SUBSET SEMILINEAR ALGEBRAS

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Subset Semilinear Algebras

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PREFACE

In this book we introduce, develop and study the new notion of subset semilinear algebras. We use the subset semigroups over the semifields to build semilinear algebras of both finite order and infinite order. The concept of subset linear independence and subset linear dependence leads to the dimension and basis of subset semilinear algebras.

Next the concept of Smarandache special strong subset semilinear algebras is defined over subset semirings. We study the substructures of them. We give examples of subspaces of these spaces. The concept of special semi linear transformation is developed and described in this book.

We define the new notion of subset A in a subset semivector space viz, the sum in A. If in the set A = (37, 12, 5, 8, 9) the subset sum of A denoted by A_S as 37 + 12 + 5 + 8 + 9 = 71. This notion is defined mainly to define the concept of subset semi inner product on subset semivector spaces. This concept is described and illustrated by examples.

This new concept helps the authors to define the notion of subset semilinear functional. The concept of subset semilinear operator and subset semiprojections are described and developed. These concepts give many innovative ideas which can find nice applications in all the places where semirings find their applications.

Finally several problems are suggested some of which are at research level.

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W.B.VASANTHA KANDASAMY
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Chapter One

INTRODUCTION

In this book we introduce the notion of subset semilinear algebras (subset semivector spaces) defined over a semifield. This study is both interesting and innovative. We also introduce the notion of subset semiinner product spaces. We need the basic concept of subset semigroup to build subset semivector spaces over a semifield.

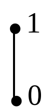
Also the notion of subset semivector spaces over subset semirings which are Smarandache subset semifields is described and some new algebraic structures are developed.

It is pertinent to keep on record that finding subset basis of a subset semivector space is also a difficult job. It is an open question to find the number of subset basis of a subset semivector space defined over a semifield.

We also leave this as an open problem, “Characterize those subset semivector spaces which has only one subset basis” or equivalently characterize those subset semivector spaces which has more than one subset basis.

Finally we wish to study under what conditions we can have the classical spectral theorem to be true in case of subset semivector spaces. When instead of the semifield $Z^+ \cup \{0\}$ or $Q^+ \cup \{0\}$ or $R^+ \cup \{0\}$ or $\langle Z^+ \cup I\rangle \cup \{0\}$ or $\langle Q^+ \cup I\rangle \cup \{0\}$ or $\langle R^+ \cup \rangle \cup \{0\}$ we use distributive lattices / chain lattices we study all these problems / properties.

Study of the subset semivector spaces when the semifield is



the Boolean algebra of order two is interesting.

If S is a subset semivector space defined over the chain lattice C_2 study or find out whether there are chances of the subset semivector space to have more than one subset semiinner product defined on it?

Can those subset semivector spaces defined over C_2 have subset semi unitary operator or subset semi normal operator to be defined on it?

Further when is it possible to have Gram Schmidt orthogonalization property to be true for the subset semivector spaces defined over C_n, a chain lattice with n elements? For more about subset algebraic structures please refer [25-6].

Chapter Two

SUBSET SEMILINEAR ALGEBRAS

In this chapter we for the first time define the notion of subset semi linear algebras defined over the semifield. We then generalize this concept over S-semirings. We describe, develop and define these new concepts.

DEFINITION 2.1: *Let S = {Collection of all subsets of a semigroup} be the subset semigroup. F be a semifield.*

(i) *If for all* $s \in S$ *and* $a \in F$; *as and sa* $\in S$
(ii) $\{0\} + s = s + \{0\} = s$ *for all* $s \in S$.
(iii) $s_1 + s_2 = s_2 + s_1 \in S$ *for all* $s_1, s_2 \in S$
(iv) $\{0\} \in S$ *and* $a \in F$; $\{0\} \times a = \{0\}$
(v) *For all* $a, b \in F$ *and* $s \in S$ *we have* $(ab)s = a(bs)$
(vi) $a(s_1 + s_2) = as_1 + as_2$ *for all* $a \in F$ *and* $s_1, s_2 \in S$.
(vii) $(a + b)s = as + bs$ *for all* $a, b \in F$ *and* $s \in S$.
(viii) $1.s = s$ *for all* $s \in S$ *and* $1 \in F$.

Then we define S to be a subset semivector space over the semifield F.

We will illustrate this situation by an example.

Example 2.1: Let S = {Collection of all subsets from the semigroup $Z^+ \cup \{0\}$} be the subset semigroup. F = $Z^+ \cup \{0\}$ be the semifield. S is a subset semivector space over the semifield F.

If A = {3, 4, 5, 7, 0} and B = {10, 2, 6, 12, 1} $\in$ S.

We see for 0 $\in$ F .

0.A = 0 $\times$ {3, 4, 5, 7, 0} = 0.

Also for {0} $\in$ S we have {0} $\times$ a = {0}
for all a $\in Z^+ \cup \{0\}$.

Consider 5 (A + B)

= 5 ({3, 4, 5, 7, 0} + {10, 2, 6, 12, 1})
= 5 ({13, 14, 15, 17, 10, 5, 6, 9, 2, 11, 16, 19, 12, 8, 1, 4})
= {65, 70, 75, 85, 50, 25, 30, 50, 10, 45, 55, 80, 60, 95, 40, 5, 20} … I

Consider 5A + 5B =

5 ({3, 4, 5, 7, 0}) + ({10, 2, 6, 12, 1})
= {15, 20, 25, 35, 0} + {50, 10, 30, 60, 5}
= {65, 70, 75, 85, 50, 25, 30, 35, 45, 10, 55, 80, 95, 60, 20, 40, 5} … II

I and II are equal hence 5 (A + B) = 5A + 5B.

Other properties can be easily checked.

Clearly the number of elements in S is infinite.

We will later describe / define the linear independence, dependence, basis etc of the subset semivector spaces.

Example 2.2: Let S = {Collection of all subsets from the semigroup $Q^+ \cup \{0\}$, under +} be the subset semigroup. S is a subset semivector space defined over the semifield $F = Z^+ \cup \{0\}$.

We see the subset semivector space in example 2.1 is different from that of the one given in example 2.2. Infact the subset semivector space in example 2.1 is contained in the subset semivector space given in example 2.2.

It is also interesting to note that S the subset semivector space given in example 2.2 will continue to be a subset semivector space defined over the semifield $Q^+ \cup \{0\}$ however the subset semivector space defined in example 2.1 will not be a subset semivector space over the semifield $Q^+ \cup \{0\}$. For if A = {2, 5, 0} is in S of example 2.1 when we multiply by the scalar $1/7 \in Q^+ \cup \{0\}$ we see 1/7 A = 1/7 {2, 5, 0} = {2/7, 5/7, 0} $\notin$ S in example 2.1.

Thus we see in general a subset semivector space defined over a semifield may not continue to be a semivector space defined over every other or any other semifield.

We see the subset semivector space given in example 2.1 defined over the semifield $Z^+ \cup \{0\}$ is not a subset semivector space defined over the semifield $R^+ \cup \{0\}$.

For if A = {0, 9/2, 1, 15/11} $\in$ S in example 2.1, we see if $\sqrt{17}/\sqrt{13} \in R^+ \cup \{0\}$ then

$$\sqrt{17}/\sqrt{13} \times A = \sqrt{17}/\sqrt{13}\ \{0, 9/2, 15/11, 1\}$$

$$= \left\{0, \sqrt{17}/13, \frac{9\sqrt{17}}{2\sqrt{13}}, \frac{15\sqrt{17}}{15\sqrt{11}}\right\} \text{ is not in S.}$$

Hence the claim.

Example 2.3: Let S_1 = {Collection of all subsets from the semigroup $(Z^+ \cup \{0\})[x]$, under +} be the subset semigroup. S_1 is a subset semivector space over the semifield $F = Z^+ \cup \{0\}$.

For if $A = \{1 + 3x^2 + 5x^3, 9x^7 + 2, 10x^9 + 12\} \in S_1$, take $10 \in F$ we see $10A = 10 \times \{1 + 3x^2 + 5x^3, 9x^7 + 2, 10x^9 + 12\} = \{10 + 30x^2 + 50x^3, 90x^7 + 20, 100x^9 + 120\} \in S_1$.

However S_1 is not a subset semivector space over the semifield $Q^+ \cup \{0\}$ but S_1 is a subset semivector space over the semifield $(Z^+ \cup \{0\})[x]$.

Example 2.4: Let S_2 = {Collection of all subsets from the semigroup $(R^+ \cup \{0\})[x]$} be the subst semigroup. S_2 is a subset semivector space over the semifield $Q^+ \cup \{0\}$.

Example 2.5: Let S_3 = {Collection of all subsets from the semigroup $(Z^+ \cup \{0\})(g)$ where $g^2 = 0$ under, +} be a subset semigroup. S is a subset semivector space over the semifield $F = Z^+ \cup \{0\}$.

Example 2.6: Let S_4 = {Collection of all subsets from the semigroup $P = [Z^+ \cup \{0\}][x_1, x_2]$ where x_1, x_2 are indeterminates and the semigroup P is taken under '+'} be the subset semigroup.

S_4 is subset semivector space over the semifield $Z^+ \cup \{0\}$ (or over the semifield $(Z^+ \cup \{0\})[x_1]$ or over the semifield $[Z^+ \cup \{0\}][x_2]$ or over the semifield $Z^+ \cup \{0\}[x_1, x_2]$).

Thus we see subset semivector spaces can be defined over other semifields still the S_4 continues to be a subset semivector space.

Example 2.7: Let S_5 = {Collection of all subsets from the semigroup, $P = \{[a_1, a_2, a_3] \mid a_i \in Z^+ \cup \{0\}; 1 \leq i \leq 3\}$ under '+'} be the subset semigroup. S_5 is a subset semivector space over the semifield $F = Z^+ \cup \{0\}$.

Take

$A = \{(3, 0, 7), (9, 2, 0), (0, 0, 1), (1, 1, 5)\}$ and

$B = \{(8, 1, 1), (0, 0, 0), (1, 5, 0)\} \in S_5$.

We see $A + B = \{(3, 0, 7), (9, 2, 0), (0, 0, 1), (1, 1, 5)\} + \{(8, 1, 1), (0, 0, 0), (1, 5, 0)\}$

$= \{(11, 1, 8), (17, 3, 1), (8, 1, 2), (13, 2, 6), (1, 1, 5), (2, 6, 5), (3, 0, 7), (9, 2, 0), (0, 0, 1), (4, 5, 7), (10, 7, 0), (1, 5, 1)\} \in S_5$.

Now take $12 \in Z^+ \cup \{0\} = F$ and $A \in S$.

$12A = 12 \{(3, 0, 7), (9, 2, 0), (0, 0, 1), (1, 1, 5)\}$
$= \{(36, 0, 84), (108, 24, 0), (0, 0, 12), (12, 12, 60)\} \in S_5$.

This is the way operations on S_5 are performed. Infact we can call S_5 as the subset row matrix semivector space over the semifield F.

Example 2.8: Let S_6 = {Collection of all subsets from the column matrix semigroup

$$P_1 = \left\{\left.\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix}\right| a_i \in Q^+ \cup \{0\}; 1 \le i \le 5\right\}$$

be the semigroup under addition} be the subset column matrix semigroup. S_6 is a subset semivector space over the semifield $F = Q^+ \cup \{0\}$.

S_6 is also known as the subset column matrix semivector space over F.

Let

$$A = \left\{ \begin{bmatrix}0\\2\\5\\0\\1\end{bmatrix}, \begin{bmatrix}0\\0\\0\\1\\3\end{bmatrix}, \begin{bmatrix}8\\2\\0\\5\\0\end{bmatrix}, \begin{bmatrix}2\\0\\2\\0\\1\end{bmatrix} \right\} \text{ and } B = \left\{ \begin{bmatrix}4\\0\\2\\3\\0\end{bmatrix}, \begin{bmatrix}0\\7\\0\\8\\0\end{bmatrix}, \begin{bmatrix}1\\1\\1\\1\\1\end{bmatrix}, \begin{bmatrix}4\\4\\4\\4\\4\end{bmatrix} \right\} \in S_6.$$

$$A + B = \left\{ \begin{bmatrix}0\\2\\5\\0\\1\end{bmatrix}, \begin{bmatrix}0\\0\\0\\1\\3\end{bmatrix}, \begin{bmatrix}8\\2\\0\\5\\0\end{bmatrix}, \begin{bmatrix}2\\0\\2\\0\\1\end{bmatrix} \right\} + \left\{ \begin{bmatrix}4\\0\\2\\3\\0\end{bmatrix}, \begin{bmatrix}0\\7\\0\\8\\0\end{bmatrix}, \begin{bmatrix}1\\1\\1\\1\\1\end{bmatrix}, \begin{bmatrix}4\\4\\4\\4\\4\end{bmatrix} \right\}$$

$$= \left\{ \begin{bmatrix}4\\2\\7\\3\\1\end{bmatrix}, \begin{bmatrix}4\\0\\2\\4\\3\end{bmatrix}, \begin{bmatrix}12\\2\\2\\8\\0\end{bmatrix}, \begin{bmatrix}6\\0\\4\\3\\1\end{bmatrix}, \begin{bmatrix}0\\9\\5\\8\\1\end{bmatrix}, \begin{bmatrix}0\\7\\0\\9\\3\end{bmatrix}, \begin{bmatrix}8\\9\\0\\13\\0\end{bmatrix}, \begin{bmatrix}2\\7\\2\\8\\1\end{bmatrix}, \right.$$

$$\left. \begin{bmatrix}1\\3\\6\\1\\2\end{bmatrix}, \begin{bmatrix}1\\1\\1\\2\\4\end{bmatrix}, \begin{bmatrix}9\\3\\1\\6\\1\end{bmatrix}, \begin{bmatrix}3\\1\\3\\1\\2\end{bmatrix}, \begin{bmatrix}4\\6\\9\\4\\5\end{bmatrix}, \begin{bmatrix}4\\4\\4\\5\\7\end{bmatrix}, \begin{bmatrix}12\\6\\4\\9\\4\end{bmatrix}, \begin{bmatrix}6\\4\\6\\4\\5\end{bmatrix} \right\} \in S_6.$$

Take 5/3 $\in Q^+ \cup \{0\}$ = F we find

$$5/3 \times A = 5/3 \times \left\{ \begin{bmatrix} 0 \\ 2 \\ 5 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 8 \\ 2 \\ 0 \\ 5 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 2 \\ 0 \\ 1 \end{bmatrix} \right\}$$

$$= \left\{ \begin{bmatrix} 0 \\ 10/3 \\ 25/3 \\ 0 \\ 5/3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 5/3 \\ 5 \end{bmatrix}, \begin{bmatrix} 40/3 \\ 10/3 \\ 0 \\ 25/3 \\ 0 \end{bmatrix}, \begin{bmatrix} 10/3 \\ 0 \\ 10/3 \\ 0 \\ 5/3 \end{bmatrix} \right\} \in S_6.$$

S_6 is also a subset semivector space over $Z^+ \cup \{0\}$.

Example 2.9: Let S = {Collection of all subsets from the matrix semiring

$$M = \left\{ \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} \middle| \; a_i \in R^+ \cup \{0\}; 1 \leq i \leq 4 \right\}\}$$

be a subset semigroup. Clearly S is a subset semivector space over the semifield $F = Z^+ \cup \{0\}$.

Example 2.10: Let S_7 = {Collection of all subsets from the matrix semigroup

$$P_2 = \left\{ \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \\ \vdots & \vdots \\ a_{13} & a_{14} \end{bmatrix} \middle| \; a_i \in Q^+ \cup \{0\}; 1 \leq i \leq 14 \right\}\}$$

be the subset semigroup. Clearly S_7 is a subset semivector space over F = $Q^+ \cup \{0\}$.

Example 2.11: Let S = {Collection of all subsets from the matrix semigroup

$$P = \left\{\begin{bmatrix} a_1 & a_2 & \dots & a_5 \\ a_6 & a_7 & \dots & a_{10} \end{bmatrix} \middle| a_i \in Q^+ \cup \{0\}; 1 \le i \le 10\}\right\}$$

be the subset semigroup. S is a subset semivector space over the semifield $Q^+ \cup \{0\}$.

For take

$$A = \left\{\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 2 & 0 & 2 & 0 & 2 \\ 0 & 2 & 0 & 2 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 & 10 \end{bmatrix}\right\}$$

and

$$B = \left\{\begin{bmatrix} 9 & 4 & 0 & 2 & 0 \\ 0 & 0 & 2 & 0 & 5 \end{bmatrix}, \begin{bmatrix} 1 & 2 & 0 & 1 & 2 \\ 5 & 0 & 7 & 0 & 0 \end{bmatrix}\right\} \in S.$$

A + B =

$$\left\{\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 2 & 0 & 2 & 0 & 2 \\ 0 & 2 & 0 & 2 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 & 10 \end{bmatrix}\right\} +$$

$$\left\{\begin{bmatrix} 9 & 4 & 0 & 2 & 0 \\ 0 & 0 & 2 & 0 & 5 \end{bmatrix}, \begin{bmatrix} 1 & 2 & 0 & 1 & 2 \\ 5 & 0 & 7 & 0 & 0 \end{bmatrix}\right\}$$

$$= \left\{\begin{bmatrix} 10 & 5 & 1 & 3 & 1 \\ 0 & 0 & 2 & 0 & 5 \end{bmatrix}, \begin{bmatrix} 11 & 4 & 2 & 2 & 2 \\ 0 & 2 & 2 & 2 & 5 \end{bmatrix}, \begin{bmatrix} 10 & 6 & 3 & 6 & 5 \\ 6 & 7 & 10 & 9 & 15 \end{bmatrix},\right.$$

$$\left.\begin{bmatrix} 2 & 3 & 1 & 2 & 3 \\ 5 & 0 & 7 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 3 & 2 & 2 & 1 & 4 \\ 5 & 2 & 7 & 2 & 0 \end{bmatrix}, \begin{bmatrix} 2 & 4 & 3 & 5 & 7 \\ 11 & 7 & 15 & 9 & 10 \end{bmatrix}\right\} \in S.$$

This is the way operations are performed on the subset semivector space.

Take $1/11 \in F = Q^+ \cup \{0\}$ and $A \in S$.

$11/1 \times A$

$$=1/11\times\left\{\begin{bmatrix}1&1&1&1&1\\0&0&0&0&0\end{bmatrix},\begin{bmatrix}2&0&2&0&2\\0&2&0&2&0\end{bmatrix},\begin{bmatrix}1&2&3&4&5\\6&7&8&9&10\end{bmatrix}\right\}$$

$$=\left\{\begin{bmatrix}1/11&1/11&1/11&1/11&1/11\\0&0&0&0&0\end{bmatrix},\right.$$

$$\begin{bmatrix}2/11&0&2/11&0&2/11\\0&2/11&0&2/11&0\end{bmatrix},$$

$$\left.\begin{bmatrix}1/11&2/11&3/11&4/11&5/11\\6/11&7/11&8/11&9/11&10/11\end{bmatrix}\right\} \in S.$$

S is also a subset semivector space over the semifield $Z^+ \cup \{0\}$.

Example 2.12: Let S_1 = {Collection of all subsets from the matrix semigroup

$$P = \left\{\begin{bmatrix}a_1&a_2&a_3&a_4\\a_5&a_6&a_7&a_8\\a_9&a_{10}&a_{11}&a_{12}\\a_{13}&a_{14}&a_{15}&a_{16}\end{bmatrix} \middle| \; a_i \in Q^+ \cup \{0\}; 1 \leq i \leq 16\right\}\}$$

be the subset semigroup.

S_1 is a subset semivector space over the semifield $F = Z^+ \cup \{0\}$.

For take

$$A = \left\{ \begin{bmatrix} 0 & 1 & 2 & 3 \\ 0 & 0 & 2 & 3 \\ 1 & 0 & 0 & 0 \\ 2 & 3 & 1 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 2 & 0 & 0 \\ 1 & 0 & 3 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 & 1 \\ 2 & 0 & 2 & 0 \\ 0 & 3 & 0 & 3 \\ 4 & 0 & 4 & 0 \end{bmatrix} \right\}$$

$$\text{and } B = \left\{ \begin{bmatrix} 0 & 1 & 6 & 1 \\ 0 & 0 & 2 & 3 \\ 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 2 & 4 & 1 \\ 3 & 1 & 3 & 0 \\ 1 & 0 & 0 & 1 \\ 4 & 1 & 1 & 0 \end{bmatrix} \right\} \in S_1.$$

We see

$$A + B = \left\{ \begin{bmatrix} 0 & 1 & 2 & 3 \\ 0 & 0 & 2 & 3 \\ 1 & 0 & 0 & 0 \\ 2 & 3 & 1 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 2 & 0 & 0 \\ 1 & 0 & 3 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 & 1 \\ 2 & 0 & 2 & 0 \\ 0 & 3 & 0 & 3 \\ 4 & 0 & 4 & 0 \end{bmatrix} \right\} +$$

$$\left\{ \begin{bmatrix} 0 & 1 & 6 & 1 \\ 0 & 0 & 2 & 3 \\ 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 2 & 4 & 1 \\ 3 & 1 & 3 & 0 \\ 1 & 0 & 0 & 1 \\ 4 & 1 & 1 & 0 \end{bmatrix} \right\}$$

$$= \left\{ \begin{bmatrix} 0 & 2 & 8 & 7 \\ 0 & 0 & 4 & 6 \\ 1 & 0 & 0 & 5 \\ 2 & 3 & 1 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 2 & 6 & 2 \\ 0 & 2 & 2 & 3 \\ 1 & 0 & 3 & 5 \\ 0 & 0 & 0 & 5 \end{bmatrix}, \begin{bmatrix} 0 & 2 & 6 & 2 \\ 2 & 0 & 4 & 3 \\ 0 & 3 & 0 & 8 \\ 4 & 0 & 4 & 0 \end{bmatrix}, \right.$$

$$\left. \begin{bmatrix} 1 & 3 & 6 & 4 \\ 3 & 1 & 5 & 3 \\ 2 & 0 & 0 & 1 \\ 6 & 4 & 2 & 2 \end{bmatrix}, \begin{bmatrix} 2 & 3 & 4 & 2 \\ 3 & 3 & 3 & 0 \\ 2 & 0 & 3 & 1 \\ 4 & 1 & 1 & 5 \end{bmatrix}, \begin{bmatrix} 1 & 3 & 4 & 2 \\ 5 & 1 & 5 & 0 \\ 1 & 3 & 0 & 4 \\ 8 & 1 & 5 & 0 \end{bmatrix} \right\} \in S_1.$$

Also $12 \in Z^+ \cup \{0\}$; $12 \times A =$

$$12 \times \left\{ \begin{bmatrix} 0 & 1 & 2 & 3 \\ 0 & 0 & 2 & 3 \\ 1 & 0 & 0 & 0 \\ 2 & 3 & 1 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 2 & 0 & 0 \\ 1 & 0 & 3 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 & 1 \\ 2 & 0 & 2 & 0 \\ 0 & 3 & 0 & 3 \\ 4 & 0 & 4 & 0 \end{bmatrix} \right\}$$

$$= \left\{ \begin{bmatrix} 0 & 12 & 24 & 36 \\ 0 & 0 & 24 & 36 \\ 12 & 0 & 0 & 0 \\ 24 & 36 & 12 & 24 \end{bmatrix}, \begin{bmatrix} 12 & 12 & 0 & 12 \\ 0 & 24 & 0 & 0 \\ 12 & 0 & 36 & 0 \\ 0 & 0 & 0 & 60 \end{bmatrix}, \begin{bmatrix} 0 & 12 & 0 & 12 \\ 24 & 0 & 24 & 0 \\ 0 & 36 & 0 & 36 \\ 48 & 0 & 48 & 0 \end{bmatrix} \right\} \in S_1.$$

This is the way operations are performed on S_1 as a subset semivector space.

Example 2.13: Let S = {Collection of all subsets from the semigroup $\langle Z^+ \cup I \rangle \cup \{0\}$} be the subset semigroup. S is a subset semivector space over the semifield $F = Z^+ \cup \{0\}$.

Let A = {3I + 2, 5I, 10I + 1, 20, 0, 1, 1+2I} and
B = {7I + 8, 10I, 7} $\in$ S.

We see

A + B = {3I + 2, 5I, 10I + 1, 20, 1, 0, 1 + 2I} + {7I + 8, 10 I, 7}

= {10I + 10, 12I + 8, 17I + 9, 7I + 28, 7I + 9, 9I + 9, 7I + 8, 13I + 2, 15I, 20I + 1, 10I + 20, 10I + 1, 10I, 12I + 1, 3I+9, 5I + 7, 10I + 8, 27, 8, 7, 8 + 2I} $\in$ S.

Suppose 12 $\in$ F = $Z^+ \cup \{0\}$ then
12 × A = 12 {3I + 2, 5I, 10I + 1, 20, 0, 1, 1+2I}
= {36 + 24, 60I, 120I + 12, 240, 0, 12, 12 + 24I} $\in$ S.

This is the way operations are performed on S. S will also be known as the subset neutrosophic semivector space.

We can use the neutrosophic semifields $\langle Z^+ \cup I\rangle \cup \{0\}$ to construct subset neutrosophic semivector spaces.

We will give an example or a two before we proceed on to develop substructure property of these structures.

Example 2.14: Let S = {Collection of all subsets from the neutrosophic polynomial semigroup

$$P = \left\{ \sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in \langle Q^+ \cup \{0\} \cup I\rangle [x] \right\}\}$$

be the subset neutrosophic polynomial semigroup.

S is a subset neutrosophic semivector space over the semifield $Q^+ \cup \{0\}$.

If we consider S as a subset neutrosophic semivector space over the neutrosophic semifield then we define S to be a subset strong neutrosophic semivector space over the neutrosophic semifield $\langle Q^+ \cup I \cup \{0\}\rangle$ or $\langle Z^+ \cup I \cup \{0\}\rangle$.

Example 2.15: Let S = {Collection of all subsets from the neutrosophic semigroup $(\langle Q^+ \cup I\rangle \cup \{0\} \times \langle Z^+ \cup I\rangle \cup \{0\} \times \langle R^+ \cup I\rangle \cup \{0\})\}$ be the subset neutrosophic semigroup. S is a subset neutrosophic semivector space over $Z^+ \cup \{0\}$ and S is a strong neutrosophic subset semivector space over the neutrosophic semifield $\langle Z^+ \cup I \cup \{0\}\rangle$.

However S is not a subset neutrosophic semivector space over $Q^+ \cup \{0\}$ or $R^+ \cup \{0\}$ and is not a strong subset neutrosophic semivector space over $\langle Q^+ \cup I \cup \{0\}\rangle$ or $\langle R^+ \cup \{0\} \cup I\rangle$.

Example 2.16: Let S = {Collection of all subsets from the neutrosophic row matrix semigroup $P = \{(x_1, x_2, \ldots, x_9) \mid x_i \in (Z^+ \cup I \cup \{0\}), 1 \leq i \leq 9\}\}$ be the subset neutrosophic semigroup.

S is a subset neutrosophic semivector space over the semifield $Z^+ \cup \{0\}$ and is a strong subset neutrosophic semivector space over the neutrosophic semifield $\langle Z^+ \cup I \cup \{0\}\rangle$.

Example 2.17: Let S = {Collection of all subsets from the neutrosophic 3 × 8 matrix semigroup

$$P = \left\{\left.\begin{bmatrix} a_1 & a_2 & \ldots & a_8 \\ a_9 & a_{10} & \ldots & a_{16} \\ a_{17} & a_{18} & \ldots & a_{24} \end{bmatrix}\right| a_i \in \langle Q^+ \cup I \cup \{0\}\rangle; 1 \leq i \leq 24\}\right\}$$

be the subset neutrosophic matrix semivector space over the semifield $Z^+ \cup \{0\}$ (or $Q^+ \cup \{0\}$).

S can also be defined as a strong subset neutrosophic semivector space over the neutrosophic semifield $\langle Q^+ \cup I \cup \{0\}\rangle$ (or $\langle Z^+ \cup I \cup \{0\}\rangle$).

Example 2.18: Let S = {Collection of all subsets from the column neutrosophic matrix semigroup

$$P = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_{12} \end{bmatrix} \middle| a_i \in \langle R^+ \cup I \cup \{0\}\rangle; 1 \leq i \leq 12 \right\}\}$$

be the subset neutrosophic semivector space over semifield $Q^+ \cup \{0\}$ (or $Z^+ \cup \{0\}$ or $R^+ \cup \{0\}$).

S will be a strong subset neutrosophic column matrix semivector space over the neutrosophic semifield $\langle Q^+ \cup I \cup \{0\}\rangle$ (or $\langle Z^+ \cup I \cup \{0\}\rangle$ or $\langle R^+ \cup I \cup \{0\}\rangle$).

Example 2.19: Let S = {Collection of all subsets from the 8 × 4 matrix neutrosophic semigroup

$$M = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ \vdots & \vdots & \vdots & \vdots \\ a_{29} & a_{30} & a_{31} & a_{32} \end{bmatrix} \middle| a_i \in \langle Q^+ \cup I \cup \{0\}\rangle; 1 \leq i \leq 32 \right\}\}$$

be the subset neutrosophic matrix semivector space over the semifield $Z^+ \cup \{0\}$ (or $Q^+ \cup \{0\}$). S will be a strong subset neutrosophic matrix semivector space over the neutrosophic semifield $\langle Z^+ \cup I \cup \{0\}\rangle$ (or $\langle Q^+ \cup I \cup \{0\}\rangle$).

Example 2.20: Let S = {Collection of all subsets from the 5 × 5 neutrosophic matrix semiring

$$M = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \end{bmatrix} \middle| \, a_i \in \langle Z^+ \cup I \cup \{0\}\rangle; \; 1 \leq i \leq 25 \right\}\}$$

be the subset neutrosophic matrix semigroup. S is a subset neutrosophic matrix semivector space over the semifield $Z^+ \cup \{0\}$ or S can be realized as the subset strong neutrosophic matrix semivector space over the neutrosophic semifield $(\langle Z^+ \cup I \cup \{0\}\rangle)$.

Example 2.21: Let S = {Collection of all subsets from the group neutrosophic semigroup $(\langle Z^+ \cup I \cup \{0\}\rangle)S_3$ under the operation '+'} be the subset neutrosophic semigroup. S is a subset neutrosophic semivector space over the semifield $Z^+ \cup \{0\}$. Further S is also a subset strong neutrosophic semivector space over the neutrosophic semifield $\langle Z^+ \cup I \cup \{0\}\rangle$.

Example 2.22: Let S = {Collection of all subsets from the neutrosophic semigroup $(\langle Q^+ \cup I \cup \{0\}\rangle)S(3)\}$ under '+'} be the subset neutrosophic semigroup. S is a subset neutrosophic semivector space over the semifield $Q^+ \cup \{0\}$ (or $Z^+ \cup \{0\}$). S is also a strong subset neutrosophic semivector space over the neutrosophic semifield $\langle Q^+ \cup \{0\} \cup I\rangle$ (or $\langle Z^+ \cup I \cup \{0\}\rangle$).

Now having seen examples of subset semivector spaces of different types now we just give examples using subset super matrix semigroups.

Example 2.23: Let S = {Collection of all subsets from the row super matrix semigroup $M = \{(a_1\ a_2\ a_3 \mid a_4 \mid a_5\ a_6 \mid a_7) \mid a_i \in Z^+ \cup \{0\};\ 1 \leq i \leq 7\}\}$ be the subset row super matrix semigroup under +. S is the subset super row matrix semivector space over the semifield $Z^+ \cup \{0\}$.

Let A = {(0 0 0 | 1 | 5 6 | 2), (1 1 1 | 0 | 2 1 | 5), (2 3 4 | 0 | 5 0 | 0), (1 2 3 | 11 | 6 6 | 2)} and B = {(1 2 3 | 4 | 5 6 | 7)} $\in$ S.

We now find

A + B = {(0 0 0 | 1 | 5 6 | 2), (1 1 1 | 0 | 2 1 | 5),
(2 3 4 | 0 | 5 0 | 0), (1 2 3 | 11 | 6 6 | 2)} +
{(1 2 3 | 4 | 5 6 | 7)}

= {(1 2 3 | 5 | 10, 12 | 9), (2 3 4 | 4 | 7 7 | 12),
(3 5 7 | 4 | 10, 6 | 7), (2 4 6 | 15 | 11 12 | 9)} $\in$ S.

This is the way operations are performed on S. Suppose 30 $\in Z^+ \cup \{0\}$ we find

30 × A = 30 × {(0 0 0 | 1 | 5 6 | 2), (1 1 1 | 0 | 2 1 | 5),
(2 3 4 | 0 | 5 0 | 0), (1 2 3 | 11 | 6 6 | 2)}

= {(0 0 0 | 30 | 150 180 | 60), (30 30 30 | 0 | 60 30
| 150), (60 90 120 | 0 | 150 0 | 0), (30 60 90 |
330 | 180 180 | 60)} $\in$ S.

Thus we have super matrix subset semivector spaces.

Example 2.24: Let S = {Collection of all subsets from the super row matrix semigroup

$$M = \left\{ \left(\begin{array}{cc|c|cccc} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 \\ a_8 & a_9 & a_{10} & a_{11} & a_{12} & a_{13} & a_{14} \\ a_{15} & a_{16} & a_{17} & a_{18} & a_{19} & a_{20} & a_{21} \end{array} \right) \middle| \; a_i \in Q^+ \cup \{0\}; \; 1 \leq i \leq 21 \right\}\}$$

be the subset super row matrix semigroup.

S is a subset super row matrix semivector space over the semiifield $Q^+ \cup \{0\}$ (or $Z^+ \cup \{0\}$).

We develop these concepts, subset super semivector spaces over semifields.

Example 2.25: Let S = {Collection of all subsets from the super column matrix semigroup

$$P = \left\{ \left[\begin{array}{c} a_1 \\ \hline a_2 \\ a_3 \\ \hline a_4 \\ a_5 \\ a_6 \\ \hline a_7 \\ a_8 \\ a_9 \\ a_{10} \end{array} \right] \middle| \; a_i \in Q^+ \cup \{0\}; 1 \leq i \leq 10 \right\}\}$$

be the subset super column matrix semigroup.

S is a subset super column matrix semivector space over the semifield $Q^+ \cup \{0\}$ (or $Z^+ \cup \{0\}$).

$$\text{Let } A = \left\{ \left[\begin{array}{c} 1 \\ \hline 2 \\ 0 \\ \hline 3 \\ 4 \\ 0 \\ \hline 5 \\ 0 \\ 6 \\ 0 \end{array} \right], \left[\begin{array}{c} 0 \\ \hline 1 \\ 0 \\ \hline 2 \\ 2 \\ 6 \\ \hline 0 \\ 0 \\ 4 \\ 5 \end{array} \right], \left[\begin{array}{c} 2 \\ \hline 0 \\ 3 \\ \hline 1 \\ 0 \\ 2 \\ \hline 4 \\ 5 \\ 0 \\ 1 \end{array} \right] \right\} \text{ and } B = \left\{ \left[\begin{array}{c} 0 \\ \hline 1 \\ 2 \\ \hline 3 \\ 4 \\ 5 \\ \hline 0 \\ 3 \\ 1 \\ 0 \end{array} \right], \left[\begin{array}{c} 4 \\ \hline 0 \\ 2 \\ \hline 1 \\ 2 \\ 0 \\ \hline 6 \\ 0 \\ 3 \\ 4 \end{array} \right] \right\} \in S.$$

We find first

$$A + B = \left\{ \begin{bmatrix} 1 \\ \hline 2 \\ 0 \\ \hline 3 \\ 4 \\ 0 \\ \hline 5 \\ 0 \\ 6 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ \hline 1 \\ 0 \\ \hline 2 \\ 2 \\ 6 \\ \hline 0 \\ 0 \\ 4 \\ 5 \end{bmatrix}, \begin{bmatrix} 2 \\ \hline 0 \\ 3 \\ \hline 1 \\ 0 \\ 2 \\ \hline 4 \\ 5 \\ 0 \\ 1 \end{bmatrix} \right\} + \left\{ \begin{bmatrix} 0 \\ \hline 1 \\ 2 \\ \hline 3 \\ 4 \\ 5 \\ \hline 0 \\ 3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ \hline 0 \\ 2 \\ \hline 1 \\ 2 \\ 0 \\ \hline 6 \\ 0 \\ 3 \\ 4 \end{bmatrix} \right\}$$

$$= \left\{ \begin{bmatrix} 1 \\ \hline 3 \\ 2 \\ \hline 6 \\ 8 \\ 5 \\ \hline 5 \\ 3 \\ 7 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ \hline 2 \\ 2 \\ \hline 5 \\ 6 \\ 11 \\ \hline 0 \\ 3 \\ 5 \\ 5 \end{bmatrix}, \begin{bmatrix} 2 \\ \hline 1 \\ 5 \\ \hline 4 \\ 4 \\ 7 \\ \hline 4 \\ 8 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ \hline 2 \\ 2 \\ \hline 4 \\ 6 \\ 0 \\ \hline 11 \\ 0 \\ 9 \\ 4 \end{bmatrix}, \begin{bmatrix} 4 \\ \hline 1 \\ 2 \\ \hline 3 \\ 4 \\ 6 \\ \hline 6 \\ 0 \\ 7 \\ 9 \end{bmatrix}, \begin{bmatrix} 6 \\ \hline 0 \\ 5 \\ \hline 2 \\ 2 \\ 2 \\ \hline 10 \\ 5 \\ 3 \\ 5 \end{bmatrix} \right\} \in S.$$

We find for

$8/7 \in Q^+ \cup \{0\}$; $8/7 \times A$

$$= 8/7 \times \left\{ \begin{bmatrix} 1 \\ \hline 2 \\ 0 \\ \hline 3 \\ 4 \\ 0 \\ \hline 5 \\ 0 \\ 6 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ \hline 1 \\ 0 \\ \hline 2 \\ 2 \\ 6 \\ \hline 0 \\ 0 \\ 4 \\ 5 \end{bmatrix}, \begin{bmatrix} 2 \\ \hline 0 \\ 3 \\ \hline 1 \\ 0 \\ 2 \\ \hline 4 \\ 5 \\ 0 \\ 1 \end{bmatrix} \right\}$$

$$= \left\{ \begin{bmatrix} 8/7 \\ \hline 16/7 \\ 0 \\ \hline 24/7 \\ 32/7 \\ 0 \\ \hline 40/7 \\ 0 \\ 48/7 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ \hline 8/7 \\ 0 \\ \hline 16/7 \\ 16/7 \\ 48/7 \\ \hline 0 \\ 0 \\ 32/7 \\ 40/7 \end{bmatrix}, \begin{bmatrix} 16/7 \\ \hline 0 \\ 24/7 \\ \hline 8/7 \\ 0 \\ 16/7 \\ \hline 32/7 \\ 40/7 \\ 0 \\ 8/7 \end{bmatrix} \right\} \in S.$$

It is easily verified S is a subset super column matrix semivector space over the semifield $Q^+ \cup \{0\}$.

Example 2.26: Let S = {Collection of all subsets from the super column matrix semiring

$$M = \left\{ \left[\begin{array}{ccc} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ \hline a_7 & a_8 & a_9 \\ \hline a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & a_{18} \\ \hline a_{19} & a_{20} & a_{21} \\ a_{22} & a_{23} & a_{24} \\ a_{25} & a_{26} & a_{27} \\ a_{28} & a_{29} & a_{30} \\ a_{31} & a_{32} & a_{33} \end{array} \right] \middle| \; a_i \in Z^+ \cup \{0\}; 1 \leq i \leq 33 \right\}$$

be the subset super column matrix semigroup. S is a subset super column matrix semivector space over semifield $Z^+ \cup \{0\}$ (or $Q^+ \cup \{0\}$).

Example 2.27: Let S = {Collection of all subsets from the super matrix semigroup under addition

$$P = \left\{ \left[\begin{array}{cc|cccc|c} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 \\ \hline a_8 & a_9 & a_{10} & a_{11} & a_{12} & a_{13} & a_{14} \\ a_{15} & a_{16} & a_{17} & a_{18} & a_{19} & a_{20} & a_{21} \\ \hline a_{22} & a_{23} & a_{24} & a_{25} & a_{26} & a_{27} & a_{28} \\ a_{29} & a_{30} & a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{36} & a_{37} & a_{38} & a_{39} & a_{40} & a_{41} & a_{42} \\ a_{43} & a_{44} & a_{45} & a_{46} & a_{47} & a_{48} & a_{49} \end{array} \right] \middle| \; a_i \in Q^+ \cup \{0\}; \right.$$

$$\left. 1 \leq i \leq 49 \right\}\}$$

be the subset semigroup of super matrices.

S is a subset super matrix semivector space over the semifield $Q^+ \cup \{0\}$.

Example 2.28: Let S = {Collection of all subsets from the super matrix

$$M = \left\{ \left[\begin{array}{cc|cccc} a_1 & a_2 & & (0) & & \\ a_3 & a_4 & & & & \\ \hline & & a_5 & a_6 & a_7 & a_8 \\ & & a_9 & a_{10} & a_{11} & a_{12} \\ (0) & & a_{13} & a_{14} & a_{15} & a_{16} \\ & & a_{17} & a_{18} & a_{19} & a_{20} \end{array} \right] \right| a_i \in Z^+ \cup \{0\};$$

$1 \leq i \leq 20\}\}$

be the subset super matrix semigroup.

S is a subset super matrix semivector space over the semifield F = $Z^+ \cup \{0\}$.

We see these subset super matrices are very much useful in the study of semivector spaces.

We see this study can also be from the neutrosophic semifields like $\langle Q^+ \cup \{0\} \cup I\rangle$, $\langle Z^+ \cup \{0\} \cup I\rangle$ and $\langle R^+ \cup I \cup \{0\}\rangle$. Strong subset neutrosophic semivector spaces can be got over neutrosophic semifields.

Next we proceed onto study the notion subset semilinear algebra over the semifield.

We give examples of them.

Example 2.29: Let S = {Collection of all subsets from the semigroup P = $(Z^+ \cup \{0\})[x]$} be the subset semigroup. S is a subset semivector space over the semifield $Z^+ \cup \{0\}$, we can give a operation product on P so that S becomes closed under the product $\times$, so S can be a subset semilinear algebra over the semifield $Z^+ \cup \{0\}$.

For if A = $\{9x^2 + 3x + 1, 6x^3 + 5x^2 + 8, 2x^3 + 1, 8x^7\}$ and B = $\{10x^2, 11x + 1, 9x^3 + 4\} \in S$.

We see

$$A + B = \{9x^2 + 3x + 1, 6x^3 + 5x^2 + 8, 2x^3 + 1, 8x^7\} + \{10x^2, 11x + 1, 9x^3 + 4\}$$

$$= \{19x^2 + 3x + 1, 6x^3 + 15x^2 + 8, 2x^3 + 10x^2 + 1, 8x^7 + 10x^2, 9x^2 + 14x + 2, 6x^3 + 5x^2 + 11x + 9, 2x^3 + 11x + 2, 8x^7 + 11x + 1, 9x^3 + 9x^2 + 3x + 5, 15x^3 + 5x^2 + 12, 11x^3 + 5, 8x^7 + 9x^3 + 4\} \in S.$$

We find

$$A \times B = \{9x^2 + 3x + 1, 6x^3 + 5x^2 + 8, 2x^3 + 1, 8x^7\} \times \{10x^2, 11x + 1, 9x^3 + 4\}$$

$$= \{90x^4 + 30x^3 + 10x^2, 60x^5 + 50x^4 + 80x^2, 20x^5 + 10x^2, 80x^9, 99x^3 + 33x^2 + 11x + 9x^2 + 3x + 1, 66x^4 + 55x^3 + 88x + 6x^3 + 5x^2 + 8, 22x^4 + 11x + 2x^3 + 1, 88x^8 + 8x^7, 81x^5 + 27x^4 + 36x^2 + 9x^3 + 12x + 4, 54x^6 + 45x^5 + 72x^3 + 24x^3 + 20x^2 + 32, 72x^{10} + 32x^7 + 9x^3 + 4 + 18x^6 + 8x^3\} \in S.$$

Thus we have seen examples of subset semilinear algebra over the semifield.

Example 2.30: Let
S = {Collection of all subsets from the semigroup $Q^+ \cup \{0\}$} be the subset semigroup. S is a subset semivector space over the semifield $Z^+ \cup \{0\}$.

We see $Q^+ \cup \{0\}$ is also a semigroup under $\times$ also.

Let A = {3, 8/7, 10, 43/2, 2/11} and
B = {0, 2, 7, 1, 23/5} $\in$ S

$$A \times B = \{3, 8/7, 10, 43/2, 2/11\} \times \{0, 2, 7, 1, 23/5\}$$

$= \{0, 6, 16/7, 20, 43, 4/11, 21, 8, 70, 301/2, 14/11, 3, 8/7, 10, 43/2, 2/11, 69/5, 184/35, 46, 43 \times 23/10, 46/55\} \in S.$

It is easily verified S is a subset semilinear algebra over the semifield.

Example 2.31: Let S = {Collection of all subsets from the semigroup M = $\{(a, b) \mid a, b \in Z^+ \cup \{0\}\}$} be the subset semigroup. S is a subset semivector space over the semifield.

We see S can be made into a subset semilinear algebra over the semifield $Z^+ \cup \{0\}$.

Let A = {(0, 9), (6, 8), (11, 2), (3, 10)} and
B = {(14, 0), (2, 3), (1, 4), (5, 2)} $\in$ S.

We find

$A \times B = \{(0, 9), (6, 8), (11, 2), (3, 10)\} \times \{(14, 0), (1, 4), (2, 3), (5, 2)\}$

$= \{(0, 0), (84, 0), (154, 0), (42, 0), (0, 36), (6, 32), (11, 8), (3, 40), (0, 27), (12, 24), (22, 6), (6, 30), (0, 18), (30, 16), (55, 4), (15, 20)\} \in S.$

S is a subset semilinear algebra over the semifield.

Example 2.32: Let S = {Collection of all subsets from the column matrix semigroup

$$\left\{\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix} \middle| \; a_i \in Q^+ \cup \{0\}; 1 \le i \le 6\right\}\}$$

be the subset semigroup. S is also a subset column matrix semivector space over the semifield $Q^+ \cup \{0\}$.

Infact S is a subset column matrix semilinear algebra over $Q^+ \cup \{0\}$ under natural product. Let A, B $\in$ S where

$$A = \left\{\begin{bmatrix}3\\2\\5\\0\\1\\2\end{bmatrix}, \begin{bmatrix}1\\2\\3\\4\\5\\0\end{bmatrix}, \begin{bmatrix}0\\1\\2\\0\\0\\0\end{bmatrix}, \begin{bmatrix}1\\0\\2\\0\\3\\0\end{bmatrix}\right\} \text{ and } B = \left\{\begin{bmatrix}0\\1\\2\\0\\0\\4\end{bmatrix}, \begin{bmatrix}4\\0\\10\\0\\0\\7\end{bmatrix}, \begin{bmatrix}5\\2\\1\\0\\0\\07\end{bmatrix}\right\} \text{ are in S.}$$

$$\text{Now } A \times_n B = \left\{\begin{bmatrix}3\\2\\5\\0\\1\\2\end{bmatrix}, \begin{bmatrix}1\\2\\3\\4\\5\\0\end{bmatrix}, \begin{bmatrix}0\\1\\2\\0\\0\\0\end{bmatrix}, \begin{bmatrix}1\\0\\2\\0\\3\\0\end{bmatrix}\right\} \times_n \left\{\begin{bmatrix}0\\1\\2\\0\\0\\4\end{bmatrix}, \begin{bmatrix}4\\0\\10\\0\\0\\7\end{bmatrix}, \begin{bmatrix}5\\2\\1\\0\\0\\7\end{bmatrix}\right\}$$

$$= \left\{\begin{bmatrix}0\\2\\10\\0\\0\\8\end{bmatrix}, \begin{bmatrix}0\\2\\6\\0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\4\\0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\4\\0\\0\\0\end{bmatrix}, \begin{bmatrix}12\\0\\50\\0\\0\\4\end{bmatrix}, \begin{bmatrix}4\\0\\30\\0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\20\\0\\0\\0\end{bmatrix}, \begin{bmatrix}4\\0\\20\\0\\0\\0\end{bmatrix}, \right.$$

$$\left.\begin{bmatrix}15\\4\\5\\0\\0\\14\end{bmatrix},\begin{bmatrix}5\\4\\3\\0\\0\\0\end{bmatrix},\begin{bmatrix}0\\2\\2\\0\\0\\0\end{bmatrix},\begin{bmatrix}5\\0\\2\\0\\0\\0\end{bmatrix}\right\} \in S.$$

Thus (S, +, $\times_n$) is a subset semilinear algebra over the semifield $Q^+ \cup \{0\}$.

Example 2.33: Let S = {Collection of all subsets from the matrix semigroup

$$M = \left\{\begin{bmatrix}a_1 & a_2 & a_3 & a_4 & a_5\\ a_6 & a_7 & a_8 & a_9 & a_{10}\end{bmatrix} \middle| \, a_i \in Z^+ \cup \{0\}; 1 \leq i \leq 10\right\}\}$$

be the subset matrix semigroup. S is a subset matrix semivector space which is also a subset matrix semilinear algebra under the natural product $\times_n$ over the semifield $F = Z^+ \cup \{0\}$.

Clearly S is a commutative subset semilinear algebra over F.

Let A, B $\in$ S where

$$A = \left\{\begin{bmatrix}2 & 0 & 1 & 0 & 5\\ 0 & 4 & 0 & 7 & 0\end{bmatrix},\begin{bmatrix}0 & 0 & 0 & 0 & 0\\ 1 & 2 & 3 & 0 & 4\end{bmatrix},\right.$$

$$\left.\begin{bmatrix}1 & 1 & 1 & 2 & 2\\ 0 & 0 & 0 & 0 & 0\end{bmatrix},\begin{bmatrix}1 & 2 & 0 & 0 & 0\\ 0 & 0 & 0 & 2 & 1\end{bmatrix}\right\} \text{ and}$$

$$B = \left\{\begin{bmatrix}1 & 2 & 0 & 0 & 0\\ 0 & 1 & 2 & 0 & 0\end{bmatrix},\begin{bmatrix}1 & 1 & 1 & 1 & 1\\ 0 & 0 & 0 & 0 & 0\end{bmatrix},\begin{bmatrix}0 & 0 & 0 & 0 & 0\\ 1 & 1 & 1 & 1 & 1\end{bmatrix}\right\} \in S.$$

$$A \times_n B = \left\{\begin{bmatrix}2&0&1&0&5\\0&4&0&7&0\end{bmatrix},\begin{bmatrix}0&0&0&0&0\\1&2&3&0&4\end{bmatrix},\right.$$

$$\left.\begin{bmatrix}1&1&1&2&2\\0&0&0&0&0\end{bmatrix},\begin{bmatrix}1&2&0&0&0\\0&0&0&2&1\end{bmatrix}\right\} \times$$

$$\left\{\begin{bmatrix}1&2&0&0&0\\0&1&2&0&0\end{bmatrix},\begin{bmatrix}1&1&1&1&1\\0&0&0&0&0\end{bmatrix},\begin{bmatrix}0&0&0&0&0\\1&1&1&1&1\end{bmatrix}\right\}$$

$$= \left\{\begin{bmatrix}2&0&0&0&0\\0&4&0&0&0\end{bmatrix},\begin{bmatrix}0&0&0&0&0\\0&2&6&0&0\end{bmatrix},\begin{bmatrix}1&2&0&0&0\\0&0&0&0&0\end{bmatrix},\right.$$

$$\begin{bmatrix}1&4&0&0&0\\0&0&0&0&0\end{bmatrix},\begin{bmatrix}2&0&1&0&5\\0&0&0&0&0\end{bmatrix},\begin{bmatrix}0&0&0&0&0\\0&0&0&0&0\end{bmatrix},$$

$$\begin{bmatrix}1&1&1&2&2\\0&0&0&0&0\end{bmatrix},\begin{bmatrix}1&2&0&0&0\\0&0&0&0&0\end{bmatrix},\begin{bmatrix}0&0&0&0&0\\0&4&0&7&0\end{bmatrix},$$

$$\left.\begin{bmatrix}0&0&0&0&0\\1&2&3&0&4\end{bmatrix},\begin{bmatrix}0&0&0&0&0\\0&0&0&2&1\end{bmatrix}\right\} \in S.$$

We see S is a subset matrix semilinear algebra over the semifield $Z^+ \cup \{0\}$.

Example 2.34: Let S = {Collection of all subsets from the matrix semigroup

$$M = \left\{\left.\begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \end{bmatrix}\right| a_i \in Q^+ \cup \{0\}; 1 \leq i \leq 16\}\right\}$$

be the subset matrix semigroup. Clearly S is a subset matrix semivector space over the semifield F = $Z^+ \cup \{0\}$.

We can define two products on S the natural product $\times_n$ and the usual product $\times$ on S. Both are different. Thus under $\times$, S is a non commutative subset matrix semilinear algebra over F and however under $\times_n$ S is a commutative subset matrix semilinear algebra.

$$\text{Let } A = \left\{\begin{bmatrix} 0 & 0 & 2 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 \\ 4 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 2 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & 4 \\ 0 & 6 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\right\} \text{ and}$$

$$B = \left\{\begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 0 & 4 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}\right\} \in S.$$

We find

$$A \times B = \left\{\begin{bmatrix} 0 & 0 & 2 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 \\ 4 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 2 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & 4 \\ 0 & 6 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\right\} \times$$

$$\left\{\begin{bmatrix}0&0&0&0\\1&0&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix},\begin{bmatrix}2&0&4&0\\0&1&0&2\\0&0&0&1\\0&1&1&0\end{bmatrix}\right\}$$

$$= \left\{\begin{bmatrix}0&0&2&0\\1&0&0&2\\0&0&0&1\\0&0&0&0\end{bmatrix},\begin{bmatrix}0&0&0&2\\0&3&2&2\\0&1&1&0\\8&0&16&0\end{bmatrix},\begin{bmatrix}0&0&0&0\\3&0&0&0\\0&0&0&1\\2&0&0&0\end{bmatrix},\right.$$

$$\left.\begin{bmatrix}4&0&8&0\\0&3&0&6\\0&1&1&0\\2&2&4&4\end{bmatrix},\begin{bmatrix}0&0&0&4\\6&0&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix},\begin{bmatrix}0&4&4&0\\0&6&0&12\\4&0&8&0\\0&0&0&0\end{bmatrix}\right\} \quad \ldots \text{I}$$

$$B \times A = \left\{\begin{bmatrix}0&0&0&0\\1&0&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix},\begin{bmatrix}2&0&4&0\\0&1&0&2\\0&0&0&1\\0&1&1&0\end{bmatrix}\right\} \times$$

$$\left\{\begin{bmatrix}0&0&2&0\\0&1&0&2\\0&0&0&1\\4&0&0&0\end{bmatrix},\begin{bmatrix}2&0&0&0\\0&3&0&0\\0&0&0&1\\1&2&0&0\end{bmatrix},\begin{bmatrix}0&0&0&4\\0&6&0&0\\2&0&0&0\\0&0&0&0\end{bmatrix}\right\}$$

$$= \left\{\begin{bmatrix}0&0&0&0\\0&0&2&0\\0&0&0&1\\4&0&0&0\end{bmatrix},\begin{bmatrix}0&0&0&0\\2&0&0&0\\0&0&0&1\\1&2&0&0\end{bmatrix},\begin{bmatrix}0&0&0&0\\0&0&0&4\\2&0&0&0\\0&0&0&0\end{bmatrix},\right.$$

$$\left.\begin{bmatrix} 0 & 0 & 4 & 4 \\ 8 & 1 & 0 & 2 \\ 4 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 \end{bmatrix}, \begin{bmatrix} 4 & 0 & 0 & 4 \\ 2 & 7 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 3 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 8 & 0 & 0 & 8 \\ 0 & 6 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 2 & 6 & 0 & 0 \end{bmatrix}\right\} \qquad \ldots \text{ II}$$

Clearly A × B ≠ B × A but both A × B and B × A are in S. Thus (S, +, ×) is a non commutative subset matrix semilinear algebra over the semifield F.

We now find

$$A \times_n B = \left\{\begin{bmatrix} 0 & 0 & 2 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 \\ 4 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 2 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & 4 \\ 0 & 6 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\right\} \times_n$$

$$\left\{\begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 0 & 4 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}\right\}$$

$$= \left\{\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 8 & 0 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix},\right.$$

$$\left.\begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 2 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 6 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\right\} \in S;$$

$A \times_n B = B \times_n A \in S$.

We see $\{S, +, \times_n\}$ is a subset commutative semilinear algebra over the semifield F.

Example 2.35: Let S = {Collection of all subsets from the semigroup $(Z^+ \cup \{0\})D_{2,5}$} be the subset semigroup under +. S is also a subset semivector space over the semifield $Z^+ \cup \{0\}$. We see $(S, \times)$ is a subset semilinear algebra which is non commutative over the semifield $Z^+ \cup \{0\}$.

Let $A = \{3a + 5b + 6b^3 + 7ab^2, 5ab^4 + ab\}$ and
$B = \{4a, 3ab^2, 6b^3, ab^4\} \in S$.

We find both $A \times B$ and $B \times A$.

Now

$$A \times B = \{3a + 5b + 6b^3 + 7ab^2, 5ab^4 + ab\} \times \{4a, 6b^3, 3ab^2, ab^4\}$$

$$= \{12 + 20ba + 24b^3a + 28ab^2a, 20ab^4a + 4aba, 18ab^3 + 30b^4 + 36b + 42a, 30ab^2 + 6ab^4, 9b^2 + 15bab^2 + 18b^3ab^2 + 21ab^2\, ab^2, 15ab^4ab^2 + 3abab^2, 3b^4 + 5bab^4 + 6b^3ab^4 + 7ab^2ab^4 + 7b, 5ab^4ab^4 + abab^4\}$$

We now find

$$B \times A = \{4a, 3ab^2, 6b^3, ab^4\} \times \{3a + 5b + 6b^3 + 7ab^2, 5ab^4 + ab\}$$

$$= \{12 + 20ab + 24ab^3 + 28b^2, 9ab^2a + 15ab^3 + 18a + 21ab^2ab^2, 18b^3a + 30b^4 + 36b + 42b^3ab^2, 3ab^4a + 5ab^5 + 6ab^2 + 7ab^4ab^2, 20b^4 + 4b, 15ab^2ab^4 + 3ab^2ab, 30b^3ab^4 + 6b^3ab, 5ab^4ab^4 + ab^4ab\}.$$

It is clear $A \times B \neq B \times A$ in S.

Thus {S, +, ×} is a subset non commutative semilinear algebra over the semifield.

Example 2.36: Let S = {Collection of all subsets from the semigroup $(Q^+ \cup \{0\})(D_{2,3} \times A_4)$} be the subset semigroup.

S is a subset semivector space over the semifield F = $Q^+ \cup \{0\}$. S is a non commutative subset semilinear algebra over the semifield F.

Thus we see the concept of non commutativity arises only in case of subset semilinear algebras.

In view of all this we have the following theorem.

THEOREM 2.1: *Let S be the subset semivector space over a semifield F. S is a non commutative semilinear algebra if and only if the basic semigroup used in constructing S is non commutative.*

Proof follows from the fact that if
S = {Collection of all subsets from the additive semigroup P} and if P is non commutative under product × so will be S under ×; so that S is non commutative subset semilinear algebra over F.

Conversely if S is commutative then for {a}, {b} ∈ S we have $\{a\} \times \{b\} \neq \{b\} \times \{a\}$ for some a, b ∈ P so P is also non commutative hence the claim.

We have seen both examples of commutative and non commutative subset semilinear algebras defined over the semifields.

Example 2.37: Let S = {Collection of all subsets from the matrix semigroup M = $\{(a_1, a_2, a_3) \mid a_i \in (Z^+ \cup \{0\})S_3, 1 \leq i \leq 3\}$} be the subset semigroup.

M is non commutative under product. S is a subset semivector space over $Z^+ \cup \{0\} = F$, the semifield. Clearly M is a non commutative subset semilinear algebra over the semifield F.

Let

$$A = \left\{\left(3\begin{pmatrix}1 & 2 & 3\\ 2 & 3 & 1\end{pmatrix} + 5\begin{pmatrix}1 & 2 & 3\\ 3 & 2 & 1\end{pmatrix}\right),\right.$$

$$\left.4\begin{pmatrix}1 & 2 & 3\\ 2 & 1 & 3\end{pmatrix} + 6, 1 + \left(\begin{pmatrix}1 & 2 & 3\\ 3 & 1 & 2\end{pmatrix}\right)\right\}$$

and

$$B = \left\{\left(6\begin{pmatrix}1 & 2 & 3\\ 2 & 1 & 3\end{pmatrix} + 7\begin{pmatrix}1 & 2 & 3\\ 2 & 3 & 1\end{pmatrix} + 10,\right.\right.$$

$$3\begin{pmatrix}1 & 2 & 3\\ 1 & 3 & 2\end{pmatrix} + 4\begin{pmatrix}1 & 2 & 3\\ 3 & 2 & 1\end{pmatrix},$$

$$\left.\left.6 + \begin{pmatrix}1 & 2 & 3\\ 2 & 1 & 3\end{pmatrix} + 7\begin{pmatrix}1 & 2 & 3\\ 1 & 3 & 2\end{pmatrix}\right)\right\} \in S.$$

We now find

$$A \times B = \left\{\left(3\begin{pmatrix}1 & 2 & 3\\ 2 & 3 & 1\end{pmatrix} + 5\begin{pmatrix}1 & 2 & 3\\ 3 & 2 & 1\end{pmatrix}\right),\right.$$

$$\left.4\begin{pmatrix}1 & 2 & 3\\ 2 & 1 & 3\end{pmatrix} + 6, 1 + \left(\begin{pmatrix}1 & 2 & 3\\ 3 & 1 & 2\end{pmatrix}\right)\right\} \times$$

$$\left\{\left(6\begin{pmatrix}1&2&3\\2&1&3\end{pmatrix}+7\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix}+10,\right.\right.$$

$$\left.\left.3\begin{pmatrix}1&2&3\\1&3&2\end{pmatrix}+4\begin{pmatrix}1&2&3\\3&2&1\end{pmatrix},\ 6+\begin{pmatrix}1&2&3\\2&1&3\end{pmatrix}+7\begin{pmatrix}1&2&3\\1&3&2\end{pmatrix}\right)\right\}$$

$$=\left\{\left(18\begin{pmatrix}1&2&3\\1&3&2\end{pmatrix}+21\begin{pmatrix}1&2&3\\3&1&2\end{pmatrix}+30\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix}+\right.\right.$$

$$30\begin{pmatrix}1&2&3\\3&1&2\end{pmatrix}+35\begin{pmatrix}1&2&3\\1&3&2\end{pmatrix}+50\begin{pmatrix}1&2&3\\3&2&1\end{pmatrix},$$

$$12\begin{pmatrix}1&2&3\\3&1&2\end{pmatrix}+16\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix}+18\begin{pmatrix}1&2&3\\1&3&2\end{pmatrix}+24\begin{pmatrix}1&2&3\\3&2&1\end{pmatrix},$$

$$6+\ 6\begin{pmatrix}1&2&3\\2&1&3\end{pmatrix}+7\begin{pmatrix}1&2&3\\1&3&2\end{pmatrix}+6\begin{pmatrix}1&2&3\\3&1&2\end{pmatrix}$$

$$\left.\left.+\begin{pmatrix}1&2&3\\3&2&1\end{pmatrix}+7\begin{pmatrix}1&2&3\\2&1&3\end{pmatrix}\text{ and so on}\right)\right\}$$

$$=\quad\left\{\left(53\begin{pmatrix}1&2&3\\1&3&2\end{pmatrix}+51\begin{pmatrix}1&2&3\\3&1&2\end{pmatrix}+\right.\right.$$

$$30\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix}+50\begin{pmatrix}1&2&3\\3&2&1\end{pmatrix},$$

$$12\begin{pmatrix}1&2&3\\3&1&2\end{pmatrix}+16\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix}+$$

$$18\begin{pmatrix}1 & 2 & 3\\1 & 3 & 2\end{pmatrix}+24\begin{pmatrix}1 & 2 & 3\\3 & 2 & 1\end{pmatrix},$$

$$6+8\begin{pmatrix}1 & 2 & 3\\2 & 1 & 3\end{pmatrix}+7\begin{pmatrix}1 & 2 & 3\\1 & 3 & 2\end{pmatrix}$$

$$\left.+6\begin{pmatrix}1 & 2 & 3\\3 & 1 & 2\end{pmatrix}+\begin{pmatrix}1 & 2 & 3\\3 & 2 & 1\end{pmatrix}\text{and so on}\right)\Bigg\} \quad \ldots \text{ I}$$

Consider

$$B \times A \quad = \quad \Bigg\{\Bigg(6\begin{pmatrix}1 & 2 & 3\\2 & 1 & 3\end{pmatrix}+7\begin{pmatrix}1 & 2 & 3\\2 & 3 & 1\end{pmatrix}+10,$$

$$3\begin{pmatrix}1 & 2 & 3\\1 & 3 & 2\end{pmatrix}+4\begin{pmatrix}1 & 2 & 3\\3 & 2 & 1\end{pmatrix},$$

$$6+\begin{pmatrix}1 & 2 & 3\\2 & 1 & 3\end{pmatrix}+7\begin{pmatrix}1 & 2 & 3\\1 & 3 & 2\end{pmatrix}\Bigg)\Bigg\} \times$$

$$\Bigg\{\Bigg(3\begin{pmatrix}1 & 2 & 3\\2 & 3 & 1\end{pmatrix}+5\begin{pmatrix}1 & 2 & 3\\3 & 2 & 1\end{pmatrix},$$

$$4\begin{pmatrix}1 & 2 & 3\\2 & 1 & 3\end{pmatrix}+6,1+\begin{pmatrix}1 & 2 & 3\\3 & 1 & 2\end{pmatrix}\Bigg)\Bigg\}$$

$$= \quad \Bigg\{\Bigg(18\begin{pmatrix}1 & 2 & 3\\3 & 2 & 1\end{pmatrix}+30\begin{pmatrix}1 & 2 & 3\\2 & 3 & 1\end{pmatrix}+21\begin{pmatrix}1 & 2 & 3\\3 & 1 & 2\end{pmatrix}+$$

$$35\begin{pmatrix}1&2&3\\2&1&3\end{pmatrix}+30\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix}+50\begin{pmatrix}1&2&3\\3&2&1\end{pmatrix},12\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix}+$$

$$18\begin{pmatrix}1&2&3\\1&3&2\end{pmatrix}+16\begin{pmatrix}1&2&3\\3&1&2\end{pmatrix}+24\begin{pmatrix}1&2&3\\3&2&1\end{pmatrix},$$

$$\left.\left.6+6\begin{pmatrix}1&2&3\\3&1&2\end{pmatrix}+\begin{pmatrix}1&2&3\\2&1&3\end{pmatrix}+\begin{pmatrix}1&2&3\\1&3&2\end{pmatrix}\text{and so on}\right)\right\}$$

$$=\left\{\left(68\begin{pmatrix}1&2&3\\3&2&1\end{pmatrix}+60\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix}+\right.\right.$$

$$21\begin{pmatrix}1&2&3\\3&1&2\end{pmatrix}+35\begin{pmatrix}1&2&3\\2&1&3\end{pmatrix},$$

$$12\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix}+18\begin{pmatrix}1&2&3\\1&3&2\end{pmatrix}+$$

$$16\begin{pmatrix}1&2&3\\3&1&2\end{pmatrix}+24\begin{pmatrix}1&2&3\\3&2&1\end{pmatrix},$$

$$\left.\left.6+6\begin{pmatrix}1&2&3\\3&1&2\end{pmatrix}+\begin{pmatrix}1&2&3\\2&1&3\end{pmatrix}+\begin{pmatrix}1&2&3\\1&3&2\end{pmatrix}\text{and so on}\right)\right\}.$$

Clearly it can be verified I and II are not equal, so (S, +, ×) is a non commutative subset semilinear algebra over the semifield.

Example 2.38: Let S = {Collection of all subsets from the column matrix semigroup

$$M = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} \middle| \; a_i \in (Q^+ \cup \{0\})\,(D_{2,7});\ 1 \leq i \leq 4 \right\}$$

be the subset column matrix semigroup. S is the subset matrix semivector space over the semifield $Z^+ \cup \{0\} = F$.

Clearly $(S, +, \times_n)$ is a subset matrix semilinear algebra which is non commutative.

$$A = \left\{ \begin{bmatrix} a+3b \\ 2ab+5b^2 \\ 7b^3 \\ 8ab+3ab^3 \end{bmatrix} \right\} \text{ and } B = \left\{ \begin{bmatrix} 7b^2+a \\ ab^3 \\ ab^2 \\ a+3ab^2 \end{bmatrix} \right\} \text{ be in S.}$$

$$\text{We find } A \times_n B = \left\{ \begin{bmatrix} a+3b \\ 2ab+5b^2 \\ 7b^3 \\ 8ab+3ab^3 \end{bmatrix} \right\} \times_n \left\{ \begin{bmatrix} 7b^2+a \\ ab^3 \\ ab^2 \\ a+3ab^2 \end{bmatrix} \right\}$$

$$= \left\{ \begin{bmatrix} a^2+3ba+7ab^2+21b^3 \\ 2abab^3+5b^2ab^3 \\ 7b^3ab^2 \\ 8aba+3ab^3a+24abab^2+9ab^3ab^2 \end{bmatrix} \right\}$$

$$= \left\{ \begin{bmatrix} 1 + 21b^3 + 3ab^6 + 7ab^5 \\ 2b^2 + 5ab \\ 7ab^6 \\ 17b^6 + 3b^4 + 24b \end{bmatrix} \right\} \qquad \ldots \text{ I}$$

Consider $B \times_n A = \left\{ \begin{bmatrix} 7b^2 + a \\ ab^3 \\ ab^2 \\ a + 3ab^2 \end{bmatrix} \right\} \times_n \left\{ \begin{bmatrix} a + 3b \\ 2ab + 5b^2 \\ 7b^3 \\ 8ab + 3ab^3 \end{bmatrix} \right\}$

$$= \left\{ \begin{bmatrix} 7b^2a + 1 + 21b^3 + 3ab \\ 2ab^3ab + 5ab^3b^2 \\ 7ab^5 \\ 24ab^2ab + 8b + 3b^3 + 9ab^2ab^3 \end{bmatrix} \right\}$$

$$= \left\{ \begin{bmatrix} 1 + 3ab + 21b^3 + 7ab^5 \\ 5ab^5 + 2b^5 \\ 7ab^5 \\ 17b + 3b^3 + 24ab^2 \end{bmatrix} \right\} \qquad \ldots \text{ II}$$

Clearly $A \times B \neq B \times A$ as I and II are distinct. We see $(S, +, \times)$ is a subset non commutative semilinear algebra over the semifield.

Example 2.39: Let S = {Collection of all subsets from the matrix semigroup

$$M = \left\{\begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 \\ a_8 & a_9 & a_{10} & a_{11} & a_{12} & a_{13} & a_{14} \\ a_{15} & a_{16} & a_{17} & a_{18} & a_{19} & a_{20} & a_{21} \end{bmatrix} \middle| \; a_i \in (Z^+ \cup \{0\})(A_4 \times D_{2,11});\ 1 \leq i \leq 21\right\}$$

be the subset matrix semiring. S is a subset matrix semivector space over the semifield $Z^+ \cup \{0\}$.

We see S is a non commutative subset matrix semilinear algebra over the semifield $F = Z^+ \cup \{0\}$.

Example 2.40: Let S = {Collection of all subsets from the matrix semigroup

$$P = \left\{\begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ \vdots & \vdots & \vdots & \vdots \\ a_{37} & a_{38} & a_{39} & a_{40} \end{bmatrix} \middle| \; a_i \in (Q^+ \cup \{0\})S(5);\ 1 \leq i \leq 40\right\}\}$$

be the subset semigroup. S is a subset matrix semivector space over the semifield $F = Q^+ \cup \{0\}$. S is a subset non commutative matrix semilinear algebra over the semifield $F = Q^+ \cup \{0\}$.

Now having seen examples of subset semilinear algebras which are non commutative. We now proceed onto describe the notion of substructures in a subset semivector space.

Example 2.41: Let S = {Collection of all subsets from the semigroup $M = Z^+ \cup \{0\}$} be the subset semivector space over the semifield $F = Z^+ \cup \{0\}$.

W_n = {Collection of all subsets from the subsemigroup $nZ^+ \cup \{0\}$; $n \in Z^+ \setminus \{1\}$} $\subseteq$ S is also a subset semivector subspace of S over the semifield $F = Z^+ \cup \{0\}$ for $n \in N \setminus \{1\}$.

Thus we see S has infinite number of subset semivector subspaces over the semifield $Z^+ \cup \{0\} = F$.

Example 2.42: Let S = {Collection of all subsets from the matrix semigroup $M = \{(a_1, a_2, \ldots, a_6) \mid a_i \in Z^+ \cup \{0\}; 1 \leq i \leq 6\}\}$ be the subset matrix semivector space over the semifield $F = Z^+ \cup \{0\}$.

Take W_1 = {Collection of all subsets from the matrix subsemigroup $P_1 = \{(a_1, 0, 0, \ldots, 0) \mid a_1 \in Z^+ \cup \{0\}\} \subseteq S$ be the subset matrix semivector subspace of S over the semifield $F = Z^+ \cup \{0\}$. W_2 = {Collection of all subsets from the matrix subsemigroup $P_2 = \{(0, a, 0, \ldots, 0) \mid a \in F = Z^+ \cup \{0\}\} \subseteq S$; be the subset matrix semivector subspace of S over the semifield $F = Z^+ \cup \{0\}$.

Let W_3 = {Collection of all subsets from the matrix subsemigroup $P_3 = \{(0, 0, a, 0, \ldots, 0) \mid a \in Z^+ \cup \{0\} = F\}\} \subseteq S$ be the subset matrix semivector subspace of S over the semifield $F = Z^+ \cup \{0\}$.

W_4 = {Collection of all subsets from the subsemigroup $P_4 = \{(0, 0, 0, a, 0, 0) \mid a \in Z^+ \cup \{0\}\}\} \subseteq S$ be the subset matrix semivector subspace of S over the semifield $F = Z^+ \cup \{0\}$.

W_5 = {Collection of all subsets from the subsemigroup $P_5 = \{(0, 0, 0, 0, a, 0) \mid a \in Z^+ \cup \{0\}\}\}$ be the subset semivector subspace of S over the semifield $F = Z^+ \cup \{0\}$.

Finally W_6 = {Collection of all subsets from the subsemigroup $P_6 = \{(0, 0, 0, 0, 0, a) \mid a \in Z^+ \cup \{0\}\}\} \subseteq S$ be the subset semivector subspace of S over the semifield $F = Z^+ \cup \{0\}$.

We see $W_i \cap W_j = (0, 0, 0, 0, 0, 0)$ if $i \neq j$, $1 \leq i, j \leq 6$.

Further we see $S = W_1 + W_2 + W_3 + W_4 + W_5 + W_6$.

Thus we see S is the direct sum of subset semivector subspaces of S over the semifield F.

Suppose M_n = {Collection of all subsets from the semigroup $T_n = \{(a_1, a_2, \ldots, a_6) \mid a_i \in nZ^+ \cup \{0\}\}, n \in Z^+ \setminus \{1\}; 1 \leq i \leq 6\}\}$ $\subseteq$ S be the collection of all subset semivector subspaces of S over the semifield F.

Clearly $M_i \cap M_j \neq \{(0, 0, 0, 0, 0, 0)\}$ if $i \neq j$; $2 \leq i, j \leq n < \infty$.

Hence we cannot write S as a direct sum of subset semivector subspaces; $M_2, M_3, \ldots, M_n$; $n < \infty$.

Example 2.43: Let S = {Collection of all subsets from the matrix semigroup

$$P = \left\{\left.\begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ \vdots & \vdots & \vdots & \vdots \\ a_{57} & a_{58} & a_{59} & a_{60} \end{bmatrix}\right| a_i \in Z^+ \cup \{0\}\}; 1 \leq i \leq 60\right\}$$

be the subset matrix semivector space over the semifield $F = Z^+ \cup \{0\}$. S can we written as a direct sum of subset matrix semivector subspaces.

We will just illustrate this.

Take W_1 = {Collection of all subsets from the subsemigroup

$$P_1 = \left\{\left.\begin{bmatrix} a_1 & 0 & a_2 & 0 \\ a_3 & 0 & a_4 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ a_{15} & 0 & a_{30} & 0 \end{bmatrix}\right| a_i \in Z^+ \cup \{0\}; 1 \leq i \leq 30\}\right\} \subseteq S$$

and

W_2 = {Collection of all subsets from the subsemigroup

$$P_2 = \left\{\left.\begin{bmatrix} 0 & a_1 & 0 & a_2 \\ 0 & a_3 & 0 & a_4 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & a_{15} & 0 & a_{30} \end{bmatrix}\right| a_i \in Z^+ \cup \{0\}; 1 \leq i \leq 30\}\right\} \subseteq S$$

as two subset matrix semivector subspaces of S over the semifield F = $Z^+ \cup \{0\}$.

$$\text{We see } W_1 \cap W_2 = \left\{\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 \end{bmatrix}\right\} \text{ and } W_1 + W_2 = S.$$

Thus S is the direct sum further $W_1^{\perp} = W_2$ and $W_2^{\perp} = W_1$ for we see if $A \in W_1$ and $B \in W_2$ then

$$A \times B = \left\{\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 \end{bmatrix}\right\}.$$

We can write S as a direct sum of two subspaces or three subspaces and so on and the maximum we can write S as a direct sum of 60 subspaces.

Example 2.44: Let S = {Collection of all subsets from the semigroup

$$M = \left\{ \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \\ a_5 & a_6 \end{bmatrix} \middle| \, a_i \in Z^+ \cup \{0\}; 1 \leq i \leq 6 \right\}$$

be the subset matrix semivector space over the semifield $F = Z^+ \cup \{0\}$.

$S = W_1 + W_2$ where W_1 = {Collection of all subsets from the subsemigroup

$$P_1 = \left\{ \begin{bmatrix} a_1 & 0 \\ a_3 & 0 \\ a_5 & 0 \end{bmatrix} \middle| \, a_i \in Z^+ \cup \{0\}; 1 \leq i \leq 3 \right\}\} \subseteq S$$

and W_2 = {Collection of all subsets from the subsemigroup

$$P_2 = \left\{ \begin{bmatrix} 0 & a_1 \\ 0 & a_2 \\ 0 & a_3 \end{bmatrix} \middle| \, a_i \in Z^+ \cup \{0\}; 1 \leq i \leq 3 \right\}\} \subseteq S$$

are subset matrix semivector subspaces of S over F and $W_1 + W_2 = S$.

$$W_1 \cap W_2 = \left\{ \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \right\}.$$

This is not unique for we can also take V_1 = {Collection of all subsets from the subsemigroup

$$L_1 = \left\{ \begin{bmatrix} a_1 & 0 \\ a_2 & a_3 \\ 0 & a_4 \end{bmatrix} \middle| \, a_i \in Z^+ \cup \{0\}; 1 \leq i \leq 4 \right\}\} \subseteq S$$

and V_2 = {Collection of all subsets from the subsemigroup

$$L_2 = \left\{ \begin{bmatrix} 0 & a_1 \\ 0 & 0 \\ a_2 & 0 \end{bmatrix} \middle| \, a_i \in Z^+ \cup \{0\}; 1 \leq i \leq 2 \right\}\} \subseteq S$$

be subset matrix semivector subspaces of S over the semifield $F = Z^+ \cup \{0\}$.

Clearly $V_1 \cap V_2 = \left\{ \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \right\}$ and $S = V_1 \oplus V_2$ and we see for every $A \in V_1$ we have for every $B \in V_2$.

$$A \times B = \left\{ \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \right\}.$$

Take B_1 = {Collection of all subsets from the subsemigroup

$$A_1 = \left\{ \begin{bmatrix} a_1 & a_2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \middle| \, a_i \in Z^+ \cup \{0\} \right\}\} \subseteq S$$

and

B_2 = {Collection of all subsets from the subsemigroup

$$A_2 = \left\{ \begin{bmatrix} 0 & 0 \\ a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} \middle| \, a_i \in Z^+ \cup \{0\}; 1 \leq i \leq 4 \right\}\} \subseteq S$$

be two subset matrix semivector subspaces of S over $F = Z^+ \cup \{0\}$. Clearly $B_1 + B_2 = S$ and $B_1 \cap B_2$ are such that

$$C \times D = \left\{\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}\right\} \text{ for every } C \in B_1 \text{ and } D \in B_2.$$

Let N_1 = {Collection of all subsets from the subsemigroup

$$D_1 = \left\{\begin{bmatrix} a_1 & 0 \\ 0 & a_2 \\ 0 & 0 \end{bmatrix} \middle| a_1, a_2 \in Z^+ \cup \{0\}\right\}\}$$

be the subset matrix semivector subspace of S over the semifield $F = Z^+ \cup \{0\}$. We see N_2 = {Collection of all subsets from the subsemigroup

$$D_2 = \left\{\begin{bmatrix} 0 & a_2 \\ 0 & 0 \\ a_1 & 0 \end{bmatrix} \middle| a_1, a_2 \in Z^+ \cup \{0\}\right\}\} \subseteq S$$

be the subset matrix semivector subspace of S over the semifield $F = Z^+ \cup \{0\}$. Let N_3 = {Collection of all subsets from the matrix subsemigroup

$$D_3 = \left\{\begin{bmatrix} 0 & 0 \\ a_1 & 0 \\ 0 & a_2 \end{bmatrix} \middle| a_1, a_2 \in Z^+ \cup \{0\}\right\}\}$$

be the subset matrix semivector subspace of S over the semifield $F = Z^+ \cup \{0\}$. We see $D_1 + D_2 + D_3 = S$ and

$$D_i \cap D_j = \left\{\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}\right\} \text{ if } i \neq j,\ 1 \leq i, j \leq 3.$$

Further we see for every $X \in D_i$ and $Y \in D_j$.

$$X \times Y = \left\{\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}\right\};\ i \neq j,\ 1 \leq i, j \leq 3.$$

We can maximum write S as the direct sum of six subset matrix semivector subspaces over $F = Z^+ \cup \{0\}$. However we have infinite number of subset matrix semivector subspaces W_i of S but

$$W_i \cap W_j \neq \left\{\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}\right\};\ \text{ if } i \neq j,\ 2 \leq i, j < n < \infty.$$

Take W_n = {Collection of all subsets from the subsemigroup

$$T_n = \left\{\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \\ a_5 & a_6 \end{bmatrix} \middle|\ a_i \in nZ^+ \cup \{0\};\ 1 \leq i \leq 6,\ n \in Z^+ \setminus \{1\}\right\}\} \subseteq S$$

be the subset matrix semivector subspace of S over the semifield $F = Z^+ \cup \{0\}$.

We see

$$W_i \cap W_j \neq \left\{\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}\right\} \text{ if } i \neq j,\ 2 \leq i, j \leq n < \infty;$$

so we have infinite collection of subset semivector subspaces of S and these cannot be written as a direct sum.

Now we proceed onto describe the notion of orthogonality of the elements in subset semivector spaces over a semifield.

Let S = {Collection of all subsets from a semigroup P} be a subset semivector space over the semifield F.

We call a subset in S as a subset semivector in S. We say two subset semivectors A and B are subset linearly independent if A $\neq$ aB for any a $\in$ F, the semifield.

We say A and B are subset linearly dependent if A = aB for some b $\in$ S.

We will first illustrate this situation by an example.

Example 2.45: Let

S = {Collection of all subsets from the semigroup $Z^+ \cup \{0\}$} be the subset semigroup. S is a subset semivector space over the semifield F = $Z^+ \cup \{0\}$.

Let A = {3, 9, 15, 30, 93} and B = {1, 3, 5, 10, 31} $\in$ S. We see A and B are subset linearly dependent as A = 3B.

But consider A = {2, 5, 0, 7, 8, 9, 12, 14, 16} and B = {0, 4, 17, 19, 13, 11, 23} $\in$ S. We say A and B subset linearly independent in S for we do not have a $\in$ F such that A = aB or B = aA. Let A = {17} and B = {23, 4, 5} $\in$ S. We say A and B are subset linearly independent in S over $Z^+ \cup \{0\}$ = F.

Let A = {2, 4, 6, 8, 10} and B = {1, 2, 3, 4, 5} $\in$ S. We say A and B are subset linearly dependent for 2 $\in$ $Z^+ \cup \{0\}$ is such that A = 2 $\times$ B.

Now take A = {1} and B = {43, 27, 8, 10} ∈ S we say A and B are subset linearly independent. However for A = {1} and B = {a}, a ∈ $Z^+ \cup \{0\} \setminus \{1\}$ are subset linearly dependent for B = a {1} as a ∈ $Z^+ \cup \{0\} \setminus \{1\}$.

Further if A = {a}, a ∈ $Z^+ \cup \{0\}$ and B = {d, b}, d ≠ b, d, b ∈ Z^+ then A and B are not subset linearly dependent.

For if A = {7} and B = {3, 14} we see A and B are subset linearly independent.

So one of the interesting problems is if A and B are subsets in S and if the number of distinct elements in A is n and the number of elements in B is m where m ≠ n can we have the subsets A and B to be subset linearly dependent?

Of course both A and B do not contain 0.

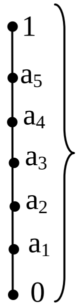
Let A = {7, 4, 5, 8, 10} and B = {14, 8, 2, 9} ∈ S. We see A and B subset linearly independent over $Z^+ \cup \{0\}$. We see A = {p} and B = {q}, p and q are distinct primes in Z^+; then A and B are subset linearly independent. For instance take A = {29} and B = {7} ∈ S. A and B are subset linearly independent over the field $Z^+ \cup \{0\}$. But A and B subset linearly dependent over $Q^+ \cup \{0\}$. For take A = $\frac{23}{41} \times$ B and B = $\frac{41}{23} \times$ A.

So the A and B subset linearly dependent over the semifield $Q^+ \cup \{0\}$ and $R^+ \cup \{0\}$, but A and B subset linearly independent over the semifield $Z^+ \cup \{0\}$.

Thus the subset linear dependence or independence also depends on the semifield over which they are defined. But however to find the subset basis is a different from usual basis of semivector spaces.

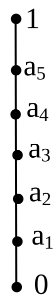
Before we define a subset basis of a subset semivector space we just give examples of finite subset semivector spaces.

Example 2.46: Let S = {Collection of all subsets from the semigroup P, under '$\cup$'. P =



be the subset semigroup of finite order.

S is a subset semivector space of finite order over the semifield P =



S = {{0}, {1}, {a_1}, {a_2}, …, {a_5}, {0, 1}, {0, a_i}, {a_i, 1}, {0, 1, a_i} {a_i, a_j, 1}, {a_i, a_j, 0}, {a_i, a_j, 1, 0}, {a_i, a_j, a_k}, …, P} where $i \neq j$, $i \neq k$, $j \neq k$, $1 \leq i, j, k \leq 5$.

We just show if B = {0, 1, a_2, a_3} and A = {a_1, a_5, a_4, 0} $\in$ S.

A $\cup$ B = {a_1, a_5, a_4, 0, 1, a_2, a_3}.

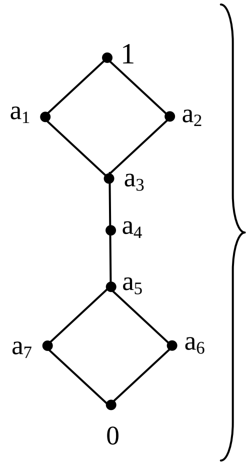
This is the way operation is performed on S.

If $a_4 \in P$ then $a_4 \times A = a_4 \times \{a_1, a_5, a_4, 0\} = \{a_1, a_4, 0\} \in S$.

Thus S is a subset semivector space of finite dimension over P.

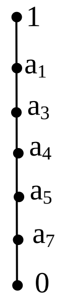
Example 2.47: Let S = {Collection of all subsets from the semilattice $(P, \cup)$ where

P =



be the subset semigroup (semilattice).

S is a subset semivector space over the semifield F which is as follows:

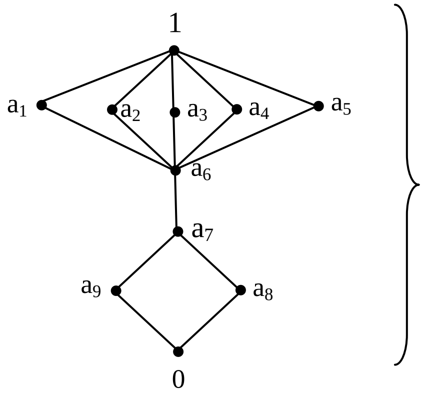


Let A = {1, a_2, a_6, a_4, a_5} and B = {0, a_7, a_3, a_1} $\in$ S.

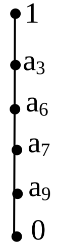
A $\cup$ B = {1, a_2, a_6, a_4, a_5} $\cup$ {0, a_7, a_3, a_1}
= {1, a_2, a_6, a_4, a_5, a_3, a_1} $\in$ S.

Let $a_4 \in$ F we find $a_4 \times$ A
= $a_4 \times$ {1, a_2, a_6, a_4, a_5}
= {a_4, a_6, a_5} $\in$ S.

Example 2.48: Let S = {Collection of all subsets from the semilattice (P, $\cup$)



be the subset semigroup. S is a subset semivector space over the semifield F.



Take A = {a_1, a_2, a_6, a_7, a_8, 0} and B = {a_3, a_4, a_5, a_9, a_7} $\in$ S.

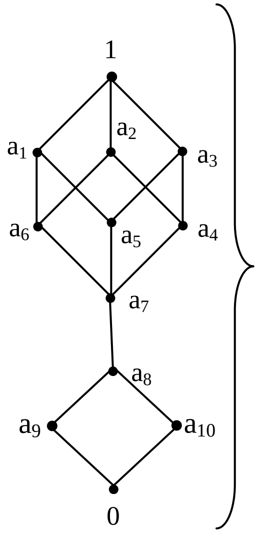
We find

$A \cup B = \{a_1, a_2, a_6, a_7, a_8, 0\} \cup \{a_3, a_4, a_5, a_9, a_7\}$
$= \{1, a_1, a_3, a_4, a_5, a_6, a_7\}$

Take $a_7 \in F$, we now find $a_7 \times A = a_7 \times \{a_1, a_2, a_6, a_7, a_8, 0\}$
$= \{a_7, a_8, 0\} \in S$.

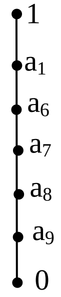
This is the way operations are carried out on S.

Example 2.49: Let S = {Collection of all subsets from the semilattice P under '∪' where P =



be the subset semigroup. S is a subset semivector space over the semifield

L =



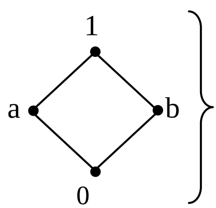
Let A = {a_2, a_5, a_6, a_7, a_4, a_{10}, a_9} and
B = {0, 1, a_6, a_5, a_3, a_2} ∈ S.

We find

A × B = {a_2, a_5, a_6, a_7, a_4, a_{10}, a_9} × {0, 1, a_6, a_5, a_3, a_2}
= {a_2, a_7, a_6, a_4, 0, a_3, a_5} ∈ S.

S is a finite subset semivector space over the semifield L.

Example 2.50: Let S = {Collection of all subsets from the semilattice



be the subset semigroup under '∪'.

S is a subset semivector space over the semifield L =

S = {{0}, {1}, {a}, {b}, {0, 1}, {0, a}, {0, b}, {a, 1}, {b, 1}, {a, b}, {0, a, b}, {0, a, 1}, {0, b, 1}, {1, a, b}, {0, 1, a, b}, ϕ} is a subset semivector space of order 16 over the semifield L.

Let A = {0, 1, a} and B = {a, b} ∈ S.

A ∪ B = {0, 1, a} ∪ {a, b} = {a, b, 1}.

Let a ∈ L; a × A = a × {0, 1, a} = {0, a} ∈ S.

We have seen subset semivector spaces of finite order over semifields.

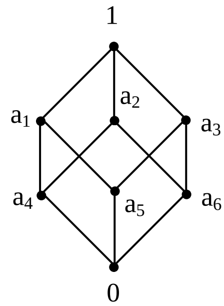
We can as in case of subset semivector spaces of infinite order define subset semilinear algebra.

We see only a few of the finite subset semivector spaces are subset semilinear algebras.

It is important to note for the semilattice must be a lattice and also in particular it must be a distributive lattice for one to get the subset semilinear algebra.

We will give an example or two of the subset semilinear algebra over a semifield.

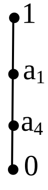
Example 2.51: Let S = {Collection of all subsets from the lattice L =



}

be the subset semigroup under '∪'.

S is a subset semivector space over the semifield F =



S is a subset semilinear algebra over the semifield F of finite order.

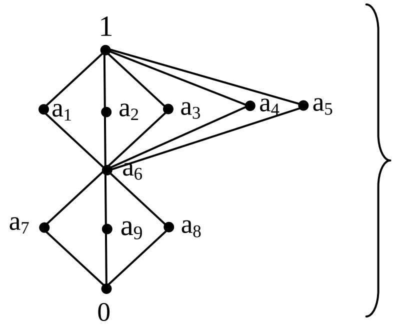
Let A = {1, a_1, a_5, a_6, a_4} and B = {a_6, 0, a_2, a_1, a_3} $\in$ S.

A $\cup$ B = {1, a_2, a_1, a_3, a_4} $\in$ S.

A $\cap$ B = {1, a_2, a_5, a_6, a_4} $\cap$ {a_6, 0, a_2, a_1, a_3}
= {a_6, 0, a_2, a_1, a_3, a_5, a_4} $\in$ S.

So S is a subset semilinear algebra over the semifield F.

Example 2.52: Let S = {Collection of all subsets from the semilattice



(S, $\cup$) is a subset semigroup.

However S is only a subset semivector space over the semifield L =

Clearly S is not a subset semilinear algebra as {S, ∪, ∩} is not a subset semiring in the first place as ∪ and ∩ do not distributive over each other.

Thus we have seen a subset semivector space over a semifield of finite order which is not a subset semilinear algebra over the semifield F.

In view of all these examples we have the following theorem.

THEOREM 2.2: *Let S = {Collection of all subsets from the semigroup} be the subset semivector space over a semifield F. S in general need not be a subset semilinear algebra over the semifield F.*

The proof is direct and hence left as an exercise to the reader.

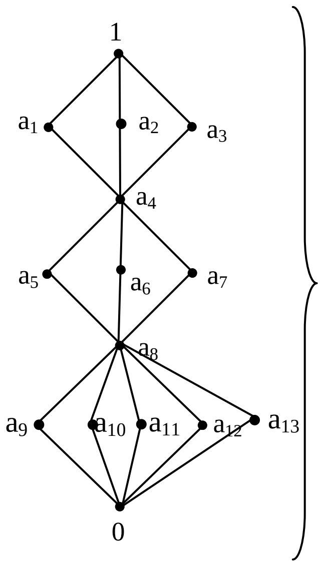
Example 2.53: Let S = {Collection of all subsets from the lattice C_8} be the subset semivector space over the chain lattice C_8.

S is a subset semilinear algebra over the chain lattice C_8.

Thus we see in general if S is a subset semilinear algebra over a semifield then S is always a subset semivector space over a semifield.

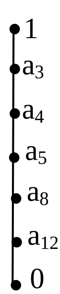
But however in general a subset semivector space is not always a subset semilinear algebra over the semifield.

Example 2.54: Let S = {Collection of all subsets from the semilattice (P, ∪) where P =



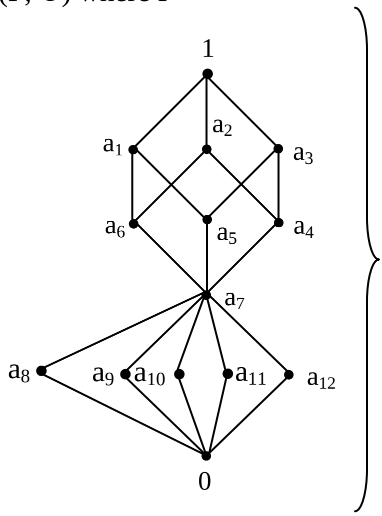
be a subset semigroup under ∪.

Clearly P is not a distributive lattice so is not a semiring. S is a subset semivector space over the semifield L =

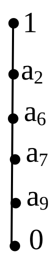


However S is not a subset semilinear algebra over L.

Example 2.55: Let S = {Collection of all subsets from the semilattice (P, $\cup$) where P =



be a subset semigroup and $\cup$. S is a subset semivector space over the semifield L =



However S is not a subset semilinear algebra over L.

Inview of all these we have the following theorem.

THEOREM 2.3: *Let S = {Collection of all subsets from a semilattice {P, $\cup$}} be the subset semivecor space over a semifield F (F $\subset$ P).*

(i) *If (P, $\cap$, $\cup$) is a not a distributive lattice then S is not a subset semilinear algebra over F.*

(ii) *S is a subset semilinear algebra over F if and only if (P, $\cup$, $\cap$) is a distributive lattice.*

The proof follows from the simple fact if (P, $\cup$, $\cap$) is a distributive lattice then S the subsets of P will be a semiring so that S can be a subset semilinear algebra over F $\subseteq$ P (F a distributive sublattice of P).

Conversely if S is a subset semilinear algebra then necessarily P must be a distributive lattice. We have seen examples of them.

It is pertinent to recall here that finding a basis for a semivector space itself was a difficult problem and we have shown [14]. Several semivector spaces had only a unique basis. Now how to find a basis of a subset semivector spaces. We have already shown that the subset linear dependence or subset linear independence is dependent on the semifield over which the subset semivector space is defined.

Suppose we have in the basic set {1}, {0, 1} using '+' we get all subsets and with these subsets we also create subsets in which we include zero for instance 7{1} + 12 {0, 1} = {7, 19} and now {7, 19} + {0, 4} = {7, 11, 19, 23} and so on now we include sets like {0, 7, 19} and {0, 7, 11, 23} also with the generated sets by {1} and {0,1}.

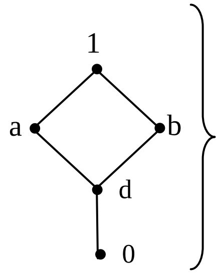
This is the case when semifields like $Z^+ \cup \{0\}$, $Q^+ \cup \{0\}$ or $R^+ \cup \{0\}$ are used.

However if we have S = {Collection of all subsets from the semigroup $R^+ \cup \{0\}$ under addition} be a subset semigroup under + and if S is a subset semivector space over the semifield $Z^+ \cup \{0\}$ the earlier mentioned method will fail for subset elements like $\{0, \sqrt{7}\}$ and $\{\sqrt{7}, \sqrt{5}, \sqrt{2}\}$ cannot be generated by {1} and {0, 1}.

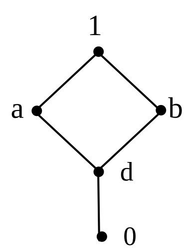
So we say in that case S is infinite dimensional subset semivector space. However if S is defined over $R^+ \cup \{0\}$ then we say it is finite dimensional for adjoining 0 with every set is taken as a finite operation. So only dimension two and the subset base elements are {1} and {0,1}.

We have to work differently in case the semifield is a distributive chain lattice.

Example 2.56: Let S = {Collection of all subsets from the semilattice (L, ∪)



be the subset semilattice under '∪'. S is a subset semivector space over the lattice L =

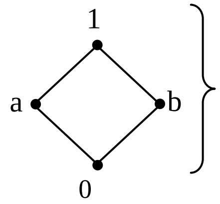


The basis of S are {0,1} and {1}. For if B = {{1}, {0,1}} we see {a}, {b}, {d}, {0} can be got using B. Further also {0,a}, {0,b}, {0,d} can be got using B.

{a,1}, {b,1}, {d,a}, {1, d}, {1,d,b}; etc. can be got however we have to add {0,1,a}, {0,1,b} and so on.

Thus B = {{1}, {0, 1}} generates S over L.

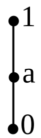
Example 2.57: Let S = {Collection of all subsets from the lattice



e the subset semigroup under '∪'. S is a subset semivector space over the lattice

Now S = {{0}, {a}, {b}, {1}, {0,1}, {0,a}, {0,b}, {1,a}, {a,b}, {1,b}, {0,a,b}, {0,a,1}, {0, 1, b}, {1,a,b}, {0,1,a,b}, ϕ}.

Can B = {{1}, {0,1}} be a subset basis of S over the lattice

L = 

{a} = a{1}, {0} = 0{1}, {a, 0} = a {0,1}.

{0,1} ∪ {a} = {a,1}, {0,1,a} by our rule of addition of zero. We can get only seven elements.

So B cannot be a subset basis of S over L.

Suppose we take B_1 = {{1}, {0,1}, {b}, {0,b}, {1, b, 0}} ⊆ S; can B_1 can a subset basis of S over L.

B_1 = {{b}, {1}, {0,1}, {0, b}}
a{b} = {0}, a{1} = a, a{0,1} = {0, a}
{0,b} ∪ {0,a} = {a, b, 0,1}.

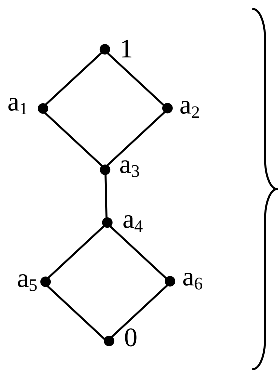
So {0, 1, a, b} is got. {0, b} ∪ {0, 1} = {0, 1, b}

{0, a} ∪ {0, 1} = {0, 1, a}
{1, a, 0} ∪ {0, 1, b} = {0, 1, a, b}
{0, a} ∪ {0, b} = {0, a, b}.

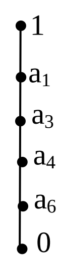
However B_1 also does not generate the whole of S. Thus we see it is not easy to find a subset basis in case the subset semivector space is defined over a sublattice of the lattice used.

Now we proceed on to describe other non commutative finite subset semilinear algebras.

Example 2.58: Let S = {Collection of all subsets from the lattice group LS_3 where L is the lattice



be the subset semigroup under '∪' = '+'. S is a subset semivector space over the semifield F =



Clearly S is a subset semilinear algebra over the semifield F.

Let A = $\{a_2p_1 + a_4p_4 + a_6p_5 + 1, a_3p_3, a_2\}$ and B = $\{p_5, a_3, a_4p_1 + a_2p_2\} \in S$ where

$$p_1 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}, p_2 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}, p_3 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix},$$

$$p_4 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}, p_5 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} \text{ and } e = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix} = 1.$$

We find A + B = $\{a_2p_1 + a_4p_4 + a_6p_5 + 1, a_3p_3, a_2\} + \{p_5, a_3, a_4p_1 + a_2p_2\}$ (Here + is the union $\cup$).

$= \{a_2p_1 + a_4p_4 + a_6p_5 + 1 + p_5, a_3p_3 + p_5, a_2 + p_5, a_2p_1 + a_4p_4 + a_6p_5 + 1 + a_3, a_3p_3 + a_3, a_2 + a_3, a_2p_1 + a_4p_4 + a_6p_5 + 1 + a_4p_1 + a_2p_2, a_3p_3 + a_4p_1 + a_2p_2, a_2 + a_4p_1 + a_2p_2\}$

$= \{a_2p_1 + a_4p_4 + 1 + p_5, a_3p_3 + p_5 + a_2+p_5, 1 + a_2p_1 + a_4p_4 + a_6p_5, a_3+a_3p_3, a_2, 1 + a_2p_1 + a_4p_4 + a_6p_5 + a_2p_2 + a_4p_1 + a_3p_3, a_2 + a_2p_2 + a_4p_1\} \in S.$

Now we find A × B =

$A \cap B = \{a_2p_1 + a_4p_4 + a_6p_5 + 1, a_3p_3, a_2\} \times \{p_5, a_3, a_4p_1 + a_2p_2\}$

$= \{a_2p_2 + a_4 + a_6p_4 + p_5, a_3p_1, a_2p_5, a_3p_1 + a_4p_4 + a_6p_5 + a_3, a_3p_3, a_3, a_4 + a_4p_1 + a_4p_2 + a_6p_3 + a_2p_5 + a_4p_3 + a_6p_1 + a_2p_2, a_4p_5 + a_3p_4, a_4p_1 + a_2p_2\}$

$= \{p_5 + a_6p_4 + a_2p_2 + a_4, a_3p_1, a_2p_5, a_3 + a_3p_1 + a_4p_4 + a_6p_5, a_3p_3, a_3, a_4 + a_4p_1 + a_2p_2 + a_4p_3 + a_2p_5, a_4 p_5 + a_3p_4, a_4p_1 + a_2p_2\}$ … I

Consider

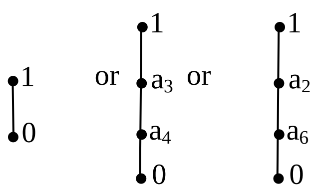
$B \times A = \{p_5, a_3, a_4p_1 + a_2p_2\} \times \{a_2p_1 + a_4p_4 + a_6p_5 + 1, a_3p_3, a_2\}$

$$= \{a_2p_3 + p_5 + a_6p_4 + a_4, a_3p_2, a_2p_5, a_3p_1 + a_4p_4 + a_6p_5 + a_3, a_3p_3, a_3, a_4 + a_4p_3 + a_6p_2 + a_4p_1 + a_2p_4 + a_4p_1 + a_2p_2 + a_6p_3, a_4p_1 + a_2p_2, a_4p_4 + a_3p_5\}$$

$$= \{a_4 + a_2p_3 + p_5 + a_6p_4, a_3p_2, a_2p_5, a_3 + a_3p_1 + a_4p_4 + a_6p_5, a_3p_3, a_3, a_4 + a_4p_3 + a_2p_2 + a_4p_1 + a_2p_4, a_4p_1 + a_2p_2, a_4p_4 + a_3p_5\} \quad \ldots \quad \text{II}$$

Clearly I and II are different but $A \cap B = A \times B$ and $B \cap A = B \times A$ are in S but $A \times B \neq B \times A$. Thus S is a finite non commutative subset semilinear algebra over the semifield.

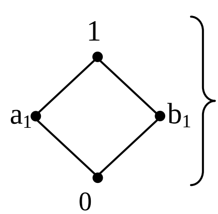
Thus we can also have finite non commutative subset semilinear algebras defined over the semifields. Infact in the example 2.58 if we take a different semifield say



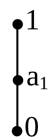
we get different subset semilinear algebras but all of them are non commutative and of finite order.

However the major difference between them would be seen when the subset basis are constructed over different semifield but for the same S.

Example 2.59: Let S = {Collection of all subsets from the lattice group $LG = LD_{2,5}$ where L =



be the subset semigroup under '+' (i.e., $\cup$) and S is a subset semivector space over the semifield F =



and S is a subset semilinear algebra of finite order over the semifield F. Clearly S is a non commutative subset semilinear algebra over F. For take A = $\{1 + a_1b^2 + a_1ab + b_1 ab^2, a_1b^3, a, b^4 + b_1\}$ and B = $\{b, b_1b^2 + a_1ab + b^3\} \in S$.

We find $A + B = A \cup B = \{1 + a_1b^2 + a_1ab + b_1ab^2, a_1b^3, a, b^4 + b_1\} \times \{b, b_1b^2 + a_1ab + b^3\}$

$= \{1 + b + a_1b^2 + a_1ab + b_1ab^2, a_1b^3 + b, a + b, b^4 + b_1 + b, 1 + b^2 + a_1ab + b_1ab^2 + b^3, b_1b^2 + a_1ab + b^3, a + b_1b^2 + a_1ab + b^3, b_1 + b_1b^2 + a_1ab + b^3 + b^4\} \in S$.

We now find $A \times B = A \cap B$ and $B \times A = B \cap A$ and show $A \times B \neq B \times A$.

Consider $A \times B = \{1 + a_1b^2 + a_1ab + b_1ab^2, a_1b^3, a, b^4 + b_1\} \times \{b, b_1b^2 + a_1ab + b^3\}$

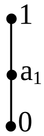
$= \{b + a_1b^3 + a_1ab^2 + b_1ab^3, a_1b^4, ab, 1 + b_1b, b_1b^2 + b_1ab^4 + a_1ab + a_1 b^2ab + a_1 + b^3 + a_1ab^4 + b_1a, a_1b^3ab + a_1b, b_1ab^2 + a_1b + ab^3, b_1b + a_1b^4 ab + b^2 + b_1b^2 + b_1b^3\}$

Consider $B \times A = \{b, b_1b^2 + a_1ab + b^3\} \times \{1 + a_1b^2 + a_1ab + b_1ab^2, a_1b^3, a, b^4 + b_1\}$

$= \{b + a_1b^3 + a_1bab + b_1bab^2, a_1b^4, ba, 1+b_1b, b_1b^2 + b_1b^2ab^2 + a_1ab + a_1ab^3 + a_1 + b^3 + a_1 + a_1b^3ab + b_1b^3 ab^2, a_1ab^4 + a_1b, bb^2a + a_1aba + b^3a, b_1b + a_1b + b + b^1b^2 + b_1b^3\}$

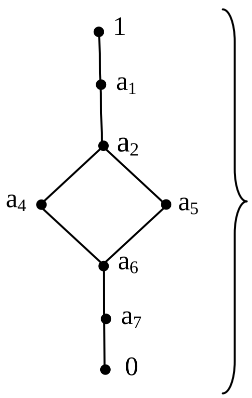
… II

It is easily verified that I and II are distinct thus we see S is a non commutative semilinear algebra over a semifield

F = 

is of finite order.

Example 2.60: Let S = {Collection of all subsets from the lattice group L (S (4) × A_5) where

L = 

be the subset semigroup under ∪ (i.e.,+) and S is also a subset semivector space over the semifield L.

S is a subset semilinear algebra over L infact S is a finite non commutative subset semilinear algebra over the semifield L.

We can define subset linear transformation of subset semivector spaces S_1 and S over a semifield F if and only if both S and S_1 are defined over the same semifield F.

Otherwise the subset linear transformation of S and S_1 cannot be defined.

We will just illustrate this by a few examples.

Example 2.61: Let S = {Collection of all subsets from the matrix semigroup M = $\{(a_1\ a_2\ a_3\ a_4) \mid a_i \in Z^+ \cup \{0\}, 1 \le i \le 4\}\}$ be the subset semivector space over the semifield F = $Z^+ \cup \{0\}$. Let S_1 = {Collection of all subsets from the matrix semigroup

$$N = \left\{\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} \middle| \ a_i \in Z^+ \cup \{0\}; 1 \le i \le 4\right\}\}$$

be the subset semivector space over the semifield F = $Z^+ \cup \{0\}$.

We can define a subset linear transformation T_s from S to S_1 as follows.

For every A = $\{(a_1, a_2, a_3, a_4)\} \in$ S and

$$A_1 = \left\{\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix}\right\} \in S_1 \text{ by}$$

$$T_S(A) = T_s(\{(a_1\ a_2\ a_3\ a_4)\} = \left\{\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix}\right\} = A_1$$

It is easily verified T_S is a subset semilinear transformation of S to S_1.

Let A_1 = {(4, 0, 2, 1), (5, 8, 9, 20), (0, 1, 0, 2), (8, 9, 11, 0)} $\in$ S.

$T_S(A_1) = T_S$ ({(4, 0, 2, 1), (5, 8, 9, 20), (0, 1, 0, 2), (8, 9, 11, 0)}

$$= B = \left\{\begin{bmatrix} 4 & 0 \\ 2 & 1 \end{bmatrix}, \begin{bmatrix} 5 & 8 \\ 9 & 20 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 8 & 9 \\ 11 & 0 \end{bmatrix}\right\} \in S_1.$$

Infact T_S is a one to one subset map from S to S_1.

Example 2.62: Let S = {Collection of all subsets from the semigroup $(Z^+ \cup \{0\})g$ where $g^2 = g$} be the subset semivector space over the semifield $F = Z^+ \cup \{0\}$.

S_1 = {Collection of all subsets from the semigroup $Z^+ \cup \{0\}$} be the subset semivector space over the semifield $F = Z^+ \cup \{0\}$.

Define $T_S : S \to S_1$ by

$T_S (\{a + bg\}) = \{a\}$ for every $a, b \in Z^+ \cup \{0\}$.

Clearly T_S is a subset semilinear transformation from S to S_1.

Example 2.63: Let S = {Collection of all subsets from the semigroup $R^+ \cup \{0\}$} be the subset semivector space over $Q^+ \cup \{0\}$. Let S_1 = {Collection of all subsets from the semigroup $Q^+ \cup \{0\}$} be subset semivector space over $Q^+ \cup \{0\}$.

Let $T_S : S \to S_1$ be such that if

$$T_S (\{a\}) = \begin{cases} \{a\} \text{ if } a \in Q^+ \cup \{0\} \\ \{0\} \text{ if } a \in R^+ \cup \{0\} \setminus Q^+ \cup \{0\} \end{cases}.$$

It is easily verified that T_S is a semilinear transformation of S to S_1.

We can as in case of usual semivector define the notion of subset semilinear operators. We give examples of subset semilinear operator of a subset semivector space.

Example 2.64: Let S = {Collection of all subsets from the semigroup

$$M = \left\{ \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \\ a_5 & a_6 \\ a_7 & a_8 \end{bmatrix} \middle| \; a_i \in Z^+ \cup \{0\}; 1 \le i \le 8 \right\}$$

be the subset semivector space over the semifield $F = Z^+ \cup \{0\}$.

Let $T_S^o : S \to S$ where $T_S^o\left(\left\{\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \\ a_5 & a_6 \\ a_7 & a_8 \end{bmatrix}\right\}\right) = \left\{\begin{bmatrix} a_1 & a_2 \\ 0 & 0 \\ a_3 & a_4 \\ 0 & 0 \end{bmatrix}\right\}$;

it is easily verified T_S^o is a subset semilinear operator on S.

We see $\ker T_S^o$ is not the zero of S.

Example 2.65: Let S = {Collection of all subsets from the semigroup

$$M = \left\{ \begin{bmatrix} a_1 & a_2 & \ldots & a_{10} \\ a_{11} & a_{12} & \ldots & a_{20} \end{bmatrix} \middle| \; a_i \in Z^+ \cup \{0\}; 1 \le i \le 20 \right\}$$

be a subset semivector space over the semifield $Z^+ \cup \{0\}$.

Let $T_S^o : S \to S$, T_S^o is a subset semilinear operator.

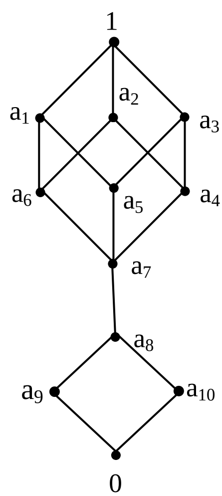
Define

$$T_S^o\left(\left\{\begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 & a_9 & a_{10} \\ a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} & a_{17} & a_{18} & a_{19} & a_{20} \end{bmatrix}\right\}\right)$$

$$= \left(\left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 & a_9 & a_{10} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \right\} \right)$$

It is easily verified T_S^o is a subset semilinear operator from S to S. $\ker T_S^o \neq \{(0)\}$ of S.

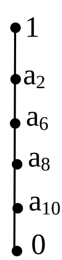
Example 2.66: Let S = {Collection of all subsets from the lattice group $LD_{2,5}$ where L is a lattice



}

be the subset semivector space over the semifield

F =



Clearly S is a subset semilinear algebra over the semifield F.

We can define $T_S^o : S \to S$ so that T_S^o is a subset semilinear operator of the S. Suppose we have two subset semilinear operators say T^1 and T^2 where

$T^1 : S \to S$ and $T^2 : S \to S$ we can define $T^1 + T^2$, T^1 o T^2 and T^2 o T^1 and all these will again be a subset semilinear operators of S.

We have defined subset semivector space using only semigroups over semifields. However we can use groups and rings and still define the notion of subset semivector spaces. We call such structures as generalized subset semivector spaces.

We will just develop and describe them.

Let S = {Collection of all subsets from the group (Z, +)} be the subset semigroup under +. S is a subset semivector space over the semifield $F = Z^+ \cup \{0\}$. We call S to be the generalized subset semivector space over the semifield F.

We first give examples of them.

Example 2.67: Let
S = {Collection of all subsets from the group (R, +)} be the subset generalized semivector space over the semifield $F = R^+ \cup \{0\}$ (or $Q^+ \cup \{0\}$ or $Z^+ \cup \{0\}$).

Take $A = \{0, 5\sqrt{3}, 7\sqrt{8} + 9, 5\sqrt{31} + 1, -3/\sqrt{7}\}$

and $B = \{1, \sqrt{3}, -\sqrt{3}/7, 8\sqrt{5}\} \in S$.

We now show how A + B is got

$$A + B = \{0, 5\sqrt{3}, 7\sqrt{8} + 9, 5\sqrt{31} + 1 - 8, -3/\sqrt{7}\} +$$

$$\{1, \sqrt{3}, -\sqrt{3}/7, 8\sqrt{5}\}$$

$$= \{1, \sqrt{3}, \sqrt{3}/7, 8/\sqrt{5}, 1 + 5\sqrt{3}, 10 + 7\sqrt{8}, 5\sqrt{31}+2, -7, 5\sqrt{3} + 8\sqrt{5}, (-3/\sqrt{7} + 8\sqrt{5}), (-8 + 8\sqrt{5}), 7\sqrt{8} + 8\sqrt{5}+9, (5\sqrt{31} + 1 + 8\sqrt{5}), -8-\sqrt{3}/7, 1-3/\sqrt{7}, 6\sqrt{3}, 7\sqrt{8} + \sqrt{3} + 9, 5\sqrt{31}+\sqrt{3}+1, -8+\sqrt{3}, -3/\sqrt{7} + \sqrt{3}, 7\sqrt{8}+9-\sqrt{3}/7, 5\sqrt{3} - \sqrt{3}/7, (-\sqrt{3}/7 - 3/\sqrt{7})$$

Let $\frac{\sqrt{3}}{5} \in F$; $\frac{\sqrt{3}}{5} \times A$

$$= \frac{\sqrt{3}}{5} \times \{0, 5\sqrt{3}, 7\sqrt{8} + 9, 5\sqrt{31}+1, -8, {-3}/{\sqrt{7}}\}$$

$$= \{0, 3, \frac{9\sqrt{3}+7\sqrt{24}}{5} \quad \frac{\sqrt{3}+5\sqrt{93}}{5}, \frac{-8\sqrt{3}}{5}, \frac{-3\sqrt{3}}{5\sqrt{7}}\} \in S.$$

Thus S is a generalized subset semivector space over the semifield $F = R^+ \cup \{0\}$.

Example 2.68: Let
S = {Collection of all subsets from the group (C, +)} be the subset semigroup under +. S is a generalized subset semivector space over the semifield $Z^+ \cup \{0\}$.

Example 2.69: Let
S = {Collection of all subsets from the group (RS_3, +)} be the subset semigroup under '+'. S is a generalized subset semivector space over the semifield $F = Q^+ \cup \{0\}$.

We can for these generalized subset semivector spaces also define the notion of substructures subset linear dependence subset linear independence, subset linear transformations and subset linear operators.

All these are considered as a matter of routine and hence left as an exercise to the reader. However we prove the following theorem which relates the subset semivector spaces and generalized subset semivector spaces.

THEOREM 2.4: *Let*
S = {Collection of all subsets from a group (G, +)} be the subset semigroup. S be a generalized subset semivector space over a semifield F (F; related to G). Then S is a subset semivector spaces. However a subset semivector space in general is not a generalized subset semivector space.

Proof. We know from the very definition that a generalized subset semivector space is a subset semivector space; but a subset semivector space in general is not generalized subset semivector space.

For consider S = {Collection of all subsets from the semigroup $Z^+ \cup \{0\}$ under +} be the subset semigroup. S is a subset semivector space over the semifield F = $Z^+ \cup \{0\}$.

It is clear that $Z^+ \cup \{0\}$ can never be a group under '+' so S can never be a generalized subset semivector space over the semifield $Z^+ \cup \{0\}$.

Now having related both we can define the notion of generalized subset semilinear algebras.

Let S = {Collection of subsets from the group (G, +)} be the generalized subset semivector space over a semifield. If on S we can define a product so that S under that product is a subset semigroup then we define S to be a subset semilinear algebra. So to define a subset semilnear algebra we need G to have product so that (G, ×) is a semigroup without which we cannot define the notion of subset semilinear algebra over the semifield.

We will illustrate this situation by a few examples.

Example 2.70: Let
S = {Collection of all subsets from the ring RS_3} be the generalized ring RS_3} be the generalized subset semilinear algebra over the semifield $Z^+ \cup \{0\}$. Clearly S is a non commutative subset semilinear algebra over the semifield $Z^+ \cup \{0\}$.

Take A = $\{-5\begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} + 9\begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} + 10,$

$$-8\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} + 19,\ 4\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}\}.$$

and

B = $\left\{3\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} + 7\begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} - 5, -10\begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}\right.,$

$$9\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} + 5\} \in S.$$

We define

$$A + B = \{-5\begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} + 9\begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} + 10, -8\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} +$$

$$19,\ 4\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}\} + \left\{3\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} + 7\begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} - 5,\right.$$

$$-10\begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix},\ 9\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} + 5\}$$

$$= \{-5\begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} + 16\begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} + 5 + 3\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix},$$

$$-15\begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} + 9\begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} + 10,\ 15 + 9\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} +$$

$$9\begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} - 5\begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix},\ 19 - 8\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} - 10\begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix},$$

$$-8\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} + 15 + 7\begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} + 3\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} + 5 +$$

$$4\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} + 9\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix},\ 7\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} + 7\begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} - 5,$$

$$19 - 8\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} - 10\begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix},\ + 4\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$$

$$- 10\begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix},\ 24 + \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}\} \in S.$$

This is the way '+' operation is performed on S.

Now we find

$$A \times B = \{-5\begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} + 9\begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} + 10,\ -8\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$$

$$+ 19,\ 4\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}\} \times \left\{3\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} + 7\begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} - 5,\right.$$

$$-10\begin{pmatrix}1 & 2 & 3\\ 3 & 1 & 2\end{pmatrix}, 9\begin{pmatrix}1 & 2 & 3\\ 2 & 1 & 3\end{pmatrix} + 5\}$$

$$= \{-15\begin{pmatrix}1 & 2 & 3\\ 1 & 2 & 3\end{pmatrix} - 35\begin{pmatrix}1 & 2 & 3\\ 2 & 1 & 3\end{pmatrix} + 25\begin{pmatrix}1 & 2 & 3\\ 3 & 1 & 2\end{pmatrix} +$$

$$27\begin{pmatrix}1 & 2 & 3\\ 2 & 1 & 3\end{pmatrix} + 63\begin{pmatrix}1 & 2 & 3\\ 1 & 2 & 3\end{pmatrix} +$$

$$30\begin{pmatrix}1 & 2 & 3\\ 2 & 3 & 1\end{pmatrix} + 70\begin{pmatrix}1 & 2 & 3\\ 1 & 3 & 2\end{pmatrix} - 50, 50\begin{pmatrix}1 & 2 & 3\\ 2 & 3 & 1\end{pmatrix}$$

$$-90\begin{pmatrix}1 & 2 & 3\\ 3 & 1 & 2\end{pmatrix} - 100\begin{pmatrix}1 & 2 & 3\\ 3 & 1 & 2\end{pmatrix},$$

$$-25\begin{pmatrix}1 & 2 & 3\\ 3 & 1 & 2\end{pmatrix} - 45\begin{pmatrix}1 & 2 & 3\\ 3 & 2 & 1\end{pmatrix} + 45\begin{pmatrix}1 & 2 & 3\\ 1 & 3 & 2\end{pmatrix} + 50 +$$

$$81\begin{pmatrix}1 & 2 & 3\\ 2 & 3 & 1\end{pmatrix}, -24\begin{pmatrix}1 & 2 & 3\\ 3 & 2 & 1\end{pmatrix} -$$

$$56\begin{pmatrix}1 & 2 & 3\\ 3 & 1 & 2\end{pmatrix} + 40\begin{pmatrix}1 & 2 & 3\\ 1 & 3 & 2\end{pmatrix} + 57\begin{pmatrix}1 & 2 & 3\\ 2 & 3 & 1\end{pmatrix} +$$

$$133\begin{pmatrix}1 & 2 & 3\\ 1 & 3 & 2\end{pmatrix} - 95, 80\begin{pmatrix}1 & 2 & 3\\ 1 & 3 & 2\end{pmatrix} -$$

$$190\begin{pmatrix}1 & 2 & 3\\ 3 & 2 & 1\end{pmatrix}, 20\begin{pmatrix}1 & 2 & 3\\ 2 & 3 & 1\end{pmatrix} + 36\begin{pmatrix}1 & 2 & 3\\ 1 & 3 & 2\end{pmatrix} \ldots \}$$

It is left for the reader to prove $A \times B \neq B \times A$.

Example 2.71: Let
S = {Collection of all subsets from the ring CS_7} be the subset semigroup under +. S is a generalized subset semilinear algebra over the semifield $R^+ \cup \{0\}$. S is clearly non commutative.

Example 2.72: Let
S = {Collection of all subsets from the ring $\langle C \cup I \rangle(S_3 \times D_{27})$} be the subset semigroup under '+'. S is a subset generalized semilinear algebra over the semifield $F = \langle Z^+ \cup I \cup \{0\}\rangle$.

Example 2.73: Let
S = {Collection of all subsets from the ring $Z(S_3 \times A_5 \times D_{2,11})$} be the subset semigroup under +. S is a subset generalized over the semifield $F = Z^+ \cup \{0\}$.

Now we next develop results about subset Smarandache semivector spaces and subset special Smarandache semivector spaces.

Example 2.74: Let
S = {Collection of all subsets from the group (Z, +)} be the subset semivector space over the semifield $F = Z^+ \cup \{0\}$. Clearly S is a Smarandache subset semivector space over the semifield F.

Example 2.75: Let
S = {Collection of all subsets from the group $\langle Q \cup I\rangle$} be the subset semivector space over the semifield $F = \langle Q^+ \cup I \cup \{0\}\rangle$. S is clearly a Smarandache subset semivector space over the semifield F.

For $P = \{\{a\} \mid a \in \langle Q \cup I\rangle\}$ is a group under + so S is a Smarandache subset semigroup.

Just we wish to recall that if P is a Smarandache semigroup then the subset semigroup S of P is a Smarandache subset semigroup.

Example 2.76: Let S = {Collection of all subsets from the semigroup $Z^+ \cup \{0\} \times Z$} be the subset semivector space over the semifield $F = Z^+ \cup \{0\}$. S is a Smarandache semivector space over the semifield $F = Z^+ \cup \{0\}$.

Example 2.77: Let S = {Collection of all subsets from the semigroup $Z^+ \cup \{0\} \times Q \times R$} be the subset semivector space over the semifield $F = Z^+ \cup \{0\}$. S is a Smarandache semivector space over the semifield $F = Z^+ \cup \{0\}$.

Example 2.78: Let S = {Collection of all subsets from the semigroup $P = (Z \times Q \times Z^+ \cup \{0\})S_3$} be the subset semivector space over the semifield $Z^+ \cup \{0\} = F$. Clearly S is a Smarandache subset semivector space over F.

Example 2.79: Let S = {Collection of all subsets from the semigroup $(Q \times Z \times R^+ \cup \{0\})$} be the subset semivector space over the semifield $F = Z^+ \cup \{0\}$. S is a Smarandache subset semivector space over the semifield F.

We have seen examples of subset Smarandahe semivector spaces other properties related with S can be derived as a matter of routine without any difficulty so this is left as an exercise for the reader.

Now we proceed onto describe the notion of quasi Smarandache subset semivector space over a Smarandache semiring.

Example 2.80: Let S = {Collection of substes from the semigroup $(Z^+ \cup \{0\} \times Q)S_3$} be the quasi Smarandache subset semivector space over the Smarandache semiring $(Z^+ \cup \{0\})S_3$.

Example 2.81: Let S = {Collection of all subsets from the semigroup $P = (Q^+ \cup \{0\} \times Z \times R)S(5)$} be the quasi Smarandache subset semivector space over the Smarandache semiring $(Z^+ \cup \{0\})\, S(5)$.

Example 2.82: Let S = {Collection of all subsets from the semigroup $(Z \times Q^+ \cup \{0\})(S_7 \times D_{2,6})$} be the quasi subset

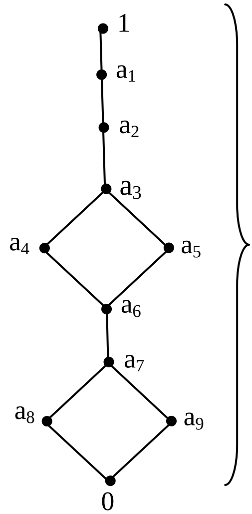
Smarandache semivector space over the Smarandache semiring $F = (Z^+ \cup \{0\})(S_7 \times \{1\})$.

Example 2.83: Let S = {Collection of all subsets from the semigroup $(Q^+ \cup \{0\} \times R)(S(5) \times A_4)$} be the subset quasi Smarandache semivector space over the Smarandache semiring $(Q^+ \cup \{0\})\ (S(5) \times \{1\})$.

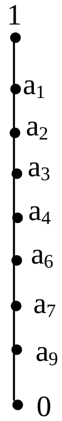
Example 2.84: Let S = {Collection of all subsets from the semigroup $(Z \times Q^+ \cup \{0\} \times R^+ \cup \{0\})S(10)$} be the quasi Smarandache subset semivector space over the Smarandache semiring $(Z^+ \cup \{0\})S_{10}$.

Having seen examples of quasi Smarandache subset semivector spaces of infinite order we now proceed onto describe finite order Smarandache quasi semivector spaces over S-semirings.

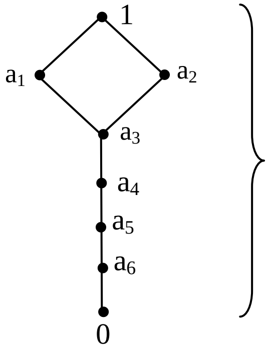
Example 2.85: Let S = {Collection of all subsets from the semigroup LS_3 where L is the lattice



be the quasi subset Smarandache semivector space over the S-semiring PS_3 where P =



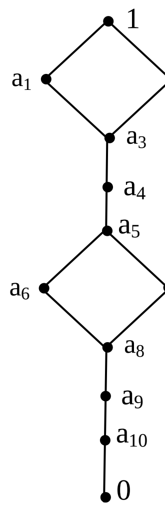
Example 2.86: Let S = {Collection of all subsets from the semigroup L_1 ($S_3 \times A_4$) where L_1 =



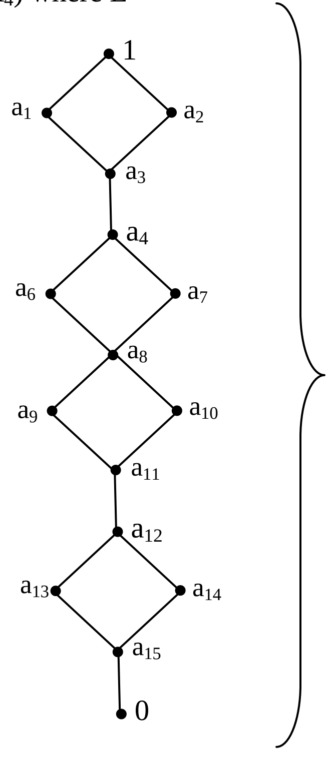
be the quasi Smarandache subset semivector space over the Smarandache semiring $L_1(S_3 \times \{1\})$. This S is a finite semivector space and S is non commutative.

S has substructures and we can define semilinear operator on S.

Example 2.87: Let S = {Collection of all subsets from the semigroup $L(S(5) \times D_{2,7})$} be the quasi Smarandache subset semivector space over the S-semiring $L(\{1\} \times D_{2,7})$ where L =

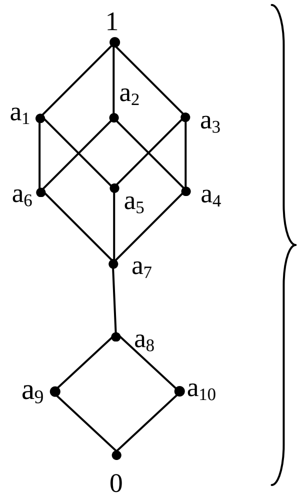


Example 2.88: Let S = {Collection of all subsets from the semigroup $L(D_{2,7} \times A_4)$ where L =

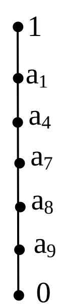


be the quasi Smarandache subset semivector space over the S–semiring $L(D_{2,7} \times A_4)$. S is also a quasi Smarandache subset semilinear algebra over $L(D_{2,7} \times A_4)$.

Example 2.89: Let S = {Collection of all subsets from the semigroup $LD_{2,9}$ where L =



be the Smarandache subset semivector space over the S-semiring $PD_{2,9}$ where P =



Clearly S is of finite order and S is non commutative quasi Smarandache subset semilinear algebra over $PD_{2,9}$.

We suggest the following problems for this chapter.

Problems

1. Give some special and interesting features enjoyed by subset semivector spaces.

2. Let S = {Collection of all subsets from the semigroup $Z^+ \cup \{0\}$} be the subset semivector space over the semifield $F = Z^+ \cup \{0\}$.

   (i) Prove S has infinite number of subset semivector subspaces.
   (ii) Find subset semilinear operator T_S^o on S so that kernel of $T_S^o \neq \{0\}$.
   (iii) Can S be written as a direct sum of subset subspaces?
   (iv) Is S a Smarandache subset semivector spaces?
   (v) What is the algebraic structure enjoyed by {Collection all subset semilinear operators on S}?
   (vi) Is S a subset semilinear algebra over F?
   (vii) Find a subset basis of S over F.

3. Let S = {Collection of all subsets from the matrix semigroup $m = \{(a_1, a_2, a_3, \ldots, a_9) \mid a_i \in Z^+ \cup \{0\}, 1 \leq i \leq 9\}$} be a subset semivector space over the semifield $F = Z^+ \cup \{0\}$.

   Study questions (i) to (vii) of problem 2 for this S.

4. Let S = {Collection of all subsets from the matrix

$$\text{semigroup } N = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ \vdots & \vdots & \vdots \\ a_{13} & a_{14} & a_{15} \end{bmatrix} \right| a_i \in \langle Q^+ \cup \{0\} \rangle;$$

   $1 \leq i \leq 15\}$} be the subset semivector space over $F = Q^+ \cup \{0\}$.

Study problems (i) to (vii) of problem 2 for this S.

If $Q^+ \cup \{0\}$ is replaced by $Z^+ \cup \{0\}$ what can be said about their subset basis?

Compare them, with S is over $Q^+ \cup \{0\}$ when S is over $Z^+ \cup \{0\}$.

5. Let S = {Collection of all subsets from the semigroup

$$P = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \end{bmatrix} \middle| \; a_i \in R^+ \cup \{0\}; \right.$$

$1 \leq i \leq 25$}} be the subset semivector space over $Z^+ \cup \{0\}$.

Study questions (i) to (vii) of problem 2 for this S.

6. Let S = {Collection of all subsets from the semigroup

$$W = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_9 \end{bmatrix} \middle| \; a_i \in Z^+ \cup \{0\}; 1 \leq i \leq 9 \right\}$$ } be the subset

semivector space over the semifield $Z^+ \cup \{0\}$.

Study questions (i) to (vii) of problem 2 for this S.

7. What are the special features enjoyed by subset semilinear algebra?

8. When will the subset basis be larger for the same S, when realized as a subset semivector space or as a subset semilinear algebra ?

9. Let S = {Collection of all subsets from the semigroup $P = \{(a_1\ a_2 \mid a_3\ a_4 \mid a_5) \mid a_i \in Z^+ \cup \{0\}; 1 \leq i \leq 5\}\}$ the subset semivector space over the semifield $Z^+ \cup \{0\}$.

   Study question (i) to (vii) of problem 2 for this S.

10. Let S = {Collection of all subsets from the semigroup

$$M = \left\{ \left[\begin{array}{c} a_1 \\ \hline a_2 \\ a_3 \\ \hline a_4 \\ a_5 \\ a_6 \\ \hline a_7 \\ a_8 \\ a_9 \end{array} \right] \middle| \; a_i \in Q^+ \cup \{0\}; 1 \leq i \leq 9 \right\}\}$$

   be the subset semivector space over the semifield $Q^+ \cup \{0\}$.

   Study questions (i) to (vii) of problem 2 for this S.

11. Let S = {Collection of all subsets from the semigroup

$$P = \left\{ \left[\begin{array}{c|cccc|cc|cc} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 & a_9 \\ a_{10} & a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} & a_{17} & a_{18} \\ a_{19} & a_{20} & \dots & \dots & \dots & \dots & \dots & \dots & a_{27} \\ a_{28} & a_{29} & \dots & \dots & \dots & \dots & \dots & \dots & a_{36} \end{array} \right] \middle| \; a_i \right.$$

   $\in Q^+ \cup \{0\}; 1 \leq i \leq 36\}\}$ be the subset semivector space over the semifield $F = Q^+ \cup \{0\}$.

Study questions (i) to (vii) of problem 2 for this S.

12. Let S = {Collection of all subsets from the semigroup

$$W = \left\{ \left[\begin{array}{cccc} a_1 & a_2 & a_3 & a_4 \\ \hline a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ \hline a_{13} & a_{14} & a_{15} & a_{16} \\ a_{17} & a_{18} & a_{19} & a_{20} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ \hline a_{25} & a_{26} & a_{27} & a_{28} \\ a_9 & a_{30} & a_{31} & a_{32} \end{array} \right] \right| a_i \in Q^+ \cup \{0\}; 1 \leq i \leq 32\}\}$$

be the subset semivector space over the semifield $F = Q^+ \cup \{0\}$.

Study questions (i) to (vii) of problem 2 for this S.

13. Let S = {Collection of all subsets from the semigroup

$$W = \left\{ \left[\begin{array}{c|c|c|c} a_1 & & (0) & (0) \\ \hline (0) & \begin{array}{cc} a_2 & a_3 \\ a_4 & a_5 \end{array} & (0) & (0) \\ \hline (0) & (0) & \begin{array}{ccc} a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} \\ a_{12} & a_{13} & a_{14} \end{array} & (0) \\ \hline (0) & (0) & (0) & \begin{array}{cccc} a_{15} & a_{16} & a_{17} & a_{18} \\ a_{19} & a_{20} & a_{21} & a_{22} \\ a_{23} & a_{24} & a_{25} & a_{26} \\ a_{27} & a_{28} & a_{29} & a_{30} \end{array} \end{array} \right] \right|$$

$a_i \in Q^+ \cup \{0\}; 1 \leq i \leq 30\}\}$ be the subset semivector space over the semifield $F = Q^+ \cup \{0\}$.

Study questions (i) to (vii) of problem 2 for this S.

Study when $Q^+ \cup \{0\}$ is replaced by $Z^+ \cup \{0\}$.

14. Let S = {Collection of all subsets from the semigroup $(Z^+ \cup \{0\})S_3$} be the subset semivector space over the semifield $F = Z^+ \cup \{0\}$.

    (i) Prove S is non commutative.
    (ii) Study questions (i) to (vii) of problem 2 for this S.

15. Evolve a method to find a subset basis of subset semivector space.

16. Give an example of a subset semivector space which has only a finite number of elements in the subset basis.

17. Give an example of a subset semivector space which has an infinite number of elements in the subset basis.

18. Do the elements of a subset basis of a subset semivector space, subset linearly dependent or subset linearly independent?

19. Prove the number of elements in a subset basis depends on the semifield over which the space is defined.

20. Give some stricking differences between the subset semivector spaces and usual semivector spaces.

21. Can a subset semivector space S have more than one subset basis?

22. Is it possible to have a subset semvector space which has more than one subset basis?

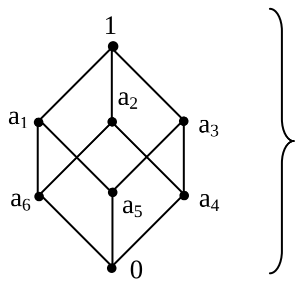
23. Let S be a subset semivector space. V_S^o = {Collection of all subsets semilinear operators on S}. Does V_S^o enjoy any nice algebraic structure?

24. Let S and S_1 be two subset semivector spaces over the same semifield F. W_s = {Collection of all subset semilinear transformations of S to S_1}.
What is the algebraic structure enjoyed by W_s?

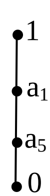
25. Let S = {Collection of all subsets from the semigroup $M = \left\{ \sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in Q^+ \cup \{0\} \right\}$} be the subset semivector space over the semifield $Q^+ \cup \{0\}$.

Study questions (i) to (vii) of problem 2 for this S.

26. Let S = {Collection of all subsets from the semigroup L where L



be the subset semivector space over the semifield F =



(i) Find o(S).
(ii) Find a subset basis for S.

(iii) Is S a subset semilinear algebra?

(iv) Find all subset subsemivector subspaces of S.

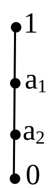
(v) Find V_S^o = {all subset semilinear operators on S}.

(vi) Can S have more than one subset basis?

27. Let S = {Collection of all subsets from the lattice L = C_{10}} be the subset semivector space over C_{10}.

Study questions (i) to (vi) of problem 26 for this S.

(i) If L is replaced by



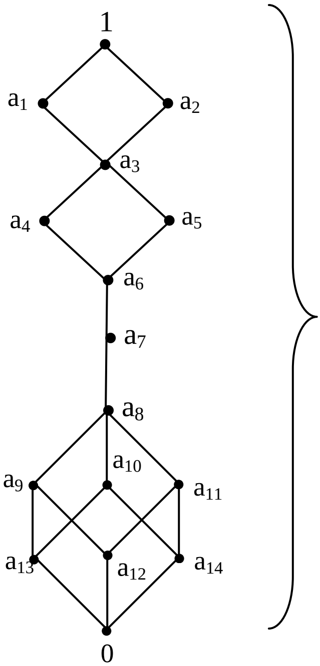
Study questions (i) to (vi) of problem 26 for this S.

28. Let S = {Collection of all subsets from the lattice group LS_4 where S is a Boolean algebra of order 16} be a subset semivector space over

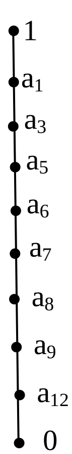
F = 

Study questions (i) to (vi) of problem 26 for this S.

29. Let S = {Collection of all subsets from the smeigroup LS(3) where L =



be the subset semivector space over the semifield P =



Study questions (i) to (vii) of problem 26 for this S.

30. Let S = {Collection of all subsets from the semigroup M = $(Z^+ \cup \{0\}$ $(S_3 \times S(7))\}$ be the subset semivector space over the semifield $Z^+ \cup \{0\}$.

    (i) S as a semilinear algebra is non commutative prove.
    (ii) Find a subset basis of S.
    (iii) Can S have more than one subset basis?
    (iv) Find all subset semivector subspaces of S.
    (v) Let V_S^o = {Collection of all subsets semilinear operators on S}.
    Find the algebraic structure enjoyed V_S^o.
    (vi) Is S a Smarandache semivector space?
    (vii) Can S be written as a direct sum of subset semivector subspaces?

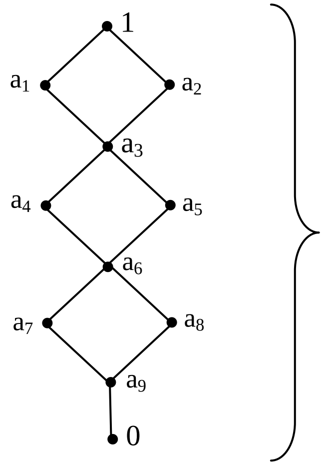
31. Let S = {Collection of all subsets from the semiring $(Z \times Q^+ \cup \{0\})D_{2,9}\}$ be the subset semivector space over the semifield $Z^+ \cup \{0\}$.

    (i) Study questions (i) to (vii) of problem 30 for this S.
    (ii) Is S a quasi Smarandache subset semivector space over $(Z^+ \cup \{0\})$ $D_{2,9}$?
    (iii) Is S a Smarandache subset semivector space over $(Z^+ \cup \{0\})$ of finite subset dimension?

32. Let S = {Collection of all subsets from the semiring $Q \times Z^+ \cup \{0\}$ be the subset semivector space over the semifield $Z^+ \cup \{0\}$.

    Study questions (i) to (vii) of problem 30 for this S.

33. Let S = {Collection of all subsets from the lattice group $L(S_3 \times S(5))$ where L =



be the quasi S-subset semivector space over the S-semiring F = L($S_3 \times \{1\}$)

(i) Find the o(S).

(ii) Find all subset semivector subspaces of S.

(iii) Can S be written as a direct sum of subset semivector subspaces?

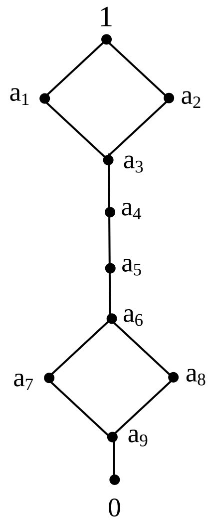
(iv) Find a subset basis of S.

(v) Can S have more subset basis?

(vi) Can S be a S-subset semilinear algebra over F = L($S_3 \times \{1\}$)?

(vii) S as a S-subset semilinear algebra have a basis different from B mentioned in iv.

34. Let S = {Collection of all subsets from the group lattice LG = LA_4 where L =



be the quasi subset semivector space over the S–semiring LA_4.

Study questions (i) to (vii) of problem 31 for this S.

35. Let S = {Collection of all subsets from the semigroup

$$M = \left\{ \left[\begin{array}{cc} a_1 & a_2 \\ a_3 & a_4 \\ \hline a_5 & a_6 \\ a_7 & a_8 \\ a_9 & a_{10} \end{array} \right] \middle| \; a_i \in (Z^+ \cup \{0\})S_3; 1 \leq i \leq 10 \right\}$$ be the

subset semivector space over the semifield $F = Z^+ \cup \{0\}$ and S_1 = {Collection of all subsets from the semigroup

$$P = \left\{ \left[\begin{array}{c} a_1 \\ \hline a_2 \\ a_3 \\ \hline a_4 \\ a_5 \\ a_6 \\ \hline a_7 \\ a_8 \\ a_9 \\ a_{10} \end{array} \right] \middle| \; a_i \in Z^+ \cup \{0\} \; (D_{2,9} \times S_3); 1 \leq i \leq 10 \right\}$$

be the subset semivector space over the semifield $F = (Z^+ \cup \{0\})$

(i) Find T_s: $S \rightarrow S_1$ so that T_S is one to one and onto subset semilinear transformation.

(ii) If W_S = {Collection of all subset semilinear transformations from S to S_1} find the algebraic structure enjoyed by W_s.

(iii) Find V_s and V_{S_1}. Is $V_S \cong V_{S_1}$ as algebraic structures?

(iv) If both S and S_1 are realized as S–quasi semivector spaces over the S-semiring $F = (Z^+ \cup \{0\})S_3 \cong Z^+ \cup \{0\} (\{1\} \times S_3)$

(v) Find W_S^q and $W_{S_1}^Q$ as a S-quasi semivector spaces.

Find V_S^q and $V_{S_1}^q$ as S-quasi semivector spaces.

(vi) Compare (i) W_S^q with W_s

(ii) $W_{S_1}^q$ with W_{S_1}

(iii) V_s with V_S^q and

(iv) V_{S_1} with $V_{S_1}^q$.

36. Enumerate some special features enjoyed by Smarandache subset semivector spaces.

37. Enumerate all the special properties associated with Smarandache quasi subset semivector spaces over S-semiring.

38. Compare S-subset semivector spaces and S-quasi subset semivector space where S is the subset from the same semiring only the semifield is contained in the S-semiring for the later structure.

39. Let S = {Collection of all subsets from the semigroup

$$M = \left\{ \left. \begin{bmatrix} a_1 & a_2 & \ldots & a_9 \\ a_{10} & a_{11} & \ldots & a_{18} \\ a_{19} & a_{20} & \ldots & a_{27} \end{bmatrix} \right| a_i \in (Z^+ \cup \{0\}) [S_8 \times A_4] \right\}$$

and

S_1 = {Collection of all subsets from the semigroup

$$N = \left\{ \left. \left[\begin{array}{ccc|cc|ccccc} a_1 & a_2 & a_3 & (0) & & & & (0) & & \\ a_4 & a_5 & a_6 & & & & & & & \\ a_7 & a_8 & a_9 & & & & & & & \\ \hline & & & a_{10} & a_{11} & & & & & \\ & (0) & & a_{12} & a_{13} & & & & & \\ & & & a_{14} & a_{15} & & & (0) & & \\ & & & a_{16} & a_{17} & & & & & \\ \hline & & & & & a_{18} & a_{19} & a_{20} & a_{21} & a_{22} \\ & (0) & & (0) & & a_{23} & a_{24} & a_{25} & a_{26} & a_{27} \end{array} \right] \right| \right.$$

$a_i \in (Z^+ \cup \{0\})[S_8 \times A_4]\}$

be the S-quasi subset semivector space over the S-semiring $F = (Z^+ \cup \{0\}) (S_8 \times A_4)\}$.

Study questions (i) to (vi) of problem 35 for this S and S_1.

40. Let S = {Collection of all subsets from $(Z^+ \cup \{0\}) (S_3 \times D_{2,7} \times A_4)\}$ and S_1 = {Collection of all subsets from $(Z^+ \cup \{0\}) (A_3 \times D_{27} \times S_4)\}$ be the S-quasi subset semivector spaces over the S-semiring $(Z^+ \cup \{0\}) (A_3 \times D_{2,7} \times A_4)$.

Study questions (i) to (vi) of problem 35 for this S and S_1.

41. Let S = {Collection of all subsets from the semiring $(Z^+ \cup \{0\}) (S_3 \times D_{2,8})\}$ be the subset semilinear algebra over the semifield $F = Z^+ \cup \{0\}$.

(i) Find a subset basis of S over F.
(ii) Can S have more than one basis over F?
(iii) Is S finite subset dimensional over F?
(iv) Let $V_s : \{T_S^o : S \to S\}$; what is the algebraic structure enjoyed by V_s?
(v) Can S be written as a finite direct sum of subset semivector subspaces of S?
(vi) Does S contain infinite number of subset subsemivector spaces?
(vii) Can S be realized as a subset semilinear algebra?
(viii) Is S a Smarandache subset semilinear algebra over F?
(ix) Find a subset basis relative to S as a subset semilinear algebra over F.

42. Let S = {Collection of all subsets from the matrix

$$\text{semigroup } M = \left\{ \begin{bmatrix} a_1 & a_2 & \dots & a_6 \\ a_7 & a_8 & \dots & a_{12} \\ a_{13} & a_{14} & \dots & a_{18} \end{bmatrix} \middle| \; a_i \in (Z^+ \cup \{0\}) \right.$$

$D_{2,11}$; $1 \leq i \leq 18\}\}$ be the subset semivector space over the semifield $F = Z^+ \cup \{0\}$.

(i) Study questions (i) to (ix) for problem 41 for this S.

(ii) Prove S can be written as a direct sum of n-subset semivector spaces n = 2, 3, …, 18.

(iii) Prove S has infinite number of subset semivector subspaces which cannot be written as a sum of subset semivector subspaces.

43. Let S = {Collection of all subsets from the matrix

$$\text{semigroup } M = \left\{\begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} \end{bmatrix} \middle|\ a_i \in (Z^+ \cup \{0\})\ A_5;\right.$$

$1 \leq i \leq 15\}\}$ be the subset semilinear algebra over the semifield $F = Z^+ \cup \{0\}$.

(i) Study question (i) to (ix) of problem 41 for this S.

(ii) Write S as a n-direct sum of subset semilinear algebras, n = 1, 2, 3, …, 15.

(iii) Find subset basis of S and compare it in case S is only realized as a subset semivector space over F.

44. Let S = {Collection of all subsets from the super matrix

$$\text{semigroup } M = \left\{ \left[\begin{array}{ccc} a_1 & a_2 & a_3 \\ \hline a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ a_{10} & a_{11} & a_{12} \\ \hline a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & a_{18} \\ \hline a_{19} & a_{20} & a_{21} \\ a_{22} & a_{23} & a_{24} \\ a_{25} & a_{26} & a_{27} \\ a_{28} & a_{29} & a_{30} \end{array} \right] \right| a_i \in (Q^+ \cup \{0\})(A_4 \times$$

$S(3)), 1 \leq i \leq 30\}\}$ be the subset semilinear algebra over the semifield $Q^+ \cup \{0\}$.

(i) Study questions (i) to (ix) for problem 41 for this S.
(ii) Write S as a n-direct sum of subset semivector subspaces over F; n = 2, …, 30.

45. Let S = {Collection of all subsets from the super matrix

$$\text{semigroup } M = \left\{ \left[\begin{array}{ccc|cc|c} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 & a_{10} & a_{11} & a_{12} \\ \hline a_{13} & a_{14} & a_{15} & a_{16} & a_{17} & a_{18} \\ a_{19} & a_{20} & a_{21} & a_{22} & a_{23} & a_{24} \\ a_{25} & a_{26} & a_{27} & a_{28} & a_{29} & a_{30} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} \end{array} \right] \right| a_i \in$$

$(Z^+ \cup \{0\} \times Q^+ \cup \{0\})S_4;\ 1 \leq i \leq 36\}\}$ be the subset semilinear algebra over the semifield $F = Z^+ \cup \{0\}$.

(i) Study questions (i) to (ix) for problem 41 for this S.
(ii) Write S as a n-direct sum of subset semilinear algebras over F; n = 2, 3, …, 36.

46. Let S = {Collection of all subsets from the super matrix

$$\text{semigroup } M = \left\{ \left[\begin{array}{cc|cccc|cc} a_1 & a_2 & a_3 & \ldots & a_8 & a_9 & a_{10} \\ a_{11} & a_{12} & a_{13} & \ldots & a_{18} & a_{19} & a_{20} \\ a_{21} & a_{22} & a_{23} & \ldots & a_{28} & a_{29} & a_{30} \\ a_{31} & a_{32} & a_{33} & \ldots & a_{38} & a_{39} & a_{40} \\ a_{41} & a_{42} & a_{43} & \ldots & a_{48} & a_{49} & a_{50} \end{array} \right] \right| a_i$$

$\in Z^+ \cup \{0\}S_9 \times Q^+ \cup \{0\}$ S(7); $1 \leq i \leq 50\}\}$ be the subset semilinear algebra over the semifield F = $Z^+ \cup \{0\}$.

(i) Study questions (i) to (ix) for problem 41 for this S.
(ii) Write S as a n-direct sum of subset semilinear algebras, n = 2, 3, …, 50.
(iii) Prove S is non commutative subst semilinear algebra over F = $Z^+ \cup \{0\}$.

47. Let S = {Collection of all subsets from the semigroup

$$M = \left\{ \left[\begin{array}{cccc} a_1 & a_2 & \ldots & a_5 \\ a_6 & a_7 & \ldots & a_{10} \\ \vdots & \vdots & & \vdots \\ a_{21} & a_{22} & \ldots & a_{25} \end{array} \right] \right| a_i \in (R^+ \cup \{0\})S(5); 1 \leq i \leq$$

25}} be the subset semilinear algebra over the semifield F = $R^+ \cup \{0\}$.

(i) Study question (i) to (ix) of problem 41 for this S.
(ii) Write S as a n-direct sum of subset semilinear algebras, n = 2, 3, …, 25.
(iii) Prove S is non commutative.
(iv) Find a subset basis of S over F.

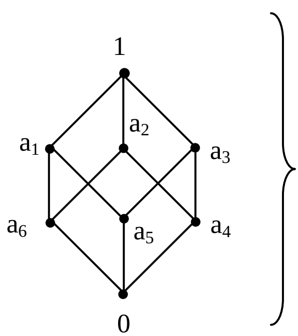
48. Let S = {Collection of all subsets from the semigroup $\langle R \cup I\rangle$ under '+'} be the S-subset semivector space over the semifield $\langle R^+ \cup \{0\} \cup I\rangle$.

    (i) Study question (i) to (ix) of problem 41 for this S.
    (ii) Prove S has a S-subsemigroup S.
    (iii) Suppose S is realized as semivector space say S_1 over $Z^+ \cup \{0\}$. Study the related problems.
    (iv) Does this change for $\langle R^+ \cup \{0\} \cup I\rangle$ to $Z^+ \cup \{0\}$ make any difference on the subset basis?

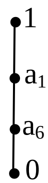
49. Let S = {Collection of all subsets from the group $R^+ \cup \{0\} \times Z^+ \cup \{0\}$} be the subset semilinear algebra over the semifield $Z^+ \cup \{0\}$.

    (i) Study question (i) to (ix) of problem 41 for this S.
    (ii) Does there exist a subset semivector subspace W of S so that $S = W + W^\perp$? ($W^\perp$ orthogonal complement of W).
    (iii) Prove S is only a commutative subset semilinear algebra.

50. Let S = {Collection of all subsets from the lattice group $LG = LA_5$ where L =



be the generalized subset semivector space over the semifield

F = 

(i) Find o(S).
(ii) Find the subset basis of S over F.
(iii) Can S have more than one subset basis?
(iv) Can S be written as a direct sum of generalized subset semivector subspaces over F?

51. Study the special features enjoyed by generalized subset semilinear algebras over semifields.

52. Let S = {Collection of all subsets from the group RS_4} be the generalized subset semivector space over the semifield $R^+ \cup \{0\}$.

Study question (ii) to (iv) of problem 50 for this S.

53. Let S = {Collection of all subsets from the group $G = \{(a_1, a_2, a_3, a_4, a_5, a_6) \mid a_i \in ZS_5; 1 \leq i \leq 6\}$} be the generalized subset semivector space over the semifield $R^+ \cup \{0\}$.

Study question (i) to (iv) of problem 50 for this S.

54. Let S = Collection of all subsets from the group

$$G = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_9 \end{bmatrix} \middle| \; a_i \in Q\,(S_4 \times S(3));\ 1 \leq i \leq 9 \right\}\}$$

be the generalized subset semilinear algebra under natural product over the semifield $F = Q^+ \cup \{0\}$.

(i) Find a subset basis of S over F.
(ii) Can S have more than one subset basis?
(iii) Is S finite subset dimensional?
(iv) If $Q^+ \cup \{0\}$ is replaced by $Z^+ \cup \{0\}$ will S be of finite dimension?
(v) Can S be represented as direct sum of generalized semilinear subalgebras?
(vi) Show $S = \{W + W^\perp$ is possible where W is a generalized subset semilinear algebra over $Q^+ \cup \{0\}$ and $W^\perp$ is the orthogonal complement of W.
(vii) Find V_S^o = {Collection of all subsets semilinear operators on S}.

What is the algebraic structure enjoyed by S?

55. Let S = {Collection of all subsets from the group

$$G = \left\{ \left[\begin{array}{cc} a_1 & a_2 \\ \hline a_3 & a_4 \\ a_5 & a_6 \\ \hline a_7 & a_8 \\ a_9 & a_{10} \\ a_{11} & a_{12} \\ a_{13} & a_{14} \end{array}\right] \middle| \; a_i \in Q(S_3 \times D_{2,7});\ 1 \le i \le 14\right\}\}$$

be the generalized subset semilinear algebra over the semifield $Z^+ \cup \{0\}$.

(i) Show S is non commutative.
(ii) Study questions (i) to (vii) of problem 54 for this S.

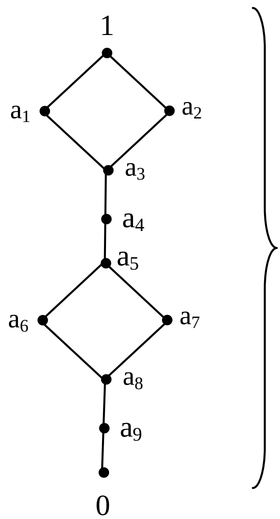
56. Let S = {Collection of all subsets from the ring

$$M = \left\{ \begin{bmatrix} a_1 & a_2 & \ldots & a_{12} \\ a_{13} & a_{14} & \ldots & a_{24} \\ a_{25} & a_{26} & \ldots & a_{36} \end{bmatrix} \middle| \; a_i \in Z\,(D_{2,7} \times S_3 \times A_4); \right.$$

$1 \leq i \leq 36\}\}$ be subset generalized semivector space over the semifield $F = Z^+ \cup \{0\}$.

Study questions (i) to (vii) of problem 54 for this S.

57. Let S = {Collection of all subsets from the lattice grouplattice LG where $G = A_5$ and L is a lattice which is as follows: L =



be the quasi subset semivector space over the S-semiring LG.

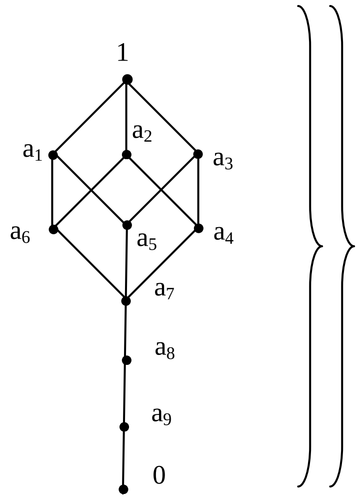
(i) Find o(S).
(ii) Find subset basis of S over LG.
(iii) Can S have more than one subset basis over LG?

(iv) Find atleast four subsets which are subset linearly independent.

(v) Find at least 5 subsets which are subset linearly dependent.

(vi) Will the subset linearly independent elements generate a subset semivector subspace over $Z^+ \cup \{0\}$?

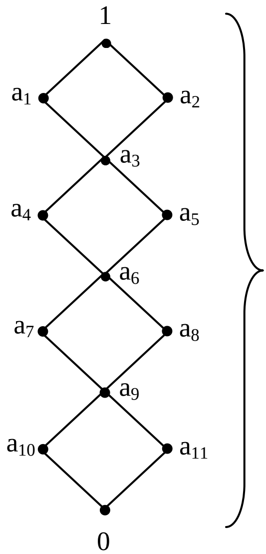
58. Let S = {Collection of all subsets from the group lattice $LD_{2,11}$ where L is the lattice



be the quasi subset semivector space of the S–semiring.

Study questions (i) to (vi) of problem 59 for this S.

59. Let S = {Collection of all subsets from the semigroup lattice LS(4) where L =



be the S-quasi subset semivector space over the S-semiring LG = LS(4).

Study questions (i) to (vi) of problem 57 for this S.

60. If S is a S-quasi subset semilinear algebra over a S-semiring F.

What is the algebraic structure enjoyed by V_S^o = {Collection of all subset semilinear operators on S}?

61. Let S_1 and S_2 be any two S-quasi subset semilinear algebras over the same S-semiring F.

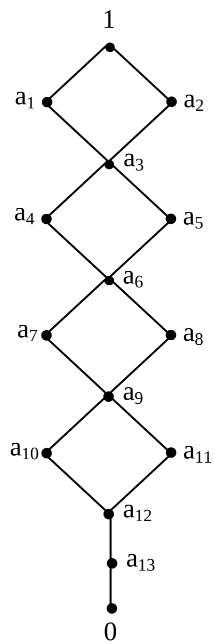
What is the algebraic structure enjoyed by W_S^q = {Collection of all quasi semilinear transformation from S_1 to S_2}?

62. Let S = {Collection of all subsets from the matrix group

$$P = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ \vdots & \vdots & \vdots \\ a_{28} & a_{29} & a_{30} \end{bmatrix} \right| a_i \in LG = L(D_{210} \times A_5);$$

$1 \leq i \leq 30\}\}$

be the S-quasi subset semilinear algebra over the S-semiring $L(D_{2,10} \times \{1\})$ where L =

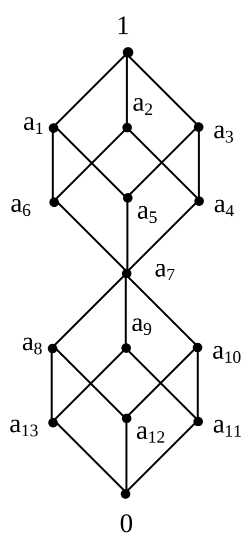


Study questions (i) to (vi) of problem 57 for this S.

63. Let S = {Collection of all subsets from the group lattice

$$\text{super matrix } M = \left\{ \left[\begin{array}{c} a_1 \\ a_2 \\ a_3 \\ a_4 \\ \hline a_5 \\ \hline a_6 \\ a_7 \\ \hline a_8 \\ a_9 \\ a_{10} \end{array} \right] \middle| \; a_i \in LS_4;\ 1 \leq i \leq 10 \text{ and L is as} \right.$$

follows;



}

be the S-quasi subset semilinear algebra over the S-semiring LS_4.

Study questions (i) to (vi) problem 57 for this S.

Chapter Three

SPECIAL STRONG SUBSET SEMILINEAR ALGEBRAS

In this chapter we for the first time define, develop and describe the new notion of Smarandache special strong subset semivector spaces and Smarandache special strong subset semi linear algebras.

All these strong special subset semilinear algebras (semivector spaces) contain as a substructure the subset semilinear algebra over the appropriate semifield of the subset semiring over which the basic structure is defined.

DEFINITION 3.1: *Let S = {Collection of all subsets from a semigroup (or a group or a semilattice under ∪)} be a subset semigroup. Suppose P be the collection of all subsets of a ring or a semiring, that is P is a subset semiring such that*

(i) *If for all* $s \in S$ *and* $p \in P$; *sp and ps* $\in S$.

(ii) $p(s_1 + s_2) = ps_1 + ps_2$

(iii) $(p_1 + p_2)s = p_1s + p_2s$

(iv) $\{0\} p = \{0\}$

(v) $S\{0\} = \{0\}$

for all $s_1, s_2, s \in S$ *and* $p, p_1, p_2 \in P$.

We define S to be a Smarandache special strong subset semivector space over the subset semiring P.

If in addition an operation product is defined on S we define S to be a Smarandache special strong subset semilinear algebra over the subset semiring P.

We use the term Smarandache strong special subset semivector space as P in most cases is only a Smarandache subset semiring and not a semifield.

We will first illustrate this situation by some examples.

Example 3.1: Let S = {Collection of all subsets from the semigroup B = ($Z^+ \cup \{0\} \times Z^+ \cup \{0\}$) under addition} be a subset semigroup of B.

P = {Collection of all subsets from the semifield $Z^+ \cup \{0\}$} be the subset semiring. P is a Smarandache subset semiring.

We see S is a Smarandache special strong subset semivector space over the S-subset semiring P.

For if A = {(3, 2), (5, 0), (0, 0), (7, 8), (0, 6), (11, 2)} and
B = {(1, 1), (2, 0), (5, 7), (8, 0)} $\in$ S.

We see

A + B = {(3, 2), (5, 0), (0, 0), (7, 8), (0, 6), (11, 2)} + {(1, 1), (2, 0), (5, 7), (8, 0)}

= {(4, 3), (5, 1), (1, 1), (8, 9), (1, 7), (5, 2), (7, 0), (2, 0), (9, 8), (2, 6), (8, 9), (10, 7), (5, 7), (12, 15), (5, 13), (12, 3), (13, 2), (19, 11), (11, 2), (13, 0), (8, 0), (15, 8), (8, 6), (19, 2)} $\in$ S.

Let M = {3, 0, 5, 9, 12, 15} $\in$ P.

M $\times$ A = {3, 0, 5, 9, 12, 15} $\times$ {(3, 2), (5, 0), (0, 0), (7, 8), (0, 6), (11, 2)}

= {(9,6), (15,0), (0,0), (21,24), (0,18), (33,6), (15,10), (25,0), (35,40), (0,30), (55,10), (27,18), (45,0), (63,72), (0,54), (99,18), (36,24), (60,0), (72,96), (0,72), (132,24), (45,30), (75,0), (105,120), (0,90), (165,30)} $\in$ S.

This is the way operations are performed on S.

Example 3.2: Let S = {Collection of all subsets from the semigroup $R^+ \cup \{0\}$ under the addition '+'} be the subset semigroup.

Let P = {Collection of all subsets from the semifield $Z^+ \cup \{0\}$} be the Smarandache subset semiring.

S is defined as a Smarandache strong subset semivector space over the S-subset semiring P.

Take A = $\{\sqrt{5}, \sqrt{3}/2, \sqrt{17}/5, 1/\sqrt{11}, 0, 25, 9\}$ and
B = $\{\sqrt{19}, \sqrt{31}, 10, 11, 12\} \in$ S and
M = {0, 1, 2, 5, 7, 10, 6} $\in$ P.

$$A + B = \{\sqrt{5}, \sqrt{3}/2, \sqrt{17}/5, 1/\sqrt{11}, 0, 25, 9\} + \{\sqrt{19}, \sqrt{31}, 10, 11, 12\}$$

$$= \{\sqrt{5} + \sqrt{19}, \sqrt{5} + \sqrt{31}, 10 + \sqrt{5}, 11+\sqrt{5}, 12 + \sqrt{5}, \sqrt{3}/2 + \sqrt{19}, \sqrt{3}/2 + \sqrt{31}, 10 + \sqrt{3}/2, 11+\sqrt{3}/2, 12 + \sqrt{3}/2, \sqrt{17}/5 + \sqrt{19}, \sqrt{17}/5 + \sqrt{31}, \sqrt{17}/5 + 10, \sqrt{17}/5 + 11, \sqrt{17}/5 + 12, 1/\sqrt{11} + \sqrt{19}, 1/\sqrt{11} + \sqrt{31}, 1/\sqrt{11} + 10, 1/\sqrt{11} + 11, 1/\sqrt{11} + 12, \sqrt{19}, \sqrt{31}, 10, 11, 12, 25 + \sqrt{19}, 25 + \sqrt{31}, 35, 36, 37, 19, 20, \sqrt{19} + 9, \sqrt{31} + 9\} \in S.$$

Now consider

$M \times A =$ {0, 1, 2, 5, 7, 10, 6} $\times$\{ $\sqrt{5}$, $\sqrt{3}$/2, $\sqrt{17}$/5, 1/$\sqrt{11}$, 0, 25, 9}

= {0, $\sqrt{17}$/5, 1/$\sqrt{11}$, $\sqrt{5}$, $\sqrt{3}$/2, 25, 9, 2$\sqrt{5}$, $\sqrt{3}$, 2$\sqrt{17}$/5, 2/$\sqrt{11}$, 50, 18, 5$\sqrt{5}$, 5$\sqrt{3}$/2, $\sqrt{17}$/$\sqrt{5}$, 5/$\sqrt{11}$, 125, 45, 7$\sqrt{5}$, 7$\sqrt{3}$/2, 7$\sqrt{17}$/5, 7/$\sqrt{11}$, 175, 63, 250, 90, 10$\sqrt{5}$, 5$\sqrt{3}$/2, 10/$\sqrt{17}$, 2/$\sqrt{17}$,6$\sqrt{5}$, 3$\sqrt{3}$, 6$\sqrt{17}$/5, 6/$\sqrt{11}$, 150, 54} $\in$ S.

This is the way operations are performed on S.

Example 3.3: Let S = {Collection of all subsets from the matrix semigroup

$$B = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix} \middle| \; a_i \in (Z^+ \cup \{0\})S_3; 1 \leq i \leq 5\} \right\}$$

be the subset semigroup under '+'.

P = {Collection of all subsets from the semifield $Z^+ \cup \{0\}$} be the subset semiring which is a Smarandache subset semiring.

$$\text{Let } A = \left\{ \begin{bmatrix} 3 \\ 0 \\ 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \\ 5 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 0 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 7 \\ 0 \\ 7 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\}$$

and B = $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ 6 \\ 7 \\ 0 \end{bmatrix}, \begin{bmatrix} 9 \\ 8 \\ 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \\ 3 \\ 4 \end{bmatrix} \right\} \in S.$

$$A + B = \left\{ \begin{bmatrix} 3 \\ 0 \\ 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \\ 5 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 0 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 7 \\ 0 \\ 7 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\} +$$

$$\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ 6 \\ 7 \\ 0 \end{bmatrix}, \begin{bmatrix} 9 \\ 8 \\ 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \\ 3 \\ 4 \end{bmatrix} \right\}$$

$$= \left\{ \begin{bmatrix} 4 \\ 0 \\ 2 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 5 \\ 6 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 0 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 8 \\ 0 \\ 8 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 4 \\ 3 \\ 1 \\ 2 \\ 6 \end{bmatrix}, \begin{bmatrix} 5 \\ 5 \\ 7 \\ 9 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 9 \\ 11 \\ 9 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 7 \\ 6 \\ 9 \\ 0 \end{bmatrix}, \right.$$

$$\begin{bmatrix}9\\5\\13\\7\\1\end{bmatrix},\begin{bmatrix}5\\7\\6\\8\\5\end{bmatrix},\begin{bmatrix}12\\8\\2\\2\\2\end{bmatrix},\begin{bmatrix}9\\12\\6\\2\\3\end{bmatrix},\begin{bmatrix}11\\10\\1\\2\\3\end{bmatrix},\begin{bmatrix}16\\0\\8\\0\\3\end{bmatrix},\begin{bmatrix}12\\10\\1\\0\\7\end{bmatrix},\begin{bmatrix}4\\2\\1\\5\\4\end{bmatrix},$$

$$\left.\begin{bmatrix}1\\6\\5\\5\\5\end{bmatrix},\begin{bmatrix}3\\4\\0\\5\\5\end{bmatrix},\begin{bmatrix}8\\2\\7\\3\\4\end{bmatrix},\begin{bmatrix}4\\4\\0\\4\\9\end{bmatrix}\right\} \in S.$$

Let M = {0, 5, 6, 7, 9, 2} $\in$ P.

$$M \times A = \{0, 5, 6, 7, 9, 2\} \times \left\{\begin{bmatrix}3\\0\\1\\2\\0\end{bmatrix},\begin{bmatrix}0\\4\\5\\2\\1\end{bmatrix},\begin{bmatrix}2\\2\\0\\2\\1\end{bmatrix},\begin{bmatrix}7\\0\\7\\0\\1\end{bmatrix},\begin{bmatrix}3\\2\\0\\1\\0\end{bmatrix}\right\}$$

$$= \left\{\begin{bmatrix}0\\0\\0\\0\\0\end{bmatrix},\begin{bmatrix}15\\0\\5\\10\\0\end{bmatrix},\begin{bmatrix}0\\20\\25\\10\\5\end{bmatrix},\begin{bmatrix}10\\10\\0\\10\\5\end{bmatrix},\begin{bmatrix}35\\0\\35\\0\\5\end{bmatrix},\begin{bmatrix}15\\10\\0\\5\\25\end{bmatrix},\begin{bmatrix}18\\0\\6\\12\\0\end{bmatrix},\begin{bmatrix}0\\24\\30\\12\\6\end{bmatrix},\begin{bmatrix}12\\12\\0\\12\\6\end{bmatrix},\begin{bmatrix}42\\0\\42\\0\\6\end{bmatrix},\right.$$

$$\begin{bmatrix}18\\12\\0\\6\\30\end{bmatrix},\begin{bmatrix}21\\0\\7\\14\\0\end{bmatrix},\begin{bmatrix}0\\28\\35\\14\\7\end{bmatrix},\begin{bmatrix}14\\14\\0\\14\\7\end{bmatrix},\begin{bmatrix}49\\0\\49\\0\\7\end{bmatrix},\begin{bmatrix}21\\14\\0\\7\\35\end{bmatrix},\begin{bmatrix}27\\0\\9\\18\\0\end{bmatrix},\begin{bmatrix}0\\36\\45\\18\\9\end{bmatrix},\begin{bmatrix}18\\18\\0\\18\\9\end{bmatrix},\begin{bmatrix}63\\0\\63\\0\\9\end{bmatrix},$$

$$\left.\begin{bmatrix}27\\18\\0\\9\\45\end{bmatrix},\begin{bmatrix}6\\0\\2\\4\\0\end{bmatrix},\begin{bmatrix}0\\8\\10\\4\\2\end{bmatrix},\begin{bmatrix}4\\4\\0\\4\\2\end{bmatrix},\begin{bmatrix}14\\0\\14\\0\\2\end{bmatrix},\begin{bmatrix}6\\4\\0\\2\\10\end{bmatrix}\right\} \in S.$$

This is the way operations are performed on S.

Example 3.4 Let S = {Collection of all subsets from the semigroup

$$M = \left\{\begin{bmatrix}a_1 & a_2 & a_3\\a_4 & a_5 & a_6\\a_7 & a_8 & a_9\end{bmatrix} \middle| a_i \in Z^+ \cup \{0\}; 1 \leq i \leq 9\right\}\}$$

be the Smarandache subset special strong semivector space over the S subset semiring;

P = {Collection of all subsets from the semifield $Z^+ \cup \{0\}$}.

We have for

$$A = \left\{\begin{bmatrix}3&0&1\\1&2&3\\4&5&0\end{bmatrix},\begin{bmatrix}2&2&0\\1&1&2\\0&0&3\end{bmatrix},\begin{bmatrix}1&1&0\\0&1&2\\0&0&1\end{bmatrix},\begin{bmatrix}3&0&0\\0&1&0\\0&0&5\end{bmatrix}\right\}$$

and

$$B = \left\{\begin{bmatrix}7&0&1\\0&2&2\\1&1&0\end{bmatrix}, \begin{bmatrix}0&2&1\\0&6&5\\9&0&0\end{bmatrix}, \begin{bmatrix}0&0&1\\0&2&0\\3&0&0\end{bmatrix}\right\} \in S.$$

$$A + B = \left\{\begin{bmatrix}3&0&1\\1&2&3\\4&5&0\end{bmatrix}, \begin{bmatrix}2&2&0\\1&1&2\\0&0&3\end{bmatrix}, \begin{bmatrix}1&1&0\\0&1&2\\0&0&1\end{bmatrix}, \begin{bmatrix}3&0&0\\0&1&0\\0&0&5\end{bmatrix}\right\} +$$

$$\left\{\begin{bmatrix}7&0&1\\0&2&2\\1&1&0\end{bmatrix}, \begin{bmatrix}0&2&1\\0&6&5\\9&0&0\end{bmatrix}, \begin{bmatrix}0&0&1\\0&2&0\\3&0&0\end{bmatrix}\right\}$$

$$= \left\{\begin{bmatrix}10&0&2\\1&4&5\\5&6&0\end{bmatrix}, \begin{bmatrix}9&2&1\\0&3&4\\1&1&3\end{bmatrix}, \begin{bmatrix}8&1&1\\1&3&4\\1&1&1\end{bmatrix}, \begin{bmatrix}10&0&1\\0&3&2\\1&1&5\end{bmatrix},\right.$$

$$\begin{bmatrix}3&2&2\\0&8&8\\13&5&0\end{bmatrix}, \begin{bmatrix}2&4&1\\1&7&7\\9&0&3\end{bmatrix}, \begin{bmatrix}1&3&1\\0&7&7\\9&0&1\end{bmatrix}, \begin{bmatrix}3&2&1\\0&7&5\\9&0&5\end{bmatrix},$$

$$\left.\begin{bmatrix}3&0&2\\1&4&3\\7&5&0\end{bmatrix}, \begin{bmatrix}1&1&1\\0&3&2\\3&0&1\end{bmatrix}, \begin{bmatrix}2&2&1\\1&3&2\\3&0&3\end{bmatrix}, \begin{bmatrix}3&0&1\\0&3&0\\3&0&5\end{bmatrix}\right\} \in S.$$

Consider for M = {0, 2, 7, 10, 20, 9} $\in$ P.

$$M \times A = \{0, 2, 7, 10, 20, 9\} \times$$

$$\left\{\begin{bmatrix}3&0&1\\1&2&3\\4&5&0\end{bmatrix},\begin{bmatrix}2&2&0\\1&1&2\\0&0&3\end{bmatrix},\begin{bmatrix}1&1&0\\0&1&2\\0&0&1\end{bmatrix},\begin{bmatrix}3&0&0\\0&1&0\\0&0&5\end{bmatrix}\right\}$$

$$= \left\{\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix},\begin{bmatrix}6&0&2\\2&4&6\\8&10&0\end{bmatrix},\begin{bmatrix}4&4&0\\2&2&4\\0&0&6\end{bmatrix},\right.$$

$$\begin{bmatrix}2&2&0\\0&2&4\\0&0&2\end{bmatrix},\begin{bmatrix}6&0&0\\0&2&0\\0&0&10\end{bmatrix},\begin{bmatrix}21&0&7\\7&14&21\\28&35&0\end{bmatrix},$$

$$\begin{bmatrix}14&14&0\\7&7&14\\0&0&7\end{bmatrix},\begin{bmatrix}7&7&0\\0&7&14\\0&0&7\end{bmatrix},\begin{bmatrix}21&0&0\\0&7&0\\0&0&35\end{bmatrix},$$

$$\begin{bmatrix}30&0&10\\10&20&30\\40&50&0\end{bmatrix},\begin{bmatrix}20&20&0\\10&10&20\\0&0&30\end{bmatrix},\begin{bmatrix}10&10&0\\0&10&20\\0&0&10\end{bmatrix},$$

$$\begin{bmatrix}30&0&0\\0&10&0\\0&0&50\end{bmatrix},\begin{bmatrix}60&0&20\\20&40&60\\80&100&0\end{bmatrix},\begin{bmatrix}40&40&0\\20&20&40\\0&0&60\end{bmatrix},$$

$$\left[\begin{array}{ccc}20 & 20 & 0 \\ 0 & 20 & 40 \\ 0 & 0 & 20\end{array}\right], \left[\begin{array}{ccc}60 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 100\end{array}\right], \left[\begin{array}{ccc}27 & 0 & 9 \\ 9 & 18 & 27 \\ 36 & 45 & 0\end{array}\right],$$

$$\left.\left[\begin{array}{ccc}18 & 18 & 0 \\ 9 & 9 & 18 \\ 0 & 0 & 27\end{array}\right], \left[\begin{array}{ccc}9 & 9 & 0 \\ 0 & 9 & 18 \\ 0 & 0 & 9\end{array}\right], \left[\begin{array}{ccc}27 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 45\end{array}\right]\right\} \in S.$$

Example 3.5: Let S = {Collection of all subsets from the matrix semigroup

$$M = \left\{\left[\begin{array}{ccc}a_1 & a_2 & a_3 \\ \hline a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ \hline a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15}\end{array}\right] \middle| \ a_i \in Q^+ \cup \{0\}; 1 \leq i \leq 15\right\}\}$$

be the Smarandache subset special strong semivector space over the S-subset semiring P where

P = {Collection of all subsets from the semifield $Q^+ \cup \{0\}$}.

$$\text{Let } A = \left\{\left[\begin{array}{ccc}2 & 0 & 1 \\ \hline 0 & 3 & 3 \\ 0 & 1 & 1 \\ \hline 1 & 0 & 1 \\ 0 & 1 & 2\end{array}\right], \left[\begin{array}{ccc}1 & 6 & 2 \\ \hline 2 & 0 & 4 \\ 0 & 5 & 0 \\ \hline 0 & 6 & 6 \\ 7 & 7 & 2\end{array}\right], \left[\begin{array}{ccc}2 & 2 & 4 \\ \hline 9 & 2 & 2 \\ 0 & 0 & 0 \\ \hline 6 & 0 & 5 \\ 1 & 2 & 3\end{array}\right]\right\} \text{ and}$$

$$B = \left\{ \left[\begin{array}{ccc} 9 & 0 & 2 \\ \hline 1 & 1 & 1 \\ 0 & 0 & 0 \\ \hline 0 & 0 & 0 \\ 1 & 0 & 0 \end{array}\right], \left[\begin{array}{ccc} 0 & 1 & 1 \\ \hline 1 & 0 & 0 \\ 0 & 1 & 0 \\ \hline 0 & 0 & 0 \\ 1 & 1 & 0 \end{array}\right] \right\} \in S.$$

$$A + B = \left\{ \left[\begin{array}{ccc} 2 & 0 & 1 \\ \hline 0 & 3 & 3 \\ 0 & 1 & 1 \\ \hline 1 & 0 & 1 \\ 0 & 1 & 2 \end{array}\right], \left[\begin{array}{ccc} 1 & 6 & 2 \\ \hline 2 & 0 & 4 \\ 0 & 5 & 0 \\ \hline 0 & 6 & 6 \\ 7 & 7 & 2 \end{array}\right], \left[\begin{array}{ccc} 2 & 2 & 4 \\ \hline 9 & 2 & 2 \\ 0 & 0 & 0 \\ \hline 6 & 0 & 5 \\ 1 & 2 & 3 \end{array}\right] \right\} +$$

$$\left\{ \left[\begin{array}{ccc} 9 & 0 & 2 \\ \hline 1 & 1 & 1 \\ 0 & 0 & 0 \\ \hline 0 & 0 & 0 \\ 1 & 0 & 0 \end{array}\right], \left[\begin{array}{ccc} 0 & 1 & 1 \\ \hline 1 & 0 & 0 \\ 0 & 1 & 0 \\ \hline 0 & 0 & 0 \\ 1 & 1 & 0 \end{array}\right] \right\}$$

$$= \left\{ \left[\begin{array}{ccc} 11 & 0 & 3 \\ \hline 1 & 4 & 4 \\ 0 & 1 & 1 \\ \hline 1 & 0 & 1 \\ 1 & 1 & 2 \end{array}\right], \left[\begin{array}{ccc} 10 & 6 & 4 \\ \hline 3 & 0 & 5 \\ 0 & 5 & 0 \\ \hline 0 & 6 & 6 \\ 8 & 7 & 2 \end{array}\right], \left[\begin{array}{ccc} 11 & 2 & 6 \\ \hline 10 & 3 & 3 \\ 0 & 0 & 0 \\ \hline 6 & 0 & 5 \\ 2 & 2 & 3 \end{array}\right], \right.$$

$$\left. \left[\begin{array}{ccc} 3 & 3 & 5 \\ \hline 10 & 2 & 2 \\ 0 & 1 & 0 \\ \hline 6 & 0 & 5 \\ 2 & 3 & 3 \end{array}\right], \left[\begin{array}{ccc} 1 & 7 & 3 \\ \hline 3 & 0 & 4 \\ 0 & 6 & 0 \\ \hline 0 & 6 & 6 \\ 8 & 8 & 2 \end{array}\right], \left[\begin{array}{ccc} 2 & 1 & 2 \\ \hline 1 & 3 & 3 \\ 0 & 2 & 1 \\ \hline 1 & 0 & 1 \\ 1 & 2 & 2 \end{array}\right] \right\} \in S$$

Let M = {7/2, 8/5, 3, 6, 0, 1/2} $\in$ P.

M $\times$ P = {7/2, 8/5, 3, 6, 0, 1/2} $\times$

$$\left\{\left[\begin{array}{ccc} 2 & 0 & 1 \\ \hline 0 & 3 & 3 \\ 0 & 1 & 1 \\ \hline 1 & 0 & 1 \\ 0 & 1 & 2 \end{array}\right], \left[\begin{array}{ccc} 1 & 6 & 2 \\ \hline 2 & 0 & 4 \\ 0 & 5 & 0 \\ \hline 0 & 6 & 6 \\ 7 & 7 & 2 \end{array}\right], \left[\begin{array}{ccc} 2 & 2 & 4 \\ \hline 9 & 2 & 2 \\ 0 & 0 & 0 \\ \hline 6 & 0 & 5 \\ 1 & 2 & 3 \end{array}\right]\right\}$$

$$= \left\{\left[\begin{array}{ccc} 7 & 0 & 7/2 \\ \hline 0 & 21/2 & 21/2 \\ 0 & 7/2 & 7/2 \\ \hline 7/2 & 0 & 7/2 \\ 0 & 7/2 & 7 \end{array}\right], \left[\begin{array}{ccc} 7/2 & 21 & 7 \\ \hline 7 & 0 & 14 \\ 0 & 35/2 & 0 \\ \hline 0 & 21 & 21 \\ 49/2 & 49/2 & 7 \end{array}\right],\right.$$

$$\left[\begin{array}{ccc} 7 & 7 & 7/2 \\ \hline 63/2 & 7 & 7 \\ 0 & 0 & 0 \\ \hline 21 & 0 & 35/2 \\ 7/2 & 7 & 21/2 \end{array}\right], \left[\begin{array}{ccc} 4/5 & 0 & 8/5 \\ \hline 0 & 24/5 & 24/5 \\ 0 & 8/5 & 8/5 \\ \hline 8/5 & 0 & 8/5 \\ 0 & 8/5 & 16/5 \end{array}\right]$$

$$\left[\begin{array}{ccc} 8/5 & 48/5 & 16/5 \\ \hline 16/5 & 0 & 32/5 \\ 0 & 8 & 0 \\ \hline 0 & 48/5 & 48/5 \\ 56/5 & 56/5 & 16/5 \end{array}\right], \left[\begin{array}{ccc} 16/5 & 16/5 & 30/5 \\ \hline 72/5 & 16/5 & 16/5 \\ 0 & 0 & 0 \\ \hline 48/5 & 0 & 8 \\ 8/5 & 16/5 & 24/5 \end{array}\right],$$

$$\left[\begin{array}{ccc} 6 & 0 & 3 \\ \hline 0 & 9 & 9 \\ 0 & 3 & 3 \\ \hline 3 & 0 & 3 \\ 0 & 3 & 6 \end{array}\right], \left[\begin{array}{ccc} 0 & 0 & 0 \\ \hline 0 & 0 & 0 \\ 0 & 0 & 0 \\ \hline 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right],$$

$$\left[\begin{array}{ccc} 12 & 0 & 6 \\ \hline 0 & 18 & 18 \\ 0 & 6 & 6 \\ \hline 6 & 0 & 6 \\ 0 & 6 & 12 \end{array}\right], \left[\begin{array}{ccc} 1 & 0 & 1/2 \\ \hline 0 & 3/2 & 3/2 \\ 0 & 1/2 & 1/2 \\ \hline 1/2 & 0 & 1/2 \\ 0 & 1/2 & 1 \end{array}\right], \left[\begin{array}{ccc} 3 & 18 & 6 \\ \hline 6 & 0 & 12 \\ 0 & 15 & 0 \\ \hline 0 & 18 & 18 \\ 21 & 21 & 6 \end{array}\right], \left[\begin{array}{ccc} 6 & 36 & 12 \\ \hline 12 & 0 & 24 \\ 0 & 30 & 0 \\ \hline 0 & 36 & 36 \\ 42 & 42 & 12 \end{array}\right],$$

$$\left.\left[\begin{array}{ccc} 1/2 & 3 & 1 \\ \hline 1 & 0 & 2 \\ 0 & 5/2 & 0 \\ \hline 0 & 3 & 3 \\ 7/2 & 7/2 & 1 \end{array}\right], \left[\begin{array}{ccc} 6 & 6 & 12 \\ \hline 27 & 6 & 6 \\ 0 & 0 & 0 \\ \hline 18 & 0 & 15 \\ 3 & 6 & 9 \end{array}\right], \left[\begin{array}{ccc} 12 & 12 & 24 \\ \hline 54 & 12 & 12 \\ 0 & 0 & 0 \\ \hline 36 & 0 & 30 \\ 6 & 12 & 18 \end{array}\right], \left[\begin{array}{ccc} 1 & 1 & 2 \\ \hline 9/2 & 1 & 1 \\ 0 & 0 & 0 \\ \hline 3 & 0 & 5/2 \\ 1/2 & 1 & 1/2 \end{array}\right]\right\}$$

$$\in S.$$

This is the way the operations are performed on S.

Example 3.6: Let S = {Collection of all subsets from the semigroup

$$M = \left\{ \sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in R^+ \cup \{0\} \right\}$$

be the subset semigroup. P = {Collection of all subsets from the semifield $Q^+ \cup \{0\}$} be the S-subset semiring.

We see S is a Smarandache special strong subset semivector space over the S-subset semiring.

Let A = {$8x^3 + 7x + 1$, $3x^7 + 4/7$ $5x^2 + 8x + 3/2$,
$10x^5 + 2x + 9/2$}

and

B = {$x^5 + 3x + 1$, $3x^{15} + 17x + 1$, $x^7 + 3x^3 + 8$} $\in$ S.

A + B = {$8x^3 + 7x + 1$, $3x^7 + 4/7$ $5x^2 + 8x + 3/2$, $10x^5 + 2x + 9/2$} + {$x^5 + 3x + 1$, $3x^{15} + 17x + 1$, $x^7 + 3x^3 + 8$}

= {$x^5 + 8x^3 + 10x + 2$, $3x^7 + x^5 + 3x + 11/7$, $5x^2 + x^5 + 11x + 5/2$, $11x^5 + 5x + 11/2$, $3x^{15}$, $8x^3 + 24x + 2$, $3x^7 + 3x^{15} + 17x + 11/7$, $3x^{15} + 5x^2 + 25x + 5/2$, $x^5 + 19x + 11/2$ $x^7 + 11x^3 + 9 + 7x$, $4x^7 + 60/7 + 3x^3$, $x^7 + 3x^3 + 5x^2 + 8x + 19/2$, $x^7 + 10x^5 + 3x^3 + 2x + 25/2$} $\in$ S.

Consider M = {7/2, 3/5, 0, 2/5, 6/7, 1, 2} $\in$ P.

M $\times$ A = {7/2, 3/5, 0, 2/5, 6/7, 1, 2} $\times${$8x^3 + 7x + 1$, $3x^7 + 4/7$ $5x^2 + 8x + 3/2$, $10x^5 + 2x + 9/2$}

= {0, $4x^3 + 49/2x + 7/2$, $21/2$ $x^7 + 2$, $35x^2/2 + 28x + 21/4$, $35x^5 + 7x + 63/4$, $35x^5 + 7x + 63/4$, $24/5x^3 + 21/5 + 3/5$, $9/5x^7 + 12/35$, $3x^2 + 24/5x + 9/10$, $6x^5 + 6/5x + 3/10$, $16/5x^3 + 14/5$ $x + 2/5$, $6/5x^7 + 8/35$, $2x^2 + 16/5x + 3/5$, $4x^5 + 4/5x + 9/5$, $42x^3/7 + 6x + 6/7$ $18/7$ $x^7 + 24/49$, $30/7x^2 + 30/7x^2 + 48/7x + 9/7$, $60/7x^5 + 12/7x + 27/7$, $8x^3 + 7x + 1$, $3x^7 + 4/7$, $5x^2 + 8x + 3/2$, $10x^5 + 2x + 9/2$, $16x^3 + 14x + 2$, $6x^7 + 8/7$, $10x^2 + 16x + 3$, $20x^5 + 4x + 9$} $\in$ S.

This is the way operations are performed on S.

Example 3.7: Let S = {Collection of all subsets from the semigroup $(Z^+ \cup \{0\})D_{2,5}$} be the subset semigroup.
P = {Collection of all subsets from the semifield $Z^+ \cup \{0\}$} be the subset semiring. P is a Smarandache subset semiring.

S is a Smarandache special strong subset semivector space over P.

Let A = {$3a + 5ab + 1$, $8ab^3 + b^2 + b$, $5ab^2 + 5ab + 3$, $10ab^3 + b^4 + 3ab$} and B = {$3ab + 5ab^2 + 5b^3 + 10a + 3b$, $5a + 2b + ab + 1$} $\in$ S.

We find A + B = {$3a + 5ab + 1$, $8ab^3 + b^2 + b$, $5ab^2 + 5ab + 3$, $10ab^3 + b^4 + 3ab$} + {$3ab + 5ab^2 + 5b^3 + 10a + 3b$, $5a + 2b + ab + 1$}

= {$8ab + 3a + 5ab^2 + 3a + 15b^3$, $13a + 3b + 5ab + 1$, $8a + 2b + 6ab + 2$, $8ab^3 + 5ab^2 + 15b^3 + b^4 + b$, $10a + 4b + b^4 + 8ab^3$, $5a + 3b + b^4 + ab + 8ab^3 + 1$, $10ab^2 + 8ab + 15b^3 + 3$, $5ab^2 + 5ab + 10a + 3b + 3$, $5a + 6ab + 2b + 4 + 5ab^2$, $10ab^3 + b^4 + 6ab + 5ab^2 + 15ab^3 + 10ab^3 + 10a + 3b + b^4 + 3ab + 10ab^3 + b^4 + 4ab + b^4 + 5a + 2b + 1$}} $\in$ S.

Now let M = {0, 1, 2, 3, 4, 5} $\in$ P.

We find M $\times$A = {0, 1, 2, 3, 4, 5} $\times$ {$3a + 5ab + 1$, $8ab^3 + b^4 + b$, $5ab^2 + 5ab + 3$, $10ab^3 + b^4 + 3ab$}

= {0, $3a + 5ab + 1$, $8ab^3 + b^4 + b$, $5ab^2 + 5ab + 3$, $10ab^3 + b^4 + 3ab$, $6a + 10ab + 2$, $16ab^3 + 2b^4 + 2b$, $10ab^2 + 10ab + 6$, $20ab^3 + 2b^4 + 6ab$, $9a + 15ab + 3$, $24ab^3 + 3b^4 + 3b$, $15ab^2 + 15ab + 9$, $30ab^3 + 3b^4 + 9ab$, $12a + 20ab + 4$, $32ab^3 + 4b^4 + 4b$, $20ab^2 + 20ab + 12$, $40ab^3 + 4b^4 + 12ab$, $15a + 25ab + 5$, $40ab^3 + 5b + 5b^4$, $25ab^2 + 25ab + 15$, $50ab^3 + 5b^4 + 15ab$} $\in$ S.

This is the way operations are performed on S.

Now we proceed to study the notion of S subset strong semilinear algebra over the S-semiring.

Example 3.8: Let S = {Collection of all subsets from the matrix semigroup

$$M = \left\{ \left[\begin{array}{cc} a_1 & a_5 \\ a_2 & a_6 \\ a_3 & a_7 \\ a_4 & a_8 \end{array}\right] \middle| \; a_i \in Q^+ \cup \{0\}; 1 \le i \le 8\} \right\}$$

be the subset semigroup. Let P = {Collection of all subsets from the semigroup $Q^+ \cup \{0\}$} be the S-subset semiring.

S is the S-subset special strong semilinear algebra over the S-subset semiring P.

We just show the product is the natural product $\times_n$ on S.

For take $A = \left\{ \begin{bmatrix} 3 & 0 \\ 1 & 2 \\ 3 & 6 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 2 \\ 0 & 0 \\ 1 & 5 \end{bmatrix}, \begin{bmatrix} 2 & 0 \\ 1 & 3 \\ 0 & 1 \\ 5 & 2 \end{bmatrix} \right\}$ and

$$B = \left\{ \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \\ 9 & 0 \end{bmatrix}, \begin{bmatrix} 9 & 2 \\ 5 & 1 \\ 0 & 1 \\ 4 & 2 \end{bmatrix} \right\} \in S.$$

We now find $A \times_n B = \left\{ \begin{bmatrix} 3 & 0 \\ 1 & 2 \\ 3 & 6 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 2 \\ 0 & 0 \\ 1 & 5 \end{bmatrix}, \begin{bmatrix} 2 & 0 \\ 1 & 3 \\ 0 & 1 \\ 5 & 2 \end{bmatrix} \right\} \times_n$

$$\left\{ \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \\ 9 & 0 \end{bmatrix}, \begin{bmatrix} 9 & 2 \\ 5 & 1 \\ 0 & 1 \\ 4 & 2 \end{bmatrix} \right\}$$

$$= \left\{ \begin{bmatrix} 3 & 0 \\ 3 & 8 \\ 16 & 36 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 27 & 0 \\ 5 & 2 \\ 0 & 6 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 2 \\ 0 & 8 \\ 0 & 0 \\ 9 & 0 \end{bmatrix}, \begin{bmatrix} 9 & 2 \\ 0 & 4 \\ 0 & 0 \\ 4 & 10 \end{bmatrix}, \begin{bmatrix} 2 & 0 \\ 3 & 12 \\ 0 & 6 \\ 45 & 0 \end{bmatrix}, \begin{bmatrix} 18 & 0 \\ 5 & 3 \\ 0 & 1 \\ 20 & 4 \end{bmatrix} \right\} \in S.$$

It is easily verified A $\times_n$ B = B $\times_n$ A. So the S-special strong subset semilinear algebra is commutative.

Example 3.9: Let S = {Collection of all subsets from the matrix semigroup

$$M = \left\{ \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \\ a_5 & a_6 \\ a_7 & a_8 \end{bmatrix} \middle| \; a_i \in (Z^+ \cup \{0\})\, D_{2,5};\ 1 \leq i \leq 8 \right\}\}$$

be the S-special strong subset semilinear algebra over the S-subset semiring
P = {Collection of all subsets from the semifield $Z^+ \cup \{0\}$}.

S is a S-special strong subset semilinear algebra under the natural product $\times_n$. We show A $\times_n$ B $\neq$ B $\times_n$ A for A, B $\in$ S.

$$\text{Let } A = \left\{ \begin{bmatrix} 3a+5b & ab \\ 2ab+1 & a \\ 5ab^2+3ab^3 & b \\ 7a & 8b^3+3a \end{bmatrix} \right\} \text{ and}$$

$$B = \left\{ \begin{bmatrix} 5ab+a & b^3a \\ 5ab^3+3 & b \\ 3ab+6ab^3 & a \\ 9b+1 & 6a+7b \end{bmatrix} \right\} \in S.$$

$$A \times_n B = \left\{\begin{bmatrix} 3a+5b & ab \\ 2ab+1 & a \\ 5ab^2+3ab^3 & b \\ 7a & 8b^3+3a \end{bmatrix}\right\} \times_n$$

$$\left\{\begin{bmatrix} 5ab+a & ba \\ 5ab^3+3 & b \\ 3ab+6ab^3 & a \\ 9b+1 & 6a+7b \end{bmatrix}\right\}$$

$$= \left\{\begin{bmatrix} 15b+3+5ba+25b & b \\ 10abab^3+3+6ab+5ab^3 & ab \\ \begin{matrix} 15ab^2ab+30ab^2ab^3 \\ +9ab^3ab+18 \end{matrix} & ba \\ 63ab+7a & 48b^3a+18+56b^3+21ab \end{bmatrix}\right\}.$$

Consider

$$B \times_n A = \left\{\begin{bmatrix} 5ab+a & b^3a \\ 5ab^3+3 & b \\ 3ab+6ab^3 & a \\ 9b+1 & 6a+7b \end{bmatrix}\right\} \times_n$$

$$\left\{\begin{bmatrix} 3a+5b & ab \\ 2ab+1 & a \\ 5ab^2+3ab^3 & b \\ 7a & 8b^3+3a \end{bmatrix}\right\}$$

$$= \left\{ \begin{bmatrix} 15aba + 25ab^2 + 5ab + 3 & b^4 \\ 5ab^3 + 3 + 6ab + 10ab^3ab & ba \\ \begin{matrix} 15abab^2 + 9abab^3 \\ + 30ab^3ab^2 + 18 \end{matrix} & ab \\ 63ba + 7a & 48ab^3 + 18 + 21a + 54b^4 \end{bmatrix} \right\}.$$

It is clear $A \times_n B \neq B \times_n A$.

Thus S is a non commutative Smarandache special strong semilinear algebra over P.

Example 3.10: Let S = {Collection of all subsets from the matrix semigroup $M = \{(a_1\ a_2\ a_3\ a_4\ a_5) \mid a_i \in (Z^+ \cup \{0\})D_{2,7}, 1 \leq i \leq 5\}$ be the S-subset special strong linear algebra over the S-subset semiring.

P = {Collection of all subsets from the semifield $Z^+ \cup \{0\}$}. We see S is a non commutative S-special strong subset semilinear algebra which is non commutative.

Let $A = \{(3a, 5b, 3a + b, 2ab, 10ab^3)\}$ and
$B = \{(b, 3a, 2a + 7b, 5ab^3, ab^4)\} \in S$.

We find $A \times_n B = \{(3a, 5b, 3a + b, 2ab, 10ab^3)\} \times_n \{(b, 3a, 2a + 7b, 5ab^3, ab^4)\}$

$= \{(3ab, 15ba, 6 + 2ba + 21ab + 7b^2\ 10abab^3, 10ab^3ab)\}$

$= \{(3ab, 15ab^6, 7b^2 + 2ab^6 + 21ab + 6, 10b^2, 10b)\}$ … I

Consider $B \times_n A = \{(b, 3a, 2a + 7b, 5ab^3, ab^4)\} \times n \{(3a, 5b, 3a + b, 2ab, 10ab^3)\}$

$= \{(3ba, 15ab, 6 + 21ba + 2ab + 7b^2, 10ab^3\ ab, 10ab^4ab^3)\}$

$= \{(3ab^6, 15ab, 6 + 7b^2 + 2ab + 21ab^6, 10b^5, 10b^6)\}$ … II

Clearly I and II are distinct. Thus $A \times_n B \neq B \times_n A$.

Hence S is a S-strong special non commutative semilinear algebra over P.

Example 3.11: Let S = {Collection of all subsets from the semipolynomial ring

$$M = \left\{ \sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in Z^+ \cup \{0\} \right\}\}$$

be the S-special strong subset semilinear algebra over the S-subset semiring. S is a commutative semilinear algebra over P.

Example 3.12: Let S = {Collection of all subsets from the semiring

$$M = \left\{ \sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in (Z^+ \cup \{0\})\, D_{2,4} \right\}$$

be the S-strong special subset semilinear algebra over the S-subset semiring
P = {Collection of all subsets from the semifield $Z^+ \cup \{0\}$}. Clearly S is on commutative S-strong special semilinear algebra over P.

For take $A = \{5abx^7 + 3bx^3 + 8ax + 3a\}$ and $B = \{6b + 7ax^3\} \in S$.

$A \times_n B = \{5abx^7 + 3bx^3 + 8ax + 3a\} \times_n \{6b + 7ax^3\}$

$= \{35abax^{10} + 21bax^6 + 21x^3 + 56x^4 + 30ab_2x^7 + 18b_2x^3 + 48abx + 18ab\}$ … I

Consider B $\times_n$ A = $\{6b + 7ax^3\} \times_n \{5abx^7 + 3bx^3 + 8ax + 3a\}$

= $\{30bx^7 + 18b_2x^3 + 48ab_3x + 18ab_3 + 35bx^{16} + 21abx^6 + 56x^4 + 21x^3\}$ … II

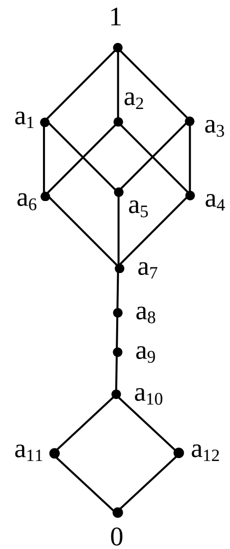
Clearly I and II are distinct so A $\times_n$ B $\neq$ B $\times_n$ A. Hence S is a S-special strong subset non commutative semilinear algebra.

Example 3.13: Let S = {Collection of all subsets from the semiring

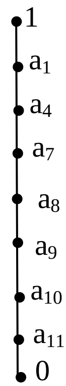
$$M = \left\{ \sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in \langle Q^+ \cup I \cup \{0\}\rangle \right\}$$

be the S-special strong semilinear algebra over the S-subset semiring. P = {Collection of all subsets from the neutrosophic semifield $\langle Q^+ \cup I \cup \{0\}\rangle$}. Cleary S is commutative.

Example 3.14: Let S = {Collection of all subsets from the semiring LS_3 where L =



be the subset semiring P = {Collection of all subsets from the semiring L} be the S-semiring. For M = {{1}, {a_1}, {a_4}, {a_7}, {a_8}, {a_9}, {a_{10}}, {a_{11}}, {0}} under the operation '$\cup$' and '$\cap$' is a subset semifield which is isomorphic with



S is a Smarandache special subset strong semilinear algebra over the S-semiring P.

Infact S is a non commutative S-strong special semilinear algebra over the S-semiring.

For if A = {$a_1p_1 + a_5p_3 + a_6p_4$} and

B = {a_7p_2} $\in$ S where $p_1 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$, $p_2 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$,

$p_3 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$, $p_4 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$, $p_5 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$ and

$$e = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix} \text{ then}$$

$$A \times B = \{a_1p_1 + a_5p_3 + a_6p_4\} \times \{a_7p_2\}$$

$$= \{a_7p_5 + a_7p_4 + a_7p_3\} \qquad \ldots \quad I$$

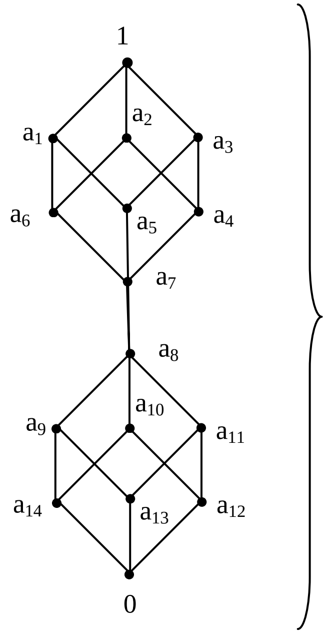
Now

$$B \times A = \{a_7p_2\} \times \{a_1p_1 + a_5p_3 + a_6p_4\}$$

$$= \{a_7p_4 + a_7p_5 + a_7p_1\} \qquad \ldots \quad II$$

Clearly I and II are different hence S is a S-special strong subset non commutative semi-linear algebra over P.

Example 3.15: Let S = {Collection of all subsets from the group lattice LA_4 where L =



be the Smarandache subset special strong semilinear algebra over the S-subset semiring.

S is a non commutative subset semilinear algebra over P.

Take A = $\{a_2\begin{pmatrix}1&2&3&4\\2&1&4&3\end{pmatrix} + a_5\begin{pmatrix}1&2&3&4\\2&3&1&4\end{pmatrix}$

$+ a_1\begin{pmatrix}1&2&3&4\\1&3&4&2\end{pmatrix} + 1\}$ and

B = $\{a_1\begin{pmatrix}1&2&3&4\\3&4&1&2\end{pmatrix} + a_4\begin{pmatrix}1&2&3&4\\4&3&2&1\end{pmatrix} +$

$\begin{pmatrix}1&2&3&4\\2&4&3&1\end{pmatrix}\} \in S$.

We find A × B = $\{a_2\begin{pmatrix}1&2&3&4\\2&1&4&3\end{pmatrix} + a_5\begin{pmatrix}1&2&3&4\\2&3&1&4\end{pmatrix}$

$+ a_1\begin{pmatrix}1&2&3&4\\1&3&4&2\end{pmatrix} + 1\} \times \{a_1\begin{pmatrix}1&2&3&4\\3&4&1&2\end{pmatrix} +$

$a_4\begin{pmatrix}1&2&3&4\\4&3&2&1\end{pmatrix} + \begin{pmatrix}1&2&3&4\\2&4&3&1\end{pmatrix}\}$

$= \{a_1\begin{pmatrix}1&2&3&4\\3&4&1&2\end{pmatrix} + a_4\begin{pmatrix}1&2&3&4\\4&3&2&1\end{pmatrix} +$

$\begin{pmatrix}1&2&3&4\\2&4&3&1\end{pmatrix} + a_6\begin{pmatrix}1&2&3&4\\4&3&2&1\end{pmatrix} +$

$a_4\begin{pmatrix}1&2&3&4\\3&4&1&2\end{pmatrix} + a_2\begin{pmatrix}1&2&3&4\\4&2&1&3\end{pmatrix} + a_5\begin{pmatrix}1&2&3&4\\4&1&3&2\end{pmatrix} +$

$$a_7\begin{pmatrix}1 & 2 & 3 & 4\\ 3 & 2 & 4 & 1\end{pmatrix} + a_5\begin{pmatrix}1 & 2 & 3 & 4\\ 4 & 3 & 2 & 1\end{pmatrix} +$$

$$a_1\begin{pmatrix}1 & 2 & 3 & 4\\ 3 & 1 & 2 & 4\end{pmatrix} + a_7\begin{pmatrix}1 & 2 & 3 & 4\\ 4 & 2 & 1 & 3\end{pmatrix} + \begin{pmatrix}1 & 2 & 3 & 4\\ 2 & 3 & 1 & 4\end{pmatrix}\}$$

… I

$$B \times A = \{a_1\begin{pmatrix}1 & 2 & 3 & 4\\ 3 & 4 & 1 & 2\end{pmatrix} + a_4\begin{pmatrix}1 & 2 & 3 & 4\\ 4 & 3 & 2 & 1\end{pmatrix} +$$

$$\begin{pmatrix}1 & 2 & 3 & 4\\ 2 & 4 & 3 & 1\end{pmatrix}\} \times \{a_2\begin{pmatrix}1 & 2 & 3 & 4\\ 2 & 1 & 4 & 3\end{pmatrix} +$$

$$a_5\begin{pmatrix}1 & 2 & 3 & 4\\ 2 & 3 & 1 & 4\end{pmatrix} + a_1\begin{pmatrix}1 & 2 & 3 & 4\\ 1 & 3 & 4 & 2\end{pmatrix} + 1\}$$

$$= \{a_6\begin{pmatrix}1 & 2 & 3 & 4\\ 4 & 3 & 2 & 1\end{pmatrix} + a_1\begin{pmatrix}1 & 2 & 3 & 4\\ 3 & 4 & 1 & 2\end{pmatrix} + a_5\begin{pmatrix}1 & 2 & 3 & 4\\ 1 & 4 & 2 & 3\end{pmatrix}$$

$$+ a_1\begin{pmatrix}1 & 2 & 3 & 4\\ 4 & 2 & 1 & 3\end{pmatrix} + a_4\begin{pmatrix}1 & 2 & 3 & 4\\ 4 & 3 & 2 & 1\end{pmatrix} + a_4\begin{pmatrix}1 & 2 & 3 & 4\\ 3 & 4 & 1 & 2\end{pmatrix}$$

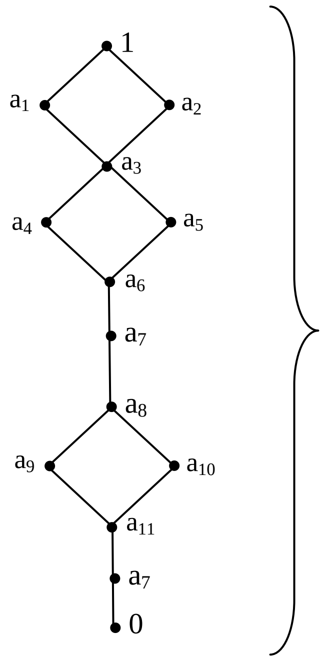
$$+ a_7\begin{pmatrix}1 & 2 & 3 & 4\\ 4 & 1 & 3 & 2\end{pmatrix} + a_7\begin{pmatrix}1 & 2 & 3 & 4\\ 2 & 4 & 3 & 1\end{pmatrix} + \begin{pmatrix}1 & 2 & 3 & 4\\ 2 & 4 & 3 & 1\end{pmatrix} +$$

$$a_2\begin{pmatrix}1 & 2 & 3 & 4\\ 1 & 3 & 4 & 2\end{pmatrix} + a_5\begin{pmatrix}1 & 2 & 3 & 4\\ 3 & 4 & 1 & 2\end{pmatrix} + a_1\begin{pmatrix}1 & 2 & 3 & 4\\ 3 & 2 & 4 & 1\end{pmatrix}\}$$

… II

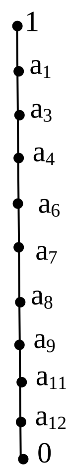
I and II are distinct and S is a S-special strong subset non commutative semilinear algebra over P.

Example 3.16: Let S = {Collection of all subsets from the lattice group L ($S_3 \times D_{2,7}$) where L =



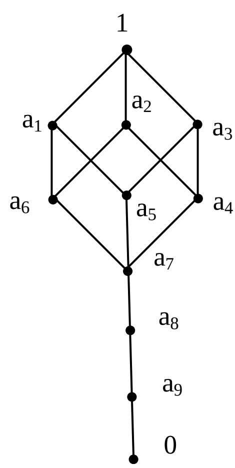
be the S-subset special strong semilinear algebra over the Smarandache subset semiring P = {Collection of all subsets from the semiring L}.

P is a S-subset semiring as D = {{1}, {a_1}, {a_3}, {a_4}, {a_6}, {a_7}, {a_8}, {a_9}, {a_{11}}, {a_{12}}, {0}} $\subseteq$ P is a subset semiring isomorphic to the semifield B =

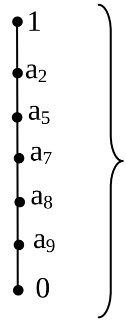


So P a subset semifield; hence P is a S-subset semiring. It is easily verified S is a S-strong special subset semilinear algebra which is non commutative.

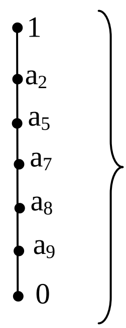
Example 3.17: Let S = {Collection of all subsets from the group lattice $(L_1 \times L_2)$ $(S_3 \times A_5)$ where L_1 =



and L_2 is a Boolean algebra of order 16} be the S-subset special strong semilinear algebra over the S-subset semiring P = {Collection of all subsets from the lattice P × {1} where P =



Clearly P contains the subset T = {{(0, 1)}, {(a_2, 1)}, {(a_5, 1)}, {(a_7, 1)}, {a_8, 1)}, {(a_9, 1), {(1, 1)}} is a subset semifield isomorphic with the semifield



However S is a non commutative subset strong special semilinear algebra over the S-subset semiring P.

Now having seen finite order, infinite order, commutative and non commutative S- special strong subset semilinear algebras we now proceed onto give examples of the notion of S-strong special substructure in them.

Example 3.18: Let S = {Collection of all subsets from the semigroup B = $Z^+ \cup \{0\} \times Z^+ \cup \{0\}$} be the subset semigroup. P = {Collection of all subsets from the semifield F = $Z^+ \cup \{0\}$} be the Smarandache subset semiring.

S is a Smarandache subset special strong semivector space over the S-subset semiring P.

Now consider M_1 = {Collection of all subsets from the subsemigroup $T_1 = (Z^+ \cup \{0\} \times \{0\})$} be the Smarandache strong special subset semivector subspace of S over the S-subset semiring P.

Take N_t = {Collection of all subsets from the subsemigroup $L = \{0\} \times t\, Z^+ \cup \{0\} \subseteq Z^+ \cup \{0\} \times Z^+ \cup \{0\}$} $\subseteq$ S; N_t is again a S-strong special subset semivector subspace of S over the S-subset semiring.

Infact as $2 \leq t < \infty$ we have infinite number of S-subset strong special subsemivector subspaces. If we take N_t = {Collection of all subsets from the subsemigroup $L_t = \{(t\, Z^+ \cup \{0\} \times \{0\})\} \subseteq (Z^+ \cup \{0\}) \times \{Z^+ \cup \{0\}\} \subseteq S$. N_t is a S-strong special subset semivector subspace of S over the S-subset semiring P.

We see S has infinite number of S-subset strong special semivector subspaces $(2 \leq t < \infty)$.

Now consider N_q^t = {Collection of all subsets from the subsemigroup $tZ^+ \cup \{0\} \times q\, Z^+ \cup \{0\}$; $2 \leq t, q < \infty$} $\subseteq$ S be the S-subset special strong semivector subspaces of S over the S-subset semiring.

Hence we can associate with this S-strong special subset semivector subspace which are infinite in number. However we can write S also as a direct sum of S-strong special subset semivector subspaces over the S-subset semiring P.

Take M_2 = {Collection of all subsets from the subsemigroup $T = \{(\{0\} \times Z^+ \cup \{0\}) \subseteq \{Z^+ \cup \{0\} \times Z^+ \cup \{0\}\}\} \subseteq S$, M_2 is a S-subset special strong semivector subspace of S over the S-subset semiring.

It is easily verified S = $M_1 \oplus M_2$. Further M_1 is the orthogonal complement of M_2 and vice versa, thus for every A $\in M_1$ and for every B $\in M_2$; is such that A × B = {0} so

$M_1^{\perp} = M_2$ and $M_2 = M_1^{\perp}$.

We have no other way of represent this S as a direct sum.

Example 3.19: Let S = {Collection of all subsets from the matrix semigroup M = $\{(a_1, a_2, \ldots, a_6) \mid a_i \in Z^+ \cup \{0\}; 1 \leq i \leq 6\}\}$ be the S-subset special strong semivector space over the S-subset semiring; P = {Collection of all subsets from the semifield F = $Z^+ \cup \{0\}\}$.

S has infinite number of S-subset special strong semivector subspaces given by L_t = {Collection of all subsets from the matrix subsemigroup; $B_t = \{(a_1, a_2, \ldots, a_6) \mid a_i \in t Z^+ \cup \{0\}; 1 \leq i \leq 6, 2 \leq t < \infty\}\}$ be the S-strong special subset semivector subspaces of S for varying t over the S-subset semiring.

Now S can also be written as a n-direct sum of S-strong special subset semivector subspaces of S over the S-subset semiring P when $2 \leq n \leq 6$.

We will just show if we take M_1 = {Collection of all subsets from the subsemigroup $N_1 = \{(a_1, a_2, 0, 0, 0, 0) \mid a_1, a_2 \in Z^+ \cup \{0\})\}\}$ be the S-strong special subset semivector subspace of S over the S-subset semiring P.

Let M_2 = {Collection of all subsets from the subsemigroup $N_2 = \{(0, 0, a_1, a_2, 0, 0) \mid a_1, a_2 \in Z^+ \cup \{0\}\}\}$ be the S-strong special subset semivector subspace of S over the S-subset semiring P. Let M_3 = {Collection of all substes from the subsemigroup $N_3 = \{(0, 0, 0, 0, a_1, a_2) \mid a_1, a_2 \in Z^+ \cup \{0\}\} \subseteq M\}\} \subseteq S$ be the S-strong special subset semivector subspace of S over the S-subset semiring P.

We see S = $M_1 + M_2 + M_2$ and $M_i \cap M_j = \{0\}$ if $i \neq j$, $1 \leq i, j \leq 3$ where {0} = {(0, 0, 0, 0, 0, 0)}. Also M_1 is orthogonal to

M_2 but $S \neq M_1 + M_2$. Further M_1 is also orthogonal with M_3 but $S \neq M_1 + M_2$ and M_2 is orthogonal with M_3 and $S \neq M_2 + M_3$.

Thus S is the 3-direct sum of S-strong special subset semivector subspaces of S over the S-subset semiring P.

Now if C_1 = {Collection of all subsets from the subsemigroup $L_1 = \{(0, a_1, 0, a_2, 0, 0\} \mid a_1, a_2 \in Z^+ \cup \{0\}\} \subseteq M\} \subseteq S$ be the S-subset special strong semivector subspace of S over the S-subset semiring P.

C_2 = {Collection of all subset from the subsemigroup $L_2 = \{(a_1, 0, 0, 0, 0, a_2) \mid a_1, a_2 \in Z^+ \cup \{0\}\} \subseteq M\} \subseteq S$ be the S-subset strong special semivector subspace of S over the S-semiring P. Let C_3 = {Collection of all subsets form the subsemigroup $L_3 = \{(0, 0, a, 0, a_2, 0) \mid a_1, a_2 \in Z^+ \cup \{0\}\} \subseteq M\}\} \subseteq S$ be the S-subset strong special semivector subspace of S over the S-subset semiring P.

We see $C_1 + C_2 + C_3 = S$ and infact C_1 is orthogonal to both C_2 and C_3. However $C_1 + C_2 \neq S$, $C_1 + C_3 \neq S$ and $C_3 + C_2 \neq S$ but $C_i \cap C_j = \{(0\ 0\ 0\ 0\ 0\ 0)\}$ if $i \neq j$, $1 \leq i, j \leq 3$. So we see $S = C_1 + C_2 + C_3$ is the 3-direct sum of S-special strong subset semivector subspaces of S over the S-subset semiring P.

Now consider V_1 = {Collection of all subsets from the subsemigroup $W_1 = \{(0, a_1, 0, a_2, 0, a_3) \mid a_1, a_2, a_3 \in Z^+ \cup \{0\}\} \subseteq M\}\}$ be the S-strong special subset semivctor subspace of S over the S-subset semiring P.

Let V_2 = {Collection of all subsets from the subsemigroup $W_1 = \{(a_1, 0, a_2, 0, a_3, 0) \mid a_1, a_2, a_3 \in Z^+ \cup \{0\} \subseteq M\}\}$ be the S-strong special subset semivector subspace of S over the S-subset semiring P. We see $W_1 + W_2 = S$.

Infact W_1 is the orthogonal complement of W_2 and vice versa for $W_1 \cap W_2 = \{(0\ 0\ 0\ 0\ 0\ 0)\}$.

Example 3.20: Let S = {Collection of all subsets from the matrix semigroup

$$M = \left\{ \left. \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_{10} \end{bmatrix} \right| a_i \in Q^+ \cup \{0\}; 1 \leq i \leq 10 \right\}\}$$

be the S-subset special strong semivector space over the S-subset semiring P = {Collection of all subsets from the semifield $Q^+ \cup \{0\}$}.

We see S has infinite number of S-subset special strong semivector space of S over the S-subset semiring P.

We see S can be written as a direct sum of S-subset special strong semivector subspaces of S in many ways $2 \leq n \leq 10$.

We will just indicate it by writing S as a 4-direct sum of S-subset special direct semivector subspaces.

Let T_1 = {Collection of all subsets from the subsemigroup

$$P_1 = \left\{ \left. \begin{bmatrix} a_1 \\ a_2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \right| a_1, a_2 \in Q^+ \cup \{0\} \right\} \subseteq M\} \subseteq S$$

be the S-strong subset special semivector subspace of S over the S-subset semiring P.

Let T_2 = {Collection of all subsets from the subsemigroup

$$P_2 = \left\{\left.\begin{bmatrix} 0 \\ 0 \\ a_1 \\ a_2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}\right| a_1, a_2 \in Q^+ \cup \{0\}\right\} \subseteq M\} \subseteq S$$

be the S-subset strong special semivector subspace of S over the S-subset semiring P.

Let T_3 = {Collection of all subsets from the subsemigroup

$$P_3 = \left\{\left.\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ a_1 \\ a_2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}\right| a_1, a_2 \in Q^+ \cup \{0\}\right\} \subseteq M\} \subseteq S$$

be the S- strong subset special semivector subspace of S over the S-subset semiring P.

Let T_4 = {Collection of all subsets from the subsemigroup

$$P_4 = \left\{ \left[\begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ a_1 \\ a_2 \\ a_3 \end{array}\right] \middle| \; a_1, a_2, a_3 \in Q^+ \cup \{0\} \right\} \subseteq M\} \subseteq S$$

be the S- subset special strong semivector subspace of S over the S-subset semiring.

$$\text{We see } T_i \cap T_j = \left\{ \left[\begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array}\right] \right\} \text{ if } i \neq j,\ 1 \leq i, j \leq 4.$$

Further S = $T_1 + T_2 + T_3 + T_4$ is the 4-direct sum of the S-special strong subset semivector subspaces.

Infact S can be made into a S-strong special subset semivector subspace as direct summand.

Example 3.21: Let S = {Collection of all subsets from the matrix semigroup

$$M = \left\{ \begin{bmatrix} a_1 & a_2 & \ldots & a_6 \\ a_7 & a_8 & \ldots & a_{12} \\ a_{13} & a_{14} & \ldots & a_{18} \end{bmatrix} \middle| \, a_i \in \langle Z^+ \cup I \cup \{0\}\rangle; 1 \leq i \leq 18 \right\}\}$$

be the S-strong subset special semivector space over the S-subset semiring P = {Collection of all subsets from the semifield $\langle Z^+ \cup I \cup \{0\}\rangle$}.

S can be written as a n-direct sum of S-subset special strong semivector subspaces $2 \leq n \leq 18$.

Apart from these S-special strong subset semivector subspaces we have an infinite number a S-special strong subset semivector subspaces.

Example 3.22: Let S = {Collection of all subsets from the matrix semigroup

$$M = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ \vdots & \vdots & \vdots \\ a_{28} & a_{29} & a_{30} \end{bmatrix} \middle| \, a_i \in Z^+ \cup \{0\}; 1 \leq i \leq 30 \right\}\}$$

be the S-special strong subset semivector space over the S-subset semiring; P = {Collection of all subsets from the semifield $Z^+ \cup \{0\}$}.

S has infinite number of S subset special strong semivector subspaces over P.

S has n-S-strong special subset semivector subspaces of S over the S-subset semiring P, $2 \leq n \leq 30$ and S can be written as a n-direct sum of S-subset special strong semivector subspaces over P.

Example 3.23: Let S = {Collection of all subsets from the super matrix semigroup

$$M = \left\{ \left[\begin{array}{c} a_1 \\ \hline a_2 \\ a_3 \\ \hline a_4 \\ a_5 \\ a_6 \\ \hline a_7 \\ a_8 \\ \hline a_9 \end{array}\right] \middle| \; a_i \in (Z^+ \cup \{0\})S_7;\ 1 \leq i \leq 9 \right\}\}$$

be the S-subset special strong semivector space over P = {Collection of all subsets from the semifield $Z^+ \cup \{0\}$}.

S has infinite number of S-subset special strong semivector subspaces and S can be written as a n-direct sum S-subset special strong semivector subspaces of S over P, $2 \leq n \leq 9$.

Example 3.24: Let S = {Collection of all subsets from the super matrix semigroup $M = \{(a_1 \mid a_2\ a_3\ a_4 \mid a_5\ a_6\ a_7\ a_8) \mid a_i \in (Z^+ \cup \{0\}, 1 \leq i \leq 8\}\}$ be the subset special strong semivector space over the S-subset semiring. P = {Collection of all subsets from the semifield $Z^+ \cup \{0\}$}}.

Clearly S is a non commutative S-subset special strong semilinear algebra over P.

Infact S has infinite number of S-strong special subset semilinear subalgebras some of which are commutative and some which are non commutative.

S can also be written as a n-direct sum $(2 \leq n \leq 8)$ of S-subset special strong semivector subspaces (semilinear subalgebras) over P.

Example 3.25: Let S = {Collection of all subsets from the semigroup

$$M = \left\{ \left[\begin{array}{cc} a_1 & a_2 \\ a_3 & a_4 \\ \hline a_5 & a_6 \\ a_7 & a_8 \\ a_9 & a_{10} \\ a_{11} & a_{12} \\ \hline a_{13} & a_{14} \end{array} \right] \middle| a_i \in (Z^+ \cup \{0\})D_{2,11}; 1 \leq i \leq 14 \right\}\}$$

be the subset strong special semivector space over the S-subset semiring;

P = {Collection of all subsets from the semifield $F = Z^+ \cup \{0\}$}.

S is a non commutative S-special strong semilinear algebra under the natural product $\times_n$ over P.

$$\text{Let } A = \left\{ \left[\begin{array}{cc} ab & 2a \\ 4a & 5b+1 \\ \hline 7 & 8b \\ a & 4ab \\ 5 & 2ab+3 \\ b & ab^2 \\ \hline 7b & 9a \end{array} \right] \right\} \text{ and}$$

$$B = \left\{ \left[\begin{array}{cc} 3a+5b & 0 \\ 7b+a & a+ab \\ \hline 3a+2ab & 5b \\ 5ab & 6a+3ab^3 \\ 7ab+b^3 & b^3+5ab^2 \\ 3ab^3 & 5ab \\ \hline 0 & 2ab+a \end{array} \right] \right\} \in S.$$

We show $A \times_n B \neq B \times_n A$.

$$\text{Take } A \times_n B = \left\{ \left[\begin{array}{cc} ab & 2a \\ 4a & 5b+1 \\ \hline 7 & 8b \\ a & 4ab \\ 5 & 2ab+3 \\ b & ab^2 \\ \hline 7b & 9a \end{array} \right] \right\} \times_n \left\{ \left[\begin{array}{cc} 3a+5b & 0 \\ 7b+a & a+ab \\ \hline 3a+2ab & 5b \\ 5ab & 6a+3ab^3 \\ 7ab+b^3 & b^3+5ab^2 \\ 3ab^3 & 5ab \\ \hline 0 & 2ab+a \end{array} \right] \right\}$$

$$= \left\{ \left[\begin{array}{cc} 3ab + 5ab^2 & 0 \\ 4 + 28ab & 5ba + a + ab + 5bab \\ \hline 21a + 14ab & 40b^2 \\ 5b & 24b + 4abab^3 \\ 35ab + 5b^3 & 3b^3 + 15ab^2 + 2ab^4 + 10abab^2 \\ 3ab^4 & 5ab^2ab \\ \hline 0 & 9 + 18b \end{array} \right] \right\}$$

$$= \left\{ \left[\begin{array}{cc} 3ab + 5ab^2 & 0 \\ 4 + 28ab & 6a + ab + 5ab^{10} \\ \hline 21a + 14ab & 40b^2 \\ 5b & 24b + 4b^2 \\ 35ab + 5b^3 & 3b^3 + 15ab^2 + 2ab^4 + 10b \\ 3ab^4 & 5b^{10} \\ \hline 0 & 9 + 18b \end{array} \right] \right\} \quad \ldots \quad I$$

Now we find

$$B \times_n A = \left\{ \left[\begin{array}{cc} 3a + 5b & 0 \\ 7b + a & a + ab \\ \hline 3a + 2ab & 5b \\ 5ab & 6a + 3ab^3 \\ 7ab + b^3 & b^3 + 5ab^2 \\ 3ab^3 & 5ab \\ \hline 0 & 2ab + a \end{array} \right] \right\} \times_n$$

$$\left\{\left[\begin{array}{cc} ab & 2a \\ 4a & 5b+1 \\ \hline 7 & 8b \\ a & 4ab \\ 5 & 2ab+3 \\ b & ab^2 \\ \hline 7b & 9a \end{array}\right]\right\}$$

$$= \left\{\left[\begin{array}{cc} 3ab+5a & 0 \\ 4+28ab^{10} & a+ab+5ab+5ab^2 \\ \hline 21a+14ab & 40b^2 \\ 5aba & 24a^2b+4ab^3ab \\ 35ab+5b^3 & 3b^3+15ab^2+2b^3ab+10ab^2ab \\ 3ab^3b & 5abab^2 \\ \hline 0 & 9+18aba \end{array}\right]\right\}$$

$$= \left\{\left[\begin{array}{cc} 3ab+5a & 0 \\ 4+28ab^{10} & 6ab+a+5ab^2 \\ \hline 21a+14ab & 40b^2 \\ 5b^{10} & 24b+4b^9 \\ 35ab+5b^3 & 15ab^2+3b^3+2b^2a+10ab^{10} \\ 3ab^4 & 5b \\ \hline 0 & 9+18ab^{10} \end{array}\right]\right\} \quad \ldots \text{ II}$$

Clearly I and II are distinct, hence $A \times_n B \neq B \times_n A$.

Thus S is a non commutative S-subset special strong semilinear algebra over P.

Example 3.26: Let S = {Collection of all subsets from the super matrix semigroup

$$M = \left\{ \left(\begin{array}{c|ccc|c} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & a_7 & a_8 & a_9 & a_{10} \end{array} \right) \middle| \; a_i \in (Q^+ \cup \{0\})A_4;\ 1 \leq i \leq 10\} \right\}$$

be the S-strong special subset semivector space over the S-subset semiring
P = {Collection of all subsets from the semifield $Q^+ \cup \{0\}$}.

S is non commutative S-special subset strong semilinear algebra over P.

Inview of all these examples we can say the following.

THEOREM 3.1: *Let*
S = {Collection of all subsets from a semigroup M} be a S-special strong subset semilinear algebra over the S-subset semiring. S is a non commutative S-subset special strong semilinear algebra if and only if the semigroup M is a non commutative semiring under ×.

Proof follows from the fact if on (M, +) the additive semigroup we have a product × defined on M such that (M, ×) is a non commutative semigroup.

To this end we have seen several examples.

Example 3.27: Let S = {Collection of all subsets from the super matrix semigroup

$$B = \left\{ \left[\begin{array}{c|cc|ccc} a_1 & (0) & & & (0) & \\ \hline & a_2 & a_3 & & & \\ (0) & a_4 & a_5 & & (0) & \\ \hline & & & a_6 & a_7 & a_8 \\ (0) & (0) & & a_9 & a_{10} & a_{11} \\ & & & a_{12} & a_{13} & a_{14} \end{array} \right] \middle| \right.$$

$a_i \in (Z^+ \cup \{0\})\ (S_7 \times D_{2,8});\ 1 \leq i \leq 14\}\}$

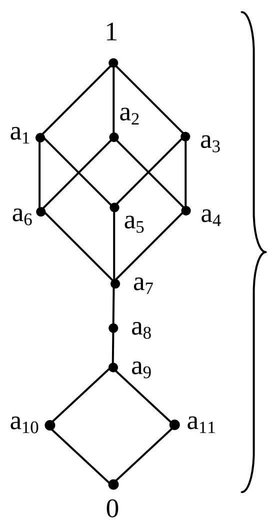
be the S-strong special subset semivector space over the S-subset semiring P where P = {Collection of all subsets from the semifield $Z^+ \cup \{0\}$}.

S is a S-subset strong special semilinear algebra over P which is non commutative.

All these S-subset strong special semilinear algebras happens to be of infinite order.

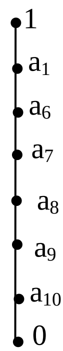
Now we proceed onto give examples of S-subset special strong semilinear algebras over the S-subset semiring P.

Example 3.28: Let S = {Collection of all subsets from the semigroup LS_3 where L is the lattices



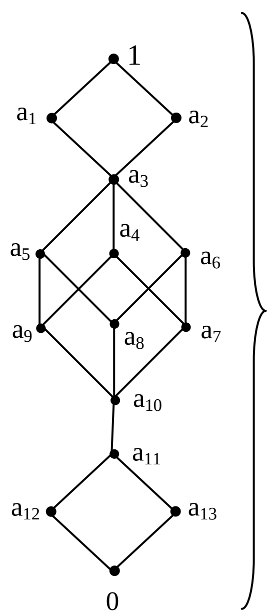
be a S-strong special subset semilinear algebra over the S-subset semiring, P = {Collection of all subsets from the lattice L}.

P is a S-subset semiring as

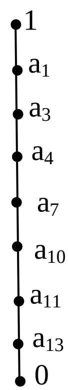
T = 

where T ⊆ L. We see o(S) < ∞. S is a non commutative S-subset special strong semilinear algebra over P.

Example 3.29: Let S = {Collection of all subsets from the lattice group $LD_{2,7}$ where L is a lattice given by



be the S-subset special strong semilinear algebra over the S-subset semiring. P = {Collection of all subsets from the lattice L}}. P is a S-subset semiring has the semifield T =

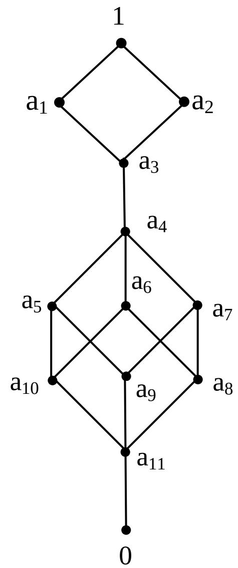


S is a S-subset special strong semilinear algebra over S.

Example 3.30: Let S = {Collection of all subsets from the semigroup

$$M = \left\{ \left[\begin{array}{c} a_1 \\ a_2 \\ \hline a_3 \\ \hline a_4 \\ a_5 \\ a_6 \\ \hline a_7 \\ a_8 \\ \hline a_9 \end{array} \right] \middle| \; a_i \in LD_{2,7};\ 1 \leq i \leq 9 \right\}\}$$

where L =

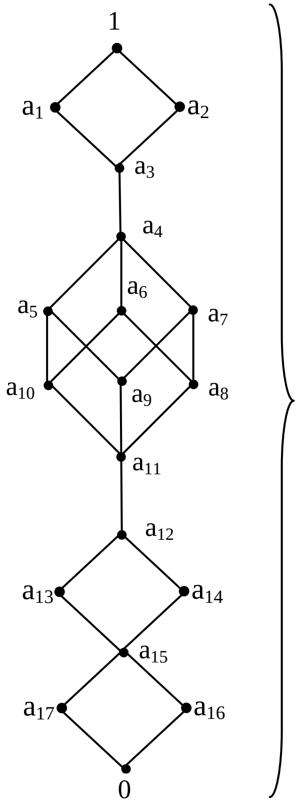


be the S-strong special subset semilinear algebra of finite order over the subset semiring P = {Collection of all subsets from the semifield L}}. S is a non commutative S subset special strong semilinear algebra over P.

Example 3.31: Let S = {Collection of all subsets from the semigroup LS_4 where L is a Boolean algebra of order 64} be the S-strong subset special semilinear algebra over the S-subset semifield P.

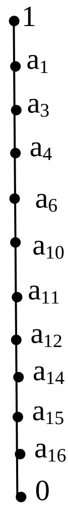
Clearly $o(S) < \infty$ and S is a non commutative S-subset special strong semilinear algebra over P, where P = {Collection of all subsets from the S-semiring L a Boolean algebra of order 64}.

Example 3.32: Let S = {Collection of all subsets from the group lattice LA_5 where L is a lattice which is as follows:



be the S-subset special strong semilinear algebra over the S-subset semiring
P = {Collection of all subsets from the lattice L}}.

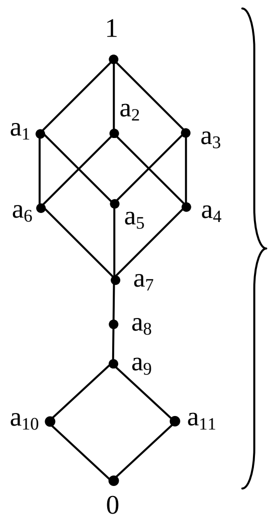
P is a S-subset semiring for it contains the semifield

T = 

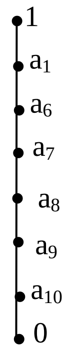
S is a finite S-subset special strong semilinear algebra which is non commutative.

Example 3.33: Let S = {Collection of all subsets from the group lattice $L(S_3 \times D_{29})$} be the S-subset strong special semilinear algebra over the S-subset semiring

P = {Collection of all subsets from the lattice L where L is as follows:

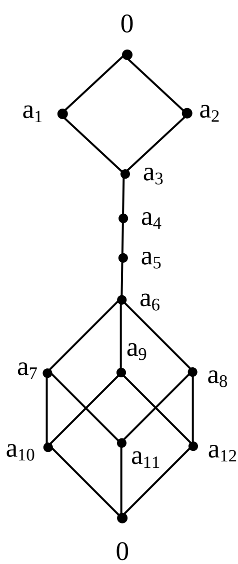


L contains a semifield T =

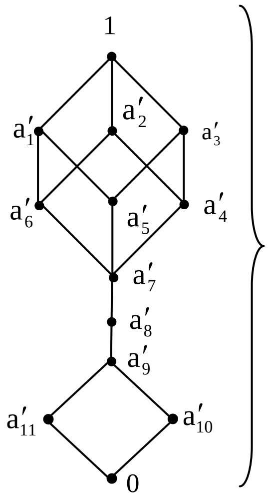


o(S) < ∞ and S is a non commutative S-subset special strong semilinear algebra over P.

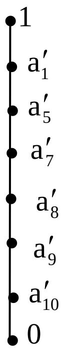
Example 3.34: Let S = {Collection of all subsets from the group lattice ($L_1 \times L_2$) ($S_3 \times S_4$) where L_1 =



and L_2 =



be the S-strong special subset semilinear algebra over the S-subset semiring P = {Collection of all subsets from the semifield $\{1\} \times T_1$ where T_1 is as follows:



$o(S) < \infty$ and S is a non commutative S-subset strong special semilinear algebra over P.

Now having seen examples of S-subset strong special semilinear algebra.

We now proceed onto describe strong subset linear dependence and strong subset linear independence and strong subset basis.

Let S = {Collection of all subsets from the semigroup $Z^+ \cup \{0\} \times Z^+ \cup \{0\}$} be the S-subset strong special semivector space over P = {Collection of all subsets from the semifield $Z^+ \cup \{0\}$ = F} be the S-subset semiring.

Take A = {(3, 2), (1, 5), (7, 1), (0, 2), (5, 5)} and B = {(6, 4), (0, 4), (10, 10), (2, 10), (14, 2)} $\in$ S.

Clearly {2}A = B so A and B are strong subset linearly dependent in S. Let A = {(8, 4), (5, 6), (7, 3), (2, 5), (4, 0), (1, 1), (9, 10), (11, 0)} and B = {(5, 6), (8, 0), (4, 4), (5, 2), (3, 7), (4, 9), (12, 15), (14, 9), (0, 0), (1, 0)} $\in$ S.

We see A and B strong subset linearly dependent. Given two elements A, B $\in$ S; they may be strong subset dependent or strong subset independent.

So finding strong subset basis is a difficult task and left as an exercise to the reader.

Now we use rings in the construction of S-strong special subset semivector spaces (semilinear algebras). We first analyse the special properties associated with them.

Example 3.35: Let S = {Collection of all subsets from the ring $Z \times Z \times Z$} be the S-strong special subset semivector space over P = {Collection of subsets from the semifield $Z^+ \cup \{0\}$ = F}. S has infinite number of S-strong special subset semivector subspaces and S is the 3 direct sum of S-strong special subset semivector spaces over P.

Example 3.36: Let S = {Collection of all subsets from the ring C} be the S-strong special subset semivector space over the S-subset semiring P = {Collection of all subsets from the semifield $Z^+ \cup \{0\}$}. S has infinite number of S subset special strong semivector spaces and S is not the direct sum of finite number of S-subset strong special semivector subspaces of S.

Example 3.37: Let
S = {Collection of all subsets from the ring RS_3} be the S-strong special subset semivector space over the S-subset semiring; P = {Collection of all subsets from the semifield $R^+ \cup \{0\}$}.

Example 3.38: Let
S = {Collection of all subsets from the ring Z_{12} S(5)} be the S-special super strong subset semivector space over the subset semiring; P = {Collection of all subsets from the semiring Z_{12}. We call S the S-special super strong semivector space if P is a subset semiring which has a proper subset T such that T is a ring.

Example 3.39: Let
S = {Collection of all subsets of the ring $Z_{11}A_5$} be the S-super special super strong semivector space over P = {Collection of all subsets from the field Z_{11}}; the special subset semiring as P contains a proper subset A which is isomorphic to Z_{11}.

S is a semilinear algebra of the S-super special super strong type. Infact S is of finite order.

We may not be in a position to inter relate these structures but basically all of them are built over S.

Example 3.40: Let
S = {Collection of all subsets from the ring Z_{40} $(S_3 \times D_{2,7})$} be the S-special super strong subset semilinear algebra over the subset semiring; P = {Collection of all subsets from the ring Z_{40}}.

Infact we can define S over the subset semiring P_1 = {Collection of all subsets from the ring {0, 10, 20, 30} $\subseteq$ Z_{40}} $\subseteq$ P also; or for that matter over any proper subring of Z_{40}. Thus this gives us the lineancy to build several such S-special super strong semilinear algebras all of which are of finite order but non commutative.

However if we use Q or Z or R or C or $\langle Q \cup I \rangle$ or $\langle Z \cup I \rangle$ or $\langle R \cup I \rangle$ or $\langle C \cup I \rangle$ we can have S-subset semirings as all these rings contains a subset which is a semifield.

Example 3.41: Let S = {Collection of all subsets from the group ring $(Z_{10} \times Z_{15})S_7$} be the S-strong super special subset semilinear algebra over the subset semiring; P = {Collection of all subsets from the ring $Z_{10} \times Z_{15}$}.

Clearly S is non commutative and is of finite order.

Example 3.42: Let S = {Collection of all subsets from the group ring $C(S_7 \times D_{210})$} be the S-super special super strong subset semilinear algebra of infinite order over the subset semiring, P = {Collection of all subsets from the field C}. Clearly S is non commutative.

Example 3.43: Let S = {Collection of all subsets from the matrix ring M = $\{(a_1, a_2, \ldots, a_7) \mid a_i \in Z_{15}, 1 \leq i \leq 7\}$} be the S-super special strong subset semivector space over the subset semiring, P = {Collection of all subsets from the ring Z_{15}}.

Clearly o(S) is finite, S is commutative and S can be written as a direct sum.

Example 3.44: Let
S = {Collection of all subsets from the ring $Z_{19}(S_3 \times D_{27})$} be the S-special super strong semivector space over the S-subset semiring, P = {Collection of all subsets from the field Z_{19}}.

We see $o(S) < \infty$ and S a S-subset super strong special semilinear algebra. S is non commutative over P.

Example 3.45: Let S = {Collection of all subsets from the ring $Z_{23}(S_3 \times S_7 \times D_{2,10})$} be the S-strong super special subset semilinear algebra over the subset semiring, P = {Collection of all subsets from the field Z_{23}}. o(S) < ∞. S is a non commutative semilinear algebra.

Example 3.46: Let S = {Collection of all subsets from the ring ($Z_5 \times Z_{23}$) (S(3) × $D_{2,12} \times A_4$))} be the S-special super strong subset semilinear algebra over the subset semiring. P = {Collection of all subsets from the ring $Z_5 \times Z_{23}$}. S is a non commutative semilinear algebra.

Example 3.47: Let
S = {Collection of all subsets from the ring Z_{29}S(5)} be the S-special super strong subset semilinear algebra over the S-subset semiring. S is also a non commutative S-special strong super subset semilinear algebra of finite order.

S has only finite number of S-subset super strong special semilinear subalgebras.

Example 3.48: Let
S = {Collection of all subsets from the ring RS_7} be the S-strong super special subset semilinear algebra over the S-subset semiring;

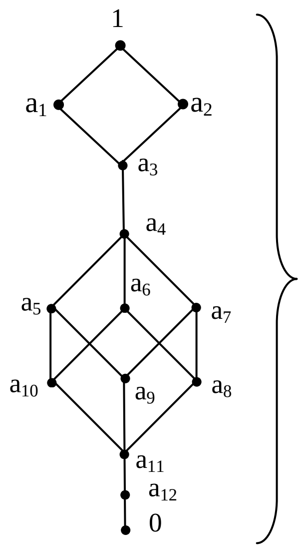
P = {Collection of all subsets from the ring Q}. o(S) = ∞ and S is a non commutative S-special super strong subset semilinear algebra over P.

It is pertinent to keep on record that we can have the subset semiring that contain a subset semifield or a subset field or a subset ring we call the later two as Smarandache super subset semiring. If this is not mentioned explicitely, one can understand from the very context.

Example 3.49: Let
S = {Collection of all subsets from the ring Z_{43}{S_3 × S(4)}} be the S-strong super special subset semilinear algebra over the S-strong super subset semiring P = {Collection of all subsets from the field Z_{43}}.

o(S) < ∞ and S is a non commutative, S-subset strong super special semilinear algebra over P.

Example 3.50: Let S = {Collection of all subsets from the lattice group LG where L is the lattice given in the following;



be the S-strong special semivector space of finite order over the S-subset semiring P.

Now having seen examples of these new structures we proceed onto give a few examples of the notion of S-strong semilinear operator and S-strong semilinear transformations of S-strong special subset semivector spaces over S-subset semirings.

Example 3.51: Let
S_1 = {Collection of all subsets from the ring $Z(S_3)$} and
S_2 = {Collection of all subsets from the ring QS(3))} be the S-strong special subset semivector spaces over the S-subset semiring.

P = {Collection of all subsets from the ring Z}.

We have $T_S : S_1 \to S_2$ such that T_S is a S-subset special strong semilinear transformation.

$T_S(A) = A$.

$$T_S(\{a_0 + a_1p_1 + a_2p_2 + a_3p_3 + a_4p_4 + a_5p_5\})$$
$$= \{a_0 + a_1p_1 + a_2p_2 + a_3p_3 + a_4p_4 + a_5p_5\}$$

Thus T_S is a embedding on S_1 onto S_2.

Suppose we wan $T'_S : S_2 \to S_1$ we can define

$$T'_S (A) = T'_S (a_0 + a_1s_1 + \ldots + a_{26}s_{26});$$
$$(s_i \in S(3) : 1 \leq i \leq 26)$$
$$= \{a_0 + \sum a_i T'_S (s_i)\}$$
where $T'_S (s_i) = 1$ if $s_i \in S(3) \setminus S_3$.
and $T'_S (s_i) = s_i$ if $s_i \in S_3$, $1 \leq i \leq 26$.

Now it is easily verified T'_S is also a S-strong super special subset semilinear transformation of S_2 to S_1.

Example 3.52: Let S_1 = {Collection of all subsets from the matrix ring

$$M = \left\{\left.\begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \end{bmatrix}\right| a_i \in Q; 1 \leq i \leq 8\}\right\}$$

and S_2 = {Collection of all subsets from the matrix ring

$$M_1 = \left\{\left.\begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix}\right| a_i \in Q; 1 \leq i \leq 9\}\right\}$$

are S-subset super special strong semivector spaces (semilinear algebras) over the S-subset semiring (Super S-subset semiring). P = {Collection of all subsets from the ring Q}.

Define $T_S : S_1 \to S_2$ by

$$T_S(A) = T_S\left(\left\{\begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \end{bmatrix}\right\}\right)$$

$$= \left\{\begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & 0 \end{bmatrix}\right\} \in S_2.$$

It is easily verified T_S is a S-special super strong subset semilinear transformation of S_1 to S_2.

$$\text{If } A = \left\{\begin{bmatrix} 3 & 4 & 5 & 0 \\ 2 & 0 & 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 1 & 0 & 6 \end{bmatrix}, \begin{bmatrix} 7 & 4 & 5 & 2 \\ 0 & 1 & 8 & 6 \end{bmatrix}\right\} \in S_1$$

then

$$T(A) = T\left(\left\{\begin{bmatrix} 3 & 4 & 5 & 0 \\ 2 & 0 & 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 1 & 0 & 6 \end{bmatrix}, \begin{bmatrix} 7 & 4 & 5 & 2 \\ 0 & 1 & 8 & 6 \end{bmatrix}\right\}\right)$$

$$= \left\{\begin{bmatrix} 3 & 4 & 5 \\ 0 & 2 & 0 \\ 1 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 6 & 0 \end{bmatrix}, \begin{bmatrix} 7 & 4 & 5 \\ 2 & 0 & 1 \\ 8 & 6 & 0 \end{bmatrix}\right\} \in S_2.$$

Suppose one is interested in defining a S-subset super special strong semilinear transformation from S_2 to S_1 say T'_S ; $T'_S : S_2 \to S_1$ is defined as follows:

$$T'_S(A) = T'_S\left\{\begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix}\right\}$$

$$= \left\{\begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \end{bmatrix}\right\} \in S_1.$$

That is if

$$A = \left\{\begin{bmatrix} 3 & 4 & 5 \\ 1 & 2 & 0 \\ 3 & 7 & 8 \end{bmatrix}, \begin{bmatrix} 2 & 0 & 1 \\ 4 & 5 & 0 \\ 1 & 1 & 7 \end{bmatrix}, \begin{bmatrix} 10 & 1 & 5 \\ 7 & 0 & 5 \\ 0 & 6 & -2 \end{bmatrix}, \begin{bmatrix} -3 & 2 & 0 \\ 0 & -9 & 11 \\ 8 & 14 & 16 \end{bmatrix}\right\} \in S_2.$$

We now find

$T'_S(A) =$

$$T'_S\left(\left\{\begin{bmatrix} 3 & 4 & 5 \\ 1 & 2 & 0 \\ 3 & 7 & 8 \end{bmatrix}, \begin{bmatrix} 2 & 0 & 1 \\ 4 & 5 & 0 \\ 1 & 1 & 7 \end{bmatrix}, \begin{bmatrix} 10 & 1 & 5 \\ 7 & 0 & 5 \\ 0 & 6 & -2 \end{bmatrix}, \begin{bmatrix} -3 & 2 & 0 \\ 0 & -9 & 11 \\ 8 & 14 & 16 \end{bmatrix}\right\}\right)$$

$$= \left\{\begin{bmatrix} 3 & 5 & 2 & 3 \\ 4 & 1 & 0 & 7 \end{bmatrix}, \begin{bmatrix} 2 & 1 & 5 & 1 \\ 0 & 4 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 10 & 5 & 0 & 0 \\ 1 & 7 & 5 & 6 \end{bmatrix}, \begin{bmatrix} -3 & 0 & -9 & 8 \\ 2 & 0 & 11 & 14 \end{bmatrix}\right\} \in S_1.$$

This is the way S-strong super special subset semilinear transformation are defined.

Example 3.53: Let S = {Collection of all subsets from the matrix ring

$$M = \left\{ \left[\begin{array}{c} a_1 \\ a_2 \\ a_3 \\ \hline a_4 \\ \hline a_5 \\ a_6 \end{array} \right] \middle| \; a_i \in Z; 1 \leq i \leq 6 \right\}\}$$

be the S-special subset super strong semivector space over the S-subset semiring.

P = {Collection of all subsets from the ring Z}.

Let S_2 = {Collection of all subsets from the matrix ring

$$M = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix} \middle| \; a_i \in Z; 1 \leq i \leq 9 \right\}\}$$

be the S-subset special strong semivector space over the S-subset semiring P = {Collection of all subsets from the ring Z}.

We now define $T_S : S_1 \to S_2$ by

$$T(\{\left[\begin{array}{c} a_1 \\ a_2 \\ a_3 \\ \hline a_4 \\ \hline a_5 \\ a_6 \end{array} \right]\}) = \{\begin{bmatrix} 0 & a_1 & a_2 \\ a_3 & 0 & a_4 \\ a_5 & a_6 & 0 \end{bmatrix}\} \text{ is in } S_2.$$

For if A = $\left\{\begin{bmatrix}3\\2\\0\\\hline 1\\\hline 1\\5\end{bmatrix},\begin{bmatrix}2\\0\\1\\\hline 0\\\hline 5\\-8\end{bmatrix},\begin{bmatrix}-2\\-7\\2\\\hline 0\\\hline -7\\9\end{bmatrix},\begin{bmatrix}8\\0\\-11\\\hline -13\\\hline 1\\2\end{bmatrix},\begin{bmatrix}-9\\6\\8\\\hline 1\\\hline 2\\7\end{bmatrix}\right\} \in S_1$ then

$$T_S(A) = \left(\left\{\begin{bmatrix}3\\2\\0\\\hline 1\\\hline 1\\5\end{bmatrix}\begin{bmatrix}-9\\6\\8\\\hline 1\\\hline 2\\7\end{bmatrix}\begin{bmatrix}2\\0\\1\\\hline 0\\\hline 5\\-8\end{bmatrix},\begin{bmatrix}-2\\-7\\2\\\hline 0\\\hline -7\\9\end{bmatrix}\begin{bmatrix}8\\0\\-11\\\hline -13\\\hline 1\\2\end{bmatrix},\right\}\right)$$

$$= \left\{\begin{bmatrix}0 & 3 & 2\\0 & 0 & 1\\1 & 5 & 0\end{bmatrix},\begin{bmatrix}0 & 2 & 0\\1 & 0 & 0\\5 & 8 & 0\end{bmatrix},\begin{bmatrix}0 & -2 & -7\\2 & 0 & 0\\-7 & 9 & 0\end{bmatrix},\right.$$

$$\left.\begin{bmatrix}0 & 8 & 0\\-11 & 0 & -13\\1 & 2 & 0\end{bmatrix},\begin{bmatrix}0 & -9 & 6\\8 & 0 & 1\\2 & 7 & 0\end{bmatrix}\right\} \in S_2,$$

It is easily verified T_S is a S-special strong super subset semilinear transformation form S_1 to S_2.

Consider $T'_S : S_2 \to S_1$ defined by

$$T'_S\left(\left\{\begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix}\right\}\right) = \left\{\begin{bmatrix} a_1 + a_2 \\ a_3 \\ a_4 + a_5 \\ \hline a_6 \\ \hline a_7 \\ a_8 + a_9 \end{bmatrix}\right\}$$

Let $A = \left\{\begin{bmatrix} 3 & 6 & 4 \\ 1 & 2 & 3 \\ 0 & 8 & -7 \end{bmatrix}, \begin{bmatrix} 2 & 0 & 1 \\ 3 & 8 & 0 \\ 6 & 0 & -2 \end{bmatrix}, \begin{bmatrix} 0 & 7 & 0 \\ 1 & 2 & 3 \\ 0 & 1 & 5 \end{bmatrix},\right.$

$$\left.\begin{bmatrix} 9 & 0 & 0 \\ 0 & 8 & 1 \\ 4 & 4 & 3 \end{bmatrix}, \begin{bmatrix} 3 & 0 & 1 \\ 4 & 2 & 3 \\ 0 & 0 & 3 \end{bmatrix}\right\} \in S_2.$$

We now find

$$T'_S(A) = T'_S\left\{\begin{bmatrix} 3 & 6 & 4 \\ 1 & 2 & 3 \\ 0 & 8 & -7 \end{bmatrix}, \begin{bmatrix} 2 & 0 & 1 \\ 3 & 8 & 0 \\ 6 & 0 & -2 \end{bmatrix}, \begin{bmatrix} 0 & 7 & 0 \\ 1 & 2 & 3 \\ 0 & 1 & 5 \end{bmatrix},\right.$$

$$\left.\begin{bmatrix} 9 & 0 & 0 \\ 0 & 8 & 1 \\ 4 & 4 & 3 \end{bmatrix}, \begin{bmatrix} 3 & 0 & 1 \\ 4 & 2 & 3 \\ 0 & 0 & 3 \end{bmatrix}\right\}$$

$$= \left\{ \begin{bmatrix} 0 \\ 4 \\ 3 \\ \hline 3 \\ \hline 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 11 \\ \hline 0 \\ \hline 6 \\ -2 \end{bmatrix}, \begin{bmatrix} 7 \\ 0 \\ 3 \\ \hline 3 \\ \hline 0 \\ 6 \end{bmatrix}, \begin{bmatrix} 9 \\ 0 \\ 8 \\ \hline 1 \\ \hline 4 \\ 7 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 6 \\ \hline 3 \\ \hline 0 \\ 3 \end{bmatrix} \right\} \in S_1.$$

Thus T'_S is a S-subset super strong subset special semilinear transformation of S-strong special subset semivector spaces over P.

Example 3.54: Let S_1 = {Collection of all subsets from the matrix ring

$$M_1 = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \end{bmatrix} \right| a_i \in Q; 1 \leq i \leq 16\}\}$$

be the special strong super subset semivector space over the S-subset semiring P = {Collection of all subsets from the ring Q}.

Let S_2 = {Collection of all subsets from the ring $M_2 = \{(a_1 \mid a_2\ a_3\ a_4 \mid a_5\ a_6 \mid a_7\ a_8) \mid a_i \in Q, 1 \leq i \leq 8\}$} be the S-strong super special subset semivector space over the S-subset semiring P.

Define $T_S : S_1 \to S_2$ by

$$T_S \left(\left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \end{bmatrix} \right\} \right)$$

$= \{(a_1 + a_2 \mid a_3 + a_4 \;\; a_5 + a_6 \;\; a_7 + a_8 \mid a_9 + a_{10} \;\; a_{11} + a_{12} \mid a_{13} + a_{14} \;\; a_{15} + a_{16})\}.$

Clearly T_S is a S-special subset super strong semilinear transformation of S_1 to S_2

Let

$$A = \left\{\begin{bmatrix} 2 & 0 & 1 & 4 \\ 0 & 5 & 0 & 7 \\ 8 & 9 & 1 & -3 \\ -7 & 8 & -5 & 3 \end{bmatrix}, \begin{bmatrix} 1 & 2 & 3 & 4 \\ -4 & -3 & -2 & -1 \\ 5 & 6 & 7 & 8 \\ 8 & 7 & 5 & 6 \end{bmatrix}, \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 \\ 4 & 4 & 4 & 4 \end{bmatrix},\right.$$

$$\left.\begin{bmatrix} 0 & 0 & 0 & 0 \\ 2 & 3 & 2 & 3 \\ 0 & 0 & 0 & 0 \\ 5 & 4 & 5 & 4 \end{bmatrix}, \begin{bmatrix} 3 & 8 & 3 & 8 \\ 8 & 3 & 8 & 3 \\ 8 & 8 & 3 & 3 \\ 3 & 3 & 8 & 8 \end{bmatrix}\right\} \in S_1.$$

We now find

$T_S(A)$

$$= T\left(\left\{\begin{bmatrix} 2 & 0 & 1 & 4 \\ 0 & 5 & 0 & 7 \\ 8 & 9 & 1 & -3 \\ -7 & 8 & -5 & 3 \end{bmatrix}, \begin{bmatrix} 1 & 2 & 3 & 4 \\ -4 & -3 & -2 & -1 \\ 5 & 6 & 7 & 8 \\ 8 & 7 & 5 & 6 \end{bmatrix}, \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 \\ 4 & 4 & 4 & 4 \end{bmatrix},\right.\right.$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 2 & 3 & 2 & 3 \\ 0 & 0 & 0 & 0 \\ 5 & 4 & 5 & 4 \end{bmatrix}, \begin{bmatrix} 3 & 8 & 3 & 8 \\ 8 & 3 & 8 & 3 \\ 8 & 8 & 3 & 3 \\ 3 & 3 & 8 & 8 \end{bmatrix}\Bigg\})$$

= $\{(2 \mid 5\ 5\ 7 \mid 17\ 4 \mid 1\ –2), (3 \mid 7\ –7\ –3 \mid 11\ 15 \mid 15\ 11),$
$(2 \mid 2\ 4\ 4 \mid 6\ 6\ 8\ 8), (0 \mid 0\ 5\ 5 \mid 0\ 0 \mid 9\ 9),$
$(11 \mid 11\ \ 11\ \ 11 \mid 16\ \ 6 \mid 6\ \ 16)\} \in S_2$.

We see $T_S : S_1 \to S_2$ so defined is a S-strong special super subset semilinear transformation.

Now define $T'_S : S_2 \to S_1$ is defined as follows:

$$T'_S\ (\{\begin{bmatrix} a_1 \\ \hline a_2 \\ a_3 \\ a_4 \\ \hline a_5 \\ a_6 \\ \hline a_7 \\ a_8 \end{bmatrix}\}) = \left\{\begin{bmatrix} a_1 & 0 & a_5 & 0 \\ a_2 & 0 & a_6 & 0 \\ a_3 & 0 & a_7 & 0 \\ a_4 & 0 & a_8 & 0 \end{bmatrix}\right\}$$

$$\text{for if A} = \left\{\begin{bmatrix} 0 \\ \hline 2 \\ 1 \\ 5 \\ \hline 6 \\ 7 \\ \hline 8 \\ 1 \end{bmatrix}, \begin{bmatrix} 9 \\ \hline 1 \\ 2 \\ 3 \\ \hline 4 \\ 5 \\ \hline 6 \\ 0 \end{bmatrix}\right\} \in S_2.$$

$$\text{then } T'_S(A) = T'_S\left(\left\{\begin{bmatrix} 0 \\ \overline{2} \\ 1 \\ 5 \\ \overline{6} \\ 7 \\ \overline{8} \\ 1 \end{bmatrix}, \begin{bmatrix} 9 \\ \overline{1} \\ 2 \\ 3 \\ \overline{4} \\ 5 \\ \overline{6} \\ 0 \end{bmatrix}\right\}\right)$$

$$= \left\{\begin{bmatrix} 0 & 0 & 6 & 0 \\ 2 & 0 & 7 & 0 \\ 1 & 0 & 8 & 0 \\ 5 & 0 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 9 & 0 & 4 & 0 \\ 1 & 0 & 5 & 0 \\ 2 & 0 & 6 & 0 \\ 3 & 0 & 0 & 0 \end{bmatrix}\right\} \in S_1.$$

Thus T'_S is a S-semi linear transformation of S_2 to S_1.

Now we proceed onto describe using examples how the Smarandache special super strong subset semilinear operators are defined on Smarandache special super strong subset semivector spaces.

Example 3.55: Let S = {Collection of all subsets from the matrix ring

$$M = \left\{\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \\ a_5 & a_6 \\ a_7 & a_8 \\ a_9 & a_{10} \\ a_{11} & a_{12} \end{bmatrix} \middle| \; a_i \in Q; 1 \leq i \leq 12\}\right\}$$

be the S-subset special super strong semivector space over the S-super subset semiring,

P = {Collection of all subsets from the ring Q}.

Let $T_S^o : S \to S$ defined by

$$T_S^o\left(\left\{\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \\ a_5 & a_6 \\ a_7 & a_8 \\ a_9 & a_{10} \\ a_{11} & a_{12} \end{bmatrix}\right\}\right) = \left\{\begin{bmatrix} a_1 & 0 \\ 0 & a_2 \\ a_3 & 0 \\ 0 & a_4 \\ a_5 & 0 \\ 0 & a_6 \end{bmatrix}\right\}; \text{ we see if}$$

$$A = \left\{\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \\ 7 & 8 \\ 9 & 1 \\ 0 & -2 \end{bmatrix}, \begin{bmatrix} 3 & 0 \\ 1 & 1 \\ -2 & 2 \\ 3 & 0 \\ -4 & 4 \\ 5 & 6 \end{bmatrix}, \begin{bmatrix} -7 & 6 \\ 6 & -1 \\ 1 & -1 \\ 2 & 3 \\ 3 & 6 \\ -1 & 8 \end{bmatrix}, \begin{bmatrix} -1 & 4 \\ 0 & 2 \\ 5 & 6 \\ 7 & -8 \\ 9 & -1 \\ 0 & -11 \end{bmatrix}\right\} \in S.$$

$$T_S^o(A) = T_S^o\left(\left\{\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \\ 7 & 8 \\ 9 & 1 \\ 0 & -2 \end{bmatrix}, \begin{bmatrix} 3 & 0 \\ 1 & 1 \\ -2 & 2 \\ 3 & 0 \\ -4 & 4 \\ 5 & 6 \end{bmatrix}, \begin{bmatrix} -7 & 6 \\ 6 & -1 \\ 1 & -1 \\ 2 & 3 \\ 3 & 6 \\ -1 & 8 \end{bmatrix}, \begin{bmatrix} -1 & 4 \\ 0 & 2 \\ 5 & 6 \\ 7 & -8 \\ 9 & -1 \\ 0 & -11 \end{bmatrix}\right\}\right)$$

$$= \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 3 \\ 5 & 0 \\ 0 & 8 \\ 9 & 0 \\ 0 & -2 \end{bmatrix}, \begin{bmatrix} 3 & 0 \\ 0 & 1 \\ -2 & 0 \\ 0 & 0 \\ -4 & 0 \\ 0 & 6 \end{bmatrix}, \begin{bmatrix} -7 & 0 \\ 0 & -1 \\ 1 & 0 \\ 0 & 3 \\ 3 & 0 \\ 0 & 8 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & 2 \\ 5 & 0 \\ 0 & -8 \\ 9 & 0 \\ 0 & -11 \end{bmatrix} \right\} \in S.$$

It is easily verified T_S^o is a S-special strong super subset semilinear operator on S.

Example 3.56: Let S ={Collection of all subsets from the matrix ring

$$M = \left\{ \left. \begin{bmatrix} a_1 & a_2 & \dots & a_{10} \\ a_{11} & a_{12} & \dots & a_{20} \\ a_{21} & a_{22} & \dots & a_{30} \end{bmatrix} \right| a_i \in Q; 1 \leq i \leq 30 \right\}\}$$

be the S-subset special super strong semivector space over the S-super subset semiring.

P = {Collection of all subsets from the ring Q}.

$T_S : S \to S$ can be defined by

$$T_S (\{ \begin{bmatrix} a_1 & a_2 & \dots & a_{10} \\ a_{11} & a_{12} & \dots & a_{20} \\ a_{21} & a_{22} & \dots & a_{30} \end{bmatrix} \})$$

$$= \{ \begin{bmatrix} a_1 & 0 & a_3 & 0 & a_5 & 0 & a_7 & 0 & a_9 & 0 \\ a_{11} & 0 & a_{13} & 0 & a_{15} & 0 & a_{17} & 0 & a_{19} & 0 \\ a_{21} & 0 & a_{23} & 0 & a_{25} & 0 & a_{27} & 0 & a_{29} & 0 \end{bmatrix} \}$$

T_S is a S-subset special super strong semilinear operator on S.

Example 3.57: Let S = {Collection of all subsets from the matrix

$$M = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ \vdots & \vdots & \vdots & \vdots \\ a_{61} & a_{62} & a_{63} & a_{64} \end{bmatrix} \middle| a_i \in \langle C \cup I \rangle; 1 \leq i \leq 64 \right\}\}$$

be the S-subset super special strong semivector space over the S super subset semiring;
P = {Collection of all subsets from the ring $\langle C \cup I \rangle$}.

We can have several S-subset special super strong semilinear operators on S.

Example 3.58: Let
S = {Collection of all subsets from the ring $Z_{45}S_7$} be the S-subset super special strong semivector space over the S-super subset semiring P. We see $o(S) < \infty$ hence the number of S-subset special super strong semilinear operators on S is finite in number.

Example 3.59: Let
S = {Collection of all subsets from the ring $\langle Z \cup I \rangle D_{2,7}$} be the S-subset special strong semivector space over the S-super subset semiring; P = {Collection of all subsets from the ring $\langle Z \cup I \rangle$}. S is also a S-subset super special strong semilinear algebra over P.

Example 3.60: Let S = {Collection of all subsets from the ring

$$M = \left\{ \left[\begin{array}{cc|ccc|c} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 & a_{10} & a_{11} & a_{12} \end{array} \right] \middle| a_i \in Z(S_4 \times D_{2,7}); 1 \leq i \leq 12 \right\}\}$$

be the S-subset super special strong semilinear algebra over the S-super subset semiring;
P = {Collection of all subsets from the ring Z}.

We can define $T_S^o : S \to S$ by

$$T_S^o\left\{\left(\left[\begin{array}{cc|ccc|c} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 & a_{10} & a_{11} & a_{12} \end{array}\right]\right)\right\}$$

$$= \left\{\left(\left[\begin{array}{cc|ccc|c} 0 & a_1 & a_2 & 0 & a_4 & a_5 \\ a_6 & 0 & 0 & a_7 & 0 & a_6 \end{array}\right]\right)\right\}$$

T_S^o is a S-subset special super strong semilinear operator from S to S.

Example 3.61: Let S = {Collection of all subsets from the ring $(\langle Z \cup I\rangle)\ [S_3 \times D_{2,7} \times A_6]\}$ be a S-special subset super strong semivector space over the S-super subset semiring P = {Collection of all subsets from the ring $\langle Z \cup I\rangle$}. We can find S-subset super special strong semilinear operators on S.

Example 3.62: Let S = {Collection of all subsets from the ring

$$M = \left\{\left[\begin{array}{ccc|cc|cccc} a_1 & a_2 & a_3 & & (0) & & & (0) & \\ a_4 & a_5 & a_6 & & & & & & \\ \hline & (0) & & a_7 & a_8 & & & (0) & \\ & & & a_9 & a_{10} & & & & \\ \hline & & & & & a_{11} & a_{12} & a_{13} & a_{14} \\ & (0) & & (0) & & a_{15} & a_{16} & a_{17} & a_{18} \\ & & & & & a_{19} & a_{20} & a_{21} & a_{22} \\ & & & & & a_{23} & a_{24} & a_{25} & a_{26} \end{array}\right]\right.$$

$a_i \in \langle Z \cup I\rangle\ (S_3 \times A_6)\}\}$

be a S-semigroup of the S-subset super special semilinear algebra over the S-subset super semiring;

P = {Collection of all subsets from the ring $\langle Z \cup I\rangle$}.

Find a few S-subset strong special semilinear operators on S.

Example 3.63: Let S = {Collection of all subsets from the matrix ring

$$M = \left\{\left.\begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \end{bmatrix}\right| a_i \in C(Z_7)S_5; 1 \leq i \leq 16\}\right\}$$

be a S-strong special super subset semilinear algebra of finite order over the S-complex subset semiring; P = {Collection of all subsets from the complex finite modulo integer ring $C(Z_7)$}. S has only a finite number of S-subset special strong semilinear operators.

Example 3.64: Let S = {Collection of all subsets from the ring $Z_{11}(g_1, g_2, g_3)$ where $g_1^2 = 0$, $g_2^2 = g_2$ and $g_3^2 = -g_3$; $g_ig_j = g_jg_i = 0$, $i \neq j$, $1 \leq i, j \leq 3$} be the S-special super strong subset semilinear algebra over the S-super subset semiring; P = {Collection of all subsets from the ring Z_{11}}. Clearly $o(S) < \infty$ and S is a commutative S-special strong subset semilinear algebra which is commutative.

Example 3.65: Let S = {Collection of all subsets from the ring $\langle Z_{42} \cup I\rangle$ $(S_7 \times A_{10})$} be the S-special super strong subset semilinear algebra over the S-super subset semiring; P = {Collection of all subsets from the neutrosophic ring $\langle Z_{42} \cup I\rangle$}. We see $o(S) < \infty$ but S is a non commutative semilinear semilinear algebra over P.

We have only finite number of S-subset special super strong semilinear operators on S.

Now having seen the usual subset substructures, subset operators and subset transformations on S we now proceed onto describe new types of substructures on S the S-strong special super subset semilinear algebras (semivector spaces).

Let S = {Collection of all subsets from the ring $\langle Z_{18} \cup I\rangle S_7$} be the S-strong special super subset semivector space over a S-super subset semiring
P = {Collection of all subsets from the ring $\langle Z_{18} \cup I\rangle$}.

Consider B = {Collection of all subsets from the subring $\langle Z_{18} \cup I\rangle A_4$} $\subseteq$ S, B is a S-special strong super subset semilinear subalgebra of S over P.

However B is also a S-special strong super subset semilinear algebra over
P_1 = {Collection of all subsets from Z_{18}} $\subseteq$ P, the S-super subset subsemiring of P.

We define B to be the quasi subsemiring S-special super strong subset semilinear subalgebra of S over the S-super subset subsemiring P_1 of P.

Infact S has several such quasi subsemiring, S-special super strong subset semilinear subalgebras of S over P_i a S-subset super subsemiring of P, $i < \infty$.

We will give more examples of the above described situation.

Example 3.66: Let
S = {Collection of all subsets from the ring $\langle (Z_{24}) \cup I\rangle S_5$} be the S-strong special super subset semilinear algebra over the S-super subset semiring
P = {Collection of all subsets from the ring $C(Z_{24})$}.

Now consider M_1 = {Collection of all subsets from the subring $\langle C(Z_{24}) \cup I\rangle A_5$} be the S-strong special super subset semilinear algebra over the S-super subset subsemiring. P_1 = {Collection of all subsets from the subring $C(Z_{24})$} we call M_1 the quasi subsemiring S-special strong subset semilinear subalgebra of S over the S-subset subsemiring P_1.

We can have several such S-strong super subset subsemilinear subalgebras over different super S-subset subsemirings.

Example 3.67: Let
S = {Collection of all subsets from the ring $C(Z_{12})$ $(S_7 \times D_{2,4})$} be the quasi super subset S-strong super special semilinear algebra over the super S-subset semiring; P = {Collection of all subsets of from the semiring $C(Z_{12})$}.

Let M_1 = {Collection of all subsets from the subring $C(Z_{12})$ $(S_7 \times \{1\})$} be the S-subsemiring quasi super subset special strong subsemilinear algebra over the super S-subset subsemiring
P_1 = {Collection of all subsets from the subsemiring Z_{12}}.

Let M_2 = {Collection of all subsets from the subring $C(Z_{12})(\{1\} \times D_{2,4})$} be the quasi super subset S-strong special semilinear subalgebra over the super subset S-strong special semilinear subalgebra over the S-super subset subsemiring P_2 = {Collection of all subsets from the subring $\{2, 0, 4, 6, 8, 10\} \subseteq Z_{12}$}.

We have more such quasi super subset S-strong special semilinear subalgebras over S-super subset subsemirings.

Example 3.68: Let S = {Collection of all subsets from the ring $\langle Q \cup I\rangle$ $(S_3 \times D_{2,5})$} be the quasi S-special strong super subset semilinear algebra over the S-super subset semiring P = {Collection of all subsets from the ring $\langle Q \cup I\rangle$}.

Consider S_1 = {Collection of all subsets from the subring $\langle Q \cup I \rangle$ ($S_3 \times \{1\}$)} be the subsemiring S-super special strong subset semilinear subalgebra of S over the S-super subset subsemiring, P_1 = {Collection of all subsets from the subring $\langle Z \cup I \rangle$}. Consider S_2 = {Collection of all subsets from the subring $\langle Q \cup I \rangle(\{1\} \times D_{2,5})$} be the subsemiring S-quasi special strong super subset semilinear subalgebra over the S-super subset semiring P_2 = {Collection of all subsets from the subring Z} and so on.

Infact S has infinite number of S-quasi special strong super subset semilinear subalgebra over the S-super subset subsemiring.

Example 3.69: Let
S = {Collection of all subsets from the ring $Z_{11}S_3$} be the S-quasi special strong subset semilinear algebra over the S-super subset semiring, P = {Collection of all subsets from Z_{11}}. We see Z_{11} has no proper subrings but P has proper subsets A which is a ring that is A = {{0}, {1}, {2}, …, {10}} $\subseteq$ P is a ring called a subset ring. Now any appropriate subset of S can also be a S-super special strong subset semivector space over A. We call that subsemivector space as subset ring quasi S-super strong special semivector subspace of S over the subset ring in P. Take W = {Collection of all subsets from the subring $Z_{11}A_3$} $\subseteq$ S. W is a subset ring quasi S-super strong special semivector subspace of S over A.

Example 3.70: Let S = {Collection of all subsets from the ring $Z_{19}G$ where G = {g | g^5 = 1} be the strong special super subset semivector space over the S-super subset semiring P = {Collection of all subsets from the field Z_{19}}. Clearly Z_{19} has no proper subring but A = {{0}, {1}, …, {18}} $\subseteq$ P is a S-subset super subsemiring which is a subset ring of P.

Take M = {n (1 + g + g_2 + g_4) | n = 0, 1, 2, …, 18} $\subseteq$ S; a subset semivector space over A which we call as quasi subset semiring S-super subset semivector subspace of S over A.

Now we proceed onto describe some results from these observations.

THEOREM 3.2: *Let S = {Collection of all subsets from the group ring Z_pG; $|G| < \infty$} be the S-special super strong subset semivector space over the S-super subset semiring P = {Collection of all subsets from Z_p}; Z_p has no subrings (as p is a prime).*
So

- *(i) S has quasi subset subsemiring S-special super strong semivector subspace of S over the S-super subset subsemiring A = {{0}, {1}, …, {p–1}} $\subseteq$ P.*
- *(ii) If |G| = q, q a prime than we see T = {{n (1 + g + …+ g^{q-1}) | n = 0, 1, 2, …, p–1}} $\subseteq$ S is the quasi subset subsemiring S-super strong special semivector subspace of S over A.*

The proof is direct and hence left as an exercise to the reader.

It is pertinent to keep on record that if Z_p is replaced by Z_n in the theorem; n a composite number we have subrings of Z_n giving way to more and more subspaces.

Likewise if Z_p is replaced by the ring $\langle Z_n \cup I\rangle$ or $\langle Z_p \cup I\rangle$ or $C(Z_p)$ or $\langle Z \cup I\rangle$, $\langle Q \cup I\rangle$ or Z or Q or R or $C(Z_n)$ or $C(\langle Z_p \cup I\rangle)$ or $C(\langle Z_n \cup I\rangle)$; we have more number of subset subsemiring semivector subspaces.

Example 3.71: Let S = {Collection of all subsets from the ring $Z_{19}G$ and G = S_7} be the S-subset super strong special semivector space over the S-super subset semiring P = {Collection of all subsets from Z_{19}}.

Now A = {{0}, {1}, …, {18}} $\subseteq$ S is a S-subset super subsemiring of P.

B = {{0}, {0, 1, 2, …, 19}} $\subseteq$ A is also a S-subset super subsemiring of P.

Thus we have several types of subset subsemivector spaces (subset semivector subspaces). These concepts are interesting and infact the notion of subset semivector spaces always contains an isomorphic copy of semivector spaces, so we see these new concepts are the more generalized one for all other types of subset semivector spaces find an isomorphic copy of this structure.

Thus the study of these notions is not only innovative and interesting but are very significant.

Indue course of time certainly these new structures will find several applications. However as claimed these subset semivector spaces (subset semilinear algebras) contain an isomorphic copy of the basic semivector space (semilinear algebra) we can rightfully claim all the applications of semivector spaces can also be extended in case of the subset semivector spaces, S-subset semivector spaces, S-subset special semivector spaces and S-subset special strong semivector spaces.

The analysis and further applications would soon be found as these algebraic subset semivector spaces become more familiar with researchers.

Finally as necessarily one needs the concept of lattices to build finite subset semivector spaces one can easily claim that applications of lattices can also be extended to these new structures.

Finally we use matrices, polynomials in these constructions so these structures can also imbibe the applications of them in these subset semivector spaces.

Further we see these structure can be non commutative as subset semilinear algebras and the product of two subsets happens to be non commutative if the basic structures used in building them happens to be non commutative.

We suggest the following problems.

Problems:

1. Find all special features enjoyed by the S-special strong subset semivector spaces which are not S-special subset semivector spaces defined over S-subset semirings.

2. Let S = {Collection of all subsets from the ring ZS_7} be the S-special strong subset semivector spaces defined over P = {Collection of all subsets from the ring Z} the S-subset semiring.

   (i) Find all S-special strong subset semivector spaces.
   (ii) Prove S is non commutative.
   (iii) If we define a product on S prove S is a S-special subset strong semilinear algebra.
   (iv) Find a S-subset basis of S.
   (v) Find a set of subset elements in S which are S-subset linearly independent.

3. Let S = {Collection of all subsets from the ring $Z(g)S_7$ with $g^2 = 0$} be the special subset strong semivector space over P, S-subset semiring.

   Study questions (i) to (v) of problem 2 for this S.

4. Let S = {Collection of all subsets from the ring $C(Z_{12})S_7$} be the S-subset super special strong semivector space over the S-subset semiring P = {Collection of all subsets from the complex modulo integer ring $C(Z_{12})$}.

   (i) Find o(S).
   (ii) Find all S-subset strong special semivector subspaces of S over P.
   (iii) Prove S is a S-subset strong special semilinear algebra over P which is non commutative.
   (iv) Find all S-subset special strong semilinear operators on S.

(v) Do the collection of (iv) have any algebraic structure?

(vi) If $C(Z_{12})$ in P is replaced by Z_{12} study questions (i) to (v).

5. Let S = {Collection of all subsets from the ring $C(Z_6)S_3 \times C(Z_{15})A_4$}be the S-subset special super strong semivector space over the S-subset semiring P = {Collection of all subsets from the ring $C(Z_6) \times C(Z_{15})$}.

(i) Study questions (i) to (v) of problem 4 for this S.

(ii) If in this P; $C(Z_6) \times C(Z_{15})$ is replaced by $Z_6 \times Z_{15}$ or $C(Z_6) \times Z_{15}$ or $Z_6 \times C(Z_{15})$, study questions (i) to (v) of problem 4 for this S.

6. Let S = {Collection of all subsets from the matrix ring

$$M = \left\{ \begin{bmatrix} a_1 & a_2 & \ldots & a_6 \\ a_7 & a_8 & \ldots & a_{12} \\ a_{13} & a_{14} & \ldots & a_{18} \end{bmatrix} \middle| a_i \in C(Z_{11}), 1 \leq i \leq 18 \right\}\}$$

be the S-subset super special strong semivector space over P = Collection of all subsets from the complex modulo integer modulo integer ring $C(Z_{11})$}.

(i) Study questions (i) to (v) of problem 4 for this S.

(ii) Write S as a direct sum of subspaces.

7. Does there exist a S-subset special strong semivector space with finite number of elements which cannot be written as a direct sum of subspaces?

8. Is it possible to write every S-subset special strong semivector space of infinite cardinality as a direct sum of subspaces?

9. Let S = {Collection of all subsets from the ring

$$B = \left\{ \left[\begin{array}{cc|c|c|cccc} a_1 & a_2 & (0) & (0) & & (0) & & \\ a_3 & a_4 & & & & & & \\ \hline (0) & & (0) & (0) & & (0) & & \\ & & & & & & & \\ \hline (0) & & (0) & a_{11} & & (0) & & \\ \hline & & & & a_{12} & a_{13} & a_{14} & a_{15} \\ (0) & & (0) & (0) & a_{16} & a_{17} & a_{18} & a_{19} \\ & & & & a_{20} & a_{21} & a_{22} & a_{23} \end{array} \right] \right|$$

$a_i \in C(Z_5)S(3)$; $1 \leq i \leq 23$}} be the S-special strong subset semivector space over P = {Collection of all subsets from the complex modulo integer ring $C(Z_5)$?

Study questions (i) to (v) of problem 4 for this S.

10. Let S be a S-strong special subset semilinear algebra over a S-subset semiring P of finite order. Find the algebraic structure enjoyed by the collection of all S-special strong subset semilinear algebra over a S-subset semiring P of finite order.

Find the algebraic structure enjoyed by the collection of all S-special strong subset semilinear operators on S.

11. Find any of the special and distinct features enjoyed by S-strong special subset semilinear algebras which are non commutative and of finite order.

12. Let S = {Collection of all subsets from the ring M =

$$\left\{\left.\begin{bmatrix} a_1 \\ \hline a_2 \\ a_3 \\ \hline a_4 \\ a_5 \\ \hline a_6 \\ a_7 \\ a_8 \end{bmatrix}\right| a_i \in C(Z_{15})\ (S_7 \times S(5));\ 1 \leq i \leq 8\right\}\}$$

be the S-subset special super strong semivector space over the S-subset semiring
P = {Collection of all subsets from the ring $C(Z_{15})$}.

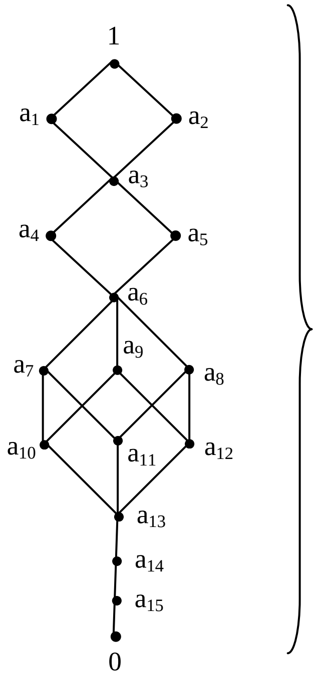
(i) Study questions (i) to (v) of problem 4 for this S.

(ii) Find $T_S^o : S \to S$ such that $\ker(T_S^o) \neq \{0\} = \left\{\begin{bmatrix} 0 \\ \hline 0 \\ 0 \\ \hline 0 \\ 0 \\ \hline 0 \\ 0 \\ 0 \end{bmatrix}\right\}$.

13. Let S = {Collection of all subsets from the ring M = {(a_1 a_2 | a_3 | a_4 a_5) | $a_i \in C(Z_9) \times C(Z_{11}) \times C(Z_{43})$; $1 \leq i \leq 5$} be the S-subset special strong semilinear algebra over the S-subset semiring; P = {Collection of all subsets from the ring $C(Z_9) \times C(Z_{11}) \times C(Z_{43})$}.

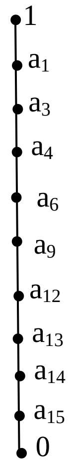
Study questions (i) to (v) of problem 4 for this S.

14. Let S = {Collection of all subsets from the group lattice LG = LS(4) where L is a lattice given below



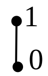
(i) Study questions (i) to (v) of problem 4 for this S.

(ii) If $L_1 \subseteq l$ where L_1 is a sublattice given by



(iii) Study questions (i) to (v) of problem 4 for this S if L_1 is taken instead of L.

(iv) If L in S is replaced by



Study questions (i) to (v) of problem 4 for this S.

15. Let S = {Collection of all subsets from the ring

$$M = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \\ a_{17} & a_{18} & a_{19} & a_{20} \\ a_{21} & a_{22} & a_{23} & a_{24} \end{bmatrix} \middle| \; a_i \in Z_{15}(S_7 \times S(3));\ 1 \leq i \leq 24 \right\}\}$$

be a S-subset special super strong semilinear algebra over the S-subset semiring
P = {Collection of all subsets from Z_{15}}.

Study questions (i) to (v) of problem 4 for this S

16. If in the above problem Z_{15} is replaced by $C(Z_{15})$.

Study questions (i) to (v) of problem 4 for this S.

17. Let S = {Collection of all subsets from the ring RS_7} be the S-special subset super strong semilinear algebra over the S-subset semiring
P = {Collection of all subsets from the ring R}.

(i) Find at least four distinct S-special strong super subset semivector subspaces over P.
(ii) Can S be written n direct sum of S-special super strong subset semivector subspaces over P ($n < \infty$)?

(iii) Find T_S^o : S → S, a S-subset strong super special semilinear operator whose null space is non zero.

(iv) Let V_s = {Collection of S-strong subset super special semilinear operators of S};
Study the algebraic structure enjoyed by V_s.

(v) Does there exist a S-special super strong subset semivector subspace over P, which is not a S-strong special super subset semilinear subalgebra over P.

18. Let S = {Collection of all subsets from the ring

$$M = \left\{ \left[\begin{array}{c|c|c|c} a_1 & a_2 & \cdots & a_{10} \\ a_{11} & a_{12} & \cdots & a_{20} \end{array} \right] \middle| a_i \in \langle C \cup I \rangle; 1 \leq i \leq 20 \right\}\}$$

be the S-subset strong super special semilinear algebra over the S-super subset semiring
P = {Collection of all subsets from the ring ⟨C ∪ I ⟩}.

Study questions (i) to (v) of problem 17 for this S.

19. Let S = {Collection of all subsets from the matrix ring

$$M = \left\{ \left[\begin{array}{ccc} a_1 & a_2 & a_3 \\ \hline a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ \hline a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & a_{18} \\ a_{19} & a_{20} & a_{21} \\ \hline a_{22} & a_{23} & a_{24} \end{array} \right] \middle| a_i \in \langle R \cup I \rangle; 1 \leq i \leq 24 \right\}\}$$

be a S-subset super strong special semivector space (semilinear algebra) over the S-super subset semiring
P = {Collection of all subsets from the ring ⟨R ∪ I⟩}.

Study questions (i) to (v) of problem 17 for this S.

20. Let S = {Collection of all subsets from the matrix ring

$$M = \left\{ \left. \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_{20} \end{bmatrix} \right| a_i \in \langle C \cup I \rangle (A_4 \times S(7));\ 1 \leq i \leq 20 \right\} \}$$

be the S-subset strong super special semilinear algebra over S.

Study questions (i) to (v) of problem 17 for this S.

21. Let S = {Collection of all subsets from the matrix ring

$$M = \left\{ \left. \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \end{bmatrix} \right| a_i \in \langle Z_{12} \cup I \rangle (S_3 \times D_{2,7} \times S(5));\right.$$

$1 \leq i \leq 12\}\}$ be the S-subset strong super special semilinear algebra over the S-super subset semiring.
P = {Collection of all subsets from $Z_{12} \cup I$}.

Study questions (i) to (v) of problem 17 for this S.

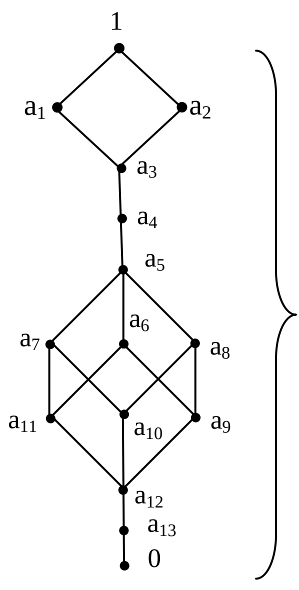
22. Let S = {Collection of all subsets from the matrix ring

$$M = \left\{ \left. \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \end{bmatrix} \right| a_i \in R(g_1, g_2, g_3, g_4) \right.$$

where $g_1^2 = 0$, $g_2^2 = 0$, $g_3^2 = g_3$ and $g_4^2 = -g_4$ with $g_ig_j = g_jg_i = 0$ if $i \neq j$, $1 \leq i, j \leq 4\}\}$ be the S-subset super special semivector space over the S-super subset semiring
P = {Collection of all subsets from the ring R}.

(i) Study questions (i) to (v) of problem 17 for this S.
(ii) Does S contain as subset super special strong semi vector subspaces W_i such that we have a W_j with $W_i^{\perp} = W_j$ and $W_j^{\perp} = W_i$ $(i \neq j)$ and $W_i + W_j = S$?

23. Let S = {Collection of all subsets from the group lattice LG where $G = S_3 \times D_{27}$ and L is as follows:



be a S-subset semivector space over the S-subset semiring
P = {Collection of all subsets from the lattice L}

(i) Find o(S).
(ii) Find all S-subset semivector subspaces of S over P.

(iii) Does S contain a S-subset semivector subspace W such that there exists a $W^{\perp}$ with $W + W^{\perp} = S$?

(iv) Find a S-subset basis of S.

(v) Find a set of subset linearly dependent elements.

(vi) Find $o(V_S^o)$ where V_S^o = {Collection of all S-subset semilinear operators on S}.

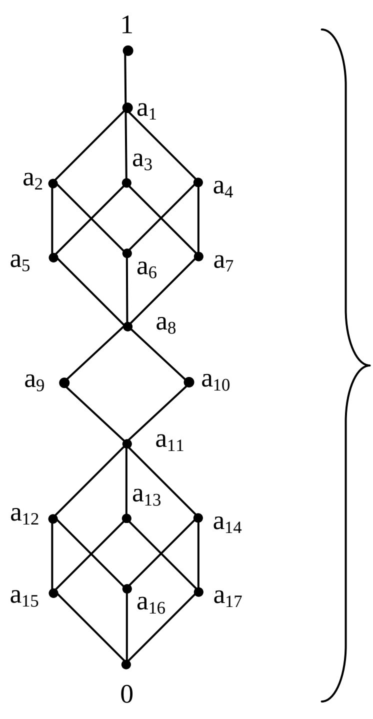
(vii) What is the algebraic structure enjoyed by V_S^o.

(viii) Prove S is non commutative as a S-subset semilinear algebra.

(ix) Can S have a S-subset semivector space which is not a S-subset semilinear algebra over P?

(x) Can S be written as n-direct sum of subspaces? What is the bound on $n < \infty$?

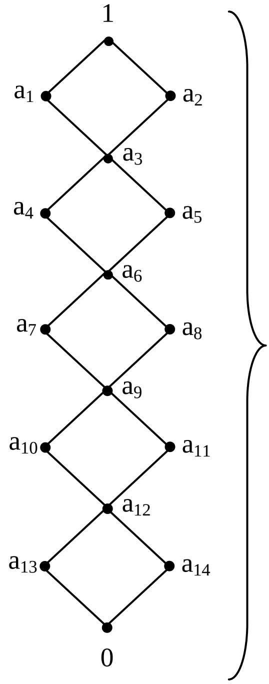
24. Let S = {Collection of all subsets from the semiring



be the S-subset semivector space over the S-subset semiring P = {Collection of all subsets from the lattice (semiring) L}.

Study questions (i) to (x) of problem 23 for this S.

25. Let S = {Collection of all subsets from the group lattice LG where L =



be the S-subset semivector space over the S-subset semiring
P = {Collection of all subsets from the lattice L}.

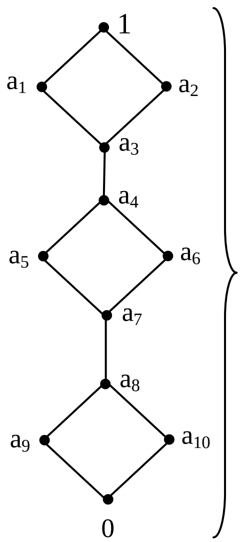
Study questions (i) to (x) of problem 23 for this S.

26. Let S = {Collection of all subsets from the semiring $L(S_3 \times D_{25})$ where L is a Boolean algebra of order 32} be the S-subset semivector space over the S-subset semiring

P = {Collection of all subsets from the Boolean algebra L of order 32}.

Study questions (i) to (x) of problem 23 for this S.

27. Let S = {Collection of all subsets from the semigroup lattice LS(4) where L is the lattice given in the following.

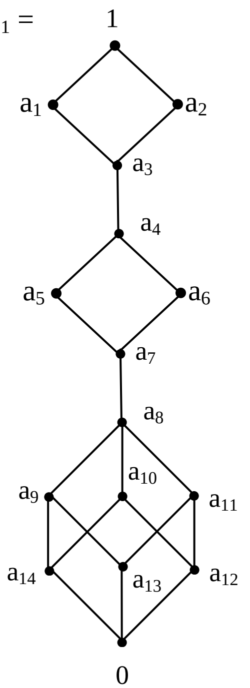


be the S-subset semivector space over the S-subset semiring
P = {Collection of all subsets from the lattice L}.

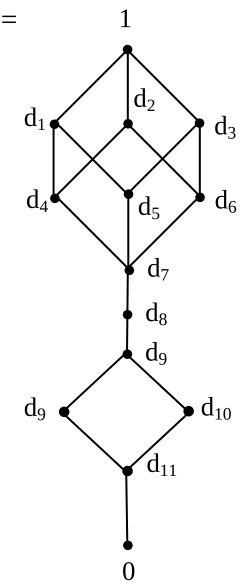
Study questions (i) to (x) of problem 23 for this S.

28. Let S = {Collection of all subsets from the group lattice $(L_1 \times L_2)$ $S_3 \times A_7$ where

L_1 =



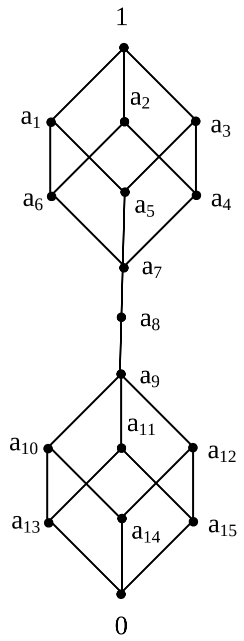
and L_2 =



be the S-subset semivector space over the S-subset semiring
P = {Collection of all subsets from the semiring $L_1 \times L_2$}.

Study questions (i) to (x) of problem 23 for this S.

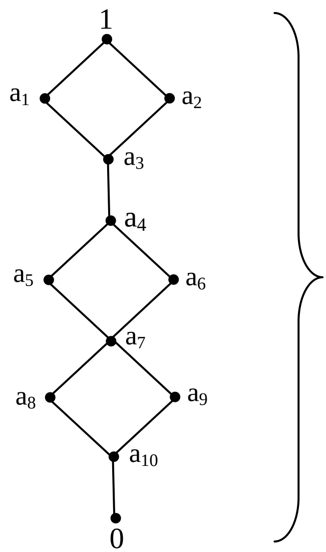
29. Let S = {Collection of all subsets from the semiring $L(S_7 \times D_{2,5})$ where L is a lattice given in the following L =



be the S-subset semivector space over the S-subset semiring.
P = {Collection of all subsets from the semiring L} be the S-subset semivector space over the S-subset semiring P.

Study questions (i) to (x) of problem 23 for this S.

30. Let S = {Collection of all subsets from the semiring LS(7) where L =



be the S-subset semivector space over the S-subset semiring
P = {Collection of all subsets from the lattice L}.

Study questions (i) to (x) of problem 23 for this S.

31. Let S = {Collection of all subsets from the semiring ($L_1 \times L_2$) ($S_3 \times A_4 \times D_{2,7}$)} be the S-subset semivector space over the S-subset semiring
P = {Collection of all subsets from the semiring $L_1 \times L_2$}.

Study questions (i) to (x) of problem 23 for this S.

32. Let A = {Collection of all subsets of the ring Z_p} (p a prime) be the S-super subset semiring.

(i) Find all S-super subset subsemiring of A.
(ii) Can A have more than two subset subsemiring?

33. Let S = {Collection of all subsets of the ring Z_{24}} be S-super subset semiring.

(i) Find all S-super subset subsemirings of S.

34. Let S = {Collection of all subsets from the ring $C(Z_{36})$} be the S-super subset semiring.

(i) Find all S-subset super subset subsemirings.

(ii) If n is the number of S-subset super subset subsemirings and m = number of S-subset super subsemirings of the S-subset super semiring using Z_{36}. Compare them.

(iii) Is n > m?

35. Let S = {Collection of all subsets from the ring $\langle Z_{18} \cup I\rangle$} be the S-subset super semiring.

Study questions (i) to (iii) of problem 34 for this S.

If $\langle Z_{18} \cup I\rangle$ is replaced by $C\langle Z_{18} \cup I\rangle$.

Study questions (i) to (iii) of problem 34 for this S.

36. Let
S = {Collection of all subsets from the ring $C(\langle Z_{23} \cup I\rangle)$} be the S-subset super semiring.

Study questions (i) to (iii) of problem 34 for this S.

37. Let S = {Collection of all subsets from the ring $C(\langle Z_{24} \cup I\rangle)$ (g_1g_2) | $g_1^2 = g_1$, $g_2^2 = g_2$, $g_1g_2 = g_2g_1 = 0$} be the S- super subset semiring.

Study questions (i) to (iii) of problem 34 for this S.

Prove $C(\langle Z_{24} \cup I\rangle)$ (g_1g_2) has more number of subrings than $C(\langle Z_{24} \cup I\rangle)$, Z_{24}, $C(Z_4)$, $\langle Z_{24} \cup I\rangle$.

38. Let S = {Collection of all subsets of the ring $C(\langle Z_{24} \cup I\rangle)$ $S_{20} \times D_{2,13}$} be the S-subset super semiring.

    (i) Study questions (i) to (iii) of problem 34 for this S.

    (ii) Prove T = $C(\langle Z_4 \cup I\rangle)\ S_{20} \times D_{2,13}$ has more number subring and this ring T is non commutative.

39. Find some applications of S-strong super special subset semilinear algebras which are non commutative.

40. Let S = {Collection of all subsets from the ring $C(Z_6)S_{10}$} be the S-special super strong subset semilinear algebra over the S-super subset semiring P = {Collection of all subsets from the ring $C(Z_6)$}.

    (i) Find a subset basis of S.

    (ii) Find o(S).

    (iii) Prove S is non commutative.

    (iv) Can S have more than one subset basis?

    (v) Find all subset subsemirings S-special super strong subset semilinear algebra over the S-subset super subsemiring in P.

41. Let S = {Collection of all subsets from the ring ZS_7} be the S-subset strong super special semivector space over the S-super subset semiring
    P = {Collection of all subsets from the ring Z}.

    Study questions (i) to (v) of problem 40 for this S.

42. Let S = {Collection of all subsets from the ring $(Z_7 \times Z_{31})\ (S_7 \times D_{2,5})$} be the S-subset strong super special semivector space (semilinear algebra over the S-super subset semiring

P = {Collection of all subsets from the ring
$(Z_7 \times Z_{31})\, S_7 \times Z_{2,5}$}.

(i) Study questions (i) to (v) of problem 40 for this S.

(ii) Find all S-subset super subsemirings in P.

43. Let S = {Collection of all subsets from the ring $\langle C \cup I\rangle S_{10}$} be the S-super strong special semilinear algebra over the S-super subset semiring
P = {Collection of all subsets from the ring $\langle C \cup I\rangle$}.

(i) Study questions (i) to (v) of problem 40 for this S.

(ii) Prove P has infinite number of S-super subset subsemiring.

(iii) Related to each of the S-super subset subsemiring find the subsemiring quasi S-super special strong semilinear subalgebra of S.

44. Give an example of a subset semilinear algebra with more than one subset basis.

45. Can any S-subset semilinear algebra have infinite number of subset basis?

46. Does there exists a S-special super strong subset semilinear algebra defined over a S-super subset semiring which has more than one subset basis.

Chapter Four

PROPERTIES OF SUBSET SEMILINEAR ALGEBRAS

In this chapter we just study some of the properties of subset semilinear algebras. We have introduced, developed and studied several types of subset semilinear algebras in chapter II and III of this book. We study more about their properties in this chapter.

We know if S = {Collection of subsets of a semigroup P} and F any semifield such that S is a semivector space over F then we call S to be a subset semivector space over the semifield F.

For more please refer chapter two of this book.

We have defined two subsets A and B are orthogonal if $A \times B = \{0\}$. For instance if $A = \{[a_1, 0], [a_2, 0], [a_3, 0], [a_4, 0], [a_5, 0]\}$ and $B = \{[0, b_1], [0, b_2], [0, b_3], [0, b_4], [0, b_5]\} \in S$; we see $A \times B = \{[0, 0]\}$.

Let A = {(a_1, 0, 0, 0, 0), (a_2, a_3, 0, 0, 0), (0, a_4, 0, 0, 0)} and B = {(0, 0, a_1, a_2, a_3), (0, 0, a_5, 0, 0), (0, 0, 0, 0, a_3), (0, 0, 0, a_4, 0), (0, 0, a_1, a_2, 0), (0, 0, 0, a_1, a_5), (0, 0, a_2, 0, a_3)} $\in$ S.

We see A $\times$ B = {(0, 0, 0, 0, 0)}.

We have used this concept for orthogonality of two subsets in S, S a subset semivector space over a subset semiring.

However we want to define the notion of subset semilinear product on a subset semivector space.

DEFINITION 4.1: *Let S = {Collection of all subsets form the semiring P} be a subset semivector space over the semifield F. We define sum in a subset A (sum of a subset A) in S to be sum of the elements in A (as A contains only elements under '+' the operation on the semigroup P used to build S).*

The sum in A is denoted $A_s = \sum a_i$, $a_i \in A$ is a singleton set in S.

We will illustrate this situation by the following examples.

Example 4.1: Let S = {Collection of all subsets from the semigroup $3Z^+ \cup \{0\}$ under '+'} be the subset semivector space over the semifield T = $Z^+ \cup \{0\}$.

Let A = {3, 6, 18, 27, 45, 90, 0} $\in$ S.

Now A_S = {3 + 6 + 18 + 27 + 45 + 90 + 0} = {189} $\in$ S is the sum of A.

Let A = {3, 24, 18, 15} $\in$ S.
A_S = {3 + 24 + 18 + 15} = {60} $\in$ S.

Example 4.2: Let S = {Collection of all subsets from the semigroup P = {(a_1, a_2) | a_1, a_2 $\in$ $Z^+ \cup \{0\}$}} be the subset semivector space over the semifield F = $Z^+ \cup \{0\}$.

Let A $= \{(3, 2), (6, 0), (7, 9), (1, 2), (5, 5), (2, 4)\} \in S$.
A_S $= \{(3, 2) + (6, 0) + (7, 9) + (1, 2) + (5, 5) + (2, 4)\}$
$= \{(24, 22)\} \in S$ is the sum of the subset A.

Let B $= \{(0, 9), (2, 0), (6, 0), (7, 0), (0, 0), (0, 2)\}$
B_S $= \{(0, 9) + (2, 0) + (6, 0) + (7, 0) + (0, 0) + (0, 2)\}$
$= \{(15, 11)\} \in S$ is the sum of the subset B.

A_S and B_S are the sum of the subset A and B respectively.

Example 4.3: Let S = {Collection of all subsets from the column matrix semigroup

$$P = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix} \middle| \; a_i \in Q^+ \cup \{0\}; 1 \leq i \leq 5\} \right\}$$

be the subset semivector space over the semifield $F = Q^+ \cup \{0\}$.

$$\text{Take } A = \left\{ \begin{bmatrix} 3 \\ 0 \\ 2 \\ 1 \\ 5 \end{bmatrix}, \begin{bmatrix} 4 \\ 6 \\ 5 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 8 \\ 8 \\ 2 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 6 \\ 0 \\ 6 \\ 0 \\ 6 \end{bmatrix}, \begin{bmatrix} 7 \\ 1 \\ 7 \\ 1 \\ 7 \end{bmatrix} \right\} \in S.$$

$$\text{We find } A_s = \left\{ \begin{bmatrix} 3 \\ 0 \\ 2 \\ 1 \\ 5 \end{bmatrix} + \begin{bmatrix} 4 \\ 6 \\ 5 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 8 \\ 8 \\ 2 \\ 2 \\ 0 \end{bmatrix} + \begin{bmatrix} 6 \\ 0 \\ 6 \\ 0 \\ 6 \end{bmatrix} + \begin{bmatrix} 7 \\ 1 \\ 7 \\ 1 \\ 7 \end{bmatrix} \right\}$$

$$= \left\{ \begin{bmatrix} 28 \\ 15 \\ 22 \\ 4 \\ 19 \end{bmatrix} \right\} \in S \text{ is the sum of the set A.}$$

Example 4.4: Let S = {Collection of all subsets from the semigroup of 3×4 matrices

$$M = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \end{bmatrix} \middle| \ a_i \in R^+ \cup \{0\};\ 1 \leq i \leq 12 \right\}\}$$

be the subset semivector space over the semifield F = $R^+ \cup \{0\}$.

Let

$$A = \left\{ \begin{bmatrix} 5 & 0 & 2 & 7/4 \\ \sqrt{3} & 0 & 0 & 0 \\ 0 & \sqrt{7} & 2 & \sqrt{5} \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & 5 \\ 2 & 0 & 1 & 0 \\ 0 & 9 & 0 & \sqrt{5} \end{bmatrix}, \begin{bmatrix} 3 & 2 & 0 & 0 \\ \sqrt{3} & 0 & 0 & \sqrt{2} \\ 0 & \sqrt{3} & \sqrt{2} & 0 \end{bmatrix} \right\}$$

$$A_S = \left\{ \begin{bmatrix} 5 & 0 & 2 & 7/4 \\ \sqrt{3} & 0 & 0 & 0 \\ 0 & \sqrt{7} & 2 & \sqrt{5} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 5 \\ 2 & 0 & 1 & 0 \\ 0 & 9 & 0 & \sqrt{5} \end{bmatrix} + \begin{bmatrix} 3 & 2 & 0 & 0 \\ \sqrt{3} & 0 & 0 & \sqrt{2} \\ 0 & \sqrt{3} & \sqrt{2} & 0 \end{bmatrix} \right\}$$

$$= \left\{ \begin{bmatrix} 8 & 2 & 2 & 27/4 \\ 2\sqrt{3}+2 & 0 & 1 & \sqrt{2} \\ 0 & \sqrt{3}+\sqrt{7}+9 & 2+\sqrt{2} & 2\sqrt{5} \end{bmatrix} \right\} \in S$$

is the sum in the subset A or sum of the subset A.

We use this concept in defining the subset semiinner product.

DEFINITION 4.2: *Let F be a semifield. S a subset semivector space over the semifield F.*

A subset semiinner product on S is a function which assigns to each ordered pair of subset vectors A, B $\in$ S; a scalar $(A \mid B)_s$ in F in such a way;

(i) $(A \mid B)_s = (A_S \mid B_S)_s = (\{A_S \times B_S\})_s$
(ii) $(A \mid A)_s > 0$ *if* $A \neq \{0\}$;

(where A_S and B_S are the subset sums of the subset A and B of S respectively).

$(A \mid B)_s = \sum a_i \times b_i \ (a_i \in A_S, b_i \in B_S)$.

Then we define $(\ \mid\ )_S$ is the subset semiinner product on S.

We will first illustrate this situation by some examples.

Example 4.5: Let S = {Collection of all subsets from the semigroup P = $\{(a_1, a_2, a_3)$ where $a_i \in Z^+ \cup \{0\}; 1 \leq i \leq 3\}\}$ be the subset semiring.

Let A, B $\in$ S where A = {(3, 0, 1), (5, 5, 3), (0, 0, 6), (0, 6, 2), (7, 7, 2)} and B = {(4, 0, 4), (2, 2, 2), (7, 7, 1), (1, 1, 1)} $\in$ S.

$$A_S = \{(3, 0, 1) + (5, 5, 3) + (0, 0, 6) + (0, 6, 2) + (7, 7, 2)\} = \{(15, 18, 14)\} \text{ and}$$

$$B_S = \{(4, 0, 4) + (2, 2, 2) + (7, 7, 1) + (1, 1, 1)\} = \{(14, 10, 8)\} \in S.$$

$$(A \mid B)_s = (A_S \mid B_S)_s = \sum_{i=1}^{3} a_i b_i$$
$$= ((15, 18, 14) \mid (14, 10, 8))$$
$$= 15 \times 14 + 18 \times 10 + 14 \times 8$$
$$= 210 + 180 + 112$$
$$= 402 \in Z^+ \cup \{0\}.$$

This is the way subset semiinner product is defined on S.

Example 4.6: Let S = {Collection of all subsets from the semigroup

$$M = \left\{\begin{pmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & a_7 & a_8 & a_9 & a_{10} \end{pmatrix} \middle| a_i \in Z^+ \cup \{0\}; 1 \le i \le 10\}\right\}$$

be the subset semivector space over the semifield F.

Let $(\mid)_s$ be the subset semiinner product, that is

$$(A \mid B)_s : S \to Z^+ \cup \{0\}.$$

Let

$$A = \left\{\begin{bmatrix} 3 & 0 & 1 & 0 & 1 \\ 0 & 2 & 0 & 4 & 0 \end{bmatrix}, \begin{bmatrix} 2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 1 & 1 & 0 & 1 \\ 2 & 2 & 2 & 0 & 2 \end{bmatrix}\right\}$$

and

$$B = \left\{\begin{bmatrix} 1 & 5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 & 2 \end{bmatrix},\right.$$

$$\left.\begin{bmatrix} 0 & 0 & 0 & 3 & 1 \\ 3 & 1 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 2 & 5 & 0 & 0 \\ 5 & 1 & 0 & 0 & 0 \end{bmatrix}\right\}$$

$$A_S = \left\{\begin{bmatrix} 3 & 0 & 1 & 0 & 1 \\ 0 & 2 & 0 & 4 & 0 \end{bmatrix} + \begin{bmatrix} 2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 1 & 1 & 0 & 1 \\ 2 & 2 & 2 & 0 & 2 \end{bmatrix}\right\}$$

$$= \left\{\begin{bmatrix} 6 & 2 & 2 & 0 & 2 \\ 2 & 4 & 2 & 5 & 4 \end{bmatrix}\right\} \in S.$$

and

$$B_S = \left\{\begin{bmatrix}1 & 5 & 0 & 0 & 0\\ 0 & 0 & 0 & 5 & 1\end{bmatrix} + \begin{bmatrix}1 & 1 & 1 & 1 & 1\\ 2 & 2 & 2 & 2 & 2\end{bmatrix} + \right.$$

$$\left. \begin{bmatrix}0 & 0 & 0 & 3 & 1\\ 3 & 1 & 0 & 0 & 0\end{bmatrix} + \begin{bmatrix}1 & 2 & 5 & 0 & 0\\ 5 & 1 & 0 & 0 & 0\end{bmatrix}\right\}$$

$$= \left\{\begin{bmatrix}3 & 8 & 6 & 4 & 2\\ 10 & 4 & 2 & 2 & 2\end{bmatrix}\right\} \in S.$$

$$(A \mid B)_s = (A_S \mid B_S)_s = \sum_{i=1}^{10} a_i b_i$$

$$= 18 + 16 + 12 + 0 + 4 + 20 + 16 + 4 + 10 + 8$$

$$= 108 \in Z^+ \cup \{0\}.$$

Example 4.7: Let = {Collection of all subsets from the column matrix semigroup

$$M = \left\{\begin{bmatrix}a_1\\ a_2\\ a_3\\ a_4\\ a_5\\ a_6\end{bmatrix} \middle| a_i \in \langle Q^+ \cup I \cup \{0\}\rangle; 1 \le i \le 6\}\right\}$$

be the subset semivector space over the neutrosophic semifield $F = \langle Z^+ \cup I \cup \{0\}\rangle$.

Let $(\mid)_s \to \langle Z^+ \cup I \cup \{0\}\rangle$ be the subset semiinner product on S.

Let A, B $\in$ S where

$$A = \left\{ \begin{bmatrix} 3+2I \\ 0 \\ 3 \\ 0+5I \\ 3+4I \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 6+I \\ 0 \\ 6 \\ 0+2I \\ 6 \end{bmatrix}, \begin{bmatrix} 2+I \\ 1 \\ 4+I \\ 3+2I \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1+I \\ 2+I \\ 0 \\ 3+4I \\ 4 \\ 0 \end{bmatrix} \right\}$$

and

$$B = \left\{ \begin{bmatrix} 4+I \\ 0 \\ 4+2I \\ 0 \\ 4 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 8+3I \\ 0 \\ 8 \\ 0 \\ 8+I \end{bmatrix}, \begin{bmatrix} 1+I \\ 2+I \\ I+3 \\ 1 \\ 1 \\ 1+4I \end{bmatrix} \right\} \in S.$$

$(A \mid B)_s$ where $A, B \in S$ and $(\ \mid\ )_S : S \to \langle Z^+ \cup \{0\} \cup I\rangle$.

$$A_S = \left\{ \begin{bmatrix} 3+2I \\ 0 \\ 3 \\ 0+5I \\ 3+4I \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 6+I \\ 0 \\ 6 \\ 0+2I \\ 6 \end{bmatrix} + \begin{bmatrix} 2+I \\ 1 \\ 4+I \\ 3+2I \\ 2 \\ 0 \end{bmatrix} + \begin{bmatrix} 1+I \\ 2+I \\ 0 \\ 3+4I \\ 4 \\ 0 \end{bmatrix} \right\}$$

$$= \left\{ \begin{bmatrix} 6+4I \\ 9+2I \\ 7+I \\ 12+11I \\ 9+6I \\ 6 \end{bmatrix} \right\} \text{ and}$$

$$B_S = \left\{\begin{bmatrix}4+I\\0\\4+2I\\0\\4\\0\end{bmatrix} + \begin{bmatrix}0\\8+3I\\0\\8\\0\\8+I\end{bmatrix} + \begin{bmatrix}1+I\\2+I\\I+3\\1\\1\\1+4I\end{bmatrix}\right\} = \left\{\begin{bmatrix}5+2I\\10+4I\\7+3I\\9\\5\\9+5I\end{bmatrix}\right\}$$

are in S.

$$(A \mid B)_s = (A_S \mid B_S)_s = \sum_{i=1}^{6} a_i b_i$$

$= (6 + 4I)(5 + 2I) + (9 + 2I)(10 + 4I) + (7 + I)(7 + 3I) + (12 + 11I)\,9 + (9+6I)5 + 6\,(9+5I)$

$= 30 + 8I + 32I + 90 + 8I + 56I + 49 + 3I + 28I + 108 + 99I + 45 + 30I + 54 + 30I$

$= 376 + 294I \in \langle Z^+ \cup I \cup \{0\}\rangle.$

Example 4.8: Let S = {Collection of all subsets from the semigroup

$$M = \left\{\sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in Z^+ \cup \{0\}\right\}$$

be the subset semiring over the semifield $F = Z^+ \cup \{0\}$.

If $A_S = \{5x^7 + 2x^3 + 5x + 1\}$ and $B_S = \{10x^8 + 5x^2 + 3x + 4\}$ then $(A_S \mid B_S)_s$ that is product is $10 \times 0 + 5 \times 0 + 2 \times 0 + 0 \times 5 + 5 \times 3 + 4 \times 1 = 19 \in Z^+ \cup \{0\}$.

If $A_S = \{3x^2 + 2x + 1\}$ and $B_S = \{5x^3 + 4x^4\}$ then $(A_S \mid B_S)_s = 0 \in Z^+ \cup \{0\}$.

So we see the subset semiinner product also behave in certain ways like the usual inner products.

Suppose we have subset semivector space on which is defined a subset semiinner product then we define S to be a subset semiinner product space over the semifield.

We define the semiinner product as sum of the products.

Example 4.9: Let S = {Collection of all subsets from the semigroup

$$M = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix} \middle| \; a_i \in Z^+ \cup \{0\}; 1 \leq i \leq 9 \right\}\}$$

be the subset semivector space over the semifield F = $Z^+ \cup \{0\}$.

We can define subset semiinner product which is as follows.

$$\text{If } A_S = \begin{pmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{pmatrix} \text{ and } B_S = \begin{pmatrix} b_1 & b_2 & b_3 \\ b_4 & b_5 & b_6 \\ b_7 & b_8 & b_9 \end{pmatrix} \in S \text{ then}$$

$$(A \mid B)_s = (A_S \mid B_S)_s$$

$$= \sum_{i=1}^{9} a_i b_i \in F.$$

Example 4.10: Let S = {Collection of all subsets from the 3 × 10 matrix semigroup

$$M = \left\{ \left. \begin{pmatrix} a_1 & a_2 & \ldots & a_{10} \\ a_{11} & a_{12} & \ldots & a_{20} \\ a_{21} & a_{22} & \ldots & a_{30} \end{pmatrix} \right| a_i \in Q^+ \cup \{0\};\ 1 \leq i \leq 30 \right\}$$

be the subset semiring over the semifield $F = Z^+ \cup \{0\}$.

Let $(\mid)_s : S \to Z^+ \cup \{0\}$.

Let $A, B \in S$, $A_S, B_S \in S$;

$$(A \mid B) = (A_S \mid B_S) = \sum_{i=1}^{30} a_i b_i \in Z^+ \cup \{0\}.$$

This is the way the subset semiinner product is defined. Further S is a subset semiinner product space.

We can define on any subset semiinner product space more than one subset semiinner product.

For in this case we can define

$(A_S \mid B_S)_s = a_{10}\, b_{10} + a_{20}b_{20} + a_{30}b_{30} \in Z^+ \cup \{0\}$ is also a subset semiinner product.

Likewise $(A_S \mid B_S)_s = a_1\, b_1 + a_3b_3 + \ldots + a_{29}\, b_{29}$ odd sum and so on.

Example 4.11: Let S = {Collection of all subsets from the polynomial semigroup under

$$M = \left\{ \sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in Z^+ \cup \{0\} \right\}$$

be the subset semivector space over the semifield $F = Z^+ \cup \{0\}$.

Take A = $\{3x^2 + 4x + 1, 2x + 1, 7x + 3\} \in S$.
$A_S = \{3x^2 + 4x + 1 + 2x + 1 + 7x + 3\}$
$= \{3x^2 + 13x + 5\}$.

Suppose B = $\{2x^3 + 7x + 1, 5x + 8, 8x^4\} \in S$ then $B_s = \{8x^4 + 2x^3 + 12x + 9\}$

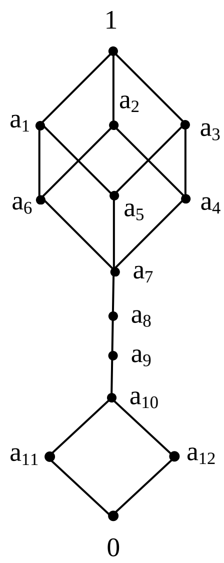
Now $(A \mid B)_s = (A_S \mid B_S)_s$
= sum of the coefficient of the even power of x in $(A_S \mid B_S)_s$.
= 24 + 40 + 26 + 27 + 156
= $173 \in Z^+ \cup \{0\}$.

This is the way this subset semiinner product is defined. It can be defined in other ways also.

For instance $(A \mid B)_s$ = Sum of the coefficient of odd terms and so on.

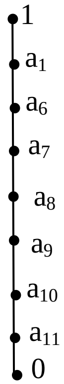
Example 4.12: Let S = {Collection of all subsets from the semigroup

L =



under the operation '$\cup$'} be the subset semivector space over the semifield

F =



$(\mid)_s : S \to F$;

Let $A = \{a_1, a_3, a_5, a_7, a_8\}$

and

$B = \{a_4, a_6, a_{10}, a_{11}\}$;

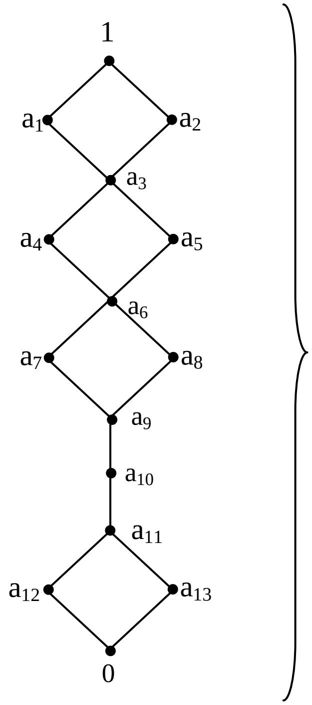
$A_S = \{1\}$ and $B_S = \{a_2\} \in S$.

$(A \mid B)_s = (A_S \mid B_S)_s = (1 \times a_2) = a_2 \in F$.

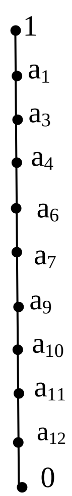
Thus $(\mid)_s$ is the subset semiinner product on S.

Example 4.13: Let S = {Collection of all subsets from the semi lattice L under $\cup$ and L is as follows:

L =



be the subset semivector space over the lattice (semifield) F =



.

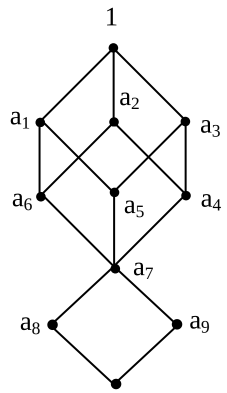
Let A = $\{a_6, a_4, a_8, a_9, a_{10}, a_{13}, a_3\}$ and B = $\{a_2, a_5, a_8, a_9, a_{12}\}$ $\in$ S.

Now $A_S = \{a_3\}$ and $B_S = \{a_2\}$ and $(A \mid B)_s = (A_S \mid B_S)_s$

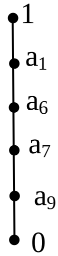
$= a_3 \times a_2 = a_3 \in F$.

Thus $(\mid)_s$ is the subset semiinner product on S.

Example 4.14: Let S = {Collection of all subsets from the semilattice L under '$\cup$' where L =

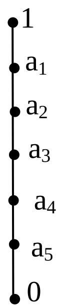


be the subset semivector space over the semifield

F = 

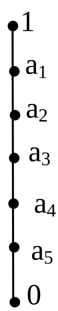
Let A = {a_6, a_7, a_8, a_9, a_5} and B = {a_2, a_3, 0, 1} $\in$ S.
A_S = {a_1} and B_S = {1} $\in$ S.
$(A \mid B)_s = (A_S \mid B_S)_s = a_1 \times 1 = a_1 \in S$.
Hence (|)$_s$ is the subset semiinner product on S.

Example 4.15: Let S = {Collection of all subsets from the semigroup LS_3 where L =



be the subset semivector space over the semifield

L =



Define a subset semiinner product (|)$_s$: S $\to$ L by
$(A \mid B)_s = (A_S \mid B_S)_s$ = sum of the coefficients of S.

Let A = {$a_1p_1 + a_2p_3 + p_4$, $p_2 + a_5p_5 + 1$} and
B = {$p_1 + a_3$, $a_1p_4 + p_5$} $\in$ S

$(A \mid B)_s = (A_S \mid B_S)_s$
(where A_S = {$1 + a_1p_1 + a_5p_5 + a_2p_3 + p_4 + p_2$})
and B_S = {$p_1 + a_3 + a_1p_4 + p_5$})

= sum of coefficient of $\{p_1 + a_3 + a_1p_4 + p_5 + a_1 + a_3p_1 + a_1p_3 + a_1p_2 + a_5 + a_5p_5 + a_5 + a_5p_4 + a_2p_3 + a_3p_3 + a_2p_2 + a_2p_1 + a_2p_5 + a_3p_3 + a_2p_2 + a_2p_1 + p_1p_4 + a_3p_4 + a_1p_5 + 1 + p_4 + a_3p_2 + a_1p_4 + p_2p_5\}$

$= 1 \in L$.

Thus $(\mid)_s$ is a subset semilinear inner product on S.

It is pertinent to keep on record that commutativity or non commutative of the structure does not depend on this defined subset semiinner product on S.

Example 4.16: Let S = {Collection of all subsets from the semigroup

$$B = \left\{ \left[\begin{array}{c|ccc|cc} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 & a_{10} & a_{11} & a_{12} \end{array} \right] \middle| \; a_i \in Z^+ \cup \{0\}; \; 1 \leq i \leq 12 \right\}\}$$

be the subset semivector space over the semifield $Z^+ \cup \{0\}$.

We define the subset semiinner product $(\mid)_s : S \to F = Z^+ \cup \{0\}$ as $(A \mid B)_s = (A_S \mid B_S)_s$ where A_S and B_S matrix sum and $(A \mid B)_s$ gives the $\sum_{i=1}^{12} a_i b_i$ where

$$A_S = \left[\begin{array}{c|ccc|cc} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 & a_{10} & a_{11} & a_{12} \end{array} \right] \text{ and}$$

$$B_S = \left[\begin{array}{c|ccc|cc} b_1 & b_2 & b_3 & b_4 & b_5 & b_6 \\ b_7 & b_8 & b_9 & b_{10} & b_{11} & b_{12} \end{array} \right].$$

Take

$$A = \left\{\left[\begin{array}{c|ccc|cc} 1 & 2 & 0 & 0 & 1 & 4 \\ 2 & 1 & 6 & 3 & 0 & 1 \end{array}\right], \left[\begin{array}{c|ccc|cc} 0 & 1 & 2 & 0 & 1 & 5 \\ 5 & 0 & 1 & 0 & 0 & 2 \end{array}\right], \left[\begin{array}{c|ccc|cc} 0 & 2 & 1 & 1 & 2 & 0 \\ 2 & 3 & 4 & 4 & 4 & 5 \end{array}\right]\right\}$$

$$\text{and } B = \left\{\left[\begin{array}{c|ccc|cc} 1 & 2 & 3 & 4 & 5 & 6 \\ 0 & 1 & 1 & 1 & 0 & 2 \end{array}\right], \left[\begin{array}{c|ccc|cc} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{array}\right],\right.$$

$$\left.\left[\begin{array}{c|ccc|cc} 1 & 2 & 0 & 0 & 1 & 3 \\ 0 & 2 & 4 & 4 & 0 & 0 \end{array}\right], \left[\begin{array}{c|ccc|cc} 2 & 0 & 3 & 4 & 0 & 1 \\ 1 & 0 & 4 & 0 & 6 & 0 \end{array}\right]\right\} \in S.$$

$$A_s = \left\{\left[\begin{array}{c|ccc|cc} 1 & 2 & 0 & 0 & 1 & 4 \\ 2 & 1 & 6 & 3 & 0 & 1 \end{array}\right] + \left[\begin{array}{c|ccc|cc} 0 & 1 & 2 & 0 & 1 & 5 \\ 5 & 0 & 1 & 0 & 0 & 2 \end{array}\right] + \left[\begin{array}{c|ccc|cc} 0 & 2 & 1 & 1 & 2 & 0 \\ 2 & 3 & 4 & 4 & 4 & 5 \end{array}\right]\right\}$$

$$= \left\{\left[\begin{array}{c|ccc|cc} 1 & 5 & 3 & 1 & 4 & 9 \\ 9 & 4 & 11 & 7 & 4 & 8 \end{array}\right]\right\} \text{ and}$$

$$B_S = \left\{\left[\begin{array}{c|ccc|cc} 1 & 2 & 3 & 4 & 5 & 6 \\ 0 & 1 & 1 & 1 & 0 & 2 \end{array}\right] + \left[\begin{array}{c|ccc|cc} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{array}\right] + \right.$$

$$\left.\left[\begin{array}{c|ccc|cc} 1 & 2 & 0 & 0 & 1 & 3 \\ 0 & 2 & 4 & 4 & 0 & 0 \end{array}\right] + \left[\begin{array}{c|ccc|cc} 2 & 0 & 3 & 4 & 0 & 1 \\ 1 & 0 & 4 & 0 & 6 & 0 \end{array}\right]\right\}$$

$$= \left\{\left[\begin{array}{c|ccc|cc} 5 & 5 & 7 & 9 & 7 & 11 \\ 1 & 3 & 10 & 5 & 6 & 2 \end{array}\right]\right\} \in S.$$

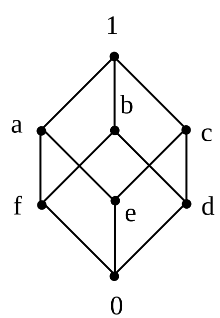
$(A \mid B)_s = (A_S \mid B_S)_s$

$= \quad 1.5 + 5.5 + 3.7 + 1.9 + 4.7 + 9.11 + 9.1 + 4.3 + 11.10 + 7.5 + 4.6 + 8.2$

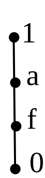
$= \quad 5 + 25 + 21 + 9 + 28 + 99 + 9 + 12 + 110 + 35 + 24 + 16$

$= \quad 393 \in F = Z^+ \cup \{0\}.$

Example 4.17: Let $M = \left\{ \left[\begin{array}{c} a_1 \\ a_2 \\ a_3 \\ a_4 \\ \hline a_5 \\ \hline a_6 \\ a_7 \\ \hline a_8 \end{array} \right] \right| a_i \in L =$ a Boolean order 8



be the subset semivector space over the semifield

F = 

$$\text{Let } A = \left[\begin{array}{c} 0 \\ a \\ b \\ c \\ \hline f \\ \hline e \\ d \\ \hline 0 \end{array}\right] \text{ and } B = \left[\begin{array}{c} a \\ f \\ e \\ d \\ \hline c \\ \hline a \\ b \\ \hline d \end{array}\right] \in S.$$

$(A \mid B)_s = 0.a + a.f + b.e + c.d + f.c + e.a + d.b + 0.d$
$= f + 0 + d + f + e + d + 0$
$= 1 \in F.$

Now having seen examples of subset semiinner product spaces we can define subset orthogonal semivectors of a subset semiinner product space.

We say A, B $\in$ S, S a subset semiinner product space $(A \mid B)_s$ to be orthogonal if $(A \mid B)_s = 0$

Let $A_S = \{(0\ 0\ 1\ 0\ 0)\}$
and $B_S = \{(1\ 1\ 0\ 0\ 0)\} \in S$

$(A \mid B)_s = (A_S \mid B_S)_s = 0.$

If we are using distributive lattices in which $a \cap b \neq 0$ if $a \neq 0$ and $b \neq 0$ or semifield $Z^+ \cup \{0\}$ or $Q^+ \cup \{0\}$ or $R^+ \cup \{0\}$ or $\langle R^+ \cup I\rangle \cup \{0\}$, $\langle Q^+ \cup I\rangle \cup \{0\}$ or $\langle Z^+ \cup I\rangle \cup \{0\}$, we can define orthogonal spaces.

We define $(A \mid A)_s$ by $||A||_s$ as the subset seminorm of the subset A $\in$ S, S a subset semiinner product space.

We can as in case of usual vector spaces define in case of subset semivector spaces also the concept orthogonal complement.

Example 4.18: Let S = {Collection of all subsets from the semigroup B = $(a_1\ a_2\ a_3\ a_4\ a_5\ a_6\ a_7) \mid a_i \in Z^+ \cup \{0\}, 1 \leq i \leq 7\}\}$ be the subset semivector space over the semifield $Z^+ \cup \{0\}$.

Let W_1 = {Collection of all subsets from the subsemigroup $B_1 = \{(0\ 0\ 0\ a_1, a_2, a_3, a_4)\ a_i \in Z^+ \cup \{0\}; 1 \leq i \leq 4\} \subseteq S$ be the subset semivector subspace of S.

We see under the subset semiinner product $W_1^\perp$ = {Collection of all subsets from the subsemigroup. $B_2 = \{(a_1, a_2, a_3, 0, 0, 0, 0)\}$ where $a_i \in Z^+ \cup \{0\}; 1 \leq i \leq 3\} \subseteq S$.

We see $(A \mid B)_s = 0$ if $A \in S_1$ and $B \in W_1^\perp$. That is $W_1^\perp$ is the subset orthogonal complement of W_1 and $S = W_1 = W_1^\perp$.

Let $M = \{(0\ 0\ 0\ a_1\ 0\ 0\ 0), (0\ 0\ 0\ 0\ a_2\ 0\ 0) \mid a_i \in 5Z^+ \cup \{0\}, 1 \leq i \leq 2\} \subseteq S$.

We see $M^\perp = \{(a_1\ a_2\ a_3\ 0\ 0\ a_4\ a_5)$ where $a_i \in Z^+ \cup \{0\}, 1 \leq i \leq 5\} \subseteq S$ we see $M^\perp$ is orthogonal to every element in M.

However $M + M^\perp \neq S$.

Take $A = \{(5, 0\ 0\ 0\ 0\ 0\ 1), (6\ 0\ 0\ 0\ 0\ 0\ 0), (0\ 0\ 0\ 0\ 0\ 0\ 1)\} \in S$.

Now {(Collection of all subsets from $A^\perp = \{(0, a_1, a_2, a_3, a_4, a_5, 0) \mid a_i \in Z^+ \cup \{0\}, 1 \leq i \leq 5\} \subseteq S$.

Clearly $A^\perp$ is a subset semivector subspace of S however A is not a subset semivector subspace only a subset from S.

Example 4.19: Let S = {Collection of all subsets from the interval semigroup under '+'. P = {[a, b] | a, b $\in Z^+ \cup \{0\}$}} be the subset semivector space over the semifield F = $Z^+ \cup \{0\}$.

Let A = {[3, 7], [5, 0], [0, 9], [2, 1], [4, 4]} and
B = {[1, 3], [4, 5], [9, 9], [7, 3], [0, 5], [0, 1]} $\in$ S

A_S = {[3, 7] + [5, 0] + [0, 9] + [2, 1] + [4, 4]} = {[14, 21]}

and

B_S = {[1, 3] + [4, 5] + [9, 9] + [7, 3] + [0, 5] + [0, 1]} = {[21, 26]} $\in$ S.

$$\begin{aligned}(A, B)_s &= (A_S, B_S)_s \\ &= \sum_{i=1}^{2} a_i b_i \\ &= 14 \times 21 + 21 \times 26 \\ &= 840 \in Z^+ \cup \{0\}.\end{aligned}$$

$(,)_s : S \to F = Z^+ \cup \{0\}$ is a subset semilinear product in S.

Example 4.20: Let S = {Collection of all subsets from the interval matrix semigroup

$$P = \left\{\left.\begin{bmatrix}[a_1, b_1] \\ [a_2, b_2] \\ [a_3, b_3] \\ [a_4, b_4] \\ [a_5, b_5]\end{bmatrix}\right| a_i \in Z^+ \cup \{0\}; 1 \le i \le 5\right\}\}$$

be the subset semivector space over the semifield F = $Z^+ \cup \{0\}$.

Let us define $(\ | \ )_s : S \to F$ by $(A \mid B)_s = (A_S \mid B_S)_s$

$$= \sum_{i=1}^{5} a_i a'_i \text{ where}$$

$$A_S = \left\{ \begin{bmatrix} [a_1 b_1] \\ [a_2 b_2] \\ \vdots \\ [a_5 b_5] \end{bmatrix} \right\} \text{ and}$$

$$B_S = \left\{ \begin{bmatrix} [a'_1 b'_1] \\ [a'_2 b'_2] \\ \vdots \\ [a'_5 b'_5] \end{bmatrix} \right\} \text{ ; } a_i, b_i, a'_1, b'_1 \in Z^+ \cup \{0\}.$$

$$\text{Let } A_S = \left\{ \begin{bmatrix} [0,1] \\ [2,5] \\ [1,9] \\ [7,2] \\ [4,4] \end{bmatrix} \right\} \text{ and } B_S = \left\{ \begin{bmatrix} [8,0] \\ [1,5] \\ [7,2] \\ [9,1] \\ [0,2] \end{bmatrix} \right\}$$

$$\begin{aligned} (A \mid B)_s = (A_S \mid B_S)_s &= 8 \times 0 + 2 \times 1 + 1 \times 7 + 7 \times 9 + 4 \times 0 \\ &= 2 + 7 + 63 \\ &= 72 \in Z^+ \cup \{0\}. \end{aligned}$$

Thus $(\mid)_s$ is the subset semiinner product on S.

Example 4.21: Let S = {Collection of all subsets from the semigroup

$$P = \left\{ \sum_{i=0}^{\infty} [a_i b_i] x^i \,\middle|\, a_i\, b_i \in Z^+ \cup \{0\} \right\}$$

be the subset semivector space over the semifield $F = Z^+ \cup \{0\}$.

Define $(A \mid B)_s = (A_S \mid B_S))_s = \sum_{i=1}^{n} a_i \times a'_i$ with

$$A_S = \left\{ \sum_{i=0}^{t} [a_i b_i]\, x^i \right\} \text{ and}$$

$$B_S = \left\{ \sum_{i=0}^{s} [a'_i b'_i]\, x^i \right\}$$

where n = t or s which ever is the greatest.

$a_i, b_i, a'_i, b'_i \in Z^+ \cup \{0\}, 0 \leq i \leq t, s.$

$(\mid)_s$ is the subset semiinner product on S.

Now having seen examples of subset semiinner product spaces we now proceed onto define subset semilinear functionals and describe them.

Let
S = {Collection of all subsets from the semigroup P under +} be a subset semivector space over a semifield F.

Let S be a finite dimensional subset semiinner product space over the semifield F.

A semilinear functional f_s on S is of the form.

$f_s(A) = (A \mid B)_s$ for some fixed subset semivector B in S.

We can define some of the properties of semivector spaces in case of subset semivector spaces with some appropriate modifications.

We will give one to two examples of this concept before we proceed onto describe other properties related with subset semivector spaces.

Example 4.22: Let S = {Collection of all subsets from the semigroup M = {$(a_1, a_2, \ldots, a_{10})$ | $a_i \in F = Z^+ \cup \{0\}$; $1 \leq i \leq 10$}} be the subset semivector space over F the semifield.

Let S be a subset innerproduct space under the inner product

$(A \mid B)_s = (A_S \mid B_S)_s = \sum_{i=1}^{10} x_i y_i$ where

$A_S = \{(x_1, x_2, \ldots, x_{10})\}$ and
$B_S = \{(y_1, y_2, \ldots, y_{10})\} \in S$.

$x_i, y_j \in F$; $1 \leq i, j \leq 10$.

Let B ∈ S be a fixed subset semivector in S.
Let f : S → F be defined by

$$\begin{aligned} f(A) &= (A \mid B)_s \\ &= (A_S \mid B_S)_s \\ &= \sum_{i=1}^{10} x_i y_i \in F. \end{aligned}$$

B the given fixed subset semivector in S.

Example 4.23: Let S = {Collection of all subsets from the semigroup

$$M = \left\{ \begin{bmatrix} a_1 & a_2 & \ldots & a_{10} \\ a_{11} & a_{12} & \ldots & a_{20} \\ a_{21} & a_{22} & \ldots & a_{30} \end{bmatrix} \middle| \; a_i \in Q^+ \cup \{0\} \right\}\}$$

be the subset semivector space over the semifield $F = Q^+ \cup \{0\}$.

$$(A \mid B)_s = (A_S \mid B_S)_s = \sum_{i=1}^{30} a_i b_i$$

where $a_i \in A_S = \left\{\begin{bmatrix} a_1 & a_2 & \ldots & a_{10} \\ a_{11} & a_{12} & \ldots & a_{20} \\ a_{21} & a_{22} & \ldots & a_{30} \end{bmatrix}\right\}$ and

$$b_i \in B_S = \left\{\begin{bmatrix} b_1 & b_2 & \ldots & b_{10} \\ b_{11} & b_{12} & \ldots & b_{20} \\ b_{21} & b_{22} & \ldots & b_{30} \end{bmatrix}\right\}, 1 \leq i \leq 30$$

and B is the fixed subset semivector of S where B_s = Sum of B.

Now having seen the concept of subset semilinear functionals we now proceed onto define the notion of T_S preserves subset semiinner products.

Let S and S_1 be two subset (semivector) semiinner product spaces over the same semifield F and let $T_S : S \to S_1$ be a subset semilinear transformation. We say that T_S preserves subset semiinner products if $(T\alpha \mid T\beta)_s = (\alpha \mid \beta)_s$ for all $\alpha, \beta \in S$.

Clearly a subset semilinear isomorphism of the subset semivector spaces also preserves subset semiinner product.

We can prove that if S = {Collection of all subsets of a semigroup '+' $P = \{(a, b) \mid a, b \in Z^+ \cup \{0\}\}$} be the subset semivector space over the semifield $F = Z^+ \cup \{0\}$ and if $V = \{(a, b) \mid a, b \in Z^+ \cup \{0\}\}$ is a semivector space over F then we have $B = \{\{(a, b)\} \mid a, b \in Z^+ \cup \{0\}\} \subseteq S$ is a subset semivector subspace of S and $B \cong V$ (B is isomorphic with V as semivector spaces).

Thus almost all the properties enjoyed by V will also be enjoyed by S with some simple and appropriate modifications.

Now we see we cannot in case of subset semivector spaces arrive at the concept of complex structures. However that is possible if we take subset semivector spaces of type I.

Let S = {Collection of all subsets from the group Z} be the subset semivector space over the semifield $Z^+ \cup \{0\} = F$.

We see for this S it is not easy to define subset semiinner product on S. However we do not say it is not possible we can always define subset semiinner product space.

We define for

$$(A \mid B)_s = (A_S \mid B_S)_s = \left|\sum_i a_i b_i\right| \in F = Z^+ \cup \{0\}.$$

If the mod is removed we see the subset semiinner product is not defined.

Let S = {Collection of all subsets from the group G = C} be the subset semivector space over the semifield $R^+ \cup \{0\}$.

We see $(A \mid B)_s$ = complex number α
= | Real part of α |;

then $(\ \mid\ )_s : S \to R^+ \cup \{0\}$ is a subset semiinner product on S.

S = {Collection of all subset from the group (a, b) where a, b $\in$ C} be the subset semivector space over the semifield $F = Z^+ \cup \{0\}$.

$(A \mid B)_s : S \to R^+ \cup \{0\}$.

$(A \mid B)_s = (A_S : B_S)_s$ = integral part and real part of $a_1b_1 + a_2b_2$.

Let S = {Collection of all subsets from group

$$G = \left\{\begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \end{bmatrix} \middle| a_i \in C; 1 \leq i \leq 8\}\right\}$$

be the subset semivector space over the semifield $Z^+ \cup \{0\}$.

$(\mid)_s : S \to Z^+ \cup \{0\}$ is defined such that

$$(A \mid B)_s = (A_S \mid B_S)_s$$

$$= \text{integral positive part of } \sum_{i=1}^{8} a_i b_i$$

(Here $A = \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \end{bmatrix}$, $B = \begin{bmatrix} b_1 & b_2 & b_3 & b_4 \\ b_5 & b_6 & b_7 & b_8 \end{bmatrix} \in G$).

We can have several such examples.

This task is left as an exercise to the reader.

We can also have the concept of subset semiinner product in case of S-subset semivector space over a S-semiring. This is considered as a matter of routine.

Other properties related with subset semivector spaces can be extended. However one of the problems is that they can have a unique basis so some times it is advantageous in certain situation and disadvantageous in situations when one needs more than one basis.

We can define subset semifunctional and derive their related properties.

However as in case of usual vector spaces (semivector spaces) we can for the case of subset semivector spaces also define the notion of subset projection. This will have more applications as we project subsets on the subsets.

Let S = {Collection of all subsets from the semigroup P under '+'} be the subset semivector space over the semivector space.

Suppose T_s is a subset semilinear operation on S and $S = W_1 + \ldots + W_k$ where the sum is a direct decompositions. T_s induces subset a semilinear operator. T_S^i on each W_i by restriction.

The k subset semilinear operators $E_S^1, E_S^2, \ldots, E_S^k$ on S is such that E_S^i is a subset projection.

We can show several of the properties are inherited in case of subset semiprojections also.

We will illustrate them by examples before we prove theorems in this direction.

Example 4.24: Let S = {Collection of all subsets from the matrix semigroup $M = \{(a_1, a_2, \ldots, a_6) \mid a_i \in Z^+ \cup \{0\}; 1 \leq i \leq 6\}\}$ be the subset semivector space over the semifield $F = Z^+ \cup \{0\}$.

Let $W_1 = \{(a_1\ a_2\ 0\ 0\ 0\ 0) \mid a_i \in Z^+ \cup \{0\}; 1 \leq i \leq 2\} \subseteq M$ and V_1 = {Collection of all subsets of the subsemigroup W_1 of M}} $\subseteq$ S; V_2 = {Collection of all subsets from the subsemigroup $W_2 = \{(0, 0, a_1, a_2, 0, 0) \mid a_i \in Z^+ \cup \{0\}; 1 \leq i \leq 2\}\} \subseteq S$ and V_3 = {Collection of all subsets from the subsemigroup $W_3 = \{(0\ 0\ 0\ 0\ a_1\ a_2) \mid a_i \in Z^+ \cup \{0\}; 1 \leq i \leq 2\}\}$ $\subseteq$ S be the three subset semivector subspaces of S over the semifield $Z^+ \cup \{0\}$. Clearly $S = V_1 + V_2 + V_3$ is the direct sum and $V_i \cap V_j = \{(0\ 0\ 0\ 0\ 0\ 0)\}$ if $i \neq j$, $1 \leq i, j \leq 3$.

Now define a subset semilinear operator $E_1 : S \to S$ by $E_1(A) = (\{(a_1, a_2, a_3, a_4, a_5, a_6)\}) = \{(a_1, a_2, 0, 0, 0, 0)\}$ for all A $\in$ S. That is if

A = {(2, 5, 0, 1, 2, 3), (4, 5, 6, 7, 8, 9), (10, 11, 0, 8, 4, 3), (9, 2, 1, 0, 1, 9)} $\in$ S.

E_1 (A) = E_1 ({(2, 5, 0, 1, 2, 3), (4, 5, 6, 7, 8, 9), (10, 11, 0, 8, 4, 3), (9, 2, 1, 0, 1, 9)})

= {(2, 5, 0, 0, 0, 0), (4, 5, 0, 0, 0, 0), (10, 11, 0, 0, 0, 0), (9, 2, 0, 0, 0, 0)} $\in V_1 \subseteq S$.

We see E_1 is a subset semilinear projection operation.

Further (E_1 o E_1) = E_1.

We define $E_2 : S \to S$ by

E_2 (B) = E_2({(a_1, a_2, a_3, a_5, a_5, a_6)}) = {(0, 0, a_3, a_5, 0, 0)} where $a_i \in Z^+ \cup \{0\}$ and A $\in$ S; $1 \leq i \leq 6$. E_2 is also a subset semilinear projection operator for E_2 o E_2 = E_2.

Take A = {(2, 3, 4, 5, 6, 7), (1, 2, 3, 4, 5, 6), (4, 5, 0, 1, 2, 3), (0, 2, 9, 6, 9, 8)} $\in$ S.

(E_1 o E_2) (A) = E_2 o E_2 ({(2, 3, 4, 5, 6, 7), (1, 2, 3, 4, 5, 6), (4, 5, 0, 1, 2, 3), (0, 2, 9, 6, 9, 8)})
E_2 = ({(0, 0, 4, 5, 0, 0), (0, 0, 3, 4, 0, 0), (0, 0, 0, 1, 0, 0), (0, 0, 9, 6, 0, 0)})

= {(0, 0, 4, 5, 0, 0), (0, 0, 3, 4, 0, 0), (0, 0, 0, 1, 0, 0), (0, 0, 9, 6, 0, 0)})

= E_2 (A). Thus E_2 o E_2 = E_2.

We can define
$E_3 : S \to S$ by E_3 {(a_1, a_2, a_3, a_4, a_5, a_6)} = {(0, 0, 0, 0, a_5, a_6)} and if
A = {(4, 2, 0, 0, 6, 8), (9, 4, 6, 8, 0, 0), (11, 0, 12, 14, 5, 0), (8, 25, 48, 56, 0, 90), (91, 48, 0, 9, 25, 126)} $\in$ S.

$E_3(A) = E_3$ ({(4, 2, 0, 0, 6, 8), (9, 4, 6, 8, 0, 0), (11, 0, 12, 14, 5, 0), (8, 25, 48, 56, 0, 90), (91, 48, 0, 9, 25, 126)}.

= {(0, 0, 0, 0, 6, 8), (0, 0, 0, 0, 0, 0), (0, 0, 0, 0, 5, 0), (0, 0, 0, 0, 0, 90), (0, 0, 0, 0, 25, 126)} $\in V_3 \subseteq S$.

We see E_3 is also a subset semilinear projection of S on W_3. Thus each E_i is a subset semilinear projection of S on W_i , i = 1, 2, 3.

We see $E_1 + E_2 + E_3 = I_S$ the identity subset semilinear identity operator on S. That is $I_S : S \to S$ is such that $I_S(A) = A$ for all $A \in S$.

That is $I_S : S \to S$ is the identity semilinear operator on S.

We can also define for an element X in a subset semivector space S over a semifield F the notion of the subset orthogonal to X in S.

We denote this by X^o and X^o is a subset semivector subspace of S even if X is not a subset semivector subspace of X.

Let S = {Collection of all subsets from the semigroup M = $\{(a_1\ a_2\ a_3\ a_4) \mid a_i \in Z^+ \cup \{0\};\ 1 \leq i \leq 4\}\}$ be the subset semivector space over the semifield $Z^+ \cup \{0\} = F$.

Take X = {(3, 0, 0, 0), (0, 5, 0, 0), (0, 7, 0, 0), (1, 2, 0, 0) (17, 5, 0, 0), (10, 0, 0, 0)} $\in$ S.

We see X is just an element in S and X is not a subset semivector subspace of S.

Consider X^o the orthogonal complement of X. X^o = {Collection of all subsets from the subsemigroup $P_1 = \{(0, 0, a_1, 0), (0, 0, 0, a_2), (0, 0, d_1, d_2) \mid d_i, a_i \in Z^+ \cup \{0\}; 1 \leq i \leq 2\} \subseteq M\}$ $\subseteq S$.

It is easily verified X^o is a subset semivector subspace of X and $X^o + X \neq S$.

However it is easily verified if the subset X in S is replaced by a subset semivector subspace say W then W^o the orthogonal subset semivector subspace of W is such that $W + W^o = S$.

We can develop almost all the properties associated semivector spaces in case of subset semivector spaces defined over a semifield. This is considered as a matter of routine and left as an exercise to the reader.

However it is also important to mention that these subset semivector spaces also find applications in all the places where semivector spaces find their applications.

Apart from this also these new structures can find more applications. This task is also left as an exercise to the reader.

We suggest the following problems.

Problems:

1. Obtain some nice and special features enjoyed by subset semiinner product spaces.

2. Given a finite dimensional subset semivector space how many subset semiinner products can be defined on it?

3. Let S = {Collection of all subsets from the semigroup

$$P = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \end{bmatrix} \middle| \, a_i \in Z^+ \cup \{0\}; 1 \leq i \leq 6\} \right\}$$

be the subset semivector space over the semifield $F = Z^+ \cup \{0\}$.

Find all possible subset semiinner products that can be defined on S.

4. Let S = {Collection of all subsets from the semigroup

$$P = \left\{ \left[\begin{array}{cc} a_1 & a_2 \\ \hline a_3 & a_4 \\ a_5 & a_6 \\ a_7 & a_8 \\ \hline a_9 & a_{10} \\ a_{11} & a_{12} \\ a_{13} & a_{14} \\ a_{15} & a_{16} \end{array} \right] \right| a_i \in Q^+ \cup \{0\}; 1 \leq i \leq 16\}\}$$

be the subset semivector space defined over the semifield $F = Q^+ \cup \{0\}$.

(i) Find all subset semiinner products that can be defined in S over $Q^+ \cup \{0\}$.

(ii) If $Q^+ \cup \{0\}$ is replaced by $Z^+ \cup \{0\}$ can you define subset semiinner products on S?

(iii) If yes for question (ii) how many such subset semiinner products can be defined?

5. Define orthogonality in subset semiinner product spaces and define some special features using it.

6. Can orthonormality be defined on subset semiinner product spaces?

7. Give some interesting and special features enjoyed by subset semilinear functionals.

8. Let S = {Collection of all subsets from the semigroup $M = \{[a_1, a_2, \ldots, a_{10}] \mid a_i \in F$, F a semifield} be the subset semivector space over the semifield F.

(i) Define a subset semiinner product on S.
(ii) How many subset semiinner products can be defined on S?
(iii) Write S as a n-direct sum of subset semivector subspaces and prove we can find T_S, $E_1 \ldots E_n$ such that $T_S = \alpha_1 E_1 + \ldots + \alpha_n E_n$, $\alpha_i \in S$; $1 \leq i \leq n$.
(iv) Define a normal semilinear operator on S.
(v) Define a semilinear functional on S.
(vi) Prove S has as many semilinear functionals as that semilinear operators defined on it.

9. Let S = {Collection of all subsets from the semigroup

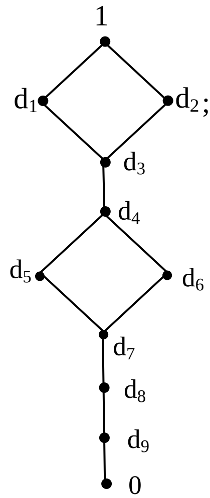
$$M = \left\{ \left[\begin{array}{cc} a_1 & a_2 \\ \hline a_3 & a_4 \\ a_5 & a_6 \\ a_7 & a_8 \\ \hline a_9 & a_{10} \\ a_{11} & a_{12} \\ \hline a_{13} & a_{14} \end{array} \right] \middle| \; a_i \in Z^+ \cup \{0\}; 1 \leq i \leq 14 \right\}\}$$

be the subset semivector space defined over the semifield $F = Z^+ \cup \{0\}$.

Study questions (i) to (vi) of problem 8 for this S.

10. Let S = {Collection of all subsets from the matrix semigroup

$$M = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & a_7 & a_8 & a_9 & a_{10} \\ a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \end{bmatrix} \middle| \; a_i \in L \right.$$

where L = 

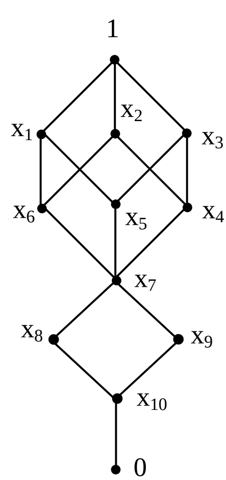
; $1 \leq i \leq 15$}}

be the subset semivector space over the semifield L.

(i) Study questions (i) to (vi) of problem 8 for this S.
(ii) Find o(S).

11. Let S = {Collection of all subsets from the semigroup

$$M = \left\{ \left[\begin{array}{c} a_1 \\ a_2 \\ \hline a_3 \\ \hline a_4 \\ a_5 \\ a_6 \\ a_7 \\ \hline a_8 \\ a_9 \end{array}\right] \middle| \; a_i \in L = \right.$$



; $1 \leq i \leq 9$}}

be the subset semivector space over the semifield L.

(i) Study questions (i) to (vi) of problem (8) for this S.
(ii) Find o(S).
(iii) How many semilinear functionals can be defined on this S?
(iv) In how many ways can S be written as a subset semidirect sum of subset semivector subspaces of S?

12. Is it possible to build the spectral theorem for subset semiinner product spaces?

13. Give an example of a subset semiinner product space so that

(i) T_s is normal.
(ii) T_s is unitary.
(iii) T_s satisfies the spectral theorem.

14. Give an example of a subset unitary operator of a subset semivector space over a semifield F.

15. Give some of the special features associated with subset semiinner product spaces of finite order.

16. Let S_1 = {Collection of all subsets from the group

$$G_1 = \left\{ \left[\begin{array}{ccc} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ a_{10} & a_{11} & a_{12} \end{array} \right] \middle| \; a_i \in Z^+ \cup \{0\}; 1 \leq i \leq 12 \right\}\}$$

be the subset semivector space over F and S_2 = {Collection of all subsets from the group

$$G_2 = \left\{\left.\begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ a_{10} & a_{11} & a_{12} \end{bmatrix}\right| a_i \in F = 2Z^+ \cup \{0\}; 1 \leq i \leq 12\}\right\}$$

be the subset semivector space over F.

(i) Define a subset semilinear transformation $T_{S_1} : S_1 \to S_2$ so that

(a) T_{S_1} preserves subset semiinner products.

(b) T_{S_1} is a subset semivector space isomorphism.

(c) T_{S_1} carries subset semiorthogonal basis for S_1 on to subset semi orthogonal basis of S_2.

(ii) Define $T_{S_2} : S_2 \to S_2$ so that

(a) to (c) of (i) is true.

Is $T_{S_1} = T_{S_2}$ or $T_{S_1} \cong T_{S_2}$?

17. Let S = {Collection of all subsets from semigroup $M = \left\{\left.\sum_{i=0}^{\infty} a_i x^i \right| a_i \in Z^+ \cup \{0\}\right\}$} be the subset semiring over the semifield $Z^+ \cup \{0\} = F$.

(i) Is it possible to write S as a n-direct summand?
(ii) Define semiinner product on S.
(iii) How many subset semiinner product can be defined on S?
(iv) Can we have subset seminormal operator on S?
(v) Define on S subset semilinear functionals.

18. Can we always define on S a subset semivector space over a semifield S a subset semiunitary operator on S?

19. Is it always possible to define on S (the subset semivector space) a subset semilinear normal operator?

20. Prove if S is a subset inner product space defined over a semifield.

    If A $\in$ S is a subset of S. Is $A^{\perp} = \{x \in S \mid (x \mid A) = (0)\}$ a subset semivector subspace of S?

21. Let S = {Collection of all subsets of the matrix semigroup

$$M = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ \vdots & \vdots & \vdots \\ a_{28} & a_{29} & a_{30} \end{bmatrix} \middle| \; a_i \in Z^+ \cup \{0\}; 1 \leq i \leq 30\right\}\}$$

    be a subset semivector space over the semifield $F = Z^+ \cup \{0\}$.

    (i) Find a subset basis B of S.

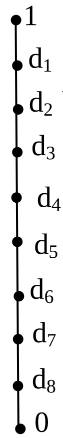
    (ii) If T = ($\alpha_1 \ldots \alpha_n$) are a subset linearly independent set in S. Find a corresponding orthogonal subset of T.

    (iii) Can T be made into a orthonormal subset linearly independent set?

    (iv) How many subset semiinner products can be defined on S?

22. Let S = {Collection of all subsets from the semigroup

$$P = \left\{ \begin{bmatrix} a_1 & a_2 & \dots & a_9 \\ a_{10} & a_{11} & \dots & a_{18} \end{bmatrix} \middle| \; a_i \in L = \right.$$



$; 1 \leq i \leq 30\}\}$

be semivector space over the semifield F = L.

Study questions (i) to (iv) of problem 21 for this S.

23. Let S = {Collection of all subsets from the semigroup

$$G = \left\{ \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \\ a_5 & a_6 \\ a_7 & a_8 \\ a_9 & a_{10} \end{bmatrix} \middle| \; a_i \in Q^+ \cup \{0\}; 1 \leq i \leq 10\} \right\}$$

be the subset semivector space over the semifield $Q^+ \cup \{0\} = F$.

Study questions (i) to (iv) of problem 21 for this S.

24. Let S = {Collection of all subsets from the matrix semigroup

$$M = \left\{ \left[\begin{array}{cccc} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \end{array} \right] \right| a_i \in R^+ \cup \{0\}; 1 \leq i \leq 16\}\}$$

be the subset semivector space over the semifield $F = R^+ \cup \{0\}$.

Study questions (i) to (iv) of problem 21 for this S.

25. Can we define a subset seminormal operator and subset semi unitary operator on the subset semivector space S = {Collection of all subsets from the semigroup

$$M = \left\{ \left[\begin{array}{c|ccc|ccc|cccc} a_1 & a_2 & a_3 & & a_4 & a_5 & a_6 & a_7 & a_8 & a_9 & a_{10} \\ a_{11} & a_{12} & a_{13} & & a_{14} & a_{15} & a_{16} & a_{17} & a_{18} & a_{19} & a_{20} \end{array} \right] \right|$$

$a_i \in Z^+ \cup \{0\}; 1 \leq i \leq 20\}\}$ be the subset semivector space over the semifield $F = Z^+ \cup \{0\}$?

26. Let S = {Collection of all subsets from the semigroup

$$P = \left\{ \left[\begin{array}{ccc} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ \vdots & \vdots & \vdots \\ a_{13} & a_{14} & a_{15} \end{array} \right] \right| a_i \in Z^+ \cup \{0\}; 1 \leq i \leq 15\}\}$$

be the subset semivector space over the semifield $F = Z^+ \cup \{0\}$.

Can we have on S the notion of subset seminormal operator and subset seminormal operator and subset semiunitary operator.

27. Does there exist a subset semivector space over a semifield such that we do not have on S for a particular subset semiinner product defined on S the notion of subset seminormal operator and subset semiunitary operator?

28. Let S = {Collection of all subsets from the semigroup

$$P = \left\{ \sum_{i=0}^{\infty} a_i x^i \;\middle|\; a_i \in Q^+ \cup \{0\} \right\}\}$$ be the subset semivector

space over the semi field $F = Q^+ \cup \{0\}$.

Find all subset seminormal operators and subset semiunitary operators on S?

29. Can we have a subset semivector space S on which it is impossible to define the notion of subset semiinner product?

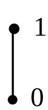
Justify your claim.

30. Does there exist a subset semivector space S on which we cannot define the notion of subset semilinear functional?

31. Let S = {Collection of all subsets from the semigroup

$$M = \left\{ \sum_{i=0}^{\infty} a_i x^i \;\middle|\; a_i \in \text{Boolean algebra of order } 64 \right\}\}$$

over the semifield

F = 

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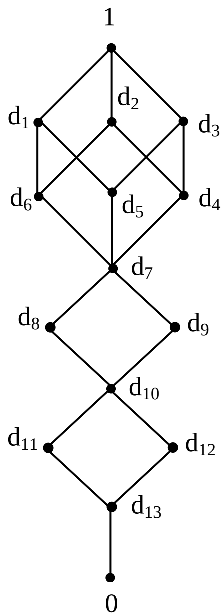
(i) Can we define on S a subset semiinner product?
(ii) Can we define on S a subset semilinear functional?
(iii) Is it possible to have on S a subset semi unitary operator?

(iv) Is it possible to have on S a subset seminormal operator?

32. Let S = {Collection of all subsets from the semigroup

$$P = \left\{ \sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in L = \right.$$



be the subset semivector space over the semifield L.

Can we define on S the notion of subset seminormal operator and subset semiunitary operator?

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Dr.W.B.Vasantha Kandasamy is a Professor in the Department of Mathematics, Indian Institute of Technology Madras, Chennai. In the past decade she has guided 13 Ph.D. scholars in the different fields of non-associative algebras, algebraic coding theory, transportation theory, fuzzy groups, and applications of fuzzy theory of the problems faced in chemical industries and cement industries. She has to her credit 646 research papers. She has guided over 100 M.Sc. and M.Tech. projects. She has worked in collaboration projects with the Indian Space Research Organization and with the Tamil Nadu State AIDS Control Society. She is presently working on a research project funded by the Board of Research in Nuclear Sciences, Government of India. This is her 87th book.

On India's 60th Independence Day, Dr.Vasantha was conferred the Kalpana Chawla Award for Courage and Daring Enterprise by the State Government of Tamil Nadu in recognition of her sustained fight for social justice in the Indian Institute of Technology (IIT) Madras and for her contribution to mathematics. The award, instituted in the memory of Indian-American astronaut Kalpana Chawla who died aboard Space Shuttle Columbia, carried a cash prize of five lakh rupees (the highest prize-money for any Indian award) and a gold medal.

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The authors use the subset semigroups over the semifields to build semilinear algebras of both finite order and infinite order. The concept of subset linear independence and subset linear dependence which leads to the dimension and basis of subset semilinear algebras is analysed here in this book.

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