`Review of Grischuk and Sachin Gravitational Wave Generator via Tokamak Physics

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Abstract

Using Grischuk and Sachin (1975) amplitude for the GW generation due to plasma in a toroid, we generalize this result for Tokamak physics. We obtain evidence for strain values up to $h_{2nd-term} \sim 10^{-23} - 10^{-24}$ in a Tokamak center, with a minimum value of $h \sim 10^{-26}$ five meters above the Tokamak center. These values are an order of magnitude sufficient to allow for possible detection of gravitational waves. The critical breakthrough is in utilizing a burning plasma drift current, which relies upon a thermal contribution to an electric field. The gravitational wave amplitude would be detectable in part also due to $n_{ion} \cdot \tau_E > .5 \times 10^{20} \cdot m^{-3} \cdot \sec$, where the n_{ion} is the numerical ion density , usually about $10^{20} \cdot m^{-3}$, i.e. about one out of a million of the present atmospheric pressure, whereas τ_E is a confinement time value for Tokamak plasma, here at least .5 seconds. This value, as given above and by Wesson (2011), is the threshold for plasma fusion burning; the temperature obtained is the main driver for how one could conceivably detect GW of amplitude as low as $\left[h_{2nd-term}\Big|_{T_{Temp} \geq 100 KeV} \right]_{5-meters-above-Tokamak} \sim 10^{-25}$ five meters above the Tokamak center.

Key words: *Tokamak physics, confinement time (of Plasma),space-time GW amplitude, Drift current*

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1. Introduction

Russian physicists Grishchuk and Sachin (1975) obtained the amplitude of a Gravitational wave (GW) in a plasma as

$$\mathrm{A(amplitude-GW) = h} \sim \frac{\mathrm{G}}{c^4} \cdot E^2 \cdot \lambda_{GW}^2 \tag{1}$$

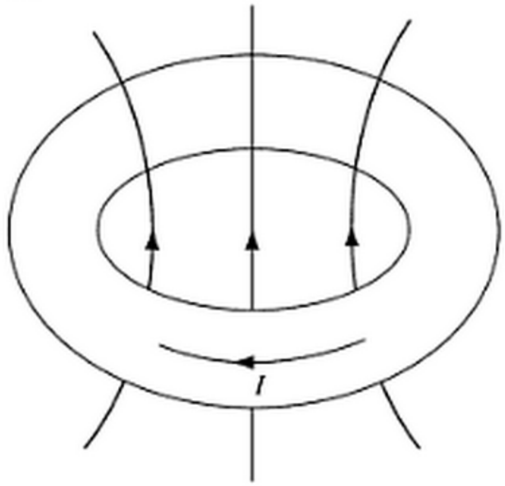

Fig. 1 *We outline the direction of Gravitational wave "flux". If the arrow in the middle of the Tokamak ring perpendicular to the direction of the current represents the z axis, we represent where to put the GW detection device as 5 meters above the Tokamak ring along the z axis.*

Note that a simple model of how to provide a current in the Toroid is provided by a transformer core. This diagram is an example of how to induce the current I, used in the simple Ohms law derivation referred to in the first part of the text

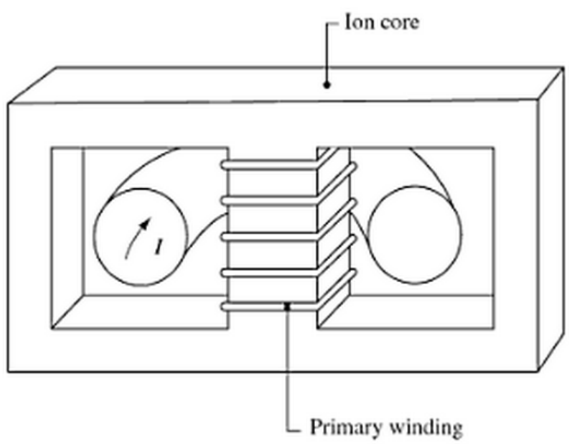

Fig. 2 *Flux change provided by a transformer core, in the simple current model first referred to in this paper*.

Here, E is the electric field whereas λ_{Gw} is the gravitational wavelength for GW generated by the Tokamak in our model. Note, if $\omega_{GW} \sim 10^6 Hz \Rightarrow \lambda_{Gw} \sim 300 \ meters$, so we will be assuming a baseline of the order of

$\omega_{GW} \sim 10^9 Hz \Rightarrow \lambda_{Gw} \sim .3 \ meters$, as a start for GW detection above the Tokamak. We will examine the would-be electric field, in ways different from the initial Ohms law .A generalized Ohm's law ties in well with Figures 1 and 2 above.

$$J = \sigma \cdot E \tag{2}$$

In order to obtain a suitable electric field, to be detected via 3DSR technology (Li et al, 2009), we will use a generalized Ohm's law as given by Wesson (2011, page 146), where E and B are electric and magnetic fields, and v is velocity. We should understand that this undercuts the use of Figure 2 above.

$$E = \sigma^{-1} J - v \times B \tag{3}$$

We will be looking for an application for radial free electric fields being applied e.g., Wesson (2011, page 120)

$$n_j e_j \cdot \left(E_r + v_{\perp j} B \right) = -\frac{dP_j}{dr} \tag{4}$$

Here, n_j = ion density, jth species, e_j = ion charge, jth species, E_r = radial electric field, $v_{\perp j}$ = perpendicular velocity, of jth species, B = magnetic field, and P_j = pressure, jth species. The results of Eq. (3) and Eq. (4) are

$$\frac{\mathrm{G}}{c^4} \cdot E^2 \cdot \lambda_{GW}^2 \sim \frac{\mathrm{G}}{c^4} \cdot \left[\frac{Const}{R} \right]^2 \cdot \lambda_{GW}^2 + \frac{\mathrm{G}}{c^4} \cdot \left[\frac{J_b}{n \cdot e} + v_R \right]^2 \cdot \lambda_{GW}^2 \quad = (1^{st}) + (2^{nd}) \tag{5}$$

Here, the 1st term is due to $\nabla \times E = 0$, and the 2nd term is due to $E_n = \frac{dP_j}{dx_n} \cdot \frac{1}{n_j \cdot e_j} - \left(v \times B \right)_n$ with the 1st term generating $h \sim 10^{-38} - 10^{-30}$ in terms of GW amplitude strain 5 meters above the Tokamak , whereas the 2nd term has an $h \sim 10^{-26}$ in terms of GW amplitude above the Tokamak. The article has contributions from amplitude from the 1st and 2nd terms separately. The second part will be tabulated separately from the first contribution assuming a minimum temperature of $T = Temp \sim 10 KeV$ as from Wesson (2011)

2. GW h strain values when the first term of Eq.(5) is used for different Tokamaks

We now look at what we can expect with the simple Ohm's law calculation for strain values. This is work which the Author with Gary Stevenson and Amara Angelica did in late 2012. As it is, the effort lead to non usable GW amplitude values of up to $h \sim 10^{-38} - 10^{-30}$ for GW wave amplitudes 5 meters above a Tokamak, and $h \sim 10^{-36} - 10^{-28}$ in the center of a Tokamak. I.e. this would be using Ohm's law and these are sample values of the Tokamak generated GW amplitude, using the first term of Eq. (5) and obtaining the following value

$$h_{First-term} \sim \frac{\mathrm{G}}{c^4} \cdot E^2 \cdot \lambda_{GW}^2 \sim \frac{\mathrm{G}}{c^4} \cdot \left[\frac{J}{\sigma} \right]^2 \cdot \lambda_{GW}^2 \tag{5a}$$

We summarize the results of such in our first table as given for when $\omega_{GW} \sim 10^9 Hz \Rightarrow \lambda_{Gw} \sim .3 \ meters$ and with conductivity $\sigma(tokamak - plasma) \sim 10 \cdot m^2 / \sec$ and with the following provisions as to initial values. What we observe are a range of Tokamak values which are, even in the case of ITER (not yet built) beyond the reach of any technological detection devices which are conceivable in the coming decade. This table and its results, assuming fixed conductivity values $\sigma(tokamak - plasma) \sim 10 \cdot m^2 / \sec$ as well as $\lambda_{Gw} \sim .3 \ meters$ is why the author, after due consideration completed his derivation of results as to the 2nd term of Eq. (5) which lead to even for when considering the results for the Chinese Tokamak in Hefei to have $h_{Second-term} \sim \frac{\mathrm{G}}{c^4} \cdot E^2 \cdot \lambda_{GW}^2 \sim \frac{\mathrm{G}}{c^4} \cdot \left[\frac{J_b}{n \cdot e} + v_R \right]^2 \cdot \lambda_{GW}^2$ values 10,000 larger than the results in ITER due to Eq.(5a).'

Table 1: ***Values of strain at center of Tokamak, and 5 meters above Tokamak***:

$\lambda_{Gw} \sim .3\ meters$, $\sigma(tokamak - plasma) \sim 10 \cdot m^2 / \sec$, **using Eq.5a above for Amplitude of GW.**

Experiment	**Site/ location**	**Plasma current, in (mili-Amps) MA**	**Strain, h, in center of the Tokamak**	**Strain, h, 5 meters above the center of the Tokamak**
JET	**Culan (UK)**	**5-7**	$h \sim 10^{-31}$	$h \sim 10^{-33}$
ASDEX	**Garching (GER)**	**2**	$h \sim 10^{-32}$	$h \sim 10^{-34}$
DIII-D	**San Diego (USA)**	**1.5-3**	$h \sim 10^{-32}$	$h \sim 10^{-34}$
HL-2A	**Chengdu (PRC)**	**.45**	$h \sim 10^{-34}$	$h \sim 10^{-36}$
HT-7U	**Hefei (PRC)**	**.25**	$h \sim 10^{-36}$	$h \sim 10^{-38}$
ITER(planned)	**Saint Paul Les-Durance (FR)**	**15**	$h \sim 10^{-28} - 10^{-29}$	$h \sim 10^{-30} - 10^{-31}$

What makes it mandatory to go the 2nd term of Eq. (5) is that even in the case of ITER, 5 meters above the Tokamak ring, the GW amplitude is 1/10,000 the size of any reasonable GW detection device, and this including the new 3DSR technology (Li et al, 2009). Hence, we need to come up with a better estimate, which is what the 2nd term of Eq.(5) is about which is derived in the next section

3 . Enhancing GW strain Amplitude via utilizing a burning Plasma drift current: Eq.(4)

The way forward is to go to Wesson, (2011, page 120) and to look at the normal to surface induced electric field contribution

$$E_n = \frac{dP_j}{dx_n} \cdot \frac{1}{n_j \cdot e_j} - \left(v \times B\right)_n \tag{6}$$

If one has for v_R as the radial velocity of ions in the Tokamak from Tokamak center to its radial distance, R, from center, and B_θ as the direction of a magnetic field in the 'face' of a Toroid containing the Plasma, in the angular θ direction from a minimal toroid radius of $R = a$, with $\theta = 0$, to $R = a + r$ with $\theta = \pi$, one has v_R for radial drift velocity of ions in the Tokamak, and B_θ having a net approximate value of:

$$\left(v \times B\right)_n \sim v_R \cdot B_\theta \tag{7}$$

Also, as a first order approximation: From Wesson (2011, page 167) the spatial change in pressure denoted

$$\frac{dP_j}{dx_n} = -B_\theta \cdot j_b \tag{8}$$

Here (ibid), the drift current, using $\xi = a/R$, and drift current j_b for Plasma charges, i.e.

$$j_b \sim -\frac{\xi^{1/2}}{B_\theta} \cdot T_{Temp} \cdot \frac{dn_{drift}}{dr} \tag{9}$$

Figure 3 below introduces the role of the drift current, in terms of Tokamaks

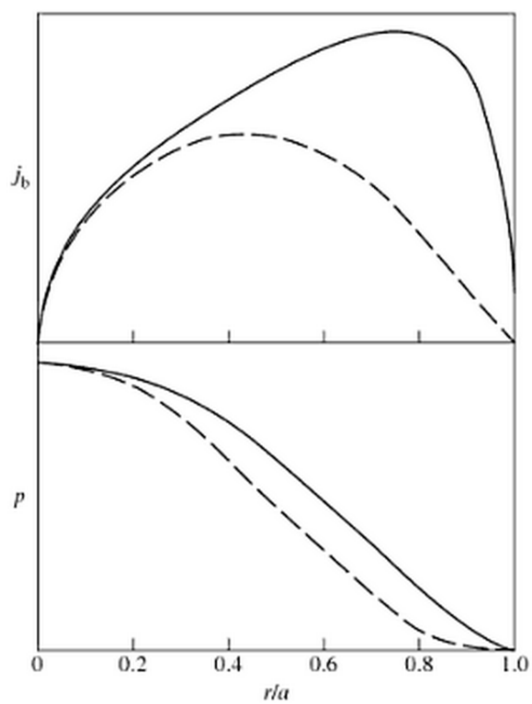

Fig. 3 *Typical bootstrap currents with a shift due to r/a where r is the radial direction of the Tokamak, and a is the inner radius of the Toroid*

Then one has

$$B_\theta{}^2 \cdot \left(j_b / n_j \cdot e_j\right)^2 \sim \frac{B_\theta{}^2}{e_j{}^2} \cdot \frac{\xi^{1/4}}{B_\theta{}^2} \cdot \left[\frac{1}{n_{drift}} \cdot \frac{dn_{drift}}{dr}\right]^2 \sim \frac{\xi^{1/4}}{e_j{}^2} \cdot \left[\frac{1}{n_{drift}} \cdot \frac{dn_{drift}}{dr}\right]^2 \tag{10}$$

Now, the behavior of the numerical density of ions, can be given as follows, namely growing in the radial direction, then

$$n_{drift} = n_{drift}\Big|_{initial} \cdot \exp\left[\tilde{\alpha} \cdot r\right] \tag{11}$$

This exponential behavior then will lead to the 2nd term in Eq.(5) having in the center of the Tokamak, for an ignition temperature of $T_{Temp} \geq 10KeV$ a value of

$$h_{2nd-term} \sim \frac{\mathrm{G}}{c^4} \cdot B_\theta^{\ 2} \cdot \left(j_b / n_j \cdot e_j \right)^2 \cdot \lambda_{GW}^2 \sim \frac{\mathrm{G}}{c^4} \cdot \frac{\xi^{1/4} \tilde{\alpha}^2 T_{Temp}^2}{e_j^{\ 2}} \cdot \lambda_{GW}^2 \sim 10^{-25} \tag{12}$$

As shown in **Fig. 4** (from Wesson 2011), there is a critical ignition temperature at its lowest point of the curve in the having $T_{Temp} \geq 30KeV$ as an optimum value of the Tokamak ignition temperature for $n_{ion} \sim 10^{20} m^{-3}$, with a still permissible temperature value of $T_{Temp}\Big|_{safe-upper-bound} \approx 100KeV$ with a value of $n_{ion} \sim 10^{20} m^{-3}$, due to from page 11, [3] the relationship of Eq.(13), where τ_E is a Tokamak confinement of plasma time of about 1-3 seconds, at least due to

$$n_{ion} \cdot \tau_E > .5 \times 10^{20} \cdot m^{-3} \cdot \sec \tag{13}$$

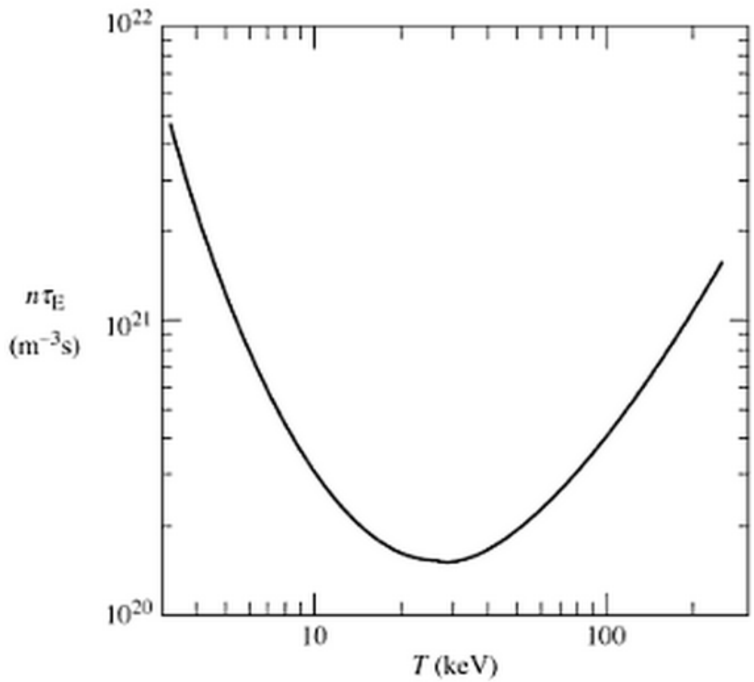

Fig. 4 *The value of* $n\,\tau_E$ *required to obtain ignition, as a function of temperature*

Also, as shown in Fig. 4, $T_{Temp}\Big|_{safe-upper-bound} \approx 100KeV$, then one could have at the Tokamak center, i.e. even the Hefei based PRC Tokamak

$$h_{2nd-term}\Big|_{T_{Temp} \geq 100\,KeV} \sim \frac{\mathrm{G}}{c^4} \cdot \frac{\xi^{1/4} \tilde{\alpha}^2 T_{Temp}^2}{e_j^{\ 2}} \cdot \lambda_{GW}^2 \sim 10^{-23} \tag{14}$$

This would lead to, for a GW reading 5 meters above the Tokamak, then lead to for then the Hefei PRC Tokamak

$$\left[h_{2nd-term}\Big|_{T_{Temp}\geq 100\,KeV} \right]_{5-meters-above-Tokamak} \sim \frac{\mathrm{G}}{c^4}\cdot\frac{\xi^{1/4}\tilde{\alpha}^2 T_{Temp}^2}{e_j^{\;2}}\cdot\lambda_{GW}^2 \sim 10^{-25} \tag{15}$$

Note that the support for up to 100 KeV for temperature can yield more stability in terms of thermal Plasma confinement as give in **Fig. 5** below, namely from [3] we have

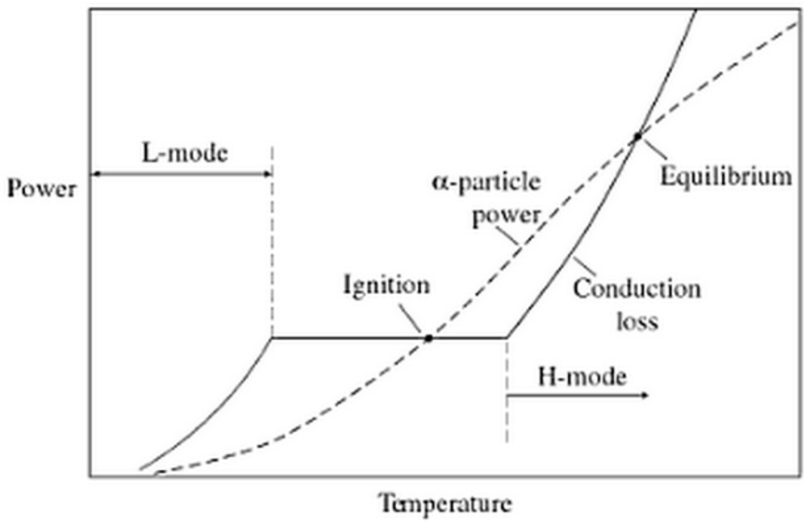

Fig 5 *Illustrating how increase in temperature can lead to the H mode region, in Tokamak physics where the designated equilibrium point, in Fig. 5 is a known way to balance conduction loss with alpha particle power, which is a known way to increase* τ_E *i.e. Tokamak confinement of plasma time*

4. Details of the model in terms of terms of adding impurities to the Plasma to get a longer confinement time (possibly to improve the chances of GW detection).

We add this detail in, due to a question raised by Dr. Li who wished for longer confinement times for the Plasma in order to allegedly improve the chances of GW detection for a detector 5 meters above the Tokamak in Hefei. Wesson (2011) stated that the confinement time may be made proportional to the numerical density of argon/ neon seeded to the plasma (page 180). This depends upon the nature of the Tokamak, but it is a known technique, and is suitable for analysis, depending upon the specifics of the Tokamak. I.e. this is a detail Dr. Li can raise with his co workers in Hefei, PRC in 2014.

5. Conclusion. GW generation due to the Thermal output of Fusion in a Tokamak, and not due to E and B field currents.

Further elaboration of this matter lies in the viability of the expression derived , namely Eq. (15) repeated

$$\left[h_{2nd-term}\Big|_{T_{Temp}\geq 100\,KeV} \right]_{5-meters-above-Tokamak} \sim \frac{\mathrm{G}}{c^4}\cdot\frac{\xi^{1/4}\tilde{\alpha}^2 T_{Temp}^2}{e_j^{\,2}}\cdot\lambda_{GW}^2 \qquad (15)$$

The importance of the formulation is in the explicit importance of temperature. i.e. a temperature range of at least $10KeV \leq T_{Temp} \leq 100KeV$. In making this range for Eq.(15), care must also be taken to obtain a sufficiently long confinement time for the fusion plasma in the Tokamak of at least 1 second or longer, and this is a matter of applied engineering dependent upon the instrumentation of the Tokamak in Hefei, PRC. The author hopes that in 2014, there will be the beginning of confirmation of this process so that some studies may commence. If so, then the next question will be finding if the instrumentation of Li and colleagues (2009) can be utilized and developed. This is expected to be extremely difficult, but the Tokamak fusion process may allow for falsifiable testing and eventual verification.

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