Communication to the National Academy of Sciences of Bolivia August 19, 2013
Translated version in English

Trajectories in space-time M4: Reinstating the Newtonian Time

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Abstract

The author analyses the difficulties in describing accelerated motion trajectories in M4 in the framework of the Special Relativity Theory due to the lack of time as an independent scalar parameter. It is proposed reinstating the Newtonian time in an Integral Relativity Theory (IRT) without detriment to the Lorentz transformations. It is shown that in the frame of IRT it is possible to exceed the speed of light in accelerated motion. The formula is deduced for the composition of relative velocities with acceleration in M4. As an illustrative example the radial velocity is calculated in the course of the cosmic cloud G2 orbiting Sagittarius A* comparing results with the observed data and the author suggests the possibility that in the G2 cloud there is a system of two or three stars. He proposes an explanation of the phenomenon of "black hole" as the passage to the hyperluminic subspace in M4. Further research is suggested towards an IRT capable of handling in M4 the Hamiltonian formulation of classical mechanics and relativistic quantum mechanics.

Resumen

El autor analiza las dificultades al describir trayectorias de movimiento acelerado en M4 en el marco de la Teoría de la Relatividad Especial (TRE), debido a la falta de un tiempo escalar independiente como parámetro. Se propone restituir el tiempo newtoniano en una Teoría de la Relatividad Integral (TRI) sin desmedro alguno de las transformaciones de Lorentz. Se demuestra que en el marco de esta TRI es posible superar la velocidad de la luz en movimiento acelerado. Se deduce la fórmula para composición de velocidades relativas con aceleración en M4. Como un ejemplo ilustrativo se calcula la velocidad radial en la trayectoria de la nube cósmica G2 que orbita *Sagittarius A** comparando resultados con los datos observados y se sugiere la posibilidad que dentro la nube G2 exista un sistema de dos o tres astros. Se propone una explicación del fenómeno de "agujero negro" como el traspaso al subespacio hiperlumínico en M4. Se sugiere mayor investigación hacia una TRI capaz de manejar en M4 una formulación hamiltoniana de la Mecánica Clásica y de la Mecánica Cuántica Relativistica.

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Subspaces en M4 and newtonian time

Let's recall that the Minkowski spacetime (M4) is defined as four-dimensional where the metric:

$$\mathbf{x}^2 = x_1^2 + x_2^2 + x_3^2 + x_4^2$$

is the invariant in M4, and where the fourth dimension includes time x_4 = ict, the constant c being the speed of light in vacuum. Here we recognize the Pythagorean theorem in M4 with the exception of the negative term x_4^2 since x_4 is an imaginary number. For this reason the modulus **|** x **|** of tetra-vector x:

$$|\, \mathbf{x} \,| \equiv [x_1^2 + x_2^2 + x_3^2 - c^2t^2]^{1/2}$$

can asumme real or imaginary values depending if the sum $x_1^2 + x_2^2 + x_3^2$ is larger or smaller than c^2t^2 resulting two subspaces in M4 and a dividing surface:

M4⁻ lumínic when	$x_1^2 + x_2^2 + x_3^2 < c^2t^2$	(velocity < c)
M4⁺ hyperluminic when	$x_1^2 + x_2^2 + x_3^2 > c^2t^2$	(velocity > c)
light-cone surface when	$x_1^2 + x_2^2 + x_3^2 = c^2t^2$	(velocity = c)

Since time t is nothing more than a coordinate in M4, we need a scalar variable, independent of the coordinates, which serves as a parameter to describe the path that traces the position four-vector **x** in M4. Therefore It becomes necessary to restore the Newtonian time t (*) as an independent scalar variable, so that a trajectory T in M4 is defined by a transformation function of the scalar variable τ:

$$\text{(1a)}\quad T:\ \mathbf{x}(\tau) = \Lambda(\tau)\, \underline{\mathbf{x}}(\tau) \qquad\qquad \text{(1b)}\quad \Lambda(\tau)\, \Lambda(\tau) = \mathbb{1}$$

where $\underline{\mathbf{x}}(\tau)$ is the position four-vector in the frame of reference not necessarily inertial where the corpuscle is at rest, while $\mathbf{x}(\tau)$ is the four-vector rotated by the transformation $\Lambda(\tau)$, which gives the position of a particle in M4 at each instant τ. This matrix meets the orthogonality condition (1b) that ensures invariance:

$$\text{(1c)}\quad |x| = |\underline{x}| \qquad [x_1^2 + x_2^2 + x_3^2 - c^2t^2]^{1/2} = [\underline{x}_1^2 + \underline{x}_2^2 + \underline{x}_3^2 - c^2\underline{t}^2]^{1/2}$$

(*) Not to be confused with the *proper time* z (Eigenzeit) that Minkowski defined as the arc length ds expressed in units of time: $(icdz)^2 \equiv (ids)^2 = dx_1^2+dx_2^2+dx_3^2+dx_4^2$, therefore, is coordinate dependent ([1]).

Coordinate rotations in M4 modify the component x_4=ict, so t has the character of a relative duration. Not so the instant τ that is the same in all reference systems. We can say that the fourth component $x_4(\tau)$ = ict(τ) represents a clock in which the scale that indicates the hour is represented by t while the current instant is given by τ. In order to synchronize clocks in rotated systems in M4, it is necessary to adjust the scale of t, not the tick rate of the needles that mark the running now instant τ which is invariant, ie τ does not vary in fixed clocks located at different reference systems.

Trajectory of a Corpuscle resting in M4

It might seem inconsistent to speak of the trajectory of something that does not move. We are accustomed to use the word *trajectory* only in relation to a line or curve that describes a particle crossing the space. For example, the parable that follows a projectile. In three-dimensional Euclidean space (E3), when we say that the corpuscle is at rest at the point of origin we mean that its coordinates are (0,0,0). But in space-time M4, in which any point has the coordinates (x_1, x_2, x_3, x_4) including time as a coordinate x_4 = ict, we can not speak of the origin of coordinates (0,0,0,0) as beeing the point at which the corpuscle is at rest, since this point is the position in M4 where it was at time $\underline{t}$ = 0, but time passes so incessantly at any time $\underline{t}$ after, it is located at the point $\underline{\mathbf{x}}$=(0,0,0, ic$\underline{t}$). The coordinate $\underline{x}_4$ could never take a constant value (past or future) and its value should perennially change to represent the current time, effectively tracing a path along the $\underline{x}_4$ axis. It would be a mathematical inconsistency if this instant we denote by the variable $\underline{t}$ also, because the variable $\underline{x}_4$ would be dependent of itself and the equations of the trajectory would become functions of the fourth coordinate (ceasing to be invariant M4).

A way to evade this difficulty is to simply deny the existence of the current time, the *arrow-time now,* following Einstein who considered the present time just a *stubborn illusion* (*). Therefore, in the Special Relativity Theory, we can not strictly speak of trajectories but simply we say *world-lines* where you can not tell at what point the corpuscle is *now* (at this moment). The orthodox approach that ignores the *present instant* causes various difficulties in the kinematic analysis in M4 which we will treat here.

Proposal: not to evade the problem, rather resolve it by reinstating the Newtonian time τ without renouncing the relativistic time x_4 = ict, thus preserving the structure of M4 and therefore the Lorentz transformations. This scheme allows us to distinguish between the time or *duration* t registered along the x_4-coordinate axis and the instant time τ as a scalar variable (independent of any reference system) representing the current instant, the now. We can view it as a cut τ on the x_4-axis always moving in the positive direction (*time-arrow*).

(*) Fragment of the farewell letter from Einstein to Mechele Besso (1955): *Nun ist er mir auch mit dem Abschied von dieser sonderbaren Welt ein wenig vorausgegangen. Dies bedeutet nichts. Für uns gläubige Physiker hat die Scheidung zwischen Vergangenheit, Gegenwart und Zukunft nur die Bedeutung einer wenn auch hartnäckigen Illusion.*

In this way the trajectory of the resting corpuscle in subspace $M4^-$ is described by a function $\underline{x}_4(\tau)$, which builds the four-vector:

$$\mathbf{\underline{x}}(\tau) = [\ 0,0,0,\underline{x}_4(\tau)\ ]$$

descriptor of the corpuscle movement along the $\underline{x}_4$-axis where is at rest, ie in the fixed position (0,0,0) in E3 in the course of time τ. It sounds contradictory to speak of motion and rest at once, but we are referring to an event in the four-space that can never be properly resting on M4 because time always passes.

The function $\underline{x}_4(\tau)$ describes the time evolution along the chronological axis in the system where the corpuscle is at rest. It is reasonable to agree that in this reference system, $\underline{x}_4(\tau)$ is a linear function of τ, it is simply $\underline{t}(\tau) = \tau$.

Trajectory of the corpuscle moving in M4

We will analyze trajectories of moving corpuscles in $M4^-$ described by $\mathbf{x}(\tau)$ as a result of the transformation $\Lambda(\tau)\ \mathbf{\underline{x}}(\tau)$. To this end consider the reference system ($\underline{x}_1$, $\underline{x}_2$, $\underline{x}_3$, $\underline{x}_4$) rotating on the plane x_4-x_1 of the observer system (x_1,x_2,x_3,x_4). The rotation angle from axis x_4 to axis $\underline{x}_4$ we designate $i\chi$ which is an imaginary number like x_4. In this way the corpuscle coordinates in the observer system are related to those of the system where the corpuscle is at rest according to the transformation equations that follows, due to a counter-clockwise rotation on the plane x_1-x_4 in $M4^-$. This is equivalent to a positive translation along x_1 and vice versa one clockwise rotation to a negative translation:

$$\mathbf{x}(\tau) = \Lambda(\chi)\ \mathbf{\underline{x}}(\tau) \qquad \chi = \chi(\tau) \qquad \underline{x}_1, \underline{x}_2, \underline{x}_3 = 0 \quad \underline{x}_4 = ic\tau$$

$$(2) \qquad \mathbf{x}(\tau) = \begin{bmatrix} \cos i\chi & -\text{sen } i\chi \\ \text{sen } i\chi & \cos i\chi \end{bmatrix} \begin{bmatrix} 0 \\ \underline{x}_4 \end{bmatrix} = \begin{bmatrix} -\underline{x}_4 \text{ sen } i\chi \\ \underline{x}_4 \cos i\chi \end{bmatrix}$$

(2a) $x_1 = -\text{sen}(i\chi)\ \underline{x}_4$ (2c) $x_1 = c\tau \sinh(\chi)$

(2b) $x_4 = \cos(i\chi)\ \underline{x}_4$ (2d) $x_4 = ic\tau \cosh(\chi)$ ($t = \tau \cosh(\chi)$)

The coordinates (x_1,x_2,x_3,x_4) represent the components of four-vector $\mathbf{x}(\tau)$ whose tip traces out the path of the corpuscle in M4. If the angular variable χ is constant, the trajectory is a straight line (constant velocity motion). But provided it is a function $\chi = \chi(\tau)$ the four-vector $\mathbf{x}(\tau)$ traces a curved path (accelerated motion).

In the case of χ = a constant, and only in this case, we see that $x_1/x_4 = -\tan(i\chi)$ or the velocity β in speed of light units is given by:

$$(3)\ \beta \equiv v/c = i \tan(i\chi) = \tanh(\chi)$$

Note: Using the following properties of the hyperbolic functions:

$\mathrm{sen}(i\chi) = i\,\mathrm{senh}(\chi)$ $\cos(i\chi) = \cosh(\chi)$ $\tan(i\chi) = i\tanh(\chi)$
$\beta \equiv \tanh(\chi)$ $\mathrm{senh}(\chi) = \beta/[1-\beta^2]^{½}$ $\cosh(\chi) = 1/[1-\beta^2]^{½}$

we can write the transformations (2c) and (2d) with well known coefficients of the Lorentz transformation:

(4a) $x_1 = c\tau\, \beta/[1-\beta^2]^{½}$
(4b) $x_4 = c\tau\, i/[1-\beta^2]^{½}$ $(t = \tau /[1-\beta^2]^{½})$

The transformation (4a) is interpreted in E3 as a positive translation along the x_1-axis with velocity β. The deduction of (4a) and (4b) from a rotation in $M4^-$ is due to Sommerfeld ([2]).

When, in general, $\chi = \chi(\tau)$ formula (3) is no longer valid, so in this case we deduce the corresponding formula for $\beta(\tau) \equiv v(\tau)/c = 1/c\, dx_1(\tau)/dt$. It is obtained by parametric differentiation (τ as a parameter) from (2c) and (2d):

$x_1(\tau) = c\tau\, \mathrm{senh}(\chi(\tau))$ $t(\tau) = \tau \cosh(\chi(\tau))$ $d_t \equiv d/dt$ $d_\tau \equiv d/d\tau$
$v(\tau) \equiv d_t x_1(\tau) = d_\tau x_1(\tau) / d_\tau t(\tau)$
$v(\tau) = \{c\, \mathrm{senh}(\chi(\tau)) + c\tau\, d_\tau\chi(\tau) \cosh(\chi(\tau))\} / \{\cosh(\chi(\tau)) + \tau\, d_\tau\chi(\tau)\, \mathrm{senh}(\chi(\tau))\}$

Dividing numerator and denominator by $\cosh \chi(\tau)$ we obtain:

$\Rightarrow$ (5a) $\beta(\tau) = \{\tanh(\chi(\tau)) + \tau\, d_\tau\chi(\tau)\} / \{1+ \tau\, d_\tau\chi(\tau) \tanh(\chi(\tau))\}$

This result is reduced to the formula (3) only when $d_\tau\chi(\tau) = 0$. Furthermore, in the case of an angular function that increases linearly with the scalar time τ so that $\chi(\tau) = \chi_0 + \alpha\tau$, we have $d_\tau\chi(\tau) = \alpha$ so that formula (5a) becomes:

$\Rightarrow$ (5b) $\beta(\tau) = \{\tanh(\chi(\tau)) + \alpha\tau\} / \{1 + \alpha\tau \tanh(\chi(\tau))\}$

Formula (5b) for accelerated movement shows an intriguing behavior: it turns out that for $\chi_0 = 0$ and $\alpha\tau \geq 1$ is the function $\beta(\alpha\tau) \geq 1$ with a maximum $\beta(1.4) = 1.02048$, as we can see in the table of Appendix A1. After the light-cone is pierced the path continues with a velocity $\beta > 1$ decreasing asymptotically towards the value β=1 always at the hyperluminic side $M4^+$.

Composition of velocities in $M4^-$

We search for velocity β_Σ' as "addition" of two relative velocities β and β'. The result is obtained by applying to (1) an new rotation $\Lambda(\chi')$, so that:

(6) $\mathbf{x'}(\tau) = \Lambda(\chi')\, \mathbf{x}(\tau) = \Lambda(\chi')\, \Lambda(\chi)\, \underline{\mathbf{x}}(\tau)$ $\chi' = \chi'(\tau)$

$$\mathbf{x'}(\tau) = \begin{bmatrix} \cos i\chi' & -\mathrm{sen}\, i\chi' \\ \mathrm{sen}\, i\chi' & \cos i\chi' \end{bmatrix} \begin{bmatrix} -\underline{x}_4\, \mathrm{sen}\, i\chi \\ \underline{x}_4 \cos i\chi \end{bmatrix} = \begin{bmatrix} -\underline{x}_4 \cos i\chi'\, \mathrm{sen}\, i\chi - \underline{x}_4\, \mathrm{sen}\, i\chi' \cos i\chi \\ -\underline{x}_4\, \mathrm{sen}\, i\chi'\, \mathrm{sen}\, i\chi + \underline{x}_4 \cos i\chi' \cos i\chi \end{bmatrix}$$

$x_1' = -\underline{x}_4 \cos i\chi'\, \mathrm{sen}\, i\chi - \underline{x}_4\, \mathrm{sen}\, i\chi' \cos i\chi = c\tau \cosh\chi'\, \mathrm{senh}\,\chi + c\tau\, \mathrm{senh}\,\chi' \cosh\chi$
$x_4' = -\underline{x}_4\, \mathrm{sen}\, i\chi'\, \mathrm{sen}\, i\chi + \underline{x}_4 \cos i\chi' \cos i\chi = ic\tau\, \mathrm{senh}\,\chi'\, \mathrm{senh}\,\chi + ic\tau \cosh\chi' \cosh\chi$

$\Rightarrow$ (6a) $x_1' = c\tau \,\mathrm{senh}\,(\chi+\chi')$ (6b) $x_4' = ic\tau \cosh(\chi+\chi')$

$\Rightarrow$ (6c) $\beta'_\Sigma \equiv x'_1/ct' = \tanh(\chi+\chi')$

This composed velocity for motion without acceleration was derived derived by Sommerfeld ([2]) taking for granted that $\chi_\Sigma' = \chi + \chi'$

Composition of velocities in M4⁻ with acceleration

$d_\tau x_1'(\tau) = c\,\mathrm{senh}\,(\chi+\chi') + c\tau\,(d_\tau\chi(\tau) + d_\tau\chi'(\tau))\cosh(\chi+\chi')$

$d_\tau x_4'(\tau) = ic\cosh(\chi+\chi') + ic\tau\,(d_\tau\chi(\tau) + d_\tau\chi'(\tau))\,\mathrm{senh}\,(\chi+\chi')$

$1/c\; d_\tau x_1'(\tau) / d_\tau t'(\tau) = \{\mathrm{senh}\,(\chi+\chi') + (d_\tau\chi(\tau) + d_\tau\chi'(\tau))\,\tau\cosh(\chi+\chi')\} /$
$\{\cosh(\chi+\chi') + (d_\tau\chi(\tau) + d_\tau\chi'(\tau))\,\tau\,\mathrm{senh}\,(\chi+\chi')\}$

dividing numerator and denominator by cosh $(\chi+\chi')$ we obtain:

$\Rightarrow$ (7) $\beta'_\Sigma = \{\tanh(\chi+\chi') + (d_\tau\chi(\tau) + d_\tau\chi'(\tau))\,\tau\} / \{1 + (d_\tau\chi(\tau) + d_\tau\chi'(\tau))\,\tau\tanh(\chi+\chi')\}$

$\downarrow$ $\chi = \chi_0$ $\chi'(\tau) = \alpha\tau$

$\rightarrow$ (7a) $\beta'_\Sigma = \{\tanh(\chi_0+\alpha\tau) + \alpha\tau\} / \{1 + \alpha\tau\tanh(\chi_0+\alpha\tau)\}$

$\downarrow$ $\chi_0 = 0$ $\rightarrow$ (7b) $\beta'_\Sigma = \{\tanh\alpha\tau + \alpha\tau\} / \{1 + \alpha\tau\tanh\alpha\tau\} = \beta$ ↵(5b)

Path length s(τ) and four-velocity u(τ)

$(ids)^2 \equiv dx_1^2 + dx_4^2$ By similar calculation to the above it is obtained:

$\chi = \chi(\tau)$ $\Rightarrow$ $ids = [(\tau\, d_\tau\chi(\tau))^2 - 1]^{1/2}\, c\, d\tau$ s(τ) es a real number in M4⁻

$d_\tau\chi(\tau) = 0$ $\Rightarrow$ $ids = ic\, d\tau$ $(dz \equiv 1/c\, ds)$ ([2]) only in this case *Eigenzeit* $z = \tau$

$\chi = \alpha\tau$ $\Rightarrow$ $ids = [(\alpha\tau)2 - 1]^{1/2}\, c\, d\tau$ For $(\alpha\tau)^2 > 1$ we over step to M4⁺

(8) $\mathbf{u}(\tau) \equiv d\mathbf{x}(\tau)/d\tau$ $|\mathbf{u}(\tau)| = ids/d\tau = c\,[(\alpha\tau)^2 - 1]^{1/2}$ For $(\alpha\tau)^2 > 1$ $\rightarrow$ $|\mathbf{u}(\tau)| > 1$

According to integration tables we obtain:

(9) $s(\tau) = c/\alpha \int_0^\tau [(\alpha\tau)^2 - 1]^{1/2}\,\alpha\, d\tau = \alpha\tau/2\,[(\alpha\tau)^2 - 1]^{1/2} - 1/2\ln(\alpha\tau + [(\alpha\tau)^2 - 1]^{1/2})$

Note: The formulas in the last two lines show the essential difference between the scalar variables **s** as dependent and **τ** as independent. Thus we see how wrong it is to use the so-called *Eigenzeit* to define velocity and acceleration in M4.

Acceleration in M4

$\mathbf{a}_1(\tau) \equiv d_t v_1(\tau) = d_\tau v_1(\tau) / d_\tau t(\tau)$ $d_\tau t(\tau) = \cosh\chi(\tau)\,\{1 + \alpha\tau\tanh\chi(\tau)\}$

$\mathbf{a}_1(\tau) = c\alpha\,\{\{1 + 1/\cosh^2(\chi)\}\{1 + \alpha\tau\tanh(\chi)\} -$
$- \{\alpha\tau + \tanh(\chi)\}\,\{\tanh(\chi) + \alpha\tau/\cosh^2(\chi)\}\} / \{1 + \alpha\tau\tanh(\chi)\}^3 \cosh(\chi)$

(10) $\Rightarrow$ $\mathbf{a}_1(\tau) = c\alpha\kappa(\chi)$ $A \equiv 1 + \alpha\tau\tanh(\chi)$

$\kappa(\tau) \equiv \{\{1 + 1/\cosh^2(\chi)\} - \{\alpha\tau + \tanh(\chi)\}\,\{\tanh(\chi) + \alpha\tau/\cosh^2(\chi)\}/A^2\} /\cosh(\chi)$

Trayectory of the cosmic cloud G2

To confirm the physical validity of the formulas here deduced we should analyze a concrete path with known observed data. Due to the constant c appearing in the formulas (2c) and (2d) a large path (au in space and years in time) is required. For this purpose, we find it appropriate to analyze the trajectory of the cosmic cloud G2 traveling thousands of astronomical units and which has been observed since 2006, when it was discovered by Stefan Gillessen and colleagues at the Max Planck Institute for Extraterrestrial Physics in Germany. The magnitudes of its accelerated movement are published in K.Phifer et al 2013 ([3]) and complemented by Gillessen et al 2013 ([4]) based on measurements collected at the European Southern Observatory in Paranal, Chile and WM Observatory Keck in Hawaii.

G2 is a huge (100 au) gas cloud with a total mass which corresponds to an estimated three times the mass of the earth. Its trajectory is determined by the gravitational field of Sagittarrius A *, an immense mass concentration M (black hole) estimated at 4.31 million times the mass of the sun located in the center of our galaxy (Milky Way) at a distance from earth 27 thousand light-years ($1.71 \cdot 10^9$ ua). According to the latest data from April 2013 ([4]), G2 had approached the center of gravity to approximately 144.4 ua taking an elongated shape. The corresponding radial velocity of the brightest point of the cloud was 2180 ± 50 km/s, while the front part was approaching 3000 km/s.

Here we want to compare the predictions of the radial velocity achieved with our formula (5b) for accelerated motion using (10) to calculate the acceleration. For this purpose, consider a simple model of a particle moving under the action of gravity caused by the mass M.

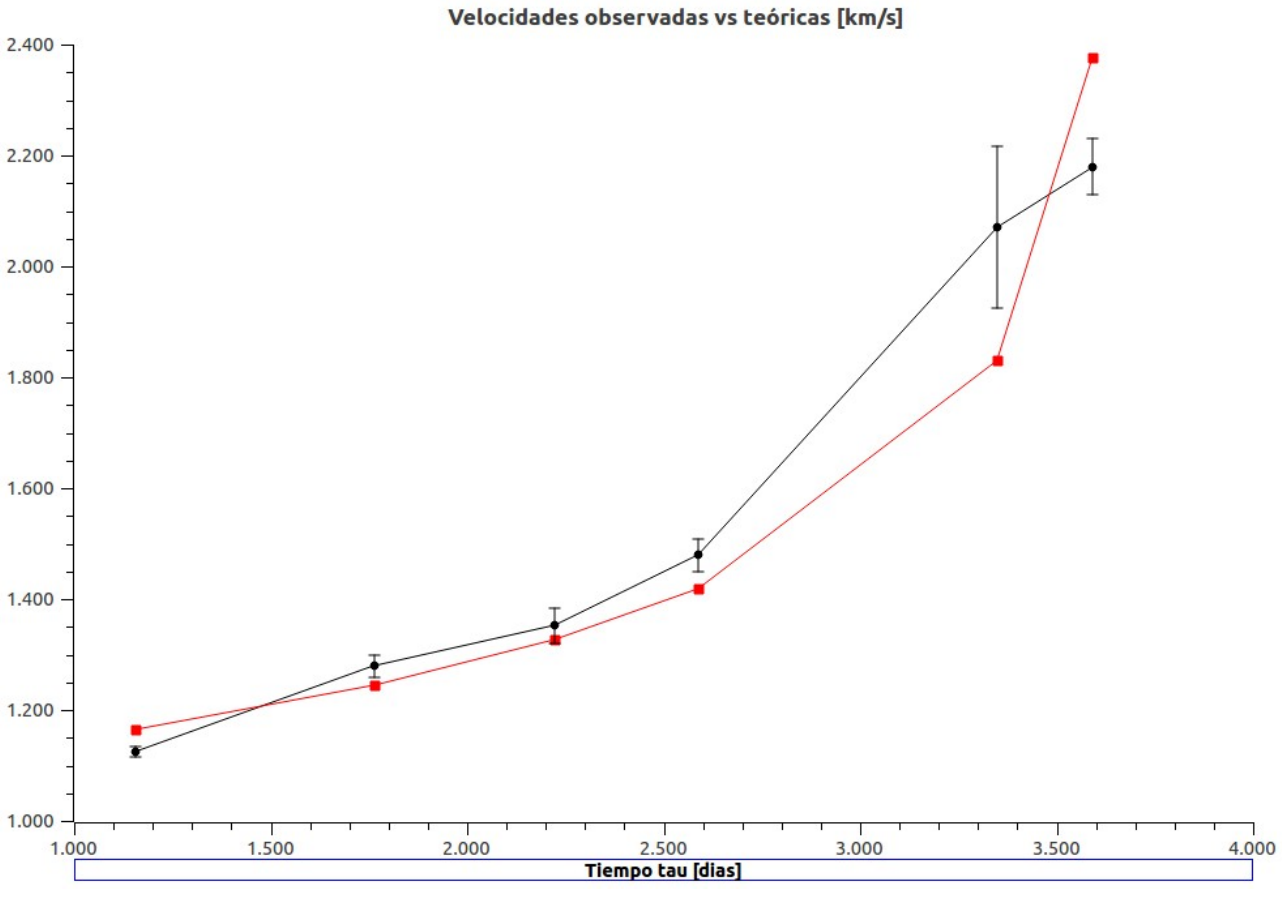
This model confronts the difficulty that the position of the center of mass of the cloud G2 does not necessarily coincide with the brightest point of the cloud, which has been used as the reference point for radial velocity measurements. This discrepancy represents a displacement of the measuring position from the position of the center of mass which is based on our theoretical model, displacement that varies with changes in the shape of the cloud along the path. This introduces an element of uncertainty in the comparison of theoretical results with the observed magnitudes plus the margin of error of the measurement itself. However the calculated values are reasonably comparable with the published measurements given in the reference papers, as shown in the following table and its corresponding graph.

In Appendix A2 we explain the calculation method and A3 shows the corresponding spreadsheet with monthly results over ten years. From this table we have obtained the displacement of the center of mass relative to the luminic center of the cloud G2, from which, because of its cyclical character we infer that within the cloud there is a system of at least two stars with masses of a similar order of magnitude, as we explain in A4.

Results – Sensitivity Analysis

P(1+ε) ua	R_0 ua	R_{min} ua	χ_0 rad	2006/06 0,003753 1125±9	2008/02 0,004266 1280±40	2009/05 0,004509 1352±32	2010/05 0,004933 1479±29	2012/06 0,006908 2071±146	2013/02 0,007272 2180±50	Month of rodeo	R_1/R_3 1,2650
1,0	5864,44	4,46	0,003566407	1165,9	1245,1	1328,3	1420,4	1829,7	2376,8	2013/06	1,2712
150,0	5788,40	151,66	0,003565414	1165,6	1245,6	1327,9	1419,8	1827,8	2366,0	2013/06	1,2760
257,4	5729,08	264,73	0,003565414	1165,7	1244,9	1328,1	1420,1	1827,5	2350,1	2013/06	1,2800
257,4	5851,00	264,73	0,003569680	1162,5	1237,2	1314,6	1398,8	1742,3	2055,4	2013/09	1,2801

This table shows only some of the iteration cycles performed for the sensitivity analysis of the initial parameters. The analysis shows that the results of the second row better approach relation R1/R3. The corresponding theoretical velocities for the whole iteration cycle is shown in A3 and is plotted against the observed values:



According to ([4]) in 2013/04 there was a front component speed of nearly 3000 km/s. This observation point is not shown in this graph because it refers to the luminous center, but it is interesting that the steep upward trend also follows the theoretical curve.

Deviations from the theoretical curve to the results of observation can be interpreted as displacements from the center of mass G2 to the luminic center suggesting that within the G2 cloud there is a system of stars or planets, as explained in A4.

Hyperluminic subspace penetration effect

In order to show the hyperluminic subspace penetration effect, applying the above derived formulas we calculate the path that would follow a corpuscle falling from the point it deviates from its orbit until it exceeds the speed barrier of light due to g_M gravitational acceleration caused by the enormous concentration of mass M of a black hole.

In order to compare results with observed data we use the mass $M = 4.31 \cdot 10^6$ solar masses of the supermassive black hole in the center of our galaxy (Milky Way) that is surrounded by a score of stars. Since 2006 the trajectory of the cosmic cloud named G2 has been observed signaling an "encounter" with the black hole. The speeds and times of observation of the path still in orbit G2 ([3.4]) we hereby use in our comparisons with the theoretical results deduced here.

We also calculate the trajectory of falling G2 (possibly only parts of this cloud) into Sagittarius A * where the speed of light is exceeded, a phenomenon still lacking confirmation.

Clarification of the phenomenon "black hole"

Allow me to propose an interpretation of the disappearance of an object that is "swallowed" by a black hole as the event in which it crosses the light cone to a radius R_N from which we can not perceive its signals from hyperluminic subspace. That is, the radius is the distance R_N from a gravitational center when a fast moving corpuscle comes to exceed the speed c due to the large acceleration caused by the proximity to a huge mass M. If we assume that M results from the accumulation of matter that is orbiting in a hyperluminic subspace region populated by tachyon stars (moving at accelerated velocities and with $\beta > 1$).

The black disk of radius R_N represents the size of the entrance channel to the hiperlumínic space where the accumulation of mass M is there occupying a volume which size we don't know, nevertheless we can assume that it is big enough to accommodate M to a density of the order of magnitude of compact star densities .

According to formula (3) we see that there can be a constant $\beta > 1$, as we very well know it agrees with the TR, therefore there can be no tachyons traveling at constant velocities. However, according to our formula (5b), a tachyon (with $\beta > 1$), to be such, must necessarily be moving with acceleration.

Towards an Integral Theory of Relativity (ITR)

The restricted TR does not explain how a path in M4 reaches its slope β (Formula 3), we only use the given values of β but we are unable to show the transition from one value to another, whether by acceleration or deceleration. When you need an angle χ that is varying with time you cannot consider it anymore as a function of the variable t that is part of x_4. It would be inconsistent to use an angle as a function of the component of the vector that defines itself. For the same reason we cannot use the arc length s. For lack of the scalar τ in the TR we cannot speak properly of curvilinear paths in the plane x_4-x_1. When we need to treat accelerated motion we are simply remitted to the Generalized Theory of Relativity.

Some authors of texts on the TR apply the Minkowsky *Eigenzeit* concept to define velocities and accelerations in M4; they even draw curvilinear paths in the plane x1-x4, which makes no sense in a space devoid of time τ as a scalar parameter. The conceptual error committed when using this trick already noted by Sommerfeld indicating that an arc length, even though artificially expressed in units of time, could not be used as a "time" since this length depends on the path that we want to describe using the *Eigenzeit*. It is interesting to point out that Einstein never tried the arc length **s** as an *Eigenzeit*.

I realize the controversial nature of this proposal of an ITR in which the Newtonian time is reinstated to handle acceleration in M4, as an alternative to GTR. However, as we have shown, it is highly advantageous to be able to analyze problems with acceleration in M4, working with two variables: t as a coordinate τ as scalar. The purpose of this paper is to show how the two variables allow us to analyze trajectories in M4, making it possible to compare results with current astronomical observations of the cloud phenomenon G2 approaching the supermassive black hole in our galaxy. With our formula (5b) we prove mathematicaly that it is possible to exceed the speed of light in an accelerated motion (fall trajectory treated in A5), by a small amount (2%) without contradicting the restricted TR valid for constant velocities.

The challenge is brought, because the recovery of an independent scalar time τ allows us to treat the four-vector momentum-energy (**p** = m**u**) as a function of τ, thus opening the feasibility of formulating coherently a Hamiltonian mechanics in M4 not possible in the TR.

The potential for further theoretical research in the field of an ITR is huge, in relation to the formulation of an integral relativistic quantum mechanics, that thanks to τ could lead to a Schrödinger wave equation in M4 with a truly L-invariant Hamiltonian operator.

References:

([1]) H. Minkowsky, 1908 - *Raum und Zeit*
([2]) A. Sommerfeld, 1909 – *Über die Zusammensetzung der Geschwindigkeiten in der Relativitäts Theorie*
([3]) K Phifer et al. 18 Apr 2013 - *Keck Observations of the Galactic Center Source G2: Gas Cloud or Star?*
([4]) S Gillessen et al 2013 – *Pericenter passage of the gas cloud G2 in the Galactic Center*

A1

Relativistic velocity and its auxiliary functions

Δ ατ = 0,1   0,001 = χo

ατ	tanh(ατ)	β(ατ)	β(χo,ατ)	A	Kapa
0	0	0,0000000	0,0010000	1,0000000	1,9999995
0,1	0,0996680	0,1976976	0,1986583	1,0996680	1,9588605
0,2	0,1973753	0,3822846	0,3831381	1,1973753	1,8510654
0,3	0,2913126	0,5437888	0,5444928	1,2913126	1,6985595
0,4	0,3799490	0,6770510	0,6775923	1,3799490	1,5211215
0,5	0,4621172	0,7815365	0,7819254	1,4621172	1,3355777
0,6	0,5370496	0,8599486	0,8602088	1,5370496	1,1552017
0,7	0,6043678	0,9165953	0,9167550	1,6043678	0,9894875
0,8	0,6640368	0,9561185	0,9562043	1,6640368	0,8443370
0,9	0,7162979	0,9827502	0,9827844	1,7162979	0,7225733
1	0,7615942	1,0000000	1,0000000	1,7615942	0,6246332
1,1	0,8004990	1,0106087	1,0105873	1,8004990	0,5492924
1,2	0,8336546	1,0166313	1,0165978	1,8336546	0,4943159
1,3	0,8617232	1,0195653	1,0195258	1,8617232	0,4569773
1,4	0,8853516	1,0204776	1,0204362	1,8853516	0,4344320
1,5	0,9051483	1,0201151	1,0200745	1,9051483	0,4239602
1,6	0,9216686	1,0189920	1,0189537	1,9216686	0,4231034
1,7	0,9354091	1,0174557	1,0174205	1,9354091	0,4297258
1,8	0,9468060	1,0157364	1,0157047	1,9468060	0,4420259
1,9	0,9562375	1,0139824	1,0139542	1,9562375	0,4585176
2	0,9640276	1,0122854	1,0122607	1,9640276	0,4779971
2,1	0,9704519	1,0106990	1,0106775	1,9704519	0,4995032
2,2	0,9757431	1,0092506	1,0092320	1,9757431	0,5222782
2,3	0,9800964	1,0079511	1,0079352	1,9800964	0,5457313
2,4	0,9836749	1,0068005	1,0067869	1,9836749	0,5694071
2,5	0,9866143	1,0057921	1,0057805	1,9866143	0,5929587
2,6	0,9890274	1,0049157	1,0049058	1,9890274	0,6161251
2,7	0,9910075	1,0041590	1,0041507	1,9910075	0,6387139
2,8	0,9926315	1,0035094	1,0035024	1,9926315	0,6605856
2,9	0,9939632	1,0029543	1,0029484	1,9939632	0,6816431
3	0,9950548	1,0024818	1,0024769	1,9950548	0,7018217
3,1	0,9959494	1,0020811	1,0020769	1,9959494	0,7210822
3,2	0,9966824	1,0017422	1,0017387	1,9966824	0,7394052
3,3	0,9972830	1,0014563	1,0014534	1,9972830	0,7567865
3,4	0,9977749	1,0012158	1,0012133	1,9977749	0,7732333
3,5	0,9981779	1,0010137	1,0010117	1,9981779	0,7887618
3,6	0,9985079	1,0008443	1,0008426	1,9985079	0,8033950
3,7	0,9987782	1,0007025	1,0007011	1,9987782	0,8171603
3,8	0,9989996	1,0005840	1,0005829	1,9989996	0,8300891
3,9	0,9991809	1,0004851	1,0004841	1,9991809	0,8422149
4	0,9993293	1,0004026	1,0004018	1,9993293	0,8535726
4,1	0,9994508	1,0003339	1,0003333	1,9994508	0,8641983
4,2	0,9995504	1,0002768	1,0002762	1,9995504	0,8741280
4,3	0,9996319	1,0002293	1,0002288	1,9996319	0,8833978
4,4	0,9996986	1,0001898	1,0001895	1,9996986	0,8920433
4,5	0,9997532	1,0001571	1,0001568	1,9997532	0,9000992
4,6	0,9997979	1,0001299	1,0001297	1,9997979	0,9075995
4,7	0,9998346	1,0001074	1,0001072	1,9998346	0,9145769
4,8	0,9998646	1,0000888	1,0000886	1,9998646	0,9210629
4,9	0,9998891	1,0000733	1,0000732	1,9998891	0,9270880
5	0,9999092	1,0000605	1,0000604	1,9999092	0,9326812
5,1	0,9999257	1,0000500	1,0000499	1,9999257	0,9378700
5,2	0,9999391	1,0000412	1,0000411	1,9999391	0,9426809
5,3	0,9999502	1,0000340	1,0000339	1,9999502	0,9471387
5,4	0,9999592	1,0000280	1,0000280	1,9999592	0,9512670
5,5	0,9999666	1,0000231	1,0000231	1,9999666	0,9550884
5,6	0,9999727	1,0000191	1,0000190	1,9999727	0,9586237
5,7	0,9999776	1,0000157	1,0000157	1,9999776	0,9618928
5,8	0,9999817	1,0000129	1,0000129	1,9999817	0,9649145
5,9	0,9999850	1,0000107	1,0000106	1,9999850	0,9677061
6	0,9999877	1,0000088	1,0000088	1,9999877	0,9702841
6,1	0,9999899	1,0000072	1,0000072	1,9999899	0,9726639
6,2	0,9999918	1,0000059	1,0000059	1,9999918	0,9748598
6,3	0,9999933	1,0000049	1,0000049	1,9999933	0,9768854
6,4	0,9999945	1,0000040	1,0000040	1,9999945	0,9787531
6,5	0,9999955	1,0000033	1,0000033	1,9999955	0,9804747
6,6	0,9999963	1,0000027	1,0000027	1,9999963	0,9820610
6,7	0,9999970	1,0000022	1,0000022	1,9999970	0,9835222
6,8	0,9999975	1,0000018	1,0000018	1,9999975	0,9848677
6,9	0,9999980	1,0000015	1,0000015	1,9999980	0,9861064
7	0,9999983	1,0000012	1,0000012	1,9999983	0,9872464
7,1	0,9999986	1,0000010	1,0000010	1,9999986	0,9882952
7,2	0,9999989	1,0000008	1,0000008	1,9999989	0,9892599
7,3	0,9999991	1,0000007	1,0000007	1,9999991	0,9901470
7,4	0,9999993	1,0000006	1,0000006	1,9999993	0,9909625
7,5	0,9999994	1,0000005	1,0000005	1,9999994	0,9917120
7,6	0,9999995	1,0000004	1,0000004	1,9999995	0,9924007
7,7	0,9999996	1,0000003	1,0000003	1,9999996	0,9930334
7,8	0,9999997	1,0000003	1,0000003	1,9999997	0,9936145
7,9	0,9999997	1,0000002	1,0000002	1,9999997	0,9941481
8	0,9999998	1,0000002	1,0000002	1,9999998	0,9946380
8,1	0,9999998	1,0000001	1,0000001	1,9999998	0,9950876
8,2	0,9999998	1,0000001	1,0000001	1,9999998	0,9955002
8,3	0,9999999	1,0000001	1,0000001	1,9999999	0,9958787
8,4	0,9999999	1,0000001	1,0000001	1,9999999	0,9962260
8,5	0,9999999	1,0000001	1,0000001	1,9999999	0,9965445
8,6	0,9999999	1,0000001	1,0000001	1,9999999	0,9968365
8,7	0,9999999	1,0000000	1,0000000	1,9999999	0,9971043
8,8	1,0000000	1,0000000	1,0000000	2,0000000	0,9973497
8,9	1,0000000	1,0000000	1,0000000	2,0000000	0,9975747

A2 Calculation method of the G2 orbital trajectory

Variables:

τ [days] newtonian time (independent escalar)
τ_ρ [dias] newtonian time (parameter) when G2 enters in round-up (R < 0)
$\Delta\tau$ [dias] time interval (parámetro)
x_1, x_4 [ua] coordinates in M4, origen at R_0 au from x_G gravitational center of Sagittarius A*
$x_4 \equiv ict$ t = relativ time at the reference system observatory [days]
χ [1] $\chi = \chi(\tau)$ iχ rotation angle of the position vector of the G2 CM in plane x_4-x_1
$x_1 = c\tau$ senh $\chi(\tau)$ according to formula (2c) follows the radial direction of R_0 to x_G
$x_4 = ic\tau \cosh\chi(\tau)$ according to formula (2d) c = speed of light (173,26 au/day)
$R = R_0 - x_1$ G2 radial position respect gravitational center of Sagittarius A*
M gravitational center of Sagittarius A* mass ($M = 4{,}31 \cdot 10^6$ sun masses)
g_M [m/s²] gravitational acceleration caused by M $g_M = GM/R^2$ (G gravitational constant)
g_f [m/s²] centrífugal acceleration due to radial gyro $g_f = GMP(1+\varepsilon)/R^3$ P perihelio
$a_1(\tau) \equiv d^2x_1/dt^2$ [m/s²] acceleration in the x_1-direction $a_1(\tau) = c\alpha\kappa(\chi)$ formula (10)
α [rads/dia] acceleration factor $\chi(\tau) = \chi_0 + \alpha\tau$ $a_1 = g \equiv g_M - g_f$ $\Rightarrow$ $\alpha = g/c\kappa(\chi)$
β_0 [1] $\beta_0 \equiv \tanh\chi_0 = 1/c\, v_0$ parameter v_0 initial velocity by entering in orbit at R_0 (x_1=0)
β [1] $\beta \equiv 1/c\ dx_1/dt$ $v = dx_1/dt$ radial velocity according fórmula (5b)

Recursive iteration cycle

The iteration cycle consists of a series of calculations of the variables starting from the parameters which are kept fixed during the cycle. Variable α is obtained using the value of the previous interval calculation and serves to give the α value to the next interval. The length of the interval is set by the parameter $\Delta\tau$ that keeps its value for all pertinent calculation cycles. It has adopted an average interval of days per month in four years $\Delta\tau$ = 30.4375 days.

A cycle starts by assigning a value to the rodeo time τ_ρ which can be estimated from the observations. In A3 are shown the corresponding cycle iterations assigning τ_ρ for June 2013 that effectively was met. We check that this allocation is the optimal prediction by comparing cycles with other parameter variations. For this purpose we take as a control variable the R_1/R_3 ratio of the radii observed in June 2006 and June 2009 compared to the ratio of the respective angles given in ([3]) $\Phi_1/\Phi_3 = 1.265$ as shown in the table below. For each iteration cycle the parameter P has been changed and thus obtained the respective values of R_1/R_3 as follows:

Sensitivity analysis for τ_ρ = June, 2013

							2013/06	2013/08	2013/05
$P(1+\varepsilon)$	257,4	175	150	100	50	10	1,0	1,0	1,0
R_1/R_3	1,2800	1,2769	1,2760	1,2741	1,2727	1,2715	1,2712	1,2748	1,2749

The best approach to $\Phi_1/\Phi_3 = 1.265$ is obtained for P (1 + e) = 1.0 and τ_ρ = 2013/06. As we can see in the last line of the calculation in A3, the interval prior to the rodeo (before the radius becomes negative) corresponds to 2013/05 so the rodeo ocurred in 2013/06, a date that has proven to be a very good prediction as this actually happened.

A3 Calculation of the orbital G2 trajectory

	Chi0 rads			f Masa	Ro ua	(1+e)P ua				
	0,00356641			1,00	5864,44	1,0				
Alfa rads/dia	Chi rads	tanh(Chi)	Beta(Tau)	Año/Mes	R(tau) ua	g(tau) m/s2	A	kapa	δR	
0	0,0035664	0,0035664	0,0035664	2003/04	5864,44	0,0007430	1,0035664	1,9999809		
107,1E-9	0,0035697	0,0035697	0,0035729		5845,63	0,0007478	1,0035697	1,9999809		
107,8E-9	0,0035730	0,0035730	0,0035795		5826,78	0,0007527	1,0035730	1,9999808		
108,5E-9	0,0035763	0,0035763	0,0035862		5807,90	0,0007576	1,0035763	1,9999807		
109,2E-9	0,0035797	0,0035797	0,0035930		5788,98	0,0007625	1,0035797	1,9999807		
109,9E-9	0,0035831	0,0035831	0,0035998		5770,02	0,0007676	1,0035831	1,9999806		
110,6E-9	0,0035866	0,0035866	0,0036068		5751,03	0,0007726	1,0035866	1,9999806		
111,3E-9	0,0035901	0,0035901	0,0036138		5732,00	0,0007778	1,0035901	1,9999805		
112,1E-9	0,0035937	0,0035937	0,0036210		5712,93	0,0007830	1,0035937	1,9999804		
112,8E-9	0,0035973	0,0035973	0,0036282	2004/01	5693,82	0,0007882	1,0035973	1,9999804		
113,6E-9	0,0036010	0,0036010	0,0036355		5674,66	0,0007936	1,0036010	1,9999803		
114,4E-9	0,0036047	0,0036047	0,0036430		5655,47	0,0007990	1,0036047	1,9999802		
115,1E-9	0,0036085	0,0036084	0,0036505		5636,24	0,0008044	1,0036084	1,9999802		
115,9E-9	0,0036123	0,0036123	0,0036581		5616,96	0,0008100	1,0036123	1,9999801		
116,7E-9	0,0036161	0,0036161	0,0036659		5597,64	0,0008156	1,0036161	1,9999800		
117,5E-9	0,0036201	0,0036200	0,0036737		5578,27	0,0008212	1,0036200	1,9999800		
118,3E-9	0,0036240	0,0036240	0,0036817		5558,86	0,0008270	1,0036240	1,9999799		
119,2E-9	0,0036281	0,0036281	0,0036897		5539,40	0,0008328	1,0036281	1,9999798		
120,0E-9	0,0036322	0,0036321	0,0036979		5519,89	0,0008387	1,0036321	1,9999797		
120,9E-9	0,0036363	0,0036363	0,0037062		5500,33	0,0008447	1,0036363	1,9999797		
121,7E-9	0,0036405	0,0036405	0,0037146		5480,72	0,0008507	1,0036405	1,9999796		
122,6E-9	0,0036448	0,0036447	0,0037231	2005/01	5461,07	0,0008568	1,0036447	1,9999795		
123,5E-9	0,0036491	0,0036491	0,0037318		5441,36	0,0008631	1,0036491	1,9999794		
124,4E-9	0,0036535	0,0036535	0,0037405		5421,59	0,0008694	1,0036535	1,9999793		
125,3E-9	0,0036579	0,0036579	0,0037494		5401,78	0,0008758	1,0036579	1,9999793		
126,2E-9	0,0036624	0,0036624	0,0037584		5381,90	0,0008822	1,0036624	1,9999792		
127,1E-9	0,0036670	0,0036670	0,0037676		5361,98	0,0008888	1,0036670	1,9999791		
128,1E-9	0,0036717	0,0036716	0,0037769		5341,99	0,0008955	1,0036716	1,9999790		
129,0E-9	0,0036764	0,0036764	0,0037863		5321,94	0,0009022	1,0036764	1,9999789		
130,0E-9	0,0036812	0,0036812	0,0037959		5301,84	0,0009091	1,0036812	1,9999788		
131,0E-9	0,0036860	0,0036860	0,0038056		5281,67	0,0009160	1,0036860	1,9999787		
132,0E-9	0,0036910	0,0036909	0,0038155		5261,44	0,0009231	1,0036909	1,9999786		
133,0E-9	0,0036960	0,0036960	0,0038255		5241,14	0,0009303	1,0036960	1,9999785		
134,1E-9	0,0037011	0,0037010	0,0038357	2006/01	5220,78	0,0009375	1,0037010	1,9999784		
135,1E-9	0,0037062	0,0037062	0,0038460		5200,35	0,0009449	1,0037062	1,9999783		
136,2E-9	0,0037115	0,0037114	0,0038565		5179,85	0,0009524	1,0037114	1,9999782		
137,2E-9	0,0037168	0,0037168	0,0038672		5159,28	0,0009600	1,0037168	1,9999781		
138,3E-9	0,0037222	0,0037222	0,0038780		5138,63	0,0009677	1,0037222	1,9999780		
139,5E-9	0,0037277	0,0037277	0,0038890	2006/06	5117,92	0,0009756	1,0037277	1,9999779	5401,78	0,0037526
140,6E-9	0,0037333	0,0037333	0,0039002		5097,12	0,0009836	1,0037333	1,9999778	5220,78	
141,7E-9	0,0037390	0,0037389	0,0039115		5076,25	0,0009917	1,0037389	1,9999777	181	
142,9E-9	0,0037447	0,0037447	0,0039231		5055,30	0,0009999	1,0037447	1,9999776		
144,1E-9	0,0037506	0,0037506	0,0039348		5034,27	0,0010083	1,0037506	1,9999775		
145,3E-9	0,0037566	0,0037566	0,0039467		5013,15	0,0010168	1,0037566	1,9999774		
146,5E-9	0,0037626	0,0037626	0,0039588		4991,94	0,0010254	1,0037626	1,9999773		
147,8E-9	0,0037688	0,0037688	0,0039712	2007/01	4970,65	0,0010342	1,0037688	1,9999771		
149,0E-9	0,0037751	0,0037751	0,0039837		4949,27	0,0010432	1,0037751	1,9999770		
150,3E-9	0,0037815	0,0037814	0,0039965		4927,79	0,0010523	1,0037814	1,9999769		
151,6E-9	0,0037880	0,0037879	0,0040095		4906,22	0,0010616	1,0037879	1,9999768		
153,0E-9	0,0037946	0,0037945	0,0040227		4884,55	0,0010710	1,0037945	1,9999766		
154,3E-9	0,0038013	0,0038013	0,0040361		4862,78	0,0010806	1,0038013	1,9999765		
155,7E-9	0,0038081	0,0038081	0,0040498		4840,91	0,0010904	1,0038081	1,9999764		
157,1E-9	0,0038151	0,0038151	0,0040638		4818,93	0,0011004	1,0038151	1,9999762		
158,6E-9	0,0038222	0,0038222	0,0040780		4796,84	0,0011105	1,0038222	1,9999761		
160,0E-9	0,0038294	0,0038294	0,0040924		4774,64	0,0011209	1,0038294	1,9999759		
161,5E-9	0,0038368	0,0038368	0,0041072		4752,32	0,0011315	1,0038368	1,9999758		
163,0E-9	0,0038443	0,0038443	0,0041222		4729,88	0,0011422	1,0038443	1,9999756		
164,6E-9	0,0038520	0,0038519	0,0041375	2008/01	4707,32	0,0011532	1,0038519	1,9999755		
166,2E-9	0,0038598	0,0038598	0,0041531		4684,64	0,0011644	1,0038598	1,9999753	4521,97	0,004266
167,8E-9	0,0038677	0,0038677	0,0041690		4661,83	0,0011758	1,0038677	1,9999751	4684,64	
169,4E-9	0,0038758	0,0038758	0,0041852		4638,88	0,0011875	1,0038758	1,9999750	-162,67	
171,1E-9	0,0038841	0,0038841	0,0042018		4615,79	0,0011994	1,0038841	1,9999748		
172,8E-9	0,0038926	0,0038925	0,0042187		4592,56	0,0012115	1,0038925	1,9999746		
174,6E-9	0,0039012	0,0039012	0,0042359		4569,18	0,0012240	1,0039012	1,9999745		
176,4E-9	0,0039100	0,0039100	0,0042535		4545,65	0,0012367	1,0039100	1,9999743		
178,2E-9	0,0039190	0,0039190	0,0042715		4521,97	0,0012496	1,0039190	1,9999741		
180,1E-9	0,0039282	0,0039281	0,0042899		4498,12	0,0012629	1,0039281	1,9999739		
182,0E-9	0,0039375	0,0039375	0,0043087		4474,11	0,0012765	1,0039375	1,9999737		
183,9E-9	0,0039471	0,0039471	0,0043278		4449,92	0,0012904	1,0039471	1,9999735		
186,0E-9	0,0039569	0,0039569	0,0043475	2009/01	4425,55	0,0013047	1,0039569	1,9999733		
188,0E-9	0,0039670	0,0039670	0,0043675		4400,99	0,0013193	1,0039670	1,9999731		
190,1E-9	0,0039773	0,0039772	0,0043881		4376,24	0,0013342	1,0039772	1,9999729		
192,3E-9	0,0039878	0,0039877	0,0044091		4351,29	0,0013496	1,0039877	1,9999726		
194,5E-9	0,0039985	0,0039985	0,0044306	2009/5	4326,14	0,0013653	1,0039985	1,9999724	4249,33	0,004510
196,7E-9	0,0040096	0,0040095	0,0044527		4300,76	0,0013815	1,0040095	1,9999722	4326,14	
199,1E-9	0,0040209	0,0040208	0,0044753		4275,16	0,0013981	1,0040208	1,9999719	-76,81	
201,5E-9	0,0040324	0,0040324	0,0044985		4249,33	0,0014151	1,0040324	1,9999717		
203,9E-9	0,0040443	0,0040443	0,0045222		4223,26	0,0014327	1,0040443	1,9999714		
206,4E-9	0,0040565	0,0040565	0,0045466		4196,93	0,0014507	1,0040565	1,9999711		
209,0E-9	0,0040691	0,0040690	0,0045717		4170,33	0,0014692	1,0040690	1,9999708		
211,7E-9	0,0040819	0,0040819	0,0045975		4143,46	0,0014884	1,0040819	1,9999706		
214,5E-9	0,0040952	0,0040952	0,0046239	2010/01	4116,29	0,0015081	1,0040952	1,9999703		
217,3E-9	0,0041088	0,0041088	0,0046512		4088,83	0,0015284	1,0041088	1,9999699		
220,2E-9	0,0041228	0,0041228	0,0046792		4061,04	0,0015494	1,0041228	1,9999696		
223,3E-9	0,0041372	0,0041372	0,0047081		4032,93	0,0015711	1,0041372	1,9999693		
226,4E-9	0,0041521	0,0041521	0,0047378	2010/05	4004,46	0,0015935	1,0041521	1,9999690	3825,14	0,0049334
229,6E-9	0,0041675	0,0041674	0,0047685		3975,62	0,0016167	1,0041674	1,9999686	4004,46	
233,0E-9	0,0041833	0,0041833	0,0048002		3946,40	0,0016407	1,0041833	1,9999682	-179,32	
236,4E-9	0,0041997	0,0041997	0,0048329		3916,76	0,0016656	1,0041997	1,9999679		
240,0E-9	0,0042166	0,0042166	0,0048668		3886,69	0,0016915	1,0042166	1,9999675		
243,7E-9	0,0042341	0,0042341	0,0049018		3856,16	0,0017184	1,0042341	1,9999670		
247,6E-9	0,0042523	0,0042522	0,0049381		3825,14	0,0017463	1,0042522	1,9999666		
251,7E-9	0,0042711	0,0042711	0,0049757		3793,60	0,0017755	1,0042711	1,9999662		
255,9E-9	0,0042907	0,0042906	0,0050149	2011/01	3761,51	0,0018059	1,0042906	1,9999657		
260,2E-9	0,0043110	0,0043110	0,0050555		3728,82	0,0018377	1,0043110	1,9999652		
264,8E-9	0,0043321	0,0043321	0,0050978		3695,51	0,0018710	1,0043321	1,9999647		
269,6E-9	0,0043542	0,0043542	0,0051420		3661,51	0,0019059	1,0043542	1,9999641		
274,6E-9	0,0043773	0,0043773	0,0051881		3626,77	0,0019426	1,0043773	1,9999635		
279,9E-9	0,0044014	0,0044014	0,0052364		3591,25	0,0019812	1,0044014	1,9999629		
285,5E-9	0,0044267	0,0044267	0,0052869		3554,85	0,0020220	1,0044267	1,9999623		
291,4E-9	0,0044533	0,0044532	0,0053401		3517,52	0,0020651	1,0044532	1,9999616		
297,6E-9	0,0044812	0,0044812	0,0053960		3479,16	0,0021109	1,0044812	1,9999609		
304,2E-9	0,0045108	0,0045108	0,0054551		3439,66	0,0021596	1,0045108	1,9999601		
311,2E-9	0,0045421	0,0045420	0,0055177		3398,91	0,0022117	1,0045420	1,9999593		
318,7E-9	0,0045753	0,0045753	0,0055841		3356,76	0,0022676	1,0045753	1,9999584		
326,8E-9	0,0046107	0,0046107	0,0056550	2012/01	3313,03	0,0023279	1,0046107	1,9999574		
335,5E-9	0,0046487	0,0046487	0,0057309		3267,53	0,0023931	1,0046487	1,9999564		
344,9E-9	0,0046895	0,0046895	0,0058126		3220,00	0,0024643	1,0046895	1,9999553		
355,1E-9	0,0047338	0,0047337	0,0059010		3170,12	0,0025424	1,0047337	1,9999540		
366,4E-9	0,0047819	0,0047819	0,0059974		3117,51	0,0026290	1,0047819	1,9999527		
378,8E-9	0,0048348	0,0048348	0,0061032	2012/06	3061,64	0,0027258	1,0048348	1,9999511	2697,84	0,0069081
392,8E-9	0,0048935	0,0048934	0,0062205		3001,84	0,0028354	1,0048934	1,9999494	3031,64	
408,6E-9	0,0049593	0,0049593	0,0063521		2937,19	0,0029616	1,0049593	1,9999474	-333,8	
426,8E-9	0,0050343	0,0050342	0,0065021		2866,41	0,0031096	1,0050342	1,9999451		
448,1E-9	0,0051213	0,0051213	0,0066761		2787,60	0,0032879	1,0051213	1,9999424		
473,8E-9	0,0052249	0,0052248	0,0068832		2697,84	0,0035103	1,0052248	1,9999391		
505,8E-9	0,0053524	0,0053524	0,0071383		2592,32	0,0038018	1,0053524	1,9999349		
547,9E-9	0,0055175	0,0055174	0,0074684	2013/01	2462,36	0,0042137	1,0055174	1,9999292		
607,2E-9	0,0057473	0,0057472	0,0079280		2290,36	0,0048702	1,0057472	1,9999208	2600	0,0072717
701,8E-9	0,0061084	0,0061084	0,0086502		2033,57	0,0061774	1,0061084	1,9999068	2290,36	
890,2E-9	0,0068179	0,0068178	0,0100690		1552,71	0,0105945	1,0068178	1,9998758	309,64	
152,7E-8	0,0091894	0,0091891	0,0148113	pre rodeo	4,46	995,0643868	1,0091891	1,9997399		

A4 Displacement [au] of G2 center of mass with respect to its luminic center

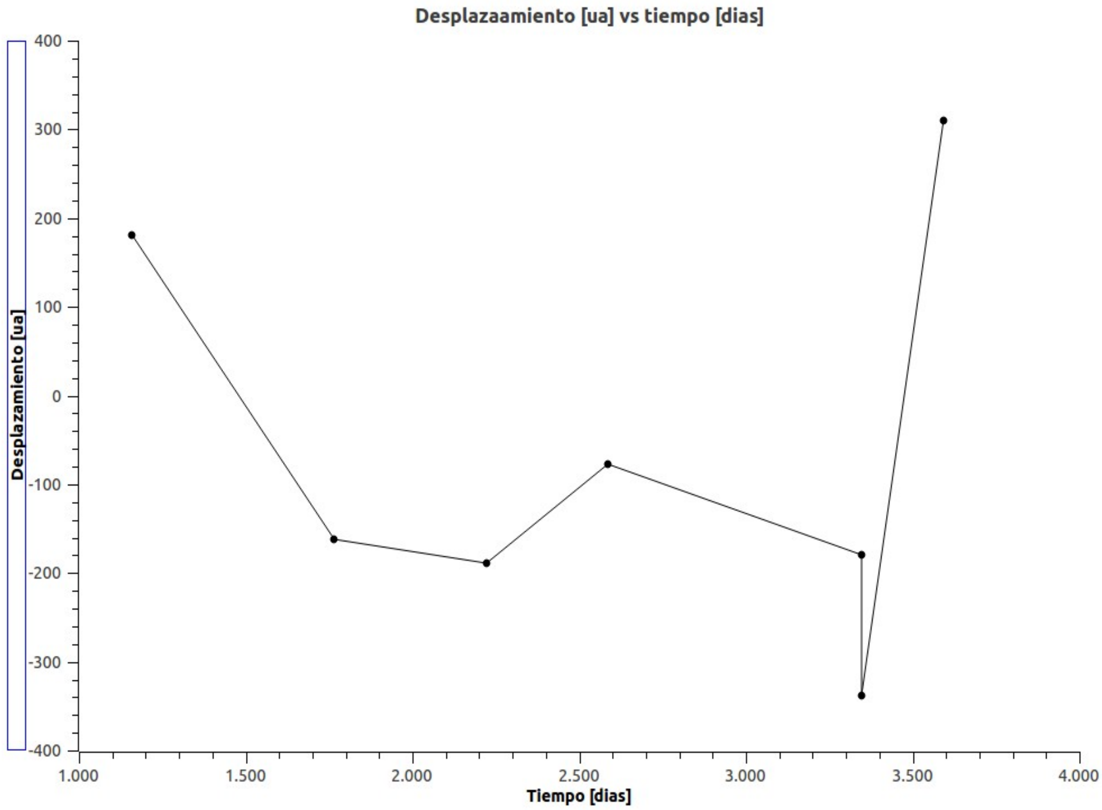
$P(1+\varepsilon)$ ua	R_0 ua	Mes del rodeo	χ_0 rad	2006/06 222,97 ±5,05 mas	2008/07 181,82 ±40 mas	2009/05 176,26 ±2,74 mas	2010/05 166,29 ±9,11 mas	2012/06 103,16 ±4,07 mas	2013/02	R_1/R_3 1,2650
1,0	5864,4	2013/06	0,00356640	5401,78 5220,78 181	4521,97 4684,64 −162,67	4249,33 4326,14 −76,81	3825,14 4004,46 −17932	2697,84 3031,64 −333,8	2600 2290,36 309,64	1,2712

The table shows the calculation of G2 center of mass deviations with respect to its respective luminous center (point of brightness) for each of the dates on which measurements have been made concerning the position of the luminous center. In each row of data there are three lines: the first shows the radial distance R (center of mass to center of gravity) for the value of the velocity of the center of mass measured on that date, the second line shows the value of the radius r (center lighting a gravitational center) corresponding to the measurement of that date. The difference $\delta =$ (R-r) shown in the third line is the displacement, which can be positive ($\delta > 0$) when the center of mass is in front of the luminous center, and negative ($\delta < 0$) when it is behind.

For the second row iteration data corresponds to the A3 spreadsheet. This iteration gives the best approximation (last column) to the radial distance relationship:

$$R_1/R_3 = 222{,}97/176{,}26 = 1{,}2650$$

that we have taken as a control variable in the sensitivity analysis of the iterations.



The quasi-cyclic behavoir of the displacement suggests that G2 contains two or three stars, so that its center of mass rotates around the luminous center where is the larger mass. For example, the repeated displacement of approximately 300 au shows the size of the center of mass orbit of the system, with a cycle of approximately 2000 days (5.5 years).

A5 **Calculation of the fall trajectory of the corpuscle near M**

Delt tau dias		Chi0 rads			f Masa	Ro ua	(1+e)P ua		
1,15741E-005		0,00000000			1,00	0,08565000	0,0		
tau	Alfa rads/dia	Chi rads	tanh(Chi)	Beta(Tau)	t segundos	R(tau) ua	g(tau) m/s2	A	kapa
000,0E-2	0	0,0000000	0,0000000	0,0000000	0,0	0,0856500	3484046,9	1,0000000	2,0000000
115,7E-7	502,1E+0	0,0058108	0,0058107	0,0116211	1,0	0,0856384	3484994,5	1,0058107	1,9998488
231,5E-7	502,2E+0	0,0116256	0,0116251	0,0232475	2,0	0,0856034	3487840,8	1,0116251	1,9993981
347,2E-7	502,7E+0	0,0174566	0,0174548	0,0349007	3,0	0,0855450	3492601,2	1,0174548	1,9986499
463,0E-7	503,6E+0	0,0233159	0,0233117	0,0466023	4,0	0,0854631	3499303,5	1,0233117	1,9976042
578,7E-7	504,9E+0	0,0292161	0,0292078	0,0583741	5,0	0,0853572	3507989,3	1,0292078	1,9962583
694,4E-7	506,4E+0	0,0351701	0,0351556	0,0702388	6,0	0,0852270	3518714,3	1,0351556	1,9946071
810,2E-7	508,4E+0	0,0411912	0,0411680	0,0822198	7,0	0,0850720	3531549,9	1,0411680	1,9926430
925,9E-7	510,8E+0	0,0472940	0,0472588	0,0943419	8,0	0,0848915	3546584,1	1,0472588	1,9903556
104,2E-6	513,5E+0	0,0534937	0,0534427	0,1066315	9,0	0,0846847	3563924,3	1,0534427	1,9877315
115,7E-6	516,7E+0	0,0598069	0,0597357	0,1191170	10,0	0,0844508	3583699,4	1,0597357	1,9847539
127,3E-6	520,4E+0	0,0662518	0,0661551	0,1318291	11,0	0,0841885	3606063,3	1,0661551	1,9814024
138,9E-6	524,5E+0	0,0728488	0,0727202	0,1448018	12,0	0,0838966	3631199,2	1,0727202	1,9776521
150,5E-6	529,2E+0	0,0796203	0,0794525	0,1580728	13,0	0,0835736	3659325,2	1,0794525	1,9734731
162,0E-6	534,4E+0	0,0865921	0,0863763	0,1716842	14,1	0,0832176	3690701,1	1,0863763	1,9688292
173,6E-6	540,3E+0	0,0937934	0,0935193	0,1856840	15,1	0,0828265	3725637,7	1,0935193	1,9636770
185,2E-6	546,8E+0	0,1012583	0,1009137	0,2001270	16,1	0,0823977	3764508,9	1,1009137	1,9579642
196,8E-6	554,1E+0	0,1090267	0,1085967	0,2150769	17,1	0,0819283	3807767,6	1,1085967	1,9516270
208,3E-6	562,3E+0	0,1171457	0,1166128	0,2306082	18,1	0,0814147	3855967,4	1,1166128	1,9445878
219,9E-6	571,5E+0	0,1256723	0,1250149	0,2468096	19,2	0,0808523	3909792,6	1,1250149	1,9367506
231,5E-6	581,8E+0	0,1346760	0,1338676	0,2637879	20,2	0,0802359	3970099,9	1,1338676	1,9279956
243,1E-6	593,5E+0	0,1442431	0,1432509	0,2816738	21,2	0,0795586	4037978,9	1,1432509	1,9181711
254,6E-6	606,7E+0	0,1544826	0,1532653	0,3006300	22,3	0,0788121	4114841,0	1,1532653	1,9070822
266,2E-6	621,8E+0	0,1655357	0,1640401	0,3208629	23,3	0,0779853	4202555,6	1,1640401	1,8944731
277,8E-6	639,3E+0	0,1775892	0,1757455	0,3426407	24,4	0,0770638	4303664,5	1,1757455	1,8800001
289,4E-6	659,7E+0	0,1908977	0,1886121	0,3663203	25,5	0,0760279	4421737,0	1,1886121	1,8631886
300,9E-6	684,0E+0	0,2058210	0,2029631	0,3923922	26,6	0,0748501	4,56E+006	1,2029631	1,8433600
312,5E-6	713,2E+0	0,2228886	0,2192696	0,4215556	27,7	0,0734899	4732420,5	1,2192696	1,8195021
324,1E-6	749,6E+0	0,2429232	0,2382549	0,4548524	28,8	0,0718847	4946131,4	1,2382549	1,7900242
335,6E-6	796,3E+0	0,2672914	0,2611027	0,4939230	30,0	0,0699306	5226426,0	1,2611027	1,7522507
347,2E-6	859,6E+0	0,2984764	0,2899177	0,5415333	31,3	0,0674381	5619897,7	1,2899177	1,7012668
358,8E-6	952,0E+0	0,3415843	0,3288911	0,6027590	32,8	0,0640145	6237098,2	1,3288911	1,6268962
370,4E-6	110,5E+1	0,4092163	0,3878071	0,6878618	34,7	0,0586694	7425333,6	1,3878071	1,5042238
381,9E-6	142,3E+1	0,5433725	0,4955366	0,8185150	38,0	0,0479213	11129638,7	1,4955366	1,2560287
393,5E-6	255,4E+1	1,0049412	0,7636616	1,0006607	52,7	0,0050565	999644800,0	1,7636616	0,6198701
405,1E-6	464,8E+3	188,2752731	1,0000000	1,0000000	###	###	0,0	2,0000000	1,0000000

The fall trajectory of any corpuscle is no longer dependent of angular momentum, so that the factor P(1 + e) = 0. We assume that the fall starts with zero speed ($\beta_0 = 0$) at a radius R_0. Due to the short fall time we take an interval of one second, so that $\Delta\tau = 1.15741 \cdot 10\text{-}5$ day = 1 s. The mass at the center of gravity is M = 4.31 · 10^6 sun masses.

The iterative cycle ends at t = 405.1 E-6 day = 35 seconds (Newtonian time). This last line shows that in 52.7 seconds (dilated time) the corpuscle at the beginning (t = 0) at 0.08565 au (12,813,240 km) from the center of gravity approaches a radius R_N = 0, 0050565 au (756 452 km) exceeding the speed of light with $\beta = 1.0006607$ to return to 1 (see A1). Beyond this radius it enters in the hyperluminic subspace, so that its signals no longer reach us, we can say metaphorically that it has penetrated into a black hole.

The density ρ of matter concentrated in the area within the radius R_N is calculated by the formula: $\rho = M/4{,}1888\ R_N^3 = 4{,}31\cdot10^6\cdot1{,}98855\cdot10^{27}ton/(4{,}1888\cdot(756452000\text{m})^3) = 4{,}73\cdot10^6$ ton/m^3, which happens to be about a million times the average density (5.52 t/m3) of our planet earth. Since the mass M is the result of the accumulation of particles which pass to the hiperlumínic subspace, the density should be calculated based on the volume that occupies in this subspace M, which data is not known. If this radius were for a density of the earth we can estimate that R_N = 7.74 au of a sphere in hyperluminic subspace. This hypothesis is more reasonable to accept as densities of the order of 10^6 ton/m^3 of material compacted by the force of gravity.