SUPERLUMINAL SPATIO-TEMPORAL TRANSFORMATIONS

First basic step toward the Superluminal Relativistic Mechanics

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Abstract: The paper provides a crucial elementary derivation of new superluminal spatio-temporal transformations based on the idea that, conceptually and kinematically, each subluminal $(0 \le v < c)$; luminal $(v = c)$ and/or superluminal inertial reference frame $v > c$ has, in addition to its relative velocity v, its proper specific kinematical parameter $\vartheta(v)$, which having the physical dimensions of a constant speed defined as: $\vartheta(v) = c$ if $0 \le v < c$; $\vartheta(v) > v$ if $v = c$; $\vartheta(v) = c\sqrt{1 + v^2/c^2}$ if $v > c$ and $\vartheta^2(-v) = \vartheta^2(v)$, $\forall v$. Consequently, the relativity principle and causality principle are coherently extended to superluminal velocities and, more importantly, this original approach constitutes the first basic step toward the formulation of superluminal relativistic mechanics in which the standard special relativity theory should be a particular case.

Keywords: superluminal inertial reference frames; superluminal spatio-temporal transformations; superluminal relativistic mechanics.

1. Introduction

It goes without saying that all physical theories of Nature must be based on internal logical coherence free from aberrations and inconsistencies. In this sense, the theoretical studies of Nature must reflect the stringent rigor of logic used in the formalism.

One of most fundamental and profound distinction between a theory of Nature and a theory of Mathematics is relative to the concept of *infinity*. While in Mathematics we can associate and attribute, in perfectly logical and coherent way, the infinite value to a parameter, a dimension, or to a limit or boundary conditions, such associations are meaningless when related as results to a physical theory. And this is because in Nature nothing is infinite; all physical parameters of phenomena and material objects (time, space, dimension, mass, energy, temperature, pressure, volume, density, force, velocity...) are defined and characterized by finite values and only finite values like: minimum, average, maximum, critical and limit values. Nature cannot be described through infinite concepts and values as they are devoid of any meaning in the physical world. Nevertheless, the infinity concept is suited only during mathematical treatment into the realm of the theories of natural sciences in order to obtain equations with finite parameters.

Indeed, any physical theory predicting, at some special upper limit conditions, infinite values for any of its physical parameters is a theory based on fundamental flawed principles and concepts, and such a theory should never be allowed to enter into the realm of theories of natural sciences.

But what Mathematics is to be used in particular study of Nature is in reality the critical question, which needs to be elucidated before embarking into any credible physical theory. Therefore, to use willy-nilly mathematical models for attempting to describe a particular phenomenon of Nature without physical justification for such an undertaking is an illogical act. So, we need constantly to be remained that all ways provided by Mathematics are abstract ways with no counterpart in the real physical world.

The clever way therefore is to be able to find a foundation of Mathematics trough which we can communicate with the real physical world and show a convincing justification for its employment.
Thus, our principal motivation behind the present work is to avoid the singularities and to show that the theoretical maximal possible velocity of an ordinary massive particle or of a physical signal is not necessary equal to that of light speed, c, in local vacuum but can be higher than c as we will see later. This consideration does not violate special relativity theory (SRT) since it is conceptually, physically and exclusively valid at subluminal kinematical level for relativistic and ultrarelativistic velocities $(v < c)$ and also because we are very convinced of the real existence of a physical world beyond the light speed as a conventional maximal limit.

The totality of the theory *'superluminal relativistic mechanics'* to be developed is based on the principle of relative motion and the principle of kinematical levels from which the superluminal spatio-temporal transformations (ST's) should be derived, consequently, their interpretation and use lead to the superluminal relativistic kinematics and superluminal relativistic dynamics that constitute the two basic parts of superluminal relativistic mechanics.

2. Problematic

Unfortunately, in spite of its remarkable success, the SRT is a classical and simple theory. Furthermore, its formalism is completely incompatible with some physical phenomena because it predicted infinite values for its physical parameters. Also this same formalism is still not free from confusions and contradictions, which are caused by the frequent usage of archaic notions, notations and equations, which have nothing to do with the essence of the theory, only with the history of its development.

It is known that the SRT –as usually understood at the present time– when applied to material bodies moving at luminal velocities leads to infinite values, which do not appear to be inherent in the phenomena. Take, for example, (1) the relativistic (total kinetic) energy formula: the object's relativistic energy reaching the infinite value when the object's velocity reaches the light speed! The same conclusion remains valid for the relativistic time dilation formula and (2) the relativistic length contraction formula: the object's length contraction reaching the zero value when the object's velocity reaches the light speed!

Now, let us focus our attention on the examples (1) and (2). We have for the first one

$$E = E_0\left(1-\frac{v^2}{c^2}\right)^{-1/2}, \quad E_0 = mc^2, \tag{i}$$

where E_0 is the object's proper energy. As noted, formula (i) shows that the relativistic energy E of the moving material object becomes infinite when $v \to c$. However, the very same formula can be written also as

$$E_0 = E\sqrt{1-\frac{v^2}{c^2}}. \tag{ii}$$

Now, in this new form of expression, we no longer find the previous result. What we obtain in this case is that the initial (proper) energy E_0 becomes zero when $v \to c$. Also, for the second example, we have for the relativistic length contraction formula

$$\ell = \ell_0\sqrt{1-\frac{v^2}{c^2}}, \tag{iii}$$

where ℓ_0 is the object's proper length. As remarked previously, the formula (iii) shows that the length ℓ of the moving material object becomes zero when $v \to c$. Also, from (iii), we can obtain the following formula

$$\ell_0 = \ell \left(1 - \frac{v^2}{c^2}\right)^{-1/2}. \qquad \text{(iv)}$$

And what we get in this case, is that the initial (proper) length ℓ_0 becomes infinite when $v \to c$.

Putting aside the absurdity of these results, the remark to be made here is that mathematics alone, as a tool in deciphering the secrets of Nature, cannot be blindly employed without the physics behind it. In other words, Physics as natural science needs to dictate the mathematics *how* to be used and this role should never be allowed to be reversed.

In order to avoid the above absurdity, SRT simply prohibited the existence of the luminal inertial reference frames, that is to say, a set of inertial frames that may be in rectilinear uniform motion at luminal velocity relative to one another. But such a prohibition seems to be entirely unreasonable because in the Nature, none can prevent any free material body from reaching or exceeding light speed in vacuum. Moreover, since any moving material object is characterized by its proper inertial reference frame (where the same object is at relative rest), hence the photon itself may be at relative rest in its proper IRF, consequently the old notion of zero-rest mass for the photon becomes meaningless [1].

2.1. Lorentz reservation about light speed as a limiting velocity

We begin this subsection by the statement of Dutch theoretical physicist Hendrick Antoon Lorentz (1853-1928) one of the principal founders of (special) relativity theory. Although he clearly understood Einstein's papers [2,3], he did not ever seem to accept their conclusion regarding light velocity as upper limit. In his theory, Einstein asserted: *"... From this we conclude that in the theory of relativity the velocity c plays the limiting part of a limiting velocity, which can neither be reached nor exceeded by any real body."*

Lorentz gave a lecture in 1913 when he remarked how rapidly relativity had been accepted. He said:
" ... Finally it should be noted that the daring assertion that one can never observe velocities larger than the velocity of light contains a hypothetical restriction of what is accessible to us, a restriction which cannot be accepted without some reservation."

Actually, it seems that the Lorentz reservation is correct, because in the last years, there has renewed interest on *superluminal velocities*, due to some new experimental evidences in different sectors of physics. Those include, *e.g.*, the apparent superluminal expansions of galactic objects [4,5] and the evidence for superluminal motions in electric and acoustic engineering [6,7]. Nevertheless, maybe the most remarkable experimental findings are those concerning the superluminal tunneling of evanescent waves and photons [8–16] and Scharnhorst effect [17–20]. Further, more recently OPERA Collaboration reported [21] the experimental evidence of superluminality of μ-neutrinos. This should be the most important experimental discovery in the area of fundamental physics. Since OPERA –Frankly, we do not have confidence in the *story* of "a fiber optic cable attached improperly, which caused the apparently faster-than-light measurements and a clock oscillator ticking too fast."– is very carefully designed experiment hence this finding may be interpreted as an additional confirmation of the previously observed superluminal neutrinos by MINOS Collaboration [22] and the FERMILAB [23].

Confronted by such a discovery, some physicists considered this experimental achievement as the end of the world because according to them, if this finding can be verified by other experiments, it would mean Einstein's SRT is wrong! This exaggerated worry shows us that these physicists have completely forgotten an important epistemological principle, which claimed that *'any well-established scientific theory should have, sooner or later, its own limit of validity.'*

The importance of such a principle resides in the dependence of science progress continuity on this limit of validity. For instance, the limit of validity of the Galilean transformations has implied the limit of validity of classical (Newtonian) mechanics, both led to the discovery of the Lorentz transformations (LT's) and relativistic mechanics, respectively. The mentioned observable and experimental superluminal motion should be explained as a tangible evidence of the limit of validity of LT's and SRT together. This means the light speed in vacuum

$$c = 299792458\,\mathrm{ms}^{-1}, \tag{1}$$

is limiting speed only in the context of SRT not for all the physical theories because LT's, which are the core of SRT, becomes meaningless when the relative velocity – of the inertial reference frame – reaches or exceeds the light speed in vacuum (1), that is when $v \geq c$ Lorentz (gamma) factor $\gamma = \left(1 - v^2/c^2\right)^{-1/2}$ becomes imaginary or infinite.

2.2. Causality Principle

What amazes us is the false assumption (inherited from Einstein) that information traveling faster than light speed in vacuum (1) represents a violation of causality principle! However, *causality* simply means that the cause of an event precedes the effect of the event. In this case, *e.g.*, a massive particle is emitted before it is absorbed in a detector. If the particle's velocity was one million times faster than c, the cause would still precede the effect, and causality principle would not be violated since, here, LT's should be replaced with the superluminal spatio-temporal transformations (ST's) because the particle in question was moving in superluminal space-time not in Minkowski space-time. Therefore, in superluminal space-time, the superluminal signals do not violate the causality principle but they can shorten the luminal vacuum time span between cause and effect.

Furthermore, it is worthwhile to note that, in his paper [3], Einstein's arrived at the 'violation of causality principle by superluminal velocity' by applying LT's which are only valid for the relative uniform motion of IRF's with subluminal velocity. However, many scientists repeatedly imitated Einstein's viewpoint by claiming that '*in the real physical world, the velocities greater than that of light in vacuum have no possibility of existence.*'

But unfortunately, the same scientists ignored one very important thing: Einstein's claim in his papers [2,3] is highly contradictory simply because a deeply critical reading of Einstein's papers on SRT has already showed more conclusively that Einstein himself [2,3] used, at the same time, the subluminal and superluminal velocities in SRT. For example, in his 1905' paper [2], he wrote: '… *Taking into consideration the principle of constancy of the velocity of light we find that*

$$t_B - t_A = \frac{r_{AB}}{c - v} \qquad \text{and} \qquad t'_A - t_B = \frac{r_{AB}}{c + v}$$

where r_{AB} *denotes the length of the moving rod- measured in the stationary system* …'

It is quite clear from the above equations, that is, since in Einstein's paper $v\ (v<c)$ is the relative velocity between the two IRF's, K and K', thus $c-v$ and $c+v$ are subluminal and superluminal velocity respectively. Therefore, forbidding the existence of superluminal velocities in the real physical world is a greatest crime against Nature and Science!

3. Superluminal Formalism

The theory here to be developed is based – like any physical model – on its proper principles as basis for organizing and facilitating our understanding of its internal structure and external consequences. The central elements that constitute the core of the present work, which will henceforth be called '*superluminal relativistic mechanics*' are the principle of relative motion, the principle of kinematical levels, the superluminal space-time geometry and the superluminal spatio-temporal transformations (ST's), respectively.

3.1. Principle of Relative Motion

If the inertial reference frame (IRF) F' moves in straight-line at a constant velocity v relative to the (IRF) F, then F moves in straight-line at a constant velocity $-v$ relative to F'.

3.2. Principle of Kinematical Levels

Conceptually, there are three kinematical levels (KL's) namely subluminal, luminal and superluminal level, such that:

a) Each KL is characterized by a set of IRF's moving with respect to each other at a subluminal velocity $(0\le v<c)$ in the first KL; at a luminal velocity $(v=c)$ in the second KL and at a superluminal velocity $(v>c)$ in the third KL.

b) Each IRF has, in addition to its relative velocity vector $\mathbf{v}$ of magnitude v, its proper specific kinematical parameter $\vartheta(v)$, which having the physical dimensions of a constant speed defined as

$$\begin{cases} \vartheta(v)=c & \text{if} \quad 0\le v<c \\ \vartheta(v)>v & \text{if} \quad v=c \\ \vartheta(v)=c\sqrt{1+v^2/c^2} & \text{if} \quad v>c \\ \vartheta^2(-v)=\vartheta^2(v), & \forall v \end{cases} . \tag{2}$$

c) All the IRF's belonging to the same KL are equivalent and their specific kinematical parameters are identical.

These two principles suffice – with the superluminal space-time geometry and ST's – for the attainment of a simple and consistent theory of superluminal relativistic mechanics in which SRT should be a particular case. Here, the above adjective '*relativistic*' is strictly speaking, relating to the Galilean relativity principle (and its Poincaré-Einsteins extension), which claimed that absolute rest does not exist; rest and uniform motion have only a *relative* character. Each material object at rest in given IRF is at the same time in uniform motion when it is observed from another IRF. And as a direct consequence, no privileged IRF can exist.

Incidentally, the definition (2) will hereafter be called "the operational definition of the specific kinematical parameter (SKP)".

3.3. Superluminal Space-Time

Now, we are arriving at the veritable heart of our subject. In addition to the mentioned OPERA-MINOS-FERMILAB experiments on superluminal neutrinos, two-dimensional modeling of the interaction with the lower ionosphere of intense electromagnetic pulses (EMP's) from lightning discharges has indicated that the optical luminosities produced at 85-95 km altitudes as result of heating by the EMP- fields [24–28] as observed from a certain distance would appear to expand laterally at superluminal velocity, 3.10 times the light speed in vacuum (1), in good agreement with the original predictions. Again, this exploit reinforces the reality of superluminal motions. Consequently, the question arises naturally: what the appropriate geometry of space-time to describe superluminal physical phenomena?

In order to answer adequately the above question, we shall take into account the principle of KL's, more precisely, the operational definition of SKP (2). Hence, we can undertake to establish the mathematical structure of superluminal space-time conceptually inspired from the existence of superluminal physical phenomena. The said mathematical structure of superluminal space-time as a *seat* of superluminal physical phenomena should be defined by the following superluminal quadratic form (superluminal metric)

$$x'^2 + y'^2 + z'^2 - \vartheta^2(v)t'^2 = x^2 + y^2 + z^2 - \vartheta^2(v)t^2 \quad . \qquad (3)$$

The velocity v in (3) is the *relative velocity* between the two inertial reference frames (IRF's) F and F'. Further, according to the operational definition of SKP (2), the superluminal quadratic form (3) may be reduced to that of Minkowski for the case $\vartheta(v) = c$ when $0 \le v < c$. The signature $(+,+,+,-)$ into (3) implies that the geometry of superluminal space-time is not completely Euclidean, it is in fact non-Euclidean because as we will see later in superluminal regime, space '*contracts*' and time '*dilates*' as in Minkowski space-time *in* subluminal KL for relativistic velocities . Therefore, According to the principle of relative motion, the superluminal quadratic form (3) should be invariant under certain superluminal spatio-temporal transformations (ST's) during any transition from a superluminal-IRF to another. For this reason, we can also define a superluminal four-vector of position as follows: relatively to (IRF) F , we call superluminal four-vector of position of a superluminal event of spatio-temporal coordinates (x, y, z, t), a vector $\mathbf{R}$ of components:

$$(x_1 = x, x_2 = y, x_3 = z, x_4 = i\,\vartheta(v)t), \quad \text{with} \quad i = \sqrt{-1}\,.$$

3.4. Superluminal Spatio-Temporal Transformations

The superluminal (spatio-temporal) transformations, the superluminal IRF's and the hypothetical tachyons as a superluminal particle have a long history and these concepts were generally related to SRT in a unfortunate way. However, the tangible evidence of superluminal motions is a fundamental question debated in modern physics. Sommerfeld [29] examined the radiation of superluminal sources in empty space; Blokhintsev [30,31] paid attention to the possibility of formulating the field theory that allows the propagation of superluminal interactions outside the light cone; Kirzhnits [32] showed that, under some conditions, a massive particle can move with a superluminal velocity; Terletsky [33] introduced into theoretical physics the particles with imaginary rest masses moving faster than light; Feinberg [34] named these particles *tachyons* and described their main properties.

Investigations on superluminal tachyon motion and superluminal transformations inaugurated additional opportunities which were studied by many researchers, for instance, by Bilaniuk and Sudarshan [35], Recami [36], Mignani [37], Kirshnits and Sazonov [38], Corben [39], Patty [40], Recami, fontan and Caravaglia [41], Parker [42], Marchildon, Antippa and Everett [43], Sutherland and Shepanski [44].

Unfortunately, all the mentioned authors' effort was a fiasco because their superluminal transformations did not form an orthogonal-orthochronous group and the hypothetical tachyon was characterized by an imaginary (rest) mass that's why many physicists remain skeptical about the possibility of formulating the superluminal physics.

With the help of the principle of relative motion and the principle of KL's, we undertake to derive the superluminal (spatio-temporal) transformations (ST's) for spatio-temporal coordinates, so that the ST's should satisfy the following principal requirements:

a) The ST's should ensure the invariance of the superluminal quadratic form (3).
b) The ST's should be real.
c) The ST's should be linear.
d) The ST's should have an algebraic structure of an orthogonal-orthochronous group[1] .

By taking into account the principle of KL's, we can derive the expected ST's in simple and lucid manner. To this end, let us consider two IRT's F and F', which are in relative uniform translational motion at superluminal velocity $\mathbf{v}$ of magnitude v such that $c < v < \vartheta(v)$. Further, let us assume that a superluminal event can be characterized by superluminal spatio-temporal coordinates $(x,\ y,\ z,\ t)$ in F and $(x',\ y',\ z',\ t')$ in F'.

To simplify the algebra let the relative superluminal velocity vector $\mathbf{v}$ of IRF's be along their common $x \mid x'$-axis with corresponding parallel planes. Also, the two origins O and O' coincide at the moment $t = t' = 0$ (henceforth, two superluminal IRF's related in this way are said to be in the standard configuration). The supposed homogeneity and isotropy of space and uniformity of time in all superluminal IRF's require that the ST's must be real and linear so that the simplest form they can take (when for example the transition operated from F to F') is:

$$F \to F' : \begin{cases} x' = \eta\left(x - vt\right) \\ y' = y \\ z' = z \\ t' = \lambda\, x + \zeta\, t \end{cases} . \tag{4}$$

In order to determine the expressions of the coefficients η, λ and ζ we must use the idea of the homogeneity and isotropy of space and uniformity of time in all superluminal IRF's, and the requirement (a).

[1] That is to say the notion of past, present and future is preserved and this implies, among other things, the preservation of the causality principle in all the IRF's.

Therefore, when Eqs.(4) are substituted in left-hand side of the superluminal quadratic form (3), we get

$$\eta^2(x-vt)^2 + y^2 + z^2 - \vartheta^2(v)\cdot(\lambda x+\zeta t)^2 = x^2 + y^2 + z^2 - \vartheta^2(v)t^2 . \tag{5}$$

From which we have

$$\begin{cases} \eta^2 - \lambda^2\vartheta^2(v) = 1 \\ \eta^2 v + \lambda\zeta\,\vartheta^2(v) = 0 \\ \zeta^2\vartheta^2(v) - \eta^2 v^2 = \vartheta^2(v) \end{cases} . \tag{6}$$

The system of three Eqs. (6) when solved for η, λ and ζ yields

$$\eta = 1\Big/\sqrt{1-v^2/\vartheta^2(v)}\ ;\quad \lambda = -\left[v/\vartheta^2(v)\right]\Big/\sqrt{1-v^2/\vartheta^2(v)};\quad \zeta = 1\Big/\sqrt{1-v^2/\vartheta^2(v)} . \tag{7}$$

Now, by substituting (7) in (4), we obtain the expressions of the expected ST's, *i.e.*, ST and its inverse $(\mathrm{ST})^{-1}$:

$$F \rightarrow F' : \begin{cases} x' = \eta(x-vt) \\ y' = y \\ z' = z \\ t' = \eta\left(t - \dfrac{vx}{\vartheta^2(v)}\right) \end{cases} , \tag{8}$$

and

$$F' \rightarrow F : \begin{cases} x = \eta(x'+vt') \\ y = y' \\ z = z' \\ t = \eta\left(t' + \dfrac{vx'}{\vartheta^2(v)}\right) \end{cases} , \tag{9}$$

where

$$\eta = 1\Big/\sqrt{1-v^2/\vartheta^2(v)} \qquad \text{and} \qquad \begin{cases} \vartheta(v) = c \quad \text{if} \quad 0 \le v < c \\ \vartheta(v) > v \quad \text{if} \quad v = c \\ \vartheta(v) = c\sqrt{1+v^2/c^2} \quad \text{if} \quad v > c \\ \vartheta^2(-v) = \vartheta^2(v), \quad \forall v \end{cases} .$$

Furthermore, we can make sure that the ST's preserve really the invariance of superluminal quadratic form (3) during, *e.g.*, any transition from F to F'. With this aim, we have

$$\begin{aligned} x'^2 + y'^2 + z'^2 - \vartheta^2(v)t'^2 &= \eta^2(x-vt)^2 + y^2 + z^2 - \vartheta^2(v)\eta^2\left(t - vx/\vartheta^2(v)\right)^2 \\ &= \eta^2\left(1-v^2/\vartheta^2(v)\right)x^2 + y^2 + z^2 - \vartheta^2(v)\eta^2\left(1-v^2/\vartheta^2(v)\right)t^2 \\ &= x^2 + y^2 + z^2 - \vartheta^2(v)t^2 \end{aligned}$$

This is in good agreement with the principle of relative motion and the principle of KL's, also it is easy to verify that the ST's (8) and (9) which depending on the parameters v and $\vartheta(v)$ form a linear orthogonal-orthochronous group since their determinant is equal to $+1$. Therefore, the ST's satisfy all the imposed requirements (a), (b), (c) and (c). Moreover, with the aid of the principle of KL's, we can easily prove that the composition of two ST's is also ST because as it is clear from the principle of KL's, all the IRF's belonging to the same KL are equivalent and their SKP's are identical. As a direct consequence, the usual LT's may be recovered from ST's for the case $\vartheta(v)=c$ when $0 \leq v < c$. From all that, we can logically affirm that the principle of relativity and the principle of causality are extended to superluminal IRF's *via* ST's. In arriving at this conclusion, we can assert to have really established the basic foundations for the *superluminal relativistic mechanics* (SupRelMec).

3.5. Specific Terminology

In order to make a clear distinction between the Superluminal Relativistic Mechanics 'SupRelMec' and the former works on the generalization of SRT, we have coined some specific keywords necessary for such a distinction, and for the internal structure of our theory. The principal derived keywords are:

-*Superluminality*: means a typical quality that is related to superluminal motions/velocities.

-*Superluminalization*: means interpreting and/or expressing some concepts and/or some classical/SRT-equations within the framework of SupRelMec.

-*Superluminalize*: is a verb that means the process of superluminalization.

-*Superluminalized*: is an adjective that characterizes all that is perfectly conformed to SupRelMec.

-*Superluminal*: is an adjective, which generally means, at the same time, faster-than-light speed in vacuum- and all that is related or proper to SupRelMec.

-*Superluminally*: is an adverb that characterizes verbs and adjectives relative to SupRelMec.

-*Superluminal relativistic*: this expression 'double adjective' means the superluminalization of the Galilean relativity principle and its Poincaré-Einstein extension.

-*Superluminal invariance*: is a key property of superluminal space-time following from the superluminal formalism. Superluminal invariance has the following meaning: a quantity that remains unchanged by a superluminal transformation (ST) is said to be superluminal invariant. Such quantities play an especially important role in superluminal relativistic mechanics and electromagnetism. The norm of any superluminal four-vector is manifestly superluminal invariant.

4. Superluminal Relativistic Mechanics 'SupRelMec'

Before the present investigation, there was a lot of research works appeared in the superluminal/tachyonic literature for almost fifty years and focalized on the possibility of extending SRT through the generalization of LT's to superluminal IRF's. As it was already noted, the attempt was a total fiasco because the concept of superluminality itself was seriously treated in the context of SRT in ill-way. The reader interested in the problem can find a more extensive study in the paper [45] where the most literature of the subject is also given.

Concerning our superluminal formalism, we have previously seen that ST's may be reduced to the usual LT's for the case $\vartheta(v) = c$ when $0 \le v < c$, that's why all the SRT-effects exist in the framework of SupRelMec except, of course, the mentioned SRT-singularities (i–iv). SupRelMec comprises two principal parts, namely the superluminal relativistic kinematics and the superluminal relativistic dynamics.

4.1. Superluminal Relativistic Kinematics

In what follows it is endeavored to establish the formulation of the superluminal relativistic kinematics which should drew from ST's. We will show the existence of the superluminal relativistic length *contraction* and time *dilation*; we will derive the transformations of superluminal velocities and accelerations. Once again, we will see, in the context of superluminal relativistic kinematics, that the causality principle is at the same time conserved and extended to superluminal velocities without any contradiction because, in superluminal space-time, the superluminal velocities do not violate the *causality* but they can only shorten the luminal vacuum *time* span between cause and effect. This is the main property of the superluminal space-time as a *seat* of superluminal physical phenomena.

4.1.1. Superluminal relativistic length contraction

Suppose two superluminal IRF's F and F' in standard configuration and consider a body at relative rest with respect to F'. Let (x_1', y_1', z_1') and (x_2', y_2', z_2') be the coordinates of its material points referred to F'. Between the coordinates (x_1, y_1, z_1) and (x_2, y_2, z_2) of these material points relative to F, there obtain at each time t of F, according to ST's (8) and (9), the relations

$$x_2 - x_1 = (x_2' - x_1')\sqrt{1 - v^2 / \vartheta^2(v)}; \;\; y_2 - y_1 = y_2' - y_1'; \;\; z_2 - z_1 = z_2' - z_1'. \qquad (10)$$

Or simply by taking account of the direction of superluminal relative motion, *i.e.*, IRF's common $x \mid x'$ - axis, the expected superluminal relativistic length contraction formula may be written as

$$\Delta x = \Delta x' \sqrt{1 - \varepsilon^2} \, , \qquad (11)$$

where $\varepsilon = v / \vartheta(v)$ is, by convention, called 'fractional superluminal velocity' –that is to say, being a superluminal velocity in $\vartheta(v)$ units– and $\Delta x'$ is the proper length of a material body in state of relative rest in F'.

Result: in superluminal relativistic kinematics, the kinematic shape of material body supposed to be in a state of uniform translation depends thus on its superluminal velocity relative to IRF, namely by differing from its proper geometric shape in being contracted in the direction of superluminal relative motion.

4.1.2. Superluminal relativistic time dilation

Like before, Suppose two superluminal IRF's F and F' in standard configuration and assume that there is a clock at relative rest at the origin of coordinates of F', which runs υ_0 times faster than the clocks used in the two IRF's F and F' for the measurement of time, *i.e.*, this clock executes υ_0 periods during a time in which the reading of a clock which is at relative rest with respect to it and is of the same nature of the other clocks used in F and F' for the measurement of time, increases by one unit. Question: How fast does the first mentioned clock run as viewed from F ?

From above considerations and with the help of ST's, we obtain the expected superluminal relativistic time dilation formula

$$\Delta t = \Delta t' \left(1 - \varepsilon^2\right)^{-1/2}, \quad \varepsilon = v / \vartheta(v). \tag{12}$$

Result: In superluminal relativistic kinematics, a clock moving uniformly with superluminal velocity relative to IRF, runs when viewed from that frame, more slowly than the same clock when is at relative rest with respect to this frame. Furthermore, in terms of frequency, we get from (12) the following important formula

$$\upsilon = \upsilon_0 \sqrt{1 - \varepsilon^2} \ . \tag{13}$$

The formula (13) may be interpreted as a superluminal relativistic Doppler effect and consequently should admit of a very interesting application, particularly, to the powerful radio quasars that exhibit the well-known superluminal motions.–Like before, the superluminal relativistic formulae (11), (12) and (13) may be reduced to the usual SRT-formulae for the case $\vartheta(v) = c$ when $0 \le v < c$.

4.1.3. Transformations of superluminal velocities

Let us call the vector $\mathbf{u}(u_x, u_y, u_z)$ of magnitude u the superluminal velocity vector of a material point in (IRF) F such that $c < u < \vartheta(v)$, and let us consider a second (IRF) F' in straight-line uniform motion at superluminal velocity of magnitude v relative to F along the x-axis. In F' the same material point is characterized by the superluminal velocity vector $\mathbf{u}'(u'_{x'}, u'_{y'}, u'_{z'})$ of magnitude u' with $c < u' < \vartheta(v)$. The two frames F and F' are connected by ST's. Thus, a direct differentiation of ST (8), gives the required transformations of superluminal velocities:

$$F \to F' : \begin{cases} u'_{x'} = (u_x - v)\left[1 - \dfrac{u_x v}{\vartheta^2(v)}\right]^{-1} \\ u'_{y'} = u_y \left[1 - \dfrac{v^2}{\vartheta^2(v)}\right]\left[1 - \dfrac{u_x v}{\vartheta^2(v)}\right]^{-1} \\ u'_{z'} = u_z \left[1 - \dfrac{v^2}{\vartheta^2(v)}\right]\left[1 - \dfrac{u_x v}{\vartheta^2(v)}\right]^{-1} \end{cases} , \tag{14}$$

and

$$F' \to F : \begin{cases} u_x = (u'_{x'} + v)\left[1 + \dfrac{u'_{x'} v}{\vartheta^2(v)}\right]^{-1} \\ u_y = u'_{y'} \left[1 - \dfrac{v^2}{\vartheta^2(v)}\right]\left[1 + \dfrac{u'_{x'} v}{\vartheta^2(v)}\right]^{-1} \\ u_z = u'_{z'} \left[1 - \dfrac{v^2}{\vartheta^2(v)}\right]\left[1 + \dfrac{u'_{x'} v}{\vartheta^2(v)}\right]^{-1} \end{cases} . \tag{15}$$

4.1.4. Addition law of superluminal velocities

Consider now the important particular case, that is, when the material point moves in F along the x-axis, we obtain from (15):

$$u_x = \frac{u'_{x'} + v}{1 + \frac{u'_{x'} v}{\vartheta^2(v)}}, \quad u_y = 0, \quad u_z = 0.$$

Consequently, if we put $u'_{x'} = u'$ and $u_x = u$, we get the following expected addition law of superluminal velocities

$$u = \frac{u' + v}{1 + \frac{u' v}{\vartheta^2(v)}} . \tag{16}$$

Remark: If we set $u' = \vartheta(v)$, we obtain from (16) the following interesting property:

$$u = \frac{\vartheta(v) + v}{1 + \frac{v}{\vartheta(v)}} = \vartheta(v). \tag{17}$$

The result (17) means that the SKP, $\vartheta(v)$, is really constant in all the superluminal IRF's.

4.1.5. Superluminal velocity four-vector

In superluminal relativistic kinematics, we define a superluminal velocity four-vector of a material point as follows: Let the vector $\mathbf{u}(u_x, u_y, u_z)$ of magnitude $u = \sqrt{u_x^2 + u_y^2 + u_z^2}$ be the superluminal velocity vector of the material point under consideration, we call a superluminal velocity four-vector, the quantity

$$\mathbf{U} = \frac{d\mathbf{R}}{dt^*} = \left(\eta u,\, i\eta\vartheta(u) \right), \quad \eta = 1 \Big/ \sqrt{1 - u^2 / \vartheta^2(u)}, \tag{18}$$

where $dt^* = \eta^{-1} dt$ is the proper time of material point and $\mathbf{R} = (x_1 = x, x_2 = y, x_3 = z, x_4 = i\,\vartheta(u)t)$ is its four-vector position. Or more explicitly

$$\mathbf{U} = (u_1 = \eta u_x,\, u_2 = \eta u_y,\, u_3 = \eta u_z,\, u_4 = i\eta\vartheta(u)) . \tag{19}$$

Also, we can remark from (19) that

$$\mathbf{U}^2 = -\vartheta^2(u) . \tag{20}$$

That is to say all the superluminal velocity four-vectors have a magnitude of $\vartheta(u)$. This is another expression of the fact that in superluminal relativistic kinematics, there is no absolute rest - at least- we are always moving forward through time! Moreover, since according to the operational definition of SKP (2), we have $\vartheta(u) = c$ when $0 \le u < c$ thus it follows that even if $\mathbf{u} = \mathbf{0}$, we get $\mathbf{U} = (0,0,0,ic)$.

4.1.6. Transformations of superluminal velocity four-vector

The superluminal velocity four-vector $\mathbf{U}$ being a superluminal four-vector, its components should be invariant under ST's during any change of superluminal IRF. Let the superluminal IRF's F and F' be in standard configuration. We call the vector $\mathbf{u}(u_x, u_y, u_z)$ of magnitude u the superluminal velocity vector of a material point relative to F such that $c < u < \vartheta(v)$. In F' the same material point is characterized by the superluminal velocity vector $\mathbf{u}'(u'_{x'}, u'_{y'}, u'_{z'})$ of magnitude u' with $c < u' < \vartheta(v)$.
We have according to ST (8)

$$F \to F' : \begin{cases} u'_1 = \eta_{(v)}(u_1 - \varepsilon u_0) \\ u'_2 = u_2 \\ u'_3 = u_3 \\ u'_0 = \eta_{(v)}(u_0 - \varepsilon u_1) \end{cases}, \tag{21}$$

where

$$\eta_{(v)} = 1\Big/\sqrt{1-\varepsilon^2}\,,\quad \varepsilon = v/\vartheta(v),$$

$$\mathbf{U} = \left(u_1 = \eta u_x,\ u_2 = \eta u_y,\ u_3 = \eta u_z,\ u_4 = iu_0 = i\eta\vartheta(u)\right),$$

$$\mathbf{U}' = \left(u'_1 = \eta' u'_{x'},\ u'_2 = \eta' u'_{y'},\ u'_3 = \eta' u'_{z'},\ u'_4 = iu'_0 = i\eta'\vartheta(u')\right),$$

with

$$\eta = 1\Big/\sqrt{1-u^2/\vartheta^2(u)}\,,\quad \eta' = 1\Big/\sqrt{1-u'^2/\vartheta^2(u')}\,.$$

By taking into account the principle of KL's, or more precisely the fact that all the IRF's belonging to the same KL are equivalent and their SKP's are identical, *i.e.*, for the present case we have $\vartheta(v) = \vartheta(u) = \vartheta(u')$, hence from all that, we deduce

$$F \to F' : \begin{cases} \eta' u'_{x'} = \eta\eta_{(v)}(u_x - v) \\ \eta' u'_{y'} = \eta u_y \\ \eta' u'_{z'} = \eta u_z \\ \eta'\vartheta(u') = \eta\eta_{(v)}\vartheta(u')\left(1 - \dfrac{u_x v}{\vartheta^2(v)}\right) \end{cases}.$$

Noting, the fourth equation in above transformation gives us $\eta/\eta' = 1/\eta_{(v)}\left(1 - u_x v/\vartheta^2(v)\right)$. Therefore, from where we can obtain, once again, the transformations of superluminal velocities. This means we have really defined a superluminal velocity four-vector invariant under ST's during any change of IRF.

4.1.7. Transformations of superluminal accelerations

For the superluminal acceleration of the moving material point treated previously, we have relatively to F and F', respectively:

$$\boldsymbol{a} = \left(a_x = du_x / dt, a_y = du_y / dt, a_z = du_z / dt\right), \tag{22}$$

and

$$\boldsymbol{a}' = \left(a'_{x'} = du'_{x'} / dt', a'_{y'} = du'_{y'} / dt', a'_{z'} = du'_{z'} / dt'\right). \tag{23}$$

The differentiation of the fourth equation in ST (8) gives $dt' = \eta\left(dt - v dx/ \vartheta^2(v)\right)$, thus we get from (22), (14) and (15) the required transformations of superluminal accelerations:

$$F \to F' : \begin{cases} a'_{x'} = \eta^{-3/2}\left[1 - \dfrac{u_x v}{\vartheta^2(v)}\right]^{-3} a_x \\ a'_{y'} = \eta^{-1}\left[1 - \dfrac{u_x v}{\vartheta^2(v)}\right]^{-3}\left[\left(1 - \dfrac{u_x v}{\vartheta^2(v)}\right)a_y + \left(\dfrac{u_y v}{\vartheta^2(v)}\right)a_x\right] \\ a'_{z'} = \eta^{-1}\left[1 - \dfrac{u_x v}{\vartheta^2(v)}\right]^{-3}\left[\left(1 - \dfrac{u_x v}{\vartheta^2(v)}\right)a_z + \left(\dfrac{u_z v}{\vartheta^2(v)}\right)a_x\right] \end{cases}, \tag{24}$$

and

$$F' \to F : \begin{cases} a_x = \eta^{-3/2}\left[1 + \dfrac{u'_{x'} v}{\vartheta^2(v)}\right]^{-3} a'_{x'} \\ a_y = \eta^{-1}\left[1 + \dfrac{u'_{x'} v}{\vartheta^2(v)}\right]^{-3}\left[\left(1 + \dfrac{u'_{x'} v}{\vartheta^2(v)}\right)a'_{y'} - \left(\dfrac{u'_{y'} v}{\vartheta^2(v)}\right)a'_{x'}\right] \\ a'_{z'} = \eta^{-1}\left[1 + \dfrac{u_x v}{\vartheta^2(v)}\right]^{-3}\left[\left(1 + \dfrac{u'_{x'} v}{\vartheta^2(v)}\right)a'_{z'} - \left(\dfrac{u'_{z'} v}{\vartheta^2(v)}\right)a'_{x'}\right] \end{cases}. \tag{25}$$

with

$$\eta = 1\Big/\sqrt{1 - v^2 / \vartheta^2(v)}.$$

4.1.8. Superluminal accelerations four-vector

Concerning the superluminal acceleration four-vector of the moving material point that previously treated; this quantity may be derived from the definition of superluminal velocity four-vector (18), namely

$$\mathbf{U} = \frac{d\mathbf{R}}{dt^*} = \left(\eta\mathbf{u}, i\eta\vartheta(u)\right), \quad dt^* = \eta^{-1}dt, \quad \eta = 1\Big/\sqrt{1 - u^2 / \vartheta^2(u)}.$$

By differentiating the above relation with respect to proper time t^* of the material point, we get

$$\mathbf{A} = \frac{d\mathbf{U}}{dt^*} \tag{26}$$

Also, from relation (20), we deduce the following important property

$$\mathbf{U}^2 = -\vartheta^2(u) \quad \Rightarrow \quad 2\mathbf{U}\frac{d\mathbf{U}}{dt^*} = 0 \quad \Rightarrow \quad \mathbf{U} \cdot \mathbf{A} = 0 \ . \tag{27}$$

Hence, in superluminal relativistic kinematics or more precisely in superluminal space-time, the superluminal velocity four-vector and acceleration four-vector are orthogonal. According to the operational definition of SKP (2), we find the SRT-property for the case $\vartheta(u) = c$ when $0 \le u < c$.

4.2. Superluminal Relativistic Dynamics

After having derived the expected laws of superluminal relativistic kinematics, now, we focus our attention on the formulation of superluminal relativistic dynamics. As we have already mentioned it explicitly or implicitly, in the superluminal formalism, all superluminal physical equations should be invariant under ST's during any transition from an IRF to another. Therefore, such equations should be defined in superluminal space-time. For example, the invariance of superluminal quadratic form (3) is the first superluminal invariance. Thus, basing on such a central idea, we will show the existence of the following physical quantities: superluminal momentum-energy four-vector; superluminal three-dimensional momentum; superluminal (total kinetic) energy; superluminal momentum-energy relation and superluminal force four-vector.

4.2.1. Superluminal momentum-energy four-vector

By definition, the combination of superluminal three-dimensional momentum $\mathbf{p}(p_x, p_y, p_z)$ and the superluminal (total kinetic) energy $\mathcal{E}$ *via* SKP $\vartheta(v)$ forms, in superluminal space-time, a superluminal momentum-energy four-vector

$$\mathbf{P} = \left(\mathbf{p}, i\frac{\mathcal{E}}{\vartheta(v)}\right), \tag{28}$$

that should characterize any material point moving at superluminal velocity. This superluminal momentum-energy four-vector (28) should be invariant under ST's during any transition from an IRF to another. Specifically, for an IRF F' in straight-line uniform motion at superluminal velocity $\mathbf{v}$ of magnitude v relative to F along the x-axis, the superluminal transformations of $\mathbf{P}$ are:

$$F \to F' : \begin{cases} p'_{x'} = \eta\left(p_x - \varepsilon\dfrac{E}{\vartheta(v)}\right) \\ p'_{y'} = p_y \\ p'_{z'} = p_z \\ \dfrac{E'}{\vartheta(v)} = \eta\left(\dfrac{E}{\vartheta(v)} - \varepsilon p_x\right) \end{cases} , \tag{29}$$

and

$$F' \to F : \begin{cases} p_x = \eta\left(p'_{x'} + \varepsilon \dfrac{E'}{\vartheta(v)}\right) \\ p_y = p'_{y'} \\ p_z = p'_{z'} \\ \dfrac{E}{\vartheta(v)} = \eta\left(\dfrac{E'}{\vartheta(v)} + \varepsilon p'_{x'}\right) \end{cases} . \tag{30}$$

To be sure that we have really defined a superluminal momentum-energy four-vector, it suffices to show from (29) or (30) the following superluminal invariance

$$p'_{x'} + p'_{y'} + p'_{z'} - \frac{\mathscr{E}'^2}{\vartheta^2(v)} = p_x + p_y + p_z - \frac{\mathscr{E}^2}{\vartheta^2(v)}, \tag{31}$$

or more compactly

$$\mathbf{P}'^2 - \frac{\mathscr{E}'^2}{\vartheta^2(v)} = \mathbf{P}^2 - \frac{\mathscr{E}^2}{\vartheta^2(v)}. \tag{32}$$

4.2.2. Superluminal three-dimensional momentum and superluminal (total kinetic) energy

If we now apply ST (30) to a material point of mass m in its proper superluminal frame F' (where the material point is at relative rest), in the observer's frame F (where the same material point is seen to move at superluminal velocity), we obtain after a simple calculation the following expected superluminal three-dimensional momentum and superluminal (total kinetic) energy

$$\mathbf{p} = \frac{\mathscr{E}}{\vartheta^2(v)}\mathbf{v}, \tag{33}$$

and

$$\mathscr{E} = \eta \mathscr{E}_0 , \tag{34}$$

where $\eta = 1\big/\sqrt{1 - v^2/\vartheta^2(v)}$, $c \le v < \vartheta(v)$ and $\mathscr{E}_0 = \mathscr{E}' = mc^2$ is the rest mass energy of the material point (the subscript 0 indicating, henceforth, quantity of the physical system referred to a comoving IRF.

4.2.3. Derivation of superluminal kinetic energy

Concerning the superluminal kinetic energy $\mathscr{E}_K$ of a material body moving at a superluminal velocity, is explicitly defined by the expression

$$\mathscr{E}_K = \mathscr{E} - \mathscr{E}_0 = \mathscr{E}_0(\eta - 1). \tag{35}$$

Rigorously, the formula (35) may be derived as follows. Firstly, recall that in classical mechanics the work done accelerating a particle during the infinitesimal time interval dt is given by the *dot* product of force $\mathbf{f}$ and displacement $d\mathbf{r}$:

$$\mathbf{f}\cdot d\mathbf{r} = \mathbf{f}\cdot \mathbf{v}dt = \frac{d\mathbf{p}}{dt}\cdot \mathbf{v}dt = \mathbf{v}\cdot d\mathbf{p} = \mathbf{v}\cdot d\left(m\mathbf{v}\right),$$

where we have assumed the well-known classical 3D-momentum expression $\mathbf{p} = m\mathbf{v}$.

Applying the *dot* product rule we see that:

$$d\left(\mathbf{v}\cdot\mathbf{v}\right) = \left(d\mathbf{v}\right)\cdot\mathbf{v} + \mathbf{v}\cdot\left(d\mathbf{v}\right) = 2\left(\mathbf{v}\cdot d\mathbf{v}\right).$$

Therefore (assuming constant mass so that dm=0), the following can be seen:

$$\mathbf{v}\cdot d\left(m\mathbf{v}\right) = \frac{m}{2}d\left(\mathbf{v}\cdot\mathbf{v}\right) = \frac{m}{2}dv^2 = d\left(\frac{mv^2}{2}\right).$$

Since this is a total differential (that is, it only depends on the final state, not how the particle got there), we can integrate it and call the result kinetic energy:

$$E_K = \int \mathbf{F}\cdot d\mathbf{r} = \int \mathbf{v}\cdot d\left(m\mathbf{v}\right) = \int d\left(\frac{mv^2}{2}\right) = \frac{mv^2}{2}.$$

Classically, this equation states that the kinetic energy E_K is equal to the integral of the *dot* product of the velocity $\mathbf{v}$ of a body and the infinitesimal change of the body's momentum $\mathbf{p}$. It is assumed that the body starts with no kinetic energy when it is at (relative) rest.

Superluminal Kinetic Energy: In superluminal relativistic dynamics, that is when the material object evolving in superluminal space-time at superluminal velocity, we must change the expression for linear momentum. Using $\mathcal{E}_0$ for rest (mass) energy, $\mathbf{v}$ with magnitude $v > c$ for the superluminal object's velocity vector and magnitude respectively, we assume according to the superluminal formulae (33) and (34) for linear momentum that $\mathbf{p} = \eta\,\vartheta^{-2}(v)\mathcal{E}_0\mathbf{v}$ where $\eta = 1\big/\sqrt{1 - v^2/\vartheta^2(v)}$ and $\mathcal{E}_0 = mc^2$.

Integrating by parts gives

$$\mathcal{E}_K = \int \mathbf{v}\cdot d\mathbf{p} = \int \mathbf{v}\cdot d\left(\eta\,\vartheta^{-2}(v)\mathcal{E}_0\mathbf{v}\right) = \eta\,\vartheta^{-2}(v)\mathcal{E}_0\mathbf{v}\cdot\mathbf{v} - \int \eta\,\vartheta^{-2}(v)\mathcal{E}_0\mathbf{v}\cdot d\mathbf{v}$$

$$= \eta\,\vartheta^{-2}(v)\mathcal{E}_0 v^2 - \int \eta\,\vartheta^{-2}(v)\mathcal{E}_0\mathbf{v}\cdot d\mathbf{v} = \eta\,\vartheta^{-2}(v)\mathcal{E}_0 v^2 - \frac{\mathcal{E}_0}{2}\int \eta\,\vartheta^{-2}(v)\,d\left(v^2\right).$$

Remembering that $\vartheta(v)$ is *conceptually* defined as a constant speed in all the IRF’s and $\eta = 1\big/\sqrt{1 - v^2/\vartheta^2(v)}$ we get:

$$\mathscr{E}_K = \eta\,\vartheta^{-2}(v)\mathscr{E}_0 v^2 - \frac{-\mathscr{E}_0}{2}\int \eta\, d\left(1 - v^2/\vartheta^2(v)\right)$$

$$= \eta\,\vartheta^{-2}(v)\mathscr{E}_0 v^2 + \mathscr{E}_0\sqrt{1 - v^2/\vartheta^2(v)} + C.$$

The integration constant C may be identified as $-\mathscr{E}_0$ that is when the material body in question is in a state of (relative) rest $\mathscr{E}_K = 0$ and $v = 0$. Thus

$$\mathscr{E}_K = \eta\,\vartheta^{-2}(v)\mathscr{E}_0 v^2 - \frac{-\mathscr{E}_0}{2}\int \eta\, d\left(1 - v^2/\vartheta^2(v)\right)$$

$$= \eta\,\vartheta^{-2}(v)\mathscr{E}_0 v^2 + \mathscr{E}_0\sqrt{1 - v^2/\vartheta^2(v)} - \mathscr{E}_0$$

$$= \mathscr{E}_0(\eta - 1).$$

4.2.4. Expression of superluminal velocity

In superluminal relativistic dynamics, and in terms of magnitude, the particle's superluminal velocity may be treated as a function of the superluminal kinetic energy of the same particle relative to the observer's IRF as we will see now. We have from the formula (35)

$$\eta = 1 + \frac{\mathscr{E}_K}{\mathscr{E}_0}. \qquad (36)$$

Remark, since the Eta-factor and SKP are, respectively, expressed by $\eta = 1\big/\sqrt{1 - v^2/\vartheta^2(v)}$ and $\vartheta(v) = c\sqrt{1 + v^2/c^2}$ when $v > c$. Hence, after substitution in (36) and performing some calculation, we find the expected expression

$$v = c\sqrt{\frac{\mathscr{E}_K}{\mathscr{E}_0}\left(\frac{\mathscr{E}_K}{\mathscr{E}_0} + 2\right)}. \qquad (37)$$

As it was thought, in superluminal relativistic dynamics, the superluminal velocity is a function of the superluminal kinetic energy, that is, $v \equiv v(\mathscr{E}_K)$. This allows us to classify –by convention– the superluminal velocities into four *categories* namely: *low* $(c < v < 10c)$, *typical* $(v = 10c)$, *high* $(v > 10c)$ and *ultra-high* $(v >> 10c)$ superluminal velocities, and all that depending, of course, on the amount of the superluminal kinetic energy $\mathscr{E}_K$ since v itself is a function of the latter. This classification is seen necessary especially to investigate the behavior of the cosmic rays according to their observed/detected kinetic energy. Also, that's why the tow formulae (36) and (37) admit a very interesting application particularly when we would investigate the superluminal velocity of the electrons and/or protons in particle accelerators.

4.2.5. Superluminal momentum-energy relation

By combining the formulae (33) and (34), we get the very expected superluminal momentum-energy relation

$$\mathscr{E}^2 = \vartheta^2(v)\mathbf{p}^2 + \mathscr{E}_0^2 \ , \tag{38}$$

or

$$\mathscr{E} = \sqrt{\vartheta^2(\mathrm{v})\mathbf{p}^2 + \mathscr{E}_0^2} \ . \tag{39}$$

Therefore, in superluminal relativistic dynamics, the superluminal momentum-energy relation (38) or (39) is a superluminal formula relating any physical object's rest mass energy, total energy and momentum. –As a direct result, the above superluminal formulae enable as to consider the photon and the tachyon as ordinary particles with nonzero (relative) rest mass. Consequently, the old notion of zero rest mass for the first and imaginary (rest) mass for the second becomes not only meaningless but simply absurd! Because according to the principle of KL's there is a set of reference systems called luminal IRF's relative to which photons behave as ordinary particles; the same thing for the tachyons, *i.e.*, the set of superluminal IRF's allows the tachyons to behave as ordinary particles[2]. Finally, like before, from (33), (34), (35) and (38), we can recover the well-known SRT-formulae for the case $\vartheta(v) = c$ when $0 \le v < c$.

4.2.6. Superluminal Lagrangian and Hamiltonian

Lagrangian and Hamiltonian as functions are so important in mechanics that's why must be adjusted in light of the new concepts presented in the superluminal formalism. We can extend the Lagrangian formalism into the realm of the superluminal relativistic mechanics in the following way.

Firstly, for a single subrelativistic[3] particle moving in a velocity-independent potential, the 3D-momentum may be classically written as

$$\mathbf{p} = \frac{\partial \mathscr{L}}{\partial v}\frac{\mathbf{v}}{v} . \tag{40}$$

According to Eq.(33), the superluminal 3D-momentum is $\mathbf{p} = \eta\,\vartheta^{-2}(v)\mathscr{E}_0\mathbf{v}$, with $\eta = 1\big/\sqrt{1 - v^2/\vartheta^2(v)}$ and $\mathscr{E}_0 = mc^2$.

We now require that the expected superluminal Lagrangian, when differentiated with respect to v as in Eq.(40), yields the superluminal 3D-momentum given by Eq.(33).

$$\frac{\partial \mathscr{L}}{\partial v}\frac{\mathbf{v}}{v} = \eta\,\vartheta^{-2}(v)\mathscr{E}_0\mathbf{v} . \tag{41}$$

This requirement involves only the superluminal velocity of the particle, so we expected that the velocity-independent part of the superluminal Lagrangian is unchanged from the classical case.

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[2] By ordinary particles, we mean bradyons, *i.e.*, particles that travel at subluminal velocities. The term 'bradyon', from Greek: βραδύς (*bradys*, 'slow'), was initially coined to contrast with tachyon, from Greek: ταχύς (*tachys*, 'rapid').

[3] Term/adjective describing a velocity (and associated properties) that is considerably less than the light speed (in vacuum) such that relativistic effects may be ignored.

The velocity-dependent part, however, may no longer be explicitly equal to the superluminal kinetic energy. We therefore write

$$\mathscr{L} = T - U \,, \tag{42}$$

where $U \equiv U(r)$ with $r = \sqrt{x^2 + y^2 + z^2}$ and $T \equiv T(v)$ with $v = \sqrt{v_x^2 + v_y^2 + v_z^2}$. Furthermore, the function T must satisfy the relation

$$\frac{\partial T}{\partial v} = \eta\, \vartheta^{-2}(v) \mathscr{E}_0 v \,. \tag{43}$$

Remembering that $\vartheta(v)$ is *conceptually* defined as a constant speed in all the IRF's, thus it can be easily verified that a suitable expression for T (apart from a possible integration constant that can be suppressed) is

$$T = -\eta^{-1} \mathscr{E}_0 \,. \tag{44}$$

Consequently, by substituting (44) in (42), we obtain the expected superluminal Lagrangian

$$\mathscr{L} = -\eta^{-1} \mathscr{E}_0 - U \,, \tag{45}$$

or more explicitly

$$\mathscr{L} = -\mathscr{E}_0 \sqrt{1 - v^2 / \vartheta^2(v)} - U \,, \quad \mathscr{E}_0 = mc^2 \,. \tag{46}$$

The equations of superluminal motion should be obtained in the standard way from Lagrange's equations. Notice that the Lagrangian should be given according to the superluminal expression for the kinetic energy (35).

The Hamiltonian can be evaluated from the following expression

$$H = \mathbf{p} \cdot \mathbf{v} - \mathscr{L} \,,$$

By using Eqs.(33), (46), (34), (38) and (39), the above expression becomes

$$H = \mathbf{p} \cdot \frac{\mathbf{p}}{\eta\, \vartheta^{-2}(v) \mathscr{E}_0} + \mathscr{E}_0 \sqrt{1 - v^2 / \vartheta^2(v)} + U = \frac{\mathbf{p}^2}{\eta\, \vartheta^{-2}(v) \mathscr{E}_0} + \eta^{-1} \mathscr{E}_0 + U$$

$$= \frac{\mathbf{p}^2 + \vartheta^{-2}(v) \mathscr{E}_0^2}{\eta\, \vartheta^{-2}(v) \mathscr{E}_0} + U = \frac{\mathbf{p}^2 + \vartheta^{-2}(v) \mathscr{E}_0^2}{\vartheta^{-2}(v) \mathscr{E}} + U$$

$$= \frac{\left(\vartheta^2(v) \mathbf{p}^2 + \mathscr{E}_0^2\right) \vartheta^{-2}(v)}{\vartheta^{-2}(v) \sqrt{\vartheta^2(v) \mathbf{p}^2 + \mathscr{E}_0^2}} + U = \sqrt{\vartheta^2(v) \mathbf{p}^2 + \mathscr{E}_0^2} + U$$

$$= \mathscr{E} + U \,. \tag{47}$$

The superluminal Hamiltonian is equal to the superluminal (total kinetic) energy (39) *plus* the potential energy. As usual, the superluminal expressions (46) and (47) may be reduced to those of SRT for the case $\vartheta(v) = c$ when $0 \le v < c$.

4.2.7. Superluminal force four-vector

In classical dynamics, to use Newton's second law of motion, the force must be defined as the rate of change of momentum with respect to the same time coordinate. That is, it requires the 3D-force vector. In superluminal relativistic dynamics, if a material point is characterized by the superluminal velocity vector $\mathbf{u}(u_x, u_y, u_z)$ of magnitude u, we can transform the superluminal 3D-force vector from the material point's comoving IRF into the observer's IRF. This yields a four-vector called 'the superluminal force four-vector'. It is, *by definition*, the rate of change of the superluminal momentum-energy four-vector with respect to material point's proper time. Hence, the superluminal invariance version of this superluminal force four-vector is

$$\mathbf{F} = \frac{d\mathbf{P}}{dt^*} = \left(\eta\mathbf{f}, i\frac{\eta(\mathbf{f}\cdot\mathbf{u})}{\vartheta(u)} \right), \tag{48}$$

where $\eta = 1\big/\sqrt{1 - u^2/\vartheta^2(u)}$, and $\mathbf{f}$ and $\mathbf{u}$ are, respectively, the superluminal 3D-force vector and superluminal velocity vector of the moving material point.

4.2.8. Transformations of superluminal force four-vector

Like for the case of superluminal velocity four-vector, that is, we use the fact that the components of $\mathbf{F}$ should transform from a superluminal IRF to another IRF *via* ST's:

$$\mathbf{F}' = \mathscr{H}\mathbf{F} \quad \text{with} \quad \mathbf{F} = \left(\eta\mathbf{f}, i\frac{\eta(\mathbf{f}\cdot\mathbf{u})}{\vartheta(u)} \right), \quad \mathbf{F}' = \left(\eta'\mathbf{f}', i\frac{\eta'(\mathbf{f}'\cdot\mathbf{u}')}{\vartheta(u')} \right), \tag{49}$$

where $\mathscr{H}$ is the matrix of ST (8), $\eta = 1\big/\sqrt{1 - u^2/\vartheta^2(u)}$, $\eta' = 1\big/\sqrt{1 - u'^2/\vartheta^2(u')}$ and $\mathbf{u}$ being the superluminal velocity vector of the material point relative to IRF F, and $\mathbf{u}'$ the superluminal velocity vector of the same material point relative to IRF F'. Let $\mathbf{v}$ of magnitude v be the superluminal velocity vector of F' with respect to F along the x-axis. Thus, we have

$$F \to F' : \begin{cases} \eta' f'_{x'} = \eta_{(v)}\left(\eta\, f_x - \varepsilon\frac{\eta(\mathbf{f}\cdot\mathbf{u})}{\vartheta(v)} \right) \\ \eta' f'_{y'} = \eta\, f_y \\ \eta' f'_{z'} = \eta\, f_z \\ \dfrac{\eta'(\mathbf{f}'\cdot\mathbf{u}')}{\vartheta(u')} = \eta_{(v)}\left(\dfrac{\eta(\mathbf{f}\cdot\mathbf{u})}{\vartheta(v)} - \varepsilon\eta\, f_x \right) \end{cases}, \tag{50}$$

where

$$\varepsilon = v/\vartheta(v), \quad \eta_{(v)} = 1\big/\sqrt{1 - v^2/\vartheta^2(v)}.$$

Recall, during the derivation of the transformation of superluminal velocity four-vector, we have found some relation between the superluminal factors η , η' and $\eta_{(v)}$, namely $\eta/\eta' = 1/\eta_{(v)}\left(1-u_x v/\vartheta^2(v)\right)$.

And, like before, by taking into account the principle of KL's, or more precisely the fact that all the IRF's belonging to the same KL are equivalent and their SKP's are identical, *i.e.*, for the present case we should have $\vartheta(v)=\vartheta(u)=\vartheta(u')$, hence we can deduce the expected transformations of superluminal force four-vector, that is, the explicit transformations of components of superluminal 3D-force vector $\mathbf{f}$ and the instantaneous superluminal power $(\mathbf{f}\cdot\mathbf{u})$:

$$F \to F' : \begin{cases} f'_{x'} = \left(1-\dfrac{u_x v}{\vartheta^2(v)}\right)^{-1}\left(f_x - \dfrac{(\mathbf{f}\cdot\mathbf{u})v}{\vartheta^2(v)}\right) \\ f'_{y'} = \left(1-\dfrac{v^2}{\vartheta^2(v)}\right)^{1/2}\left(1-\dfrac{u_x v}{\vartheta^2(v)}\right)^{-1} f_y \\ f'_{z'} = \left(1-\dfrac{v^2}{\vartheta^2(v)}\right)^{1/2}\left(1-\dfrac{u_x v}{\vartheta^2(v)}\right)^{-1} f_z \\ (\mathbf{f}'\cdot\mathbf{u}') = \left(1-\dfrac{u_x v}{\vartheta^2(v)}\right)^{-1}\left((\mathbf{f}\cdot\mathbf{u}) - v\, f_x\right) \end{cases} , \tag{51}$$

and

$$F' \to F : \begin{cases} f_x = \left(1+\dfrac{u'_{x'} v}{\vartheta^2(v)}\right)^{-1}\left(f'_{x'} + \dfrac{(\mathbf{f}'\cdot\mathbf{u}')v}{\vartheta^2(v)}\right) \\ f_y = \left(1-\dfrac{v^2}{\vartheta^2(v)}\right)^{1/2}\left(1+\dfrac{u'_{x'} v}{\vartheta^2(v)}\right)^{-1} f'_{y'} \\ f_z = \left(1-\dfrac{v^2}{\vartheta^2(v)}\right)^{1/2}\left(1+\dfrac{u'_{x'} v}{\vartheta^2(v)}\right)^{-1} f'_{z'} \\ (\mathbf{f}\cdot\mathbf{u}) = \left(1+\dfrac{u'_{x'} v}{\vartheta^2(v)}\right)^{-1}\left((\mathbf{f}'\cdot\mathbf{u}') + v\, f'_{x'}\right) \end{cases} . \tag{52}$$

It is worthwhile to note that the superluminal transformations (51) and (52) may be reduced to the well-known SRT-transformations for the case $\vartheta(v)=c$ when $0 \le v < c$. Now, we can say that we have really formulated the two principal parts of the Superluminal Relativistic Mechanics (SupRelMec), namely, the superluminal relativistic kinematics and dynamics. In this sense, we can affirm that SupRelMec is in fact a pure superluminalization of SRT which will lead, automatically, to the superluminalization of electromagnetism.

5. Superluminalization of the Differential Operators

As we know it more clearly, the four-dimensional superluminal space-time (geometry) is, by definition, the *seat* of all the superluminal physical phenomena, and in order to study such phenomena in correct and rigorous way, we should modelize these superluminal physical phenomena by using the fundamental physical equations. However, in the framework of superluminal formalism, any fundamental physical equation should be explicitly invariant under ST's, like for instance the case of the Maxwell's equations in (empty) superluminal space-time. This operation by itself is called *superluminalization* of the fundamental physical equations. Hence, to arrive at such aim, we are firstly obliged to superluminalize the most useful differential operators, such that Nabla (Del) operator and D'Alembertian operator because, as we shall see, the superluminal invariance of these operators implies automatically the very expected superluminal invariance of the fundamental physical equations under ST's and thus in all the superluminal IRF's.

5.1. Superluminalization of Nabla and d'Alembertian (Operators)

Recall that the superluminal four-vector of position of a superluminal event of spatio-temporal coordinates (x, y, z, t), is a vector $\mathbf{R}$ of components: $(x_1 = x, x_2 = y, x_3 = z, x_4 = i\vartheta(v)t)$. In 3D-Euclidean space, the operators: 'divergence' and 'Laplacian' may be easily expressed in terms of Nabla operator ∇. This operator is given in Cartesian coordinates as follows:

$$\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right).$$

Thus, in Cartesian coordinates, the divergence of a vector $\mathbf{V}(V_x, V_y, V_z)$ and the Laplacian of a scalar $\mathrm{f} \equiv \mathrm{f}(x, y, z)$ are respectively:

$$\operatorname{div}\mathbf{V} = \nabla\cdot\mathbf{V} = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} \quad \text{and} \quad \nabla^2\mathrm{f} = \frac{\partial^2\mathrm{f}}{\partial x^2} + \frac{\partial^2\mathrm{f}}{\partial y^2} + \frac{\partial^2\mathrm{f}}{\partial z^2}.$$

Therefore, the superluminalization of the divergence and the Laplacian depending on the generalization of Nabla operator by defining ∇ in 4D-superluminal space-time to get the expected superluminal four-Nabla operator:

$$\boldsymbol{\nabla} = \left(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \frac{\partial}{\partial x_3}, \frac{\partial}{\partial x_4}\right) \equiv \left(\nabla, \frac{1}{i\vartheta^2(v)}\frac{\partial}{\partial t}\right). \tag{53}$$

Now, let us show the superluminal invariance of the superluminal four-Nabla operator (53) under ST's during any transition from an IRF to another. With this aim, we have according to ST's:

$$F \to F' : \begin{cases} x'_1 = \eta(x_1 + i\varepsilon x_4) \\ x'_2 = x_2 \\ x'_3 = x_3 \\ x'_4 = \eta(-i\varepsilon x_1 + x_4) \end{cases}, \qquad F' \to F : \begin{cases} x_1 = \eta(x'_1 - i\varepsilon x'_4) \\ x_2 = x'_2 \\ x_3 = x'_3 \\ x_4 = \eta(i\varepsilon x'_1 + x'_4) \end{cases}, \qquad \eta = 1\big/\sqrt{1-\varepsilon^2}, \quad \varepsilon = v/\vartheta(v).$$

Furthermore, we should have $\frac{\partial}{\partial x'_n} = \sum_{\upsilon} \frac{\partial x_\upsilon}{\partial x'_n}\frac{\partial}{\partial x_\upsilon}$, that is

$$\nabla' : \begin{cases} \frac{\partial}{\partial x'_1} = \frac{\partial x_1}{\partial x'_1}\frac{\partial}{\partial x_1} + \frac{\partial x_4}{\partial x'_1}\frac{\partial}{\partial x_4} = \eta\left(\frac{\partial}{\partial x_1} + i\,\varepsilon\frac{\partial}{\partial x_4}\right) \\ \frac{\partial}{\partial x'_2} = \frac{\partial}{\partial x_2} \\ \frac{\partial}{\partial x'_3} = \frac{\partial}{\partial x_3} \\ \frac{\partial}{\partial x'_4} = \frac{\partial x_1}{\partial x'_4}\frac{\partial}{\partial x_1} + \frac{\partial x_4}{\partial x'_4}\frac{\partial}{\partial x_4} = \eta\left(-i\,\varepsilon\frac{\partial}{\partial x_1} + \frac{\partial}{\partial x_4}\right) \end{cases} . \tag{54}$$

From (54), we get

$$\nabla' = \mathscr{H}\nabla . \tag{55}$$

Therefore, we can define the superluminal four-(Nabla) divergence of a four-vector $\mathbf{W}$ by the relation

$$\nabla \cdot \mathbf{W} = \frac{\partial W_1}{\partial x_1} + \frac{\partial W_2}{\partial x_2} + \frac{\partial W_3}{\partial x_3} + \frac{\partial W_4}{\partial x_4} . \tag{56}$$

Let us ensure the reader that the scalar (56) is really a superluminal invariant under ST's during any transition from an IRF to another. Thus, we have

$$\nabla' \cdot \mathbf{W}' = \eta^2\left(\frac{\partial}{\partial x_1} + i\,\varepsilon\frac{\partial}{\partial x_4}\right)\left(W_1 + i\,\varepsilon W_4\right) + \frac{\partial W_2}{\partial x_2} + \frac{\partial W_3}{\partial x_3} + \eta^2\left(-i\,\varepsilon\frac{\partial}{\partial x_1} + \frac{\partial}{\partial x_4}\right)\left(-i\,\varepsilon W_1 + W_4\right) = \nabla \cdot \mathbf{W} .$$

From the definition of the superluminal four-Nabla operator (53), we can deduce the following expression for the superluminal d'Alembertian operator:

$$\square = \nabla^2 - \frac{1}{i\,\vartheta^2(v)}\frac{\partial^2}{\partial t^2} , \tag{57}$$

6. Upper Limit of Validity of the Lorentz Transformations

In this section we would investigate the *upper limit of validity* of the Lorentz transformations (LT's) in the context of the superluminal formalism. We are interested in such an investigation because LT's are, mainly, the basic foundation of special relativity theory (SRT) and relativistic quantum field theory.

In recent years, however, motivated by attempts to combine all the known forces and particles into *one* ultimate unified theory, some researchers have been investigating the possibility that SRT's postulates provides only an approximation of Nature's workings. The hope is that small SRT violations might offer the first experimental signals of the long-sought ultimate theory.

However, according to our present superluminal formalism, the search for SRT violations is conceptually meaningless since SRT is among the most fundamental and well verified of all physical theories. That is, SRT is correct in its proper *sector* namely the subluminal KL for relativistic velocities $(v < c)$. Therefore, the best thing to do is the determination of the upper limit of validity of LT's because, as it is well known, each physical theory has its own domain of application and has its own limits of validity.

6.1. Luminal (spatio-temporal) Transformations

We begin this subsection by the following quote from Dirac's Noble Lecture (12 December 1933) entitled 'Theory of electrons and positrons': *"The variables α also give rise to some rather unexpected phenomena concerning the motion of the electron. These have been fully worked out by Schrödinger. It is found that an electron which seems to us to be moving slowly, must actually have a very high frequency oscillatory motion of small amplitude superposed on the regular motion which appears to us. As a result of this oscillatory motion, the velocity of the electron at any time equals the velocity of light. This is a prediction which cannot be directly verified by experiment, since the frequency of the oscillatory motion is so high and its amplitude is so small. But one must believe in this consequence of the theory, since other consequences of the theory which are inseparably bound up with this one, such as the law of scattering of light by an electron, are confirmed by experiment."*

This passage reflects exactly the expected limit of validity of LT's *via* SRT because, here, the electron as a fundamental elementary particle has well-known mass and charge, and its oscillatory motion at the velocity of light implies that such a luminal oscillatory motion should manifest in the superluminal space-time as a *seat* of luminal and superluminal physical phenomena.

Furthermore, since each physical object has its *proper*-IRF thus, the photon should have its proper IRF. In this sense, the existence of the luminal IRF's constitutes the upper limit of validity of LT's and SRT. The luminal (spatio-temporal) transformations that ensure the link between all the luminal IRF's are in fact a special case of the superluminal (spatio-temporal) transformations, explicitly, according to the operational definition of SKP (2), ST's (8) and (9) reduce to the luminal (spatio-temporal) transformations for the case $\vartheta(c) > c$ when $v = c$. Particularly, if the IRF's F and F' are in relative motion –at luminal velocity of magnitude– with respect to each other, thus F and F' are connected by the following luminal (spatio-temporal) transformations:

$$F \to F' : \begin{cases} x' = \eta(x - ct) \\ y' = y \\ z' = z \\ t' = \eta\left(t - \dfrac{cx}{\vartheta^2(c)}\right) \end{cases}, \tag{58}$$

and

$$F' \to F : \begin{cases} x = \eta(x' + ct') \\ y = y' \\ z = z' \\ t = \eta\left(t' + \dfrac{cx'}{\vartheta^2(c)}\right) \end{cases}, \tag{59}$$

where

$$\eta = 1\Big/\sqrt{1 - c^2/\vartheta^2(c)}, \tag{60}$$

and

$$\vartheta(c) > c,\ v = c. \tag{61}$$

Now, in order to determine an upper limit for Lorenz factor (γ), we must evaluate Eta-factor (60). But firstly, remark that the numerical value $3\times10^8\ \mathrm{m\,s^{-1}}$ is *frequently* used as an approximation to the light speed in vacuum (1). However, such an approximation is in fact an exaggeration because the quantity $3\times10^8\ \mathrm{m\,s^{-1}}$ has the physical dimensions of a constant superluminal velocity since the actual and empirical value of light speed in vacuum is $c = 299792458\,\mathrm{ms^{-1}}$. To show this claim more convincingly, we have

$$3\times10^8\ \mathrm{ms^{-1}} - c = 207542\mathrm{ms^{-1}}$$

and

$$\frac{3\times10^8\ \mathrm{ms^{-1}}}{c} = 1.00069228559445614872673014 3424,$$

thus

$$3\times10^8\ \mathrm{ms^{-1}} > c.$$

For that reason and mainly for the purpose of applicability, we can consider, in the context of superluminal formalism, the quantity $3\times10^8\ \mathrm{ms^{-1}}$ as a minimal superluminal velocity. Consequently, this consideration and the definition (61) allow us to write $\vartheta_{\min} = 3\times10^8\ \mathrm{m\,s^{-1}}$ and the Eta-factor (60) for this value becomes

$$\eta_{\min} = 1\Big/\sqrt{1 - c^2/\vartheta_{\min}^2} = \sqrt{723}, \tag{62}$$

where

$$\vartheta_{\min} = 3\times10^8\ \mathrm{m\,s^{-1}}. \tag{63}$$

Now, we are arrived at our main aim, namely, the determination of an upper limit for Lorentz factor. Since the minimal Eta-factor (62) has the value $\sqrt{723}$ therefore the expected upper limit should be explicitly defined by the following inequality

$$\gamma < \eta_{\min} . \tag{64}$$

It follows from the inequality (64) that the research for possible violations of Lorentz invariance becomes unnecessary. Moreover, in the context of superluminal formalism, the Lorentz transformations will not be valid any more when $\gamma \geq \eta_{\min}$. Indeed, if for example E_0 and E_{K} are, respectively, the rest (mass) energy and the kinetic energy of the same particle in a particle accelerator, this particle could be regarded as subluminal for relativistic or ultrarelativistic velocity in laboratory-IRF and therefore could be studied by SRT-laws only if $\left(E_{\mathrm{K}}/E_0\right)+1<\sqrt{723}$.

As a direct result, the claims such as: *probably a proton detected at a speed close to* $0.999999999\mathbf{9}99999999\mathbf{9}9951c$ and/or *the Lorentz factor is about* $\gamma \approx 10^4$ becomes highly meaningless. Because, according to superluminal formalism, these claims do not belong to SRT-context.

7. Superluminality of protons in the LHC

It is always best to recall that the superluminal relativistic mechanics established in order to investigate the superluminal physical phenomena. Thus this last section is devoted to the study of the *superluminality* of protons in the Large Hadron Collider (LHC). The study is very important in view of the fact that it facilitates the comprehension of the superluminality of electrons in the Van Allen belt and the observed high and ultra-high energy cosmic rays.

Context: The Large Hadron Collider (LHC), in Switzerland, has been successfully tested as a particle accelerator on Wednesday 10 September 2009. A beam of protons was accelerated and completed several loops through the whole structure (26 659 m), clockwise and counter-clockwise.

When the power of this machine is discussed, the energy of each proton is often mentioned: The protons each have a kinetic energy of 7 TeV. Thus, colliding two protons with that energy together we get the potential for a maximum energy of 14 TeV. This new energy range at the LHC is why scientists are optimistic about finding new things – like the Higgs boson, Supersymmetry, and Extra-Dimensions – more energy means more opportunity for discovery.

Firstly, by using SRT-formalism, we will calculate the Lorentz factor from which the velocity of the protons in the LHC may be deduced. The value of Lorentz factor should be compared with the minimal Eta-factor (62) in order to see if the inequality (64) is respected or violated. Secondly, we will apply the superluminal formalism to calculate and evaluate the same quantities.

7.1. Lorentz factor and Velocity of the protons in the LHC according to SRT

At subluminal KL for subrelativistic velocities ($v << c$), the kinetic energy of material object is classically measured by

$$E_K = \frac{1}{2} m v^2 .$$

However, this formula cannot be applied at relativistic and ultrarelativistic velocities. We must use SRT, which defines the total kinetic energy as $E = \gamma m c^2$ where m is the mass at rest and γ is the Lorentz factor, defined as $\gamma = \left(1 - v^2 / c^2\right)^{-1/2}$. It is clear that when the particle is at rest $\left(v = 0\right)$, this yields the equivalence between mass and energy, *i.e.*, the well-known rest (mass) energy: $E_0 = mc^2$.

It is worthwhile to note that the energy reported by the LHC is only the kinetic energy of the particles, it doesn't include the rest energy. Indeed, the rest energy of a proton is around 938.272 MeV. Thus, with the help of SRT-formalism, we can calculate the Lorentz factor and evaluate the velocity of the protons in the LHC as following. We have

$$\begin{aligned} E &= E_K + E_0 \\ E_K &= E - E_0 \\ E_K &= \gamma E_0 - E_0 \\ \gamma &= 1 + \frac{E_K}{E_0} \end{aligned}$$

Finally, since the Lorentz factor has the explicit expression $\gamma = \left(1 - v^2 / c^2\right)^{-1/2}$, thus from the last equation, we deduce an expression for the velocity

$$v = \frac{c}{\gamma} \sqrt{\gamma^2 - 1} .$$

With $E_0 = 938.272\,\mathrm{MeV}$ and $E_K = 7\,\mathrm{TeV}$, we get, after a direct substitution and a simple calculation, the following values for the Lorentz factor and velocity, respectively:

$$\gamma = 7461,$$

and

$$v = 0.999999991c.$$

Since, here, $\gamma = 7461 > \eta_{\min} = \sqrt{723}$ therefore in this case the velocity $v = 0.999999991c$ does not have any physical content, but have to be considered as a pure asymptotic velocity –without physical foundation. Consequently, we cannot apply SRT because the LHC accelerated the protons at superluminal velocity.

7.2. Eta-factor and Superluminal Velocity of the protons in the LHC

By means of the superluminal formulae (36) and (37), we can calculate the Eta-factor and the superluminal velocity of the protons in the LHC. To this end, substituting $\mathscr{E}_0 = 938.272\,\mathrm{MeV}$ and $\mathscr{E}_K = 7\,\mathrm{TeV}$ in the above mentioned formulae and after a simple calculation, we obtain the following values for the Eta-factor and superluminal velocity, respectively:

$$\eta = 7461,$$

and

$$v = \eta c \ .$$

Result: It is clear from the last relation, the superluminal velocity of the protons in LHC is 7461 times faster than light speed in vacuum that is –according to the conventional classification– $v = \eta c$ is a high superluminal velocity. Furthermore, the ratio v/c becomes comparable to Eta-factor η when the superluminal kinetic energy $\mathscr{E}_K$ is sufficiently very large. In such situation, the superluminal velocity v should approach the SKP $\vartheta(v)$ asymptotically. The same account stays valid for high and ultra-high energy cosmic rays.

8. Conclusion

We have derived new superluminal spatio-temporal transformations based on the idea that, conceptually and kinematically, each subluminal $(0 \le v < c)$; luminal $(v = c)$ and/or superluminal inertial reference frame $v > c$ has, in addition to its relative velocity v, its proper specific kinematical parameter $\vartheta(v)$, which having the physical dimensions of a constant speed. Basing on such a derivation, we have formulated the superluminal relativistic mechanics in which the relativity principle and causality principle are coherently extended to superluminal velocities and, more importantly, the standard special relativity theory becomes a particular case.

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