EFFECTIVE ISO-RADIUS OF DYNAMIC ISO-SPHERE HOLOGRAPHIC RINGS

Nathan O. Schmidt
Department of Mathematics
Boise State University
1910 University Drive
Boise, ID 83725, USA
nathanschmidt@u.boisestate.edu

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Abstract

In this work, we introduce the "effective iso-radius" for dynamic iso-sphere Inopin holographic rings (IHR) as the iso-radius varies, which facilitates a heightened characterization of these emerging, cutting-edge iso-spheres as they vary in size and undergo "iso-transitions" between "iso-states". The initial results of this exploration fuel the construction of a new "effective iso-state" platform with a potential for future scientific application, but this emerging dynamic iso-architecture warrants further development, scrutiny, collaboration, and hard work in order to advance it as such.

Keywords: Santilli iso-number; Inopin holographic ring; Santilli iso-sphere; Dynamic iso-sphere; Iso-radius; Effective iso-state.

1 Introduction

Santilli's iso-mathematics [1, 2, 3, 4, 5] has sparked a revolution in *universal* number classification and topology. Recently, the triplex numbers and Inopin's dual 4D space-time IHR topology [6, 7] were iso-topically lifted [1, 2, 3, 4, 5] to construct the iso-triplex numbers and the iso-dual 4D space-time IHR topology for iso-fractals [8]. Subsequently, these emerging developments [8] were deployed to propose a topological iso-string theory [9] and assemble Mandelbrot iso-sets [10]. Furthermore, such implementations facilitated the *dynamic* iso-topic lifting of iso-spaces to install dynamic iso-spaces [11], which built the foundation of dynamic iso-sphere IHRs with exterior and interior iso-duality [12].

In this assignment, we focus on advancing the representation of dynamic iso-sphere IHRs [12] by forging the *effective iso-radius* platform to launch the encoding of their characteristic "iso-transitions" between "iso-states" as they vary in size. The effective iso-radius concept introduced in this paper was originally inspired by the "effective radius" from Corda's new black hole effective state framework [13, 14, 15, 16]. However, this paper is devoted to iso-mathematics rather than physics; thus, the effective representation proposed here targets spherically-symmetric iso-mathematical structures rather than spherically-symmetric physical quantities. Hence, for now, we limit our investigation to the realm of iso-mathematics [1, 2, 3, 4, 5] but recognize a significant potential for scientific application in the near future. Thus, we conduct our investigation with Section refsection:procedure by presenting a step-by-step procedure that constructs the effective iso-radius for a dynamic iso-sphere IHR [12] with dynamic iso-topic lifting [11] in the iso-dual 4D space-time IHR topology [8]. Finally, we conclude our paper with Section 3, where we recapitulate the results of Section 2 with a brief discussion and suggest future modes of research.

2 Procedure

In the venture of this section, we assemble the effective iso-radius for a dynamic iso-sphere IHR [12] and thereby introduce the notion of "effective iso-state".

2.1 Initializing the dual 4D space-time IHR topology

Here, we instantiate the dual 4D space-time IHR topology via the following procedure:

1. First, from eq. (7) of [8], let $X = \mathbb{C}$ be the set of all complex numbers, the Euclidean complex space, and the *dual 2D Cartesian-polar coordinate-vector state space*, where the complex number $\vec{x} \in X$ is a *dual 2D Cartesian-polar coordinate-vector state* that is defined by eq. (6) of [8] as
$$x = \vec{x} = \vec{x}_{\mathbb{R}} + \vec{x}_{\mathbb{I}} = (\vec{x}) = (|\vec{x}|, \langle\vec{x}\rangle)_P = (\vec{x}_{\mathbb{R}}, \vec{x}_{\mathbb{I}})_C, \quad \forall \vec{x} \in X. \tag{1}$$
In eq. (1), $(\vec{x}_{\mathbb{R}}, \vec{x}_{\mathbb{I}})_C$ is a *2D Cartesian coordinate-vector state* in the *2D Cartesian coordinate-vector state space* X_C so $(\vec{x}_{\mathbb{R}}, \vec{x}_{\mathbb{I}})_C \in X_C$, while $(|\vec{x}|, \langle\vec{x}\rangle)_P$ is a *2D polar coordinate-vector state* in the *2D polar coordinate-vector state space* X_P so $(|\vec{x}|, \langle\vec{x}\rangle)_P \in X_P$, where X_C and X_P are iso-morphic, dual, synchronized, and interlocking in X [8]. Thus, eq. (1) complies with the constraints imposed by eqs. (8–13) of [8]—see Figure 1.

2. Second, from eq. (16) of [8] we have
$$T^1 = \{\vec{x} \in X : |\vec{x}| = r\}, \tag{2}$$
where $T^1 \subset X$ is the 1-sphere IHR of amplitude-radius $r > 0$ (with corresponding curvature $\kappa = \frac{1}{r}$) that is centered on the origin $O \in X$; T^1 is the multiplicative group of all non-zero complex numbers with amplitude-radius r, which is iso-metrically embedded in X and is simultaneously dual to the two complex sub-spaces X_- and X_+ [8, 6, 7]—see Figure 2.

3. Third, from eq. (18) of [8], let $Y \equiv \mathbb{T}$ be the set of all triplex numbers, the Euclidean triplex space, and the *dual 3D Cartesian-spherical coordinate-vector state space*, where the triplex number $\vec{y} \in Y$ is a *dual 3D Cartesian-spherical coordinate-vector state* that is defined by eq. (17) of [8] as
$$y = \vec{y} = \vec{y}_{\mathbb{R}} + \vec{y}_{\mathbb{I}} + \vec{y}_Z = (\vec{y}) = (|\vec{y}|, \langle\vec{y}\rangle, [\vec{y}])_S = (\vec{y}_{\mathbb{R}}, \vec{y}_{\mathbb{I}}, \vec{y}_Z)_C, \quad \forall \vec{y} \in Y. \tag{3}$$

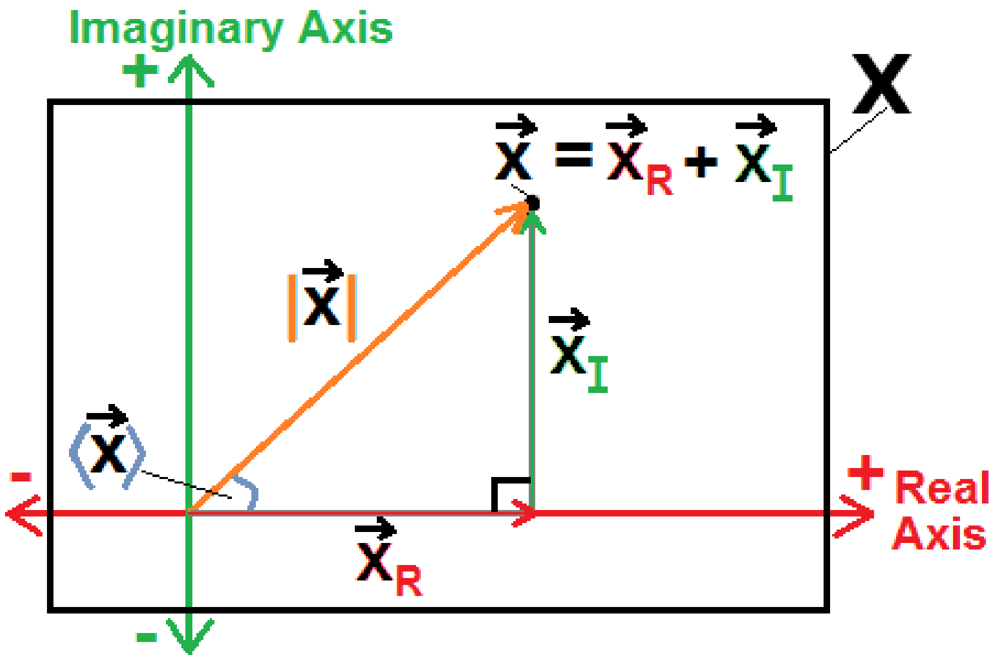

Fig. 1: Complex components for the dual 2D Cartesian-polar coordinate-vector state $\vec{x}$ in the dual 2D Cartesian-polar coordinate-vector state space (and Euclidean complex space) X, such that $\vec{x} \in X$, where $\vec{x}$ is simultaneously treated as a complex number, 2D polar coordinate-vector, and 2D Cartesian coordinate-vector [8]. Specifically, $(\vec{x}_{\mathbb{R}}, \vec{x}_{\mathbb{I}})_C$ is a 2D Cartesian coordinate-vector state in the 2D Cartesian coordinate-vector state space X_C so $(\vec{x}_{\mathbb{R}}, \vec{x}_{\mathbb{I}})_C \in X_C$, while $(|\vec{x}|, \langle\vec{x}\rangle)_P$ is a 2D polar coordinate-vector state in the 2D polar coordinate-vector state space X_P so $(|\vec{x}|, \langle\vec{x}\rangle)_P \in X_P$, where X_C and X_P are iso-morphic, dual, synchronized, and interlocking in X [8]. Note that $\vec{x}_{\mathbb{R}}$ and $\vec{x}_{\mathbb{I}}$ are treated as vectors (with axis-dependent magnitude and direction) so the vector sum is $\vec{x} = \vec{x}_{\mathbb{R}} + \vec{x}_{\mathbb{I}}$ with amplitude $|\vec{x}|$ and direction $\langle\vec{x}\rangle$ [8].

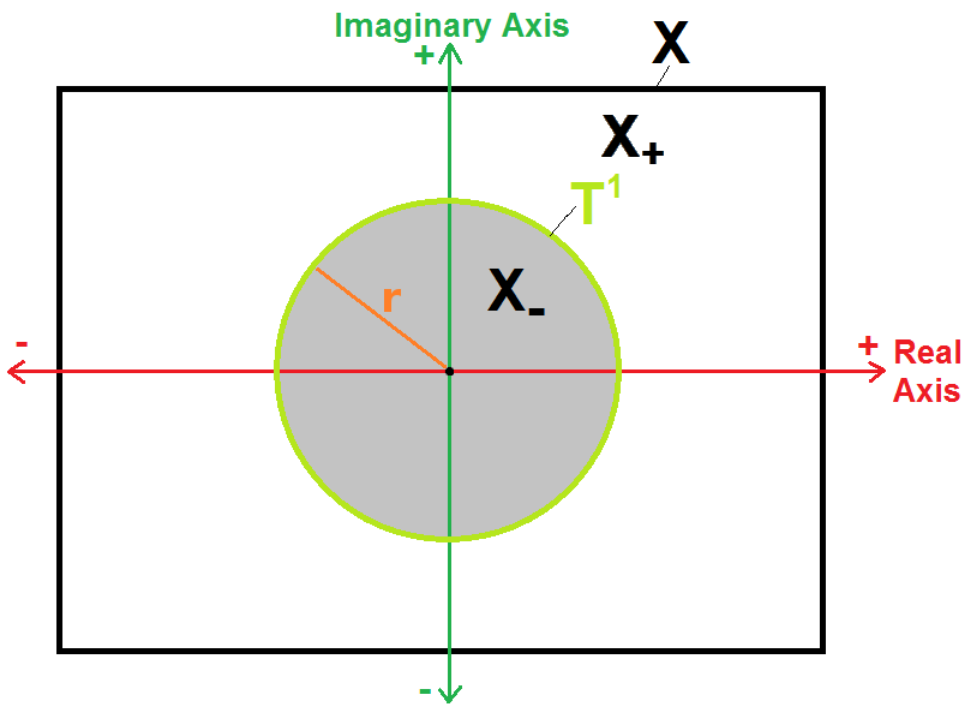

Fig. 2: The dual 3D space-time 1-sphere IHR topology for the dual 2D Cartesian-polar coordinate-vector state space (and Euclidean complex space) X, where the topological 1-sphere IHR $T^1 \subset X$ is simultaneously dual to two spatial 2-branes [8, 6]: the "2D micro sub-space zone" $X_- \subset X$ and the "2D macro sub-space zone" $X_+ \subset X$ for interior and exterior dynamical systems, respectively [8].

In eq. (3), $(\vec{y}_{\mathbb{R}}, \vec{y}_{\mathbb{I}}, \vec{y}_Z)_C$ is a *3D Cartesian coordinate-vector state* in the *3D Cartesian coordinate-vector state space* Y_C so $(\vec{y}_{\mathbb{R}}, \vec{y}_{\mathbb{I}}, \vec{y}_Z)_C \in Y_C$, while $(|\vec{y}|, \langle\vec{y}\rangle, [\vec{y}])_S$ is a *3D spherical coordinate-vector state* in the *3D spherical coordinate-vector state space* Y_S so $(|\vec{y}|, \langle\vec{y}\rangle, [\vec{y}])_S \in Y_S$, where Y_C and Y_S are iso-morphic, dual, synchronized, and interlocking in Y [8]. Thus, eq. (3) complies with the constraints imposed by eqs. (19–28) of [8]—see Figures 3 and 4.

4. Fourth, from eq. (33) of [8] we have

$$T^2 = \{\vec{y} \in Y : |\vec{y}| = r\}, \tag{4}$$

where $T^2 \subset Y$ is the 2-sphere IHR of amplitude-radius $r > 0$ (the same as T^1) that is centered on the origin $O \in X, Y$; T^2 is the multiplicative group of all non-zero triplex numbers with amplitude-radius r, which is iso-metrically embedded in Y and is simultaneously dual to the two triplex sub-spaces Y_- and Y_+ [8, 6, 7]. Here, given $X \subset Y$, $T^1 \subset T^2, X, Y$ is the great circle of T^2, such that $T^1 = X \cap T^2$ [8, 6, 7]—see Figure 5.

At this point, we've initialized the dual 4D space-time IHR topology [8, 6, 7]. Therefore, we are ready to explore the proposed the effective iso-radius encoding platform of Section 2.2.

2.2 Constructing the effective iso-radius for the iso-dual 4D space-time IHR topology

Here, we assemble the effective iso-radius encoding platform for representing iso-sphere IHR [12] iso-states and iso-transitions in the iso-dual 4D space-time topology [8] via the following procedure:

1. First, in conventional mathematics, the number 1 is the multiplicative identity that satisfies the original number field axioms [18]. Thus, the number 1 plays important and diverse roles throughout mathematics in general such as, for example, normalization in statistics. Therefore, we start by setting the amplitude-radius $r = 1$ for T^1 and T^2.

2. Second, in iso-mathematics [1, 2, 3, 4, 5, 8], Santilli demonstrated that the multiplicative unit is not limited to the number 1 and can

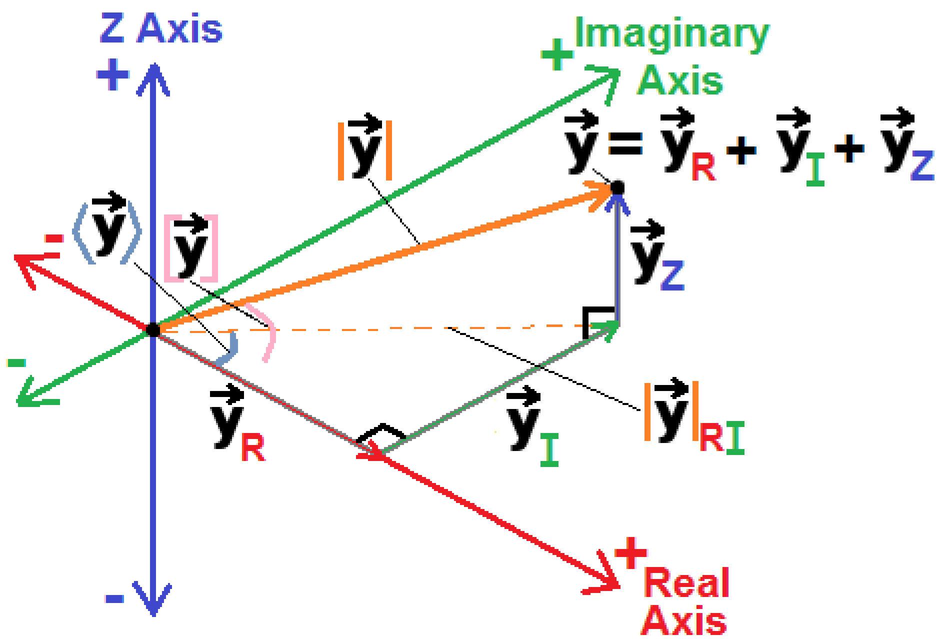

Fig. 3: Triplex components for the dual 3D Cartesian-spherical coordinate-vector state $\vec{y}$ in the dual 3D Cartesian-polar coordinate-vector state space (and Euclidean triplex space) Y, such that $\vec{y} \in Y$, where $\vec{y}$ is simultaneously treated as a triplex number, 3D spherical coordinate-vector, and 3D Cartesian coordinate-vector [8]. Specifically, $(\vec{y}_{\mathbb{R}}, \vec{y}_{\mathbb{I}}, \vec{y}_Z)_C$ is a 3D Cartesian coordinate-vector state in the 3D Cartesian coordinate-vector state space Y_C so $(\vec{y}_{\mathbb{R}}, \vec{y}_{\mathbb{I}}, \vec{y}_Z)_C \in Y_C$, while $(|\vec{y}|, \langle\vec{y}\rangle, [\vec{y}])_S$ is a 3D spherical coordinate-vector state in the 3D spherical coordinate-vector state space Y_S so $(|\vec{y}|, \langle\vec{y}\rangle, [\vec{y}])_S \in Y_S$, where Y_C and Y_S are iso-morphic, dual, synchronized, and interlocking in Y [8]. Note that $vecy_{\mathbb{R}}$, $\vec{y}_{\mathbb{I}}$, and $\vec{y}_Z$ are treated as vectors (with axis-dependent magnitude and direction) so the vector sum is $\vec{y} = \vec{y}_{\mathbb{R}} + \vec{y}_{\mathbb{I}} + \vec{y}_Z$ with amplitude $|\vec{y}|$ and two directions $\langle\vec{y}\rangle$ and $[\vec{y}]$ [8].

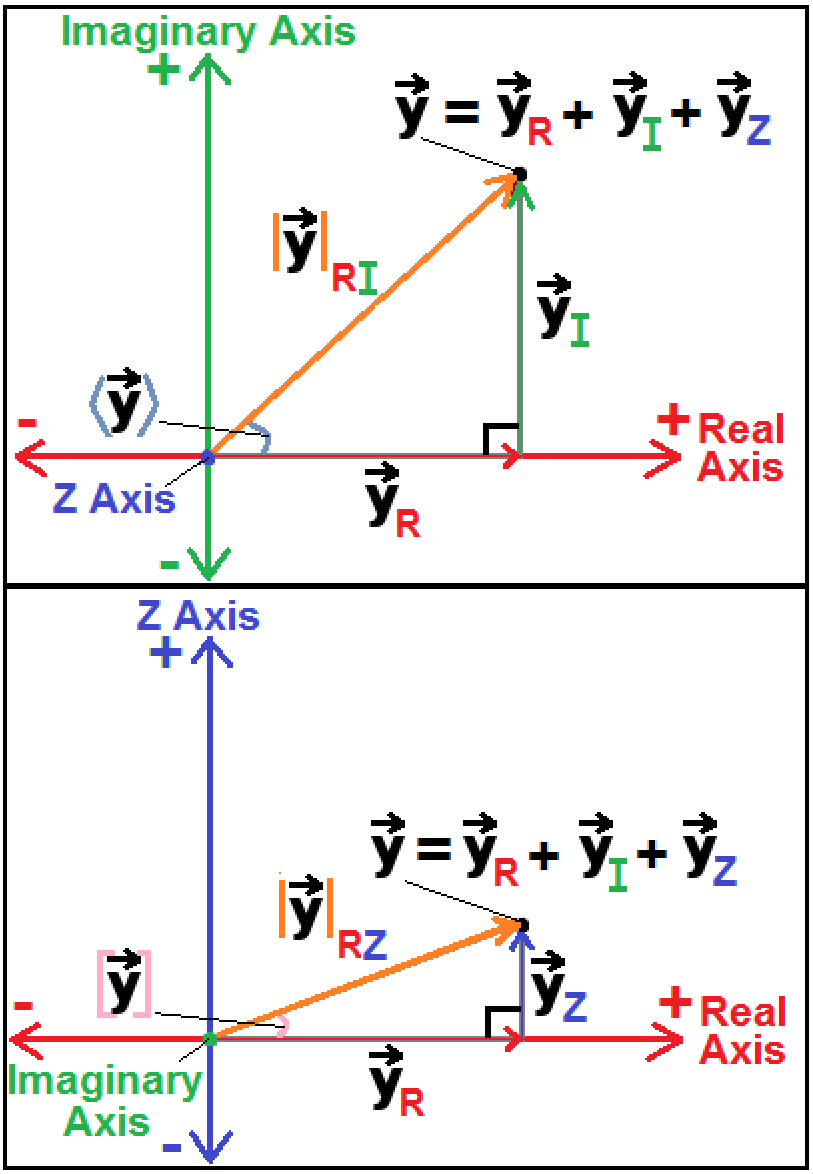

Fig. 4: Aligned perspectives of $\vec{y} \in Y$ from the $\mathbb{R}\mathbb{I}$-plane (top) and the $\mathbb{R}Z$-plane (bottom) [8].

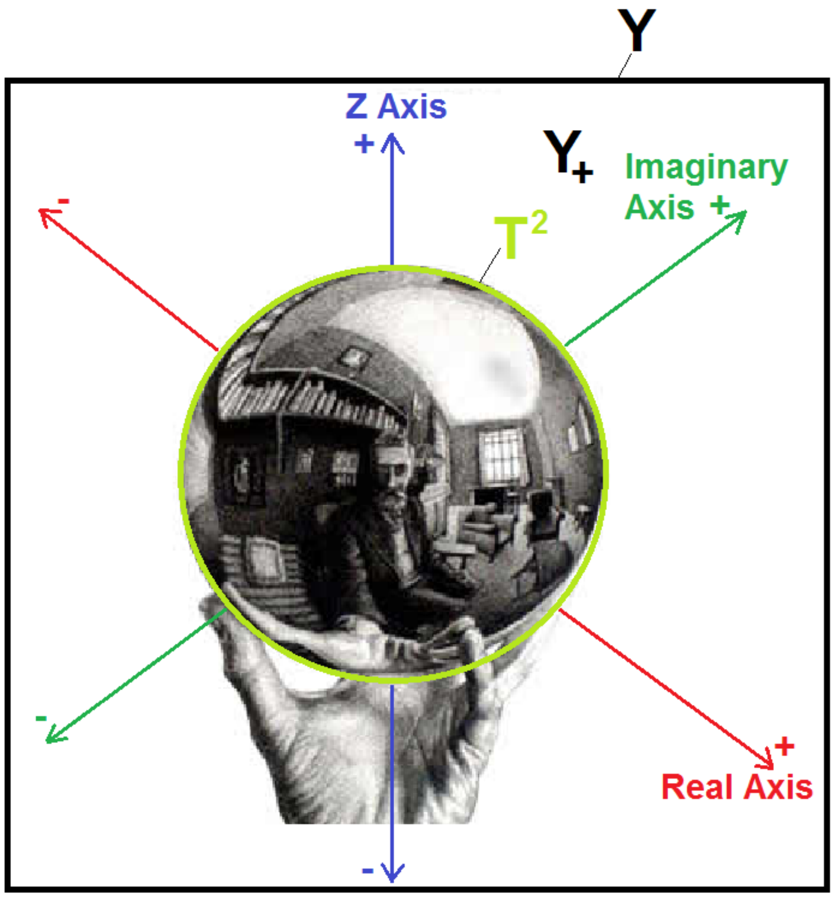

Fig. 5: The dual 4D space-time 2-sphere IHR topology in the dual 3D Cartesian-spherical coordinate-vector state space (and Euclidean triplex space) Y, where the topological 2-sphere IHR $T^2 \subset Y$ is simultaneously dual to two spatial 3-branes [8, 6]: the "3D micro sub-space zone" $Y_{-} \subset Y$ and the "3D macro sub-space zone" $Y_{+} \subset Y$ for interior and exterior dynamical systems, respectively [8]. Here, T^2 is depicted as M. C. Escher's famous reflecting sphere [17].

therefore be replaced with the positive-definite iso-multiplicative iso-unit $\hat{r} > 0$ with corresponding inverse $\hat{\kappa} = \frac{1}{\hat{r}} > 0$ for iso-numbers. Hence, for some $\hat{r}$, we employ Santilli's iso-methodology [1, 2, 3, 4, 5] to iso-topically lift T^2 via

$$\vec{y}_{\hat{r}} \equiv \vec{y} \times \hat{r}, \ \forall \vec{y} \in T^2 \rightarrow \forall \vec{y}_{\hat{r}} \in T^2_{\hat{r}}, \tag{5}$$

for the transition and its inverse

$$\begin{aligned} f(\hat{r}, T^2): \quad & T^2 \ \rightarrow \ T^2_{\hat{r}} \\ f^{-1}(\hat{r}, T^2_{\hat{r}}): \quad & T^2_{\hat{r}} \ \rightarrow \ T^2 \end{aligned} \tag{6}$$

to identify the iso-2-sphere IHR of iso-radius (or "iso-amplitude-radius") $\hat{r}$ from [8], so T^2 and $T^2_{\hat{r}}$ are *locally iso-morphic* and are both centered on the origin $O \in X, Y$. Here, note that in addition to being the iso-radius of T^1 and T^2, $\hat{r}$ also serves as the iso-unit for Santilli's iso-multiplication [1, 2, 3, 4, 5, 8], where the iso-unit inverse $\hat{\kappa}$ is also the iso-curvature of T^1 and T^2.

3. Third, given the dynamic iso-topic lifting and dynamic iso-spheres of [11, 12], we furthermore define T^2's iso-radius as an iso-function in the positive-definite form

$$T^2_{\hat{r}(m)}: \ \hat{r} \equiv \hat{r}(m) \equiv ma + b \equiv \frac{1}{\hat{\kappa}(m)} > 0, \tag{7}$$

where $\hat{r}(m)$ is the *dynamic iso-radius iso-function* (or "dynamic iso-unit iso-function") with the parameter m and $\hat{\kappa}(m)$ is the corresponding *dynamic iso-curvature iso-function*, such that m is some *mathematical* quantity while a and b are coefficients. Thus, eq. (5) is rewritten as

$$\vec{y}_{\hat{r}(m)} \equiv \vec{y} \times \hat{r}(m), \ \forall \vec{y} \in T^2 \rightarrow \forall \vec{y}_{\hat{r}(m)} \in T^2_{\hat{r}(m)} \tag{8}$$

so eq. (6) becomes

$$\begin{aligned} f(\hat{r}(m), T^2): \quad & T^2 \quad \rightarrow \ T^2_{\hat{r}(m)} \\ f^{-1}(\hat{r}(m), T^2_{\hat{r}(m)}): \quad & T^2_{\hat{r}(m)} \ \rightarrow \ T^2. \end{aligned} \tag{9}$$

4. Fourth, given that eq. (7) is a *dynamic* iso-unit iso-function, we wish to show that $\hat{r}(m)$ is characterized by constant change as m varies and takes on values from some positive-definite sequence M, such that $m \in M$ as $m \to \infty$. In [11, 12], there are *two* distinct types of dynamic iso-unit iso-functions:

   - *continuous dynamic iso-unit iso-functions*, so M may be a *continuous* sequence of positive-definite values such as, for example, the case of $M \equiv M_{\mathbb{R}+}$ for the positive-definite interval of *real numbers*

   $$M_{\mathbb{R}+} = (0, \infty_{\mathbb{R}+}),\ m \in M_{\mathbb{R}+},\ m \to \infty_{\mathbb{R}+}; \tag{10}$$

   and

   - *discrete dynamic iso-unit iso-functions*, so M may be a *discrete* sequence of positive-definite values such as, for example, the case of $M \equiv M_{\mathbb{N}}$ for the positive-definite set of *natural numbers*

   $$M_{\mathbb{N}} = \{1, 2, 3, 4, 5, ...\},\ m \in M_{\mathbb{N}},\ m \to \infty_{\mathbb{N}} \tag{11}$$

   or in the case of $M \equiv M_{Fib}$ for the positive-definite set of *Fibonacci numbers*

   $$M_{Fib} = \{1, 1, 2, 3, 5, ...\},\ m \in M_{Fib},\ m \to \infty_{Fib}. \tag{12}$$

5. Fifth, for this introductory investigation, consider a relatively simple case and suppose that $a = 2$ and $b = 0$, where we know that the $\hat{r}(m) > 0$ and $\hat{\kappa}(m) > 0$ of eq. (7) will remain positive-definite as $m > 0$ varies, regardless of whether the positive-definite M is continuous or discrete. Thus, eq. (7) is rewritten as

   $$T^2_{\hat{r}(m)}:\ \hat{r} \equiv \hat{r}(m) \equiv m2 + 0 \equiv 2m \equiv \frac{1}{\hat{\kappa}(m)} > 0. \tag{13}$$

   In the procedure of this initial thought experiment, we will operate eq. (13) with $\hat{r}(m) = 2m$, but note that eq. (13) could be rewritten again to relate $\hat{r}$ to additional *mathematical* quantities as long as it complies with Santilli's positive-definite iso-unit constraint $\hat{r}(m) > 0$ [1, 2, 3, 4, 5, 8] for the iso-topic liftings of eqs. (8–9).

6. Sixth, given the fundamental exterior and interior iso-duality establishment [12], we briefly note that

$$\begin{aligned} T^2_{\hat{r}(m)} &\equiv T^2_{\hat{r}_+(m)} \\ T^2_{\hat{\kappa}(m)} &\equiv T^2_{\hat{r}_-(m)} \end{aligned} \tag{14}$$

because in this context the $T^2_{\hat{r}(m)} \equiv T^2_{\hat{r}_+(m)}$ of "outer" iso-radius $\hat{r} \equiv \hat{r}_+(m)$ is the *exterior 2-sphere IHR* that is "outside" of T^2 because $T^2_{\hat{r}_+(m)} \subset Y_+$, while the $T^2_{\hat{\kappa}(m)} \equiv T^2_{\hat{r}_-(m)}$ of "inner" iso-radius $\hat{\kappa} \equiv \hat{r}_-(m)$ is the *interior 2-sphere IHR* that is "inside" of T^2 because $T^2_{\hat{r}_-(m)} \subset Y_-$ [12]: $T^2_{\hat{r}_+(m)}$ and $T^2_{\hat{r}_-(m)}$, or equivalently $T^2_{\hat{\kappa}(m)}$ and $T^2_{\hat{r}(m)}$, are iso-dual [12] due to the fact that

$$\hat{r}_+(m) \equiv \hat{r}(m) \equiv \frac{1}{\hat{\kappa}(m)} \equiv \frac{1}{\hat{r}_-(m)}. \tag{15}$$

7. Seventh, given eq. (13), we define the *initial iso-radius iso-state* of $T^2_{\hat{r}(m)}$ as

$$T^2_{\hat{r}(m_0)}: \ \hat{r}_0 \equiv \hat{r}(m_0) \equiv 2m_0 \equiv \frac{1}{\hat{\kappa}(m_0)} > 0, \tag{16}$$

to identify the *initial iso-2-sphere IHR iso-state* $T^2_{\hat{r}(m_0)}$, where $\hat{r}(m_0) > 0$ is the *initial iso-radius*, $\hat{\kappa}(m_0) > 0$ is the *initial iso-curvature*, and $m_0 > 0$ is the *initial quantity*, such that $m_0 \in M$, regardless of whether the positive-definite M is continuous or discrete. Therefore, for this initial case we assign $m = m_0$ for eq. (8) to establish

$$\vec{y}_{\hat{r}(m_0)} \equiv \vec{y} \times \hat{r}(m_0), \ \forall \vec{y} \in T^2 \to \forall \vec{y}_{\hat{r}(m_0)} \in T^2_{\hat{r}(m_0)} \tag{17}$$

so eq. (9) becomes

$$\begin{aligned} f(\hat{r}(m_0), T^2): \quad & T^2 && \to T^2_{\hat{r}(m_0)} \\ f^{-1}(\hat{r}(m_0), T^2_{\hat{r}(m_0)}): \quad & T^2_{\hat{r}(m_0)} && \to T^2. \end{aligned} \tag{18}$$

8. Eighth, suppose that the quantity m_0 undergoes a change that is characterized by

$$\delta_m : \; m_0 \to m_1, \tag{19}$$

which causes

$$\delta_{\hat{r}(m)} : \; \hat{r}(m_0) \to \hat{r}(m_1), \tag{20}$$

such that

$$m_0 = m_1 + \Delta_m. \tag{21}$$

Thus, a second version of eq. (16) is written to define the *final iso-radius iso-state* of $T^2_{\hat{r}(m)}$ as

$$T^2_{\hat{r}(m_1)} : \; \hat{r}_1 \equiv \hat{r}(m_1) \equiv 2m_1 \equiv \frac{1}{\hat{\kappa}(m_1)} > 0, \tag{22}$$

to identify the *final iso-2-sphere IHR iso-state* $T^2_{\hat{r}(m_1)}$, where $\hat{r}(m_1) > 0$ is the *final iso-radius*, $\hat{\kappa}(m_1) > 0$ is the *final iso-curvature*, and $m_1 > 0$ is the *final quantity*, such that $m_1 \in M$, regardless of whether the positive-definite M is continuous or discrete. Therefore, for this final case we assign $m = m_1$ for eq. (8) to establish

$$\vec{y}_{\hat{r}(m_1)} \equiv \vec{y} \times \hat{r}(m_1), \; \forall \vec{y} \in T^2 \to \forall \vec{y}_{\hat{r}(m_1)} \in T^2_{\hat{r}(m_1)} \tag{23}$$

so eq. (18) becomes

$$\begin{aligned} f(\hat{r}(m_1), T^2) : \quad & T^2 && \to \; T^2_{\hat{r}(m_1)} \\ f^{-1}(\hat{r}(m_1), T^2_{\hat{r}(m1)}) : \quad & T^2_{\hat{r}(m_1)} && \to \; T^2. \end{aligned} \tag{24}$$

9. Ninth, given the impact of eqs. (19–24), the initial iso-2-sphere IHR iso-state $T^2_{\hat{r}(m_0)}$ (of initial iso-radius $\hat{r}(m_0)$) is iso-topically lifted to the final iso-2-sphere IHR iso-state $T^2_{\hat{r}(m_1)}$ (of final iso-radius $\hat{r}(m_1)$) via

$$\vec{y}_{\hat{r}(m_1)} \equiv \vec{y}_{\hat{r}(m_0)} \times \frac{\hat{r}(m_1)}{\hat{r}(m_0)}, \; \forall \vec{y}_{\hat{r}(m_0)} \in T^2_{\hat{r}(m_0)} \to \forall \vec{y}_{\hat{r}(m_1)} \in T^2_{\hat{r}(m_1)} \tag{25}$$

for the iso-transition and its inverse

$$\begin{aligned} f(\tfrac{\hat{r}(m_1)}{\hat{r}(m_0)}, T^2_{\hat{r}(m_0)}): \quad & T^2_{\hat{r}(m_0)} \quad \rightarrow \quad T^2_{\hat{r}(m_1)} \\ f^{-1}(\tfrac{\hat{r}(m_1)}{\hat{r}(m_0)}, T^2_{\hat{r}(m_1)}): \quad & T^2_{\hat{r}(m_1)} \quad \rightarrow \quad T^2_{\hat{r}(m_0)} \end{aligned} \tag{26}$$

with the iso-radius ratio $\frac{\hat{r}(m_1)}{\hat{r}(m_0)}$ and the corresponding iso-curvature ratio $\frac{\hat{r}(m_0)}{\hat{r}(m_1)}$ characterize the iso-transition to establish that T^2, $T^2_{\hat{r}(m_0)}$, and $T^2_{\hat{r}(m_1)}$ are indeed locally iso-morphic.

10. Tenth, we note that the iso-transition between $T^2_{\hat{r}(m_0)}$ and $T^2_{\hat{r}(m_1)}$ depends on Δ_m and complies with the trichotomy:

    - **Case $\Delta_m < 0$:** $T^2_{\hat{r}(m_0)}$ is *de-magnified* to become $T^2_{\hat{r}(m_1)}$ via the iso-topic lifting $T^2_{\hat{r}(m_0)} \rightarrow T^2_{\hat{r}(m_1)}$ because $m_1 < m_0$ so $\hat{r}(m_1) < \hat{r}(m_0)$.
    - **Case $\Delta_m = 0$:** $T^2_{\hat{r}(m_0)}$ is equivalent to $T^2_{\hat{r}(m_1)}$ via the iso-topic lifting $T^2_{\hat{r}(m_0)} \rightarrow T^2_{\hat{r}(m_1)}$ because $m_1 = m_0$ so $\hat{r}(m_1) = \hat{r}(m_0)$.
    - **Case $\Delta_m > 0$:** $T^2_{\hat{r}(m_0)}$ is *magnified* to become $T^2_{\hat{r}(m_1)}$ via the iso-topic lifting $T^2_{\hat{r}(m_0)} \rightarrow T^2_{\hat{r}(m_1)}$ because $m_1 > m_0$ so $\hat{r}(m_1) > \hat{r}(m_0)$.

11. Finally, given the new and developing framework of [13, 14, 15, 16] that characterizes the effective *physical* state of black holes for an emission or absorption transition, we are motivated to define the effective *iso-mathematical* state of dynamic iso-2-sphere IHRs (which are also spherically-symmetric objects) for a transition from $T^2_{\hat{r}(m_0)}$ to $T^2_{\hat{r}(m_1)}$. Therefore, given the *physical black hole effective radius* definition from eq. (5) of [16], we implement the dynamic iso-topic lifting of [11, 12] and define the *iso-mathematical effective iso-2-sphere IHR iso-radius* as

$$T^2_{\hat{r}(m_0)} \rightarrow T^2_{\hat{r}(m_1)}: \; \hat{r}_E \equiv \hat{r}_E(m_0, m_1) \equiv 2m_E(m_0, m_1) \equiv \frac{1}{\hat{\kappa}_E(m_0, m_1)} > 0, \tag{27}$$

where $\hat{\kappa}_E(m_0, m_1)$ is the *effective iso-2-sphere IHR iso-curvature* and inverse of the iso-unit, such that the *effective iso-2-sphere IHR quantity* is defined as

$$m_E(m_0, m_1) \equiv \frac{m_0 + m_1}{2}, \tag{28}$$

which is simply the *average* of $T^2_{\hat{r}(m_0)}$'s initial quantity m_0 and $T^2_{\hat{r}(m_1)}$'s final quantity m_1.

At this point, we've assembled the effective iso-radius encoding platform for representing dynamic iso-sphere IHR [12] iso-states and iso-transitions in the iso-dual 4D space-time topology [8].

3 Conclusion

In this work, we successfully assembled the effective iso-radius for a dynamic iso-sphere IHR [12] in the iso-dual 4D space-time IHR topology [8] and introduced the corresponding notion of effective iso-state to begin encoding the iso-transition between two distinct iso-states. For this, the procedure and step-by-step developing results were presented in Section 2, and applies to both continuous and discrete dynamic iso-sphere IHRs. Also, we demonstrated that all of these outcomes comply with the exterior and interior IHR iso-duality [12]. To recapitulate the final results more precisely, we defined—for the dynamic iso-sphere IHR $T^2_{\hat{r}(m)}$—the effective iso-radius $\hat{r}_E(m_0, m_1)$ as the average of the initial iso-radius $\hat{r}(m_0)$ and the final iso-radius $\hat{r}(m_1)$ in eqs. (27–28), which correspond to the initial dynamic iso-sphere IHR $T^2_{\hat{r}(m_0)}$ and the final dynamic iso-sphere IHR $T^2_{\hat{r}(m_1)}$, respectively.

The results, constructions, and implications of this preliminary investigation are significant because they exemplify alternative modes of cutting-edge iso-mathematics research that facilitate a heightened quantifiable characterization of dynamic iso-sphere IHRs [12] in terms of effective iso-states for iso-transitions with iso-duality. Hence, given that iso-sphere IHRs are equipped with topological deformation order parameters [6, 7, 8], the next logical step of this analysis should be to implement iso-topic liftings [1, 2, 3, 4, 5] for the order parameters and then topologically incorporate these "iso-deformations" into the existing effective iso-state definition. From there, we may build on this platform and continue to develop the framework by exploring and assessing the frontiers of iso-, geno-, and hyper- mathematics [1, 2, 3, 4, 5]. Thus, this developing class of dynamic iso-sphere IHRs warrants further development, scrutiny, collaboration, and hard work in order to advance it for future application in the discipline of science.

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