mathbf{i} \propto \mathbf{e}_z \cdot q \cdot C_q \cdot \exp\left(-\frac{x^2+y^2}{w_0^2}\right) \cdot \cos(\omega t - \beta z) \quad (9)$$

where $\mathbf{e}_z$ is unit vector parallel to the z-axis. C_q is arbitrary constant which is proportional to the number of electrons, then the localized potential can be expressed as a function which produces (9).

This functional form is determined by $\mathbf{i}$. Note that (9) is just one example. Arbitrary forms which satisfy (7) can be employed for $\mathbf{A}_{\mathrm{l}}$, ϕ_{l} and $\mathbf{i}$. When $\mathbf{i} = 0$ and $\rho = 0$, the equations (7) can express the localized electromagnetic fields in free space as described in the previous section.

In contrast, the equations (8) are independent of (7). Therefore the non-localized potentials can eternally populate the whole of space as waves defined by Maxwell equations, which propagate at the speed of light.

(8) expresses the gauge invariance of the localized electro magnetic field or electron flows and the non-localized potentials are well-known gauge field introduced by gauge transformation of the second kind.

INTERFERENCE OF SINGLE PHOTON

In a quantum mechanical description, the photon interference is calculated by introducing the electric field operator $\hat{E} = \frac{1}{\sqrt{2}}\hat{a}_1 \exp(i\theta) + \frac{1}{\sqrt{2}}\hat{a}_2$ and the number state $|n\rangle$ as follows. [20] Where $\hat{a}_{1\mathrm{or}2}$ is the electric field operator in path 1 or 2 respectively, θ is the phase difference.

$$\langle\hat{I}\rangle \propto \frac{1}{2}\langle n|\hat{a}_1^\dagger\hat{a}_1|n\rangle + \frac{1}{2}\langle n|\hat{a}_2^\dagger\hat{a}_2|n\rangle + \cos\theta\langle n|\hat{a}_1^\dagger\hat{a}_2|n\rangle \quad (10)$$

Where $\langle\hat{I}\rangle$ is expectation value of the field intensity which is proportional to photon number. $\hat{a}_{1\mathrm{or}2}$ and $\hat{a}_{1\mathrm{or}2}^\dagger$ are defined as $\hat{a} = \frac{\hat{a}_1+\hat{a}_2}{\sqrt{2}}$ and $\hat{a}^\dagger = \frac{\hat{a}_1^\dagger+\hat{a}_2^\dagger}{\sqrt{2}}$ by using the electric field operators $\hat{a}$ and $\hat{a}^\dagger$ at the input with $\langle n|\hat{a}_1^\dagger\hat{a}_1|n\rangle = \langle n|\hat{a}_2^\dagger\hat{a}_2|n\rangle = \langle n|\hat{a}_1^\dagger\hat{a}_2|n\rangle = \frac{1}{2}n$. When photon number is one ($n = 1$), i.e., single photon, the above expectation value is calculated to be $\langle\hat{I}\rangle \propto \frac{1}{4}+\frac{1}{4}+\frac{1}{2}\cos\theta = \frac{1}{2}+\frac{1}{2}\cos\theta$. In this traditional treatment, the electric field operators are obtained from quantization of (6) by using Coulomb gauge under assumption of $\mathbf{i} = 0$ and $\rho = 0$.

However we can make a different description by using the concept of the above non-localized potentials as follows.

Photon number will be proportional to C_E squared in equation (3). In contrast, the non-localized potentials $\mathbf{C}$ and ϕ or (8) are not necessarily proportional to photon number. When there are a large number of photons, it is reasonable to suppose that half of photons pass through path 1 and the rest pass through path 2 by law of large numbers because the probability of "which path does each photon select" should be $\frac{1}{2}$. This concept corresponds to mixed state instead of quantum-superposition state whose probabilities are statistical and fundamental sense respectively.

However when there are only a few photons, which correspond to the localized vector potential expressed as first term of equation (3) is comparable with the non-localized potentials expressed as the rest terms or (8), we should consider greater probability that the localized vector potential which represent a photon selects and passes through path 1 and the non-localized potentials pass through path 2. This description can also be applied to (7) and (8).

According to this description, the operator $\hat{E} = \hat{a}_1 \exp(i\theta) + \hat{a}_2$ with $\langle 1|\hat{a}_1^\dagger \hat{a}_1|1\rangle = 1$ and $\langle 1|\hat{a}_2^\dagger \hat{a}_2|1\rangle \neq 1$ instead of $\hat{E} = \frac{1}{\sqrt{2}}\hat{a}_1 \exp(i\theta) + \frac{1}{\sqrt{2}}\hat{a}_2$ should be introduced because the photon passes through only path 1. Note that $\hat{a}_1$ is the electric field operator in path 1 obtained from the traditional quantization as mentioned above using (7) instead of (6). In contrast, $\hat{a}_2$ is a purely-formal operator in path 2 obtained from the traditional quantization using (8) instead of (6), which is not the electric field operator but provides some quantity related to the non-localized potentials in path 2.

Therefore the expectation value of output 1 ($\frac{\pi}{2}$ phase difference will correspond to output 2) can be expressed as follows.

$$\begin{aligned}\langle \hat{I}\rangle &\propto \langle 1|\hat{a}_1^\dagger \hat{a}_1|1\rangle + \langle 1|\hat{a}_2^\dagger \hat{a}_2|1\rangle \\ &\quad + e^{i\theta}\langle 1|\hat{a}_1^\dagger \hat{a}_2|1\rangle + e^{-i\theta}\langle 1|\hat{a}_2^\dagger \hat{a}_1|1\rangle \\ &= 1 + \langle 1|\hat{a}_2^\dagger \hat{a}_2|1\rangle \\ &\quad + e^{i\theta}\langle 1|\hat{a}_1^\dagger \hat{a}_2|1\rangle + e^{-i\theta}\langle 1|\hat{a}_2^\dagger \hat{a}_1|1\rangle \end{aligned} \tag{11}$$

If $\langle 1|\hat{a}_2^\dagger \hat{a}_2|1\rangle = -\frac{1}{2}$ and $\langle 1|\hat{a}_1^\dagger \hat{a}_2|1\rangle = \langle 1|\hat{a}_2^\dagger \hat{a}_1|1\rangle^* = \pm\frac{1}{4}e^{i\phi}$, the same interference $\langle \hat{I}\rangle \propto \frac{1}{2} \pm \frac{1}{2}\cos(\theta+\phi)$ can be observed. Where $*$ means complex conjugate.

The above calculation is based on Heisenberg picture. We can calculate the same interference based on Schrödinger picture. In Schrödinger picture, the interference can be calculated by using the output 1 (or 2: $\frac{\pi}{2}$ phase difference) state $|1\rangle + |\zeta\rangle$ and the electric field operator $\hat{E} = \hat{a}$ at the output 1 (or 2). Because the operator is fixed in Schrödinger picture, the operator is obtained from the traditional quantization using (6). Where $|1\rangle$ and $|\zeta\rangle$ represent the states of a photon passes through path 1 and non-localized potentials passes through path 2 respectively. Because nothing is observed in path 2, we should recognize $\langle \zeta|\zeta\rangle = 0$.

In this picture, the expectation value can be expressed as follows.

$$\begin{aligned}\langle \hat{I}\rangle &\propto \left(e^{i\theta}\langle 1| + \langle \zeta|\right)\hat{a}^\dagger \hat{a}\left(e^{-i\theta}|1\rangle + |\zeta\rangle\right) \\ &= 1 + \langle \zeta|\hat{a}^\dagger \hat{a}|\zeta\rangle + e^{i\theta}\langle 1|\zeta\rangle + e^{-i\theta}\langle \zeta|1\rangle \end{aligned} \tag{12}$$

Where $\hat{a}^\dagger \hat{a}|1\rangle = |1\rangle$ and $\langle 1|\hat{a}^\dagger \hat{a} = \langle 1|$ are used.

If $\langle \zeta|\hat{a}^\dagger \hat{a}|\zeta\rangle = -\frac{1}{2}$ and $\langle 1|\zeta\rangle = \langle \zeta|1\rangle^* = \pm\frac{1}{4}e^{i\phi}$, the same interference $\langle \hat{I}\rangle \propto \frac{1}{2} \pm \frac{1}{2}\cos(\theta+\phi)$ can be observed. From this expression, we can recognize that $|\zeta\rangle$ has the phase difference for the interference without substantial photons.

Note that ϕ is determined by the phase difference of the MZI paths. When there is no phase difference between the MZI paths, ϕ is determined from the normalization of probability, i.e., $(\langle 1| + \langle \zeta|)(|1\rangle + |\zeta\rangle) = \langle 1|1\rangle + \langle 1|\zeta\rangle + \langle \zeta|1\rangle + \langle \zeta|\zeta\rangle = 1 + \langle 1|\zeta\rangle + \langle \zeta|1\rangle + \langle \zeta|\zeta\rangle = 1$, and $\langle \zeta|\zeta\rangle = 0$ then $\langle 1|\zeta\rangle = -\langle \zeta|1\rangle$. Therefore $\phi = \frac{\pi}{2} + N\pi$. Where N is integer.

In the above mathematical formula for the interference by Schrödinger picture, there is no mathematical solution in usual Hilbert space. Therefore the non-localized potentials, which can not be observed alone, must be regarded as a vector in indefinite metric Hilbert space. The same kind of unobservable vector has been introduced as "ghost" in quantum field theory. [21–24] We also call $|\zeta\rangle$ "ghost" in this report though this "ghost" has a different definition. The traditional "ghost" was introduced mathematically as an auxiliary field for consistent with relativistic covariance of the theory and had no effect on physical phenomena. However, the above "ghost" is a physical field which causes the interferences or is essential for the interferences instead of the mathematical auxiliary field.

From the equation (11) and (12), the non-localized potentials pass through path 2 produce the single photon interference as if the photon passes through the both paths in cooperation with a photon field passes through path 1.

The photon number should be proportional to C_E squared as can be seen in equation (3). However non-localized potentials $\mathbf{C}$ and ϕ or (8), which express "ghost", are not proportional to it as mentioned above. Therefore, the interference effect will be drop off when there are a large number of photons. This will be the reason why quantum effects are hardly observed in macroscopic scale.

In a classical description, we can express the electric field of the interference formed by one side MZI path, i.e., $\mathbf{E}_{\text{out}} = \frac{1}{2}\mathbf{E}_1 + \frac{1}{2}\mathbf{E}_2$, as follows by using the potentials

$$\begin{aligned}\mathbf{E}_{\text{out}} &= -\frac{\partial}{\partial t}\mathbf{A}_{\text{l}} - \nabla\phi_{\text{l}} - \frac{\partial}{\partial t}\mathbf{A}_{\text{nl}} - \nabla\phi_{\text{nl}} \\ &= \mathbf{E}_1 - \frac{\partial}{\partial t}\mathbf{A}_{\text{nl}} - \nabla\phi_{\text{nl}} \end{aligned} \tag{13}$$

If the non-localized potentials configure the following electric field, the interference by one side MZI path can be produced in cooperation with a photon field passes through path 1.

$$-\frac{\partial}{\partial t}\mathbf{A}_{\text{nl}} - \nabla\phi_{\text{nl}} = \frac{1}{2}\mathbf{E}_2 - \frac{1}{2}\mathbf{E}_1 \tag{14}$$

where subscripts 1 and 2 stand for the MZI path 1 and 2 respectively.

INTERFERENCE OF SINGLE ELECTRON

In a quantum mechanical description, the 2-slits (pinholes) single electron interference is typically explained by the probability of finding the electron on the screen.

[18]

$$P_{12} = |\phi_1 + \phi_2|^2 \tag{15}$$

Where $\phi_1 = \langle x|1\rangle\langle 1|s\rangle$ and $\phi_2 = \langle x|2\rangle\langle 2|s\rangle$, which are composed of probability amplitudes

$\langle 1_{\rm or}2|s\rangle$: "$\langle$electron arrives at pinhole 1 or 2|electron leaves s (electron source)$\rangle$" and

$\langle x|1_{\rm or}2\rangle$: "$\langle$electron arrives at screen x|electron leaves pinhole 1 or 2$\rangle$".

When either pinhole 1 or 2 is closed, the each and total probabilities are calculated to be $P_1 = |\phi_1|^2$, $P_2 = |\phi_2|^2$ and $P = P_1 + P_2 \neq P_{12}$. Therefore we must admit the electron passes through both pinholes at the same time despite an electron can not be split off, which forces us to introduces a concept of quantum-superposition states .

However we can examine the states of the localized electron propagation and non-localized potentials instead of the quantum-superposition state as mentioned above.

In such a case, the electron wave functions should be expressed as follows.

$$\psi_1' = \psi_1 \cdot \exp\left[i\frac{q}{\hbar}\int_{s\to{\rm Pinhole1}\to{\rm screen}} (\phi_{\rm nl}dt - \mathbf{A}_{\rm nl}\cdot d\mathbf{x})\right]$$
$$\psi_2' = \psi_2 \cdot \exp\left[i\frac{q}{\hbar}\int_{s\to{\rm Pinhole2}\to{\rm screen}} (\phi_{\rm nl}dt - \mathbf{A}_{\rm nl}\cdot d\mathbf{x})\right] \tag{16}$$

where, ψ_1' and ψ_2' are the electron wave functions on the screen passing through pinhole 1 and 2 with the non-localized potentials respectively. ψ_1 and ψ_2 are the electron wave functions heading to pinhole 1 and 2 at the electron source without the effects of the non-localized potentials. $\phi_{\rm nl}$ and $\mathbf{A}_{\rm nl}$ include not only the non-localized potentials expressed as (8) but also the non-localized part of the potentials generated by localized potentials such as (3) and (5).

Then the probability of finding the electron on the screen by using these wave functions can be described as follows,

$$\begin{aligned} P_{12} &\propto |\psi'|^2 = |\psi_1' + \psi_2'|^2 \\ &= |\psi_1|^2 + |\psi_2|^2 \\ &-2{\rm Re}\left(\exp\left[i\frac{q}{\hbar}\oint_{s\to 1\to{\rm screen}\to 2\to s}(\phi_{\rm nl}dt - \mathbf{A}_{\rm nl}\cdot d\mathbf{x})\right]\psi_1^*\psi_2\right) \end{aligned} \tag{17}$$

where 1 and 2 of the integration path denote pinhole 1 and 2 respectively. In case of single electron interference, we can find the electron at pinhole 1 without fail but not at pinhole 2, i.e., $|\psi_1|^2 = 1$ and $|\psi_2|^2 = 0$. Although $\int |\psi_{1{\rm or}2}|^2 d\mathbf{V} = 1$ or 0 should be exact expression, we continue analysis with $|\psi_1|^2 = 1$ and $|\psi_2|^2 = 0$ for simplification.

When we introduce a phase difference θ between ψ_1 and ψ_2, P_{12} expresses the interference as follows,

$$P_{12} \propto 1 - 2{\rm Re}\left(\exp i\left[\gamma + \theta\right]\psi_1^*\psi_2\right) \tag{18}$$

where $\gamma = \frac{q}{\hbar}\oint_{s\to 1\to{\rm screen}\to 2\to s}(\phi_{\rm nl}dt - \mathbf{A}_{\rm nl}\cdot d\mathbf{x})$.

Note that when θ is fixed, the interference can be observed on the screen as a function of γ, i.e., position on the screen. When γ is fixed, the interference can be observed on a fixed position of the screen as a function of θ.

However, the wave function ψ_2 must satisfy $\psi_1^*\psi_2 \neq 0$ and $|\psi_2|^2 = 0$.

Then we introduce the states "an electron passes through pinhole 1 with the non-localized potentials" as $e^{i\gamma_1}|\psi_1\rangle$ with $P_1 = \langle\psi_1|\psi_1\rangle = 1$ and "no electron passes through pinhole 2 with the non-localized potentials" as $e^{i\gamma_2}|\psi_2\rangle$ with $P_2 = \langle\psi_2|\psi_2\rangle = 0$. In these states, γ_1, γ_2 and $\gamma = \gamma_1 - \gamma_2$ correspond to the phase terms of (16) and (17).

After the example of single photon interference as described above, [20] we introduce the charge operator $\mathbf{Q} \equiv \int d^3x j_0(x)$ defined by a conserved current $j_\mu = (q, \mathbf{i})$, i.e., $\partial^\mu j_\mu = \frac{\partial q}{\partial t} + \nabla\cdot\mathbf{i} = 0$. The charge operator satisfies $\mathbf{Q}|\psi_1\rangle = q|\psi_1\rangle$, which means the electron state incoming from pinhole 1 is the eigenstate of $\mathbf{Q}$. [25, 26]

The interference can be calculated using the charge operator as follows.

$$\begin{aligned} \langle I\rangle &= \\ &\left(e^{i(\theta-\gamma_1)}\langle\psi_1| + e^{-i\gamma_2}\langle\psi_2|\right)\mathbf{Q}\left(e^{-i(\theta-\gamma_1)}|\psi_1\rangle + e^{i\gamma_2}|\psi_2\rangle\right) \\ &= q + \langle\psi_2|\mathbf{Q}|\psi_2\rangle \\ &\quad + qe^{i(\theta-\gamma)}\langle\psi_1|\psi_2\rangle + qe^{-i(\theta-\gamma)}\langle\psi_2|\psi_1\rangle \end{aligned} \tag{19}$$

where $\langle I\rangle$ is the expectation value of charge intensity. If $\langle\psi_2|\mathbf{Q}|\psi_2\rangle = -\frac{1}{2}q$ and $\langle\psi_1|\psi_2\rangle = \langle\psi_2|\psi_1\rangle^* = \pm\frac{1}{4}e^{i\delta}$, then the single electron interference $\langle I\rangle = q\left\{\frac{1}{2} + \frac{1}{2}\cos(\theta - \gamma + \delta)\right\}$ can be obtained.

The above discussion suggests that the non-localized potentials produce phase shift of the electron wave functions or electron states and will correspond to gauge fields introduced by gauge transformation of the second kind as can be seen from (16). Hence, we can recognize the state "no electron passes through pinhole 2 with the non-localized potentials" has the phase difference for the interference without electron charges.

In the above expression for $|\psi_2\rangle$, there is no mathematical solution in usual Hilbert space. Therefore the state of "no electron passes through pinhole 2 with the non-localized potentials" must also be regarded as a vector with zero probability amplitude in indefinite metric Hilbert space and we can express the quantum state of the interference without quantum-superposition state.

Note that the calculation using the superposition state of (17) is valid in case of mixed state whose probability is statistical sense.

DISCUSSION

uncertainty principle and the reduction of the wave packet

By the existence of the non-localized potentials, Heisenberg's uncertainty principle can be explained independently of measurements. In addition, the paradox of the reduction of the wave packet typified by "Schrödinger's cat" and "Einstein, Podolsky and Rosen (EPR)" [1, 2] can be solved, because the origins of both are quantum-superposition state.

Former results insist the states of path 1 and 2 or pinhole 1 and 2 by Schrödinger picture are defined when the system is prepared expressed as a substantial single photon or electron and the non-localized potentials respectively and each state does not split off such as quantum-superposition state, which means there is no reduction of the wave packet.

"When the system is prepared" corresponds to immediately after the branching point of the optical MZI or the pinholes. Which path or pinhole does the photon or electron select is unpredictable but after the selection, the state is fixed instead of quantum-superposition state. The concept of these states is identical with mixed states rather than pure states formed by quantum-superposition, which suggests there is no concept of quantum-superposition state.

As for Heisenberg's uncertainty principle, we should recognize it as trade-offs derived from Fourier transform non-related to measurement, which correspond to the canonical commutation relation.

zero-point energy

If we calculate the equation (12) under vacuum instead of single photon, $\langle\zeta|\hat{a}^\dagger\hat{a}|\zeta\rangle = -\frac{1}{2}$ can eliminate zero-point energy as follows.

$$\begin{aligned}
&(\langle 0| + \langle\zeta|)\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right)(|0\rangle + |\zeta\rangle) \\
&= \frac{1}{2} + \frac{1}{2}(\langle 0|\zeta\rangle + \langle\zeta|0\rangle + \langle\zeta|\zeta\rangle) \\
&+\langle 0|\hat{a}^\dagger\hat{a}|\zeta\rangle + \langle\zeta|\hat{a}^\dagger\hat{a}|0\rangle + \langle\zeta|\hat{a}^\dagger\hat{a}|\zeta\rangle \\
&= \frac{1}{2} + \langle\zeta|\hat{a}^\dagger\hat{a}|\zeta\rangle = \frac{1}{2} - \frac{1}{2} = 0
\end{aligned} \tag{20}$$

where $\hat{a}|0\rangle = \left(\langle 0|\hat{a}^\dagger\right)^\dagger = 0$ and normalization of probability, i.e.,

$$\begin{aligned}
(\langle 0| + \langle\zeta|)(|0\rangle + |\zeta\rangle) &= \langle 0|0\rangle + \langle 0|\zeta\rangle + \langle\zeta|0\rangle + \langle\zeta|\zeta\rangle \\
&= 1 + \langle 0|\zeta\rangle + \langle\zeta|0\rangle + \langle\zeta|\zeta\rangle = 1
\end{aligned}$$

then $\langle 0|\zeta\rangle + \langle\zeta|0\rangle + \langle\zeta|\zeta\rangle = 0$, are used.

spontaneous symmetry breaking

Traditional treatment of the spontaneous symmetry breaking, which explores the possibility of $\mathbf{Q}|0\rangle \neq 0$ or generally "$|0\rangle$ is not an eigenstate of $\mathbf{Q}$", introduces an artificial intricate boson field such as Goldstone boson or Higgs boson. [26] Where $|0\rangle$ is vacuum state.

However, the non-localized potentials eternally populate the whole of space as mentioned above and there are no electron at pinhole 2. Therefore the state of pinhole 2, $e^{i\gamma_2}|\psi_2\rangle$, can be identified as vacuum instead of $|0\rangle$. From the relation $\langle\psi_2|\psi_2\rangle = 0$ as described above, if $e^{i\gamma_2}|\psi_2\rangle$ is an eigenstate of $\mathbf{Q}$, i.e., $\mathbf{Q}e^{i\gamma_2}|\psi_2\rangle = \alpha e^{i\gamma_2}|\psi_2\rangle$, then $\langle\psi_2|e^{-i\gamma_2}\mathbf{Q}e^{i\gamma_2}|\psi_2\rangle = \alpha\langle\psi_2|\psi_2\rangle = 0 \neq -\frac{1}{2}q$, where α is an eigenvalue. Hence the vacuum $e^{i\gamma_2}|\psi_2\rangle$ is not an eigenstate of $\mathbf{Q}$, which expresses the spontaneous symmetry breaking.

The traditional intricate bosons may correspond to the non-localized potentials.

SUMMARY

There are some unresolved paradoxes in quantum theory.

If we take advantage of the indefinite metric vectors as described in this report, the paradoxes can be removed. In addition, it can explain the uncertainty principle independently of measurements, eliminate zero-point energy and cause spontaneous symmetry breaking without complexity.

We should consistently introduce indefinite metric vectors because Maxwell equations are wave equations in Minkowski space. When we introduce state vectors in Minkowski space, indefinite metric vectors are absolutely required. The required vector should be recognized not only as an auxiliary field but also as a real physical field which is the root cause of single photon and electron interferences.

The results insist the vacuum space is filled with the non-localized potentials which can eternally exist as waves and correspond to gauge fields introduced by gauge transformation of the second kind.

This idea provides exactly the same calculation and experimental results by using quantum-superposition state because the phase difference between the photon or electron and the non-localized potentials provide the interferences as if the quantum-superposition state exists. In addition, the concept is based on an analogy from the expression of substantial localize electromagnetic fields or an electron flow and the non-localized potentials instead of curious quantum-superposition state that forces us to imagine a photon or an electron passes through the both paths or pinholes despite a photon or an electron can not be split off.

The superposition states are valid in case of mixed states whose probabilities are statistical sense. However, quantum-superposition state is not valid in case of pure state whose probability is fundamental sense. Therefor, there is no concept of quantum-superposition state in nature, which insists Quantum theory is a kind of statistical physics.

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