Exploring Prime Numbers and Modular Functions II:

On the Prime Number via Elliptic Integral Function

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In the beginning was the Word, and the Word was with God, and the Word was God.

John 1:1

ABSTRACT. The main goal this paper is to develop an asymptotic formula for the prime number, using elliptic integral function.

1. INTRODUCTION

As consequence of the prime number theorem, I put the asymptotic formula for the *n*th prime number, denoted by p_n:

(1) $$p_n \sim n \ln n.$$

In this paper, I prove that

$$p_n \sim 2nK\left(\frac{256n-4}{256n+4}\right)-\frac{\ln 2}{8}(64n+1)-\frac{\ln n}{64}.$$

2. THEOREM

THEOREM 1. *I have*

$$p_n \sim 2nK\left(\frac{256n-4}{256n+4}\right)-\frac{\ln 2}{8}(64n+1)-\frac{\ln n}{64},$$

where p_n *denotes the* *n*th prime number and $K(x)$ denotes the complete elliptic integral of first kind.

Proof. In [1], we encounter the identity

(2) $$\operatorname{agm}(x,y)=\frac{\pi}{4}\cdot\frac{x+y}{K\left(\frac{x-y}{x+y}\right)}.$$

In [2], we encounter an alternative for extremely high precision calculation is the formula

$$\ln n \approx \frac{\pi}{2\operatorname{agm}(1,4/s)}-m\ln 2,$$

where agm denotes the arithmetic-geometric mean of 1 and $4/s$, and

$$s=n\cdot 2^m>2^{p/2},$$

with m chosen so that p bits of precision is attained. Now, the value of 8 for m is sufficient. Hence,

(3) $$\ln n \approx \frac{\pi}{2\operatorname{agm}(1,4/256n)}-8\ln 2.$$

Substituting (2) into (3), I get around

$$\ln n \approx \frac{512nK\left(\frac{256n-4}{256n+4}\right)}{256n+4} - 8\ln 2 = \frac{128nK\left(\frac{256n-4}{256n+4}\right) - 8\ln 2\,(64n+1)}{64n+1}. \tag{4}$$

Wherefore,

$$(64n+1)\ln n \approx 128nK\left(\frac{256n-4}{256n+4}\right) - 8\ln 2\,(64n+1), \tag{5}$$

$$n\ln n \approx \frac{128}{64}nK\left(\frac{256n-4}{256n+4}\right) - \frac{8}{64}\ln 2\,(64n+1) - \frac{\ln n}{64},$$

$$n\ln n \approx 2nK\left(\frac{256n-4}{256n+4}\right) - \frac{\ln 2}{8}(64n+1) - \frac{\ln n}{64}.$$

I take (5) in (1), and achieve

$$p_n \sim 2nK\left(\frac{256n-4}{256n+4}\right) - \frac{\ln 2}{8}(64n+1) - \frac{\ln n}{64}.$$

This completes the proof. □

REFERENCES

[1] http://en.wikipedia.org/wiki/Arithmetic_geometric_mean, available in November 16, 2013.

[2] http://en.wikipedia.org/wiki/Natural_logarithm, available in November 16, 2013.

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