The Mass of the Photon

Leonard William Nelson

Vancouver, B.C, Canada E-mail: theory.of.infinity@gmail.com

The mass of a photon is derived. Frequencies of light are shown to represent infinitesimal differences in speed just below c. Formulas of Newton, Einstein, Planck, Lorentz, Doppler and de Broglie for relativity, frequency, energy, velocity-addition and waveforms of matter are all linked using simple mathematical terms into a single set of formulas that all describe the same phenomena: matter, movement and energy. The physical laws governing the astronomically large are the same laws governing the microscopically small.

Contents

1 Introduction

Is it possible that a photon has mass? We will see that the proposition of a massful photon carries far more positive side effects than the idea of a photon without. You will soon see that the mathematics for a photon *with* mass are greatly simplified and reinforce the concept of relativity to a point of being self-evident. The formulas contained in this paper adhere to the straightforwardness that the wisdom of Occam's Razor suggests: they are simple. We will use Occam's Razor as a guiding tool in this discussion.

This paper is organized into two sections. The first is a brief overview that is intended to quickly communicate the topics being discussed and make it clear where this dialogue is headed. The second section repeats the same topics but with full explanations and derivations of formulas. Both start with a presentation of a set of formulas and then end with the reconciliations and proofs. In a nutshell, this paper presents simple, clear-cut mathematical links between formulas that were previously considered unconnected. In the end we have complete unification and a precise measurement of the mass of the photon.

A high precision calculator is required to reproduce some of the calculations presented here. A good tool can be found by searching for "pari gp calculator" on Google. Try setting it to about 100 significant digits.

2 Quick Overview

2.1 Overview of the Formulas

2.1.1 Rate of Time

Rate of time (r_t) is the speed at which time passes for a frame of reference. Two bodies travelling at two different speeds have two different rates of time. The ratio between these two rates of time, is a *factor* that essentially describes *how much relativity* exists from one point of view to the other. In other words, this factor is the famous Lorentz factor.

$$r_t = \sqrt{c^2 - v^2} \;\; where\; \gamma = \frac{r_{t1}}{r_{t2}} = \frac{\sqrt{c^2 - {v_1}^2}}{\sqrt{c^2 - {v_2}^2}} = \frac{1}{\sqrt{1 - \frac{{v_2}^2}{c^2}}}$$

This "rate of time" view is an important concept in relativity that is generally overlooked. We will use this idea extensively in order to help derive the mass of the photon.

2.1.2 Velocity of Frequency

As demonstrated by de Broglie, all matter has a frequency that increases with speed. If a photon has mass, then it is a particle and its speed must be determined by its frequency. Changes in speeds of light would vary by only tiny amounts just below c. The following formula will calculate the velocity of any photon or the frequency of any moving particle. We will demonstrate that this formula converts into the exact same Nobel Prize winning formula that de Broglie found for calculating the wavelength of any moving particle. Use these formulas to convert between velocity and frequency:

$$v = \sqrt{\frac{c^2}{\left(\frac{1}{fc^2} + 1\right)}} \quad and \quad f = \frac{1}{\left(\frac{c^4}{v^2} - c^2\right)}$$

We will denote these formulas with the following functions:

$$f_v(f) = \sqrt{\frac{c^2}{\left(\frac{1}{fc^2} + 1\right)}} \quad and \quad f_f(v) = \frac{1}{\left(\frac{c^4}{v^2} - c^2\right)}$$

2.1.3 Moderate Speed Additive Velocities

Einstein has a formula for adding two relativistic velocities. It is based on Lorentz's length contraction hypothesis. We can completely duplicate this formula by instead using a *velocity-view of time dilation*, which effectively describes the oddity of time dilation as an *expanded speed* - instead of a contracted length (Lorentz), which is effectively the same thing. In other words, it describes time dilation in terms of variations of speed instead of variations in distance - a perfectly mathematically equivalent point of view, given that $t = \frac{d}{v}$. This newly derived formula is based on a simple intuitive construct. We arrive at Einstein's same results by simply converting the two speeds (u, v) to their *proper* speeds (i.e. "Proper Velocity"), adding them together, and then converting that result back to its *perceived* speed. It's quite simple. We "add" the speeds together by increasing one of them by the ratio that the speeds increased. Notice in the following formula that we convert one speed (u) to proper velocity by multiplying it by the Lorentz factor, then we increase it, then we reverse the Lorentz factor to covert back into perceived speed.

$$Einstein: \quad s = \frac{v + u}{1 + \frac{vu}{c^2}} \quad \approx \quad \frac{\left(\frac{v}{u} + 1\right)\gamma u}{\gamma}$$

Where all we did was include the Lorentz factor into what is otherwise classical addition:

$$v + u \ \rightarrow \ \frac{uv}{u} + u \ \rightarrow \ \left(\frac{v}{u} + 1\right)u$$

These formulas are equivalent. We later show exactly how to derive Einstein's additive formula from this formula. They are the same formula, except that this one firmly reinforces the idea that the *velocity-view* is the more accurate description of reality; not length-contraction because with the velocity view we can derive this formula in two easy sentences. Occam's Razor states that two competing theories that make identical predictions, have only one idea that is true: the simpler one. Our conditioning, however, compels us to stick to what we already believe. If we were to rewind time back one hundred years and be presented with both these views for the very first time, which one would *you* believe?

2.1.4 High Speed Additive Velocities

Einstein's additive velocity formula is a replacement for a previous formula developed by Fizeau. Fizeau performed experiments on light that was travelling through water; therefore, Einstein's formula describes how speeds add together - *when travelling through water*. In order to use this formula for *high* speeds of light that are travelling through empty space, we must substitute "refraction of index" in for the velocity of the light. This way, we can set it to "1" for light in a vacuum. Note: you will see that when in vacuum, the resulting percent change in "proper" velocity is precisely the **square root** of the change in the *perceived* velocity. This is a significant discovery because it unites the behaviour of matter with energy. Since energy is known to be tied to proper velocity, Newton's *non-linear* formula for changes in energy ($E = \frac{1}{2}mv^2$) becomes *linear* to match Planck ($E = hf$) because the square becomes nullified. Take Einstein's additive velocity:

$$s = \frac{v + u}{1 + (\frac{vu}{c^2})}$$

Substitute the light with *refraction of index*:

$$u = \frac{c}{n}$$

Which leads to:

$$s = \frac{v + \frac{c}{n}}{1 + \frac{v}{cn}}$$

But, here is the trick. This new arrangement of the formula is identical to Einstein's; however, for light in vacuum, we set the refraction of index (n) to "1" - which would imply the light ($\frac{c}{n}$) is travelling *at* "c" - which is impossible. If a photon has mass, it could never go this fast; it has to remain slightly less. We will substitute the variable L in place of c to represent the speed of the light where $L \approx c$ *and* $L < c$. The variable L is equivalent to c except that it carries the restriction that it cannot actually *equal* c:

$$s = \frac{v + \frac{L}{n}}{1 + \frac{v}{cn}}$$

For all practical purposes, $L = c$. Our *velocity of frequency* formula above tells us that the speed of green light would be:

$$L = .99999999999999999999999999999999 * c$$

We will represent this formula as functions for adding and subtracting speeds, using the following symbols:

$$u \oplus v \quad and \quad u \ominus v$$

So that:

$$\oplus(L, v) = \frac{v + \frac{L}{n}}{1 + \frac{v}{cn}}$$

and

$$\ominus(L, v) = \frac{-v + \frac{L}{n}}{1 + \frac{-v}{cn}}$$

Using these formulas, we can add two speeds (30m/s and green light) and compare the *rate* of change between perceived and proper velocities and see that one is precisely the square root of the other, telling us that changes in high particle speeds result in a linear increase of energy - just like with Planck:

$$\sqrt{\Delta perceived} = 1.0000000500346130279915570059052847215 65$$
$$\Delta proper = 1.0000000500346130279915570059052847215 65$$

This vacuum-addition formula suggests a direct connection between Planck and Newton.

2.1.5 Energy of a Fast Moving Particle

When adding the speeds of fast moving particles, the resulting percent change in proper speed is precisely the *square root* of what would be the change in perceived classical speeds. We see this above with Einstein's additive velocity formula, and we know this must be true *"if"* a photon has mass and the formulas for light and matter are describing the same phenomenon because Planck's linear ($E = hf$) formula must reconcile with Newton's non-linear ($E = 1/2mv^2$) formula. Now, energy is already known to be accurately tied to proper velocity with the following formula which basically just multiplies the speed by the Lorentz factor to get proper velocity ($v\gamma$), before converting it to energy:

$$E = \frac{1}{2}m(v\gamma)^2$$

But, due to the square root effect of how the *proper* speeds add, we can show with a little calculus that it takes exactly twice as much energy to accelerate a particle that is travelling at high speeds, losing the $\frac{1}{2}$ out of Newton's kinetic energy formula. We end up with this formula for the energy of high speed particles, such as light:

$$E = mv^2\gamma^2$$

2.1.6 Mass of the Photon

Now that we have all these formulas, we can simply put them all together to calculate the mass of a photon.

$$Newton: \quad E = mv^2\gamma^2$$
$$Planck: \quad E = hf$$
$$Lorentz: \quad \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$
$$Velocity: \quad v = \sqrt{\frac{c^2}{\left(\frac{1}{fc^2} + 1\right)}}$$

Giving us the mass of a photon, using plain algebra:

$$m = \frac{h}{c^4}$$

where m is constant for all velocities and frequencies, and equals about $8.2 * 10^{-68}$*Kg.*

2.2 Overview of the Proofs and Unifications

2.2.1 Unification 1: Planck with Newton

Now that we know the mass of a photon, we can use either Newton's formula or Planck's formula for calculating the energy of any moving body - whether it be a particle *or* a wave. Either formula will work for either task because they both describe exactly the same phenomenon.

Use Newton to find the energy of a photon ($E = mv^2\gamma^2$):

$$E = \left(\frac{h}{c^4}\right)\left(\sqrt{\frac{c^2}{\left(\frac{1}{fc^2} + 1\right)}}\right)^2 \gamma^2$$

Use Planck to find the energy of a *fast*-moving body ($E = hf$):

$$E = (mc^4)\left(\frac{1}{\left(\frac{c^4}{v^2} - c^2\right)}\right)$$

2.2.2 Unification 2: Everything with de Broglie

Above, we put all the formulas together and saw that they nicely produced the mass of a photon; a simple equation. Soon in the next section, we will put all the formulas together again and reconcile with Doppler. Here in this section, we will put the formulas together and perfectly reconcile with de Broglie's formula for the wavelength of matter - the basis for quantum physics - using only simple algebra. Our new formula for converting between velocity and frequency ***is*** de Broglie's formula for converting between velocity and wavelength, except that here we can now use it for light particles as well.

$$Lorentz: \quad \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$
$$Velocity: \quad v = \sqrt{\frac{c^2}{\left(\frac{1}{fc^2} + 1\right)}}$$
$$Photon\ Mass: \quad m = \frac{h}{c^4}$$

Putting these all together gives us the final reduced equation:

$$\lambda = \frac{h}{mv\gamma}$$

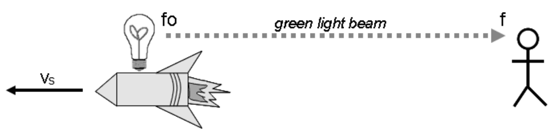
Which is identical to de Broglie:

$$de\ Broglie: \quad \lambda = \frac{h}{p} = \frac{h}{mv\gamma}$$

2.2.3 Unification 3: Einstein with Doppler

We adjusted Einstein's formula earlier for adding velocities. This adjustment allows us to add velocities of particles travelling at *very* high speeds - speeds equivalent to light. Since particles and photons are the same thing, we will now show that Einstein's formula for adding particle speeds produces the same numbers that Doppler's does for light. Start with the Doppler formula:

$$f = \left(1 - \frac{v_s}{c}\right) f_o$$



This formula has variables for two frequencies, one (f_o) for the frequency *before* being reduced by the moving body (v_s), and another for *after* (f). It describes the above scenario where a person observes light shining back from a departing ship.

We now use our velocity of frequency formula $f_v(f_o)$ to convert that frequency into speed, and then use our additive velocity formula for vacuum ($\ominus$) to subtract the spaceship velocity from the initial light velocity, and then convert that resulting total speed back into frequency $f_f(v_t)$:

$$f = f_f(\ f_v(fo) \ominus v_s\ )$$

And we find that:

$$f_f(\ f_v(fo) \ominus v_s\ ) = \left(1 - \frac{v_s}{c}\right) f_o$$

These formulas are equal, confirming that both our *velocity of frequency* formula as well as our *additive velocity in vacuum* formula are correct and that light is a particle with speeds just below c, unifying the laws of energy with the laws of matter. The two formulas are equal to the following accuracy, for a range of values of fo and v_s:

$fo = 1Hz$ and $v_s = 30m/s$	
$f_f(f_v(fo) \ominus v_s)$	0.99999989999307714405543842
$\left(1 - \frac{v_s}{c}\right) f_o$	0.99999989999307714405543851

$fo = 3000Hz$ and $v_s = 3000m/s$	
$f_f(f_v(fo) \ominus v_s)$	2999.96997923143216631553811
$\left(1 - \frac{v_s}{c}\right) f_o$	2999.96997923143216631553819

$fo = 563519657894736Hz$ and $v_s = 296794532.42m/s$	
$f_f(f_v(fo) \ominus v_s)$	5635198458646.608120297942918889
$\left(1 - \frac{v_s}{c}\right) f_o$	5635198458646.608120297942918897

This is known as the "**Champagne Formula**" because it was quite a celebration when it was discovered. It links matter to energy.

2.2.4 Unification 4: Grand Unifying Formula: Matter and Energy

Matter and energy are related in this way: to add high speed velocities together, simply convert those speeds to their energy equivalents, add them, then convert back. The results match both Doppler and Einstein's in vacuum formula. The velocity addition formula for light in vacuum we used above ($\oplus\ominus$) can be completely replaced by a simpler velocity-view version of the formula. This new formula illustrates how matter is related to energy. It also shows just why the square-root effect occurs. This is a essentially a second version of our Champagne formula. To add two high speed velocities of particles that are travelling close to c, simply convert each speed into its energy (via its proper velocity: $E = m(v\gamma)^2$), add them by using the factor of velocity change ($\frac{v_1}{v_2} + 1$), then convert back into proper speed ($v_a = \sqrt{\frac{E}{m}}$) and back into perceived speed ($v_p = \frac{v_a}{\gamma}$). That's it. The result is the same as what we saw earlier with two other formulas, from Einstein and Doppler. To start, set all mass values to "1" for simplicity since the mass doesn't affect anything here.

$$m = 1$$

So that the formula for kinetic energy is:

$$E = (1)v^2\gamma^2$$
$$E = (v\gamma)^2$$

To add or subtract two high speeds by converting to energy ($\bigoplus$ *and* $\bigominus$):

$$\bigoplus(u, v) = \frac{\sqrt{\left(\frac{v}{u} + 1\right)(u\gamma)^2}}{\gamma}$$
$$\bigominus(u, v) = \frac{\sqrt{\left(\frac{-v}{u} + 1\right)(u\gamma)^2}}{\gamma}$$

And with this *new* addition formula ($\oplus$ *and* $\ominus$), we again reconcile with Doppler:

$$f_f(\ f_v(fo) \ominus v_s) = \left(1 - \frac{v_s}{c}\right) f_o$$

And of course with Einstein:

$$f_f(\ f_v(fo) \ominus v_s) = \ f_f(\ f_v(fo) \ominus v_s) = \left(1 - \frac{v_s}{c}\right) f_o$$

In other words, Doppler's formula can easily be expressed in terms of the velocity of particles and the connection between matter and energy is simply that of *velocity* as particles take on more characteristics of a wave as they gain speed - just as de Broglie demonstrated with his Nobel Prize winning formula back in 1929.

3 Detailed Derivations of Formulas

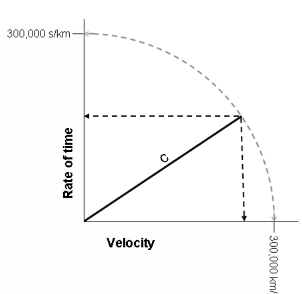
3.1 The Formulas

3.1.1 Rate of Time

"*Rate of Time*" is a crucial variable for working with relativity. The faster-flying twin has a clock that runs slower than his stationary brother's clock - implying that the faster twin's rate of time is less than that of the stationary twin. The relationship between velocity and rate of time just happens to be one involving a constant four vectors: three vectors of directional *velocity* combined with one for rate of time. These vectors have the same behaviour as Minkowski's four *space-time* vectors. Rate of time can be calculated simply by using Pythagoras's formula. We wrap this calculation up into the following function $f_{rt}(v)$.

$$f_{rt}(v) = \sqrt{c^2 - v^2} \quad where \ \ v = \sqrt{{v_x}^2 + {v_y}^2 + {v_z}^2}$$

Which simply calculates the y component of the constant vector c for any given velocity, as shown in the following graph (notice that the combination of all velocity and rate of time vectors always equals c):



By taking the ratio between the rate of time of the observer (who has a velocity of zero) compared to that of the traveller, we wind up with the Lorentz factor - which is the logical conclusion we would *expect* to find when comparing two rates of time. Here we see the Lorentz factor:

$$The\ observer\ has\ a\ velocity\ of\ zero:$$

$$r_t = \sqrt{c^2 - v^2} \tag{1}$$

$$\gamma = \frac{r_{to}}{r_{tt}} \tag{2}$$

$$\gamma = \frac{\sqrt{c^2 - 0^2}}{\sqrt{c^2 - v^2}}$$

$$\gamma = \frac{c}{\sqrt{c^2 - v^2}} \tag{3}$$

$$\gamma^2 = \frac{c^2}{\left(\sqrt{c^2 - v^2}\right)^2}$$

$$\gamma^2 = \frac{c^2}{c^2 - v^2}$$

$$c^2 - v^2 = \frac{c^2}{\gamma^2}$$

$$v^2 = c^2 - \frac{c^2}{\gamma^2}$$

$$v^2 = c^2\left(1 - \frac{1}{\gamma^2}\right)$$

$$-\frac{v^2}{c^2} = \frac{1}{\gamma^2} - 1$$

$$1 - \frac{v^2}{c^2} = \frac{1}{\gamma^2}$$

$$\gamma^2 = \frac{1}{1 - \frac{v^2}{c^2}}$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

If you look closely at the graph, you will discover how time is related to space. Rate of time is the mirror of velocity. Upon further investigation, the reason for Heisenberg's *uncertainty principle* becomes evident. To judge the position of something at a point in time, a camera could be used to catch both the object that is moving as well as the object's local time that the picture was taken. When the photo is shot, no matter how fast the shutter is on the camera, the object will still be blurred to some extent in the photo because the object was moving while the shutter was open. The exact position cannot be determined - the position is somewhere within the width of the blur. The slower the object is travelling, the less blur and the more accurately the position can be determined; however, at slower speeds the rate of time increases. The higher the rate of time, for the same reason as with velocity, the less accurately the exact time can be measured. The accuracy of one measurement is always at the expense of the other.

3.1.2 Velocity of Frequency

The formula to calculate the velocity of any frequency of light depends entirely on finding a connection between velocity and frequency. If we can find a formula that has both frequency (f) and velocity (v) in it, then we'll have our connection. Whatever this formula works out to be, it ***must*** reconcile with de Broglie's formula for the wavelength of moving particles because that is exactly what we are doing - deriving a formula that can calculate the wavelength of a fast-moving photon particle, or any other particle of matter. To achieve this, we are going to begin by working with *factors*. We will compare *changes* in frequency to *changes* in speed. These factors are based on the following simple concept:

$$x_f = \frac{x + \Delta x}{x} \tag{4}$$

$$\therefore x_f = \frac{x_{total}}{x_{initial}} \tag{5}$$

so that:

$$x_{total} = x_{initial} * x_f \tag{6}$$

which means that a 10% increase in x results in a factor (x_f) of 1.1. Given this, we can describe a change in velocity like this:

$$v_f = \frac{v + \Delta v}{v} = \frac{v_{total}}{v_{initial}}$$

And a change in frequency like this:

$$f_f = \frac{f + \Delta f}{f} = \frac{f_{total}}{f_{initial}}$$

Now, since the relationship between energy and frequency is specified by Planck in this linear relationship:

$$E = hf$$

We know that a percent change in frequency leads to the same percent change in energy:

$$E_f = f_f$$

Because (5):

$$E_f = \frac{E_{total}}{E_{initial}}$$

$$E_f = \frac{h * f_{total}}{h * f_{initial}}$$

$$E_f = \frac{h * (f_{initial} * f_f)}{h * f_{initial}} \ \ where \ \ f_{total} = f_{initial} * f_f$$

$$E_f = \frac{h * f_{initial} * f_f}{h * f_{initial}}$$

$$E_f = f_f \tag{7}$$

We now have the formula we need for the frequency side of the connection we seek. Next, we play with the velocity side of things. This will take a few steps. First, we see how energy is related to changes in speed. Newton's formula for kinetic energy is nonlinear; the velocity is squared in order to arrive at the energy. Given this, we know that a change in velocity leads to that change squared in energy as we see here:

$$E_f = {v_f}^2 \tag{8}$$

Because:
Let v_2 be the total speed resulting from a change in v_1:

$$v_2 = v_1 + \Delta v_1$$

And from earlier (4):

$$v_f = \frac{v_1 + \Delta v_1}{v_1}$$

$$v_1 + \Delta v_1 = v_f v_1$$

$$\therefore v_2 = v_1 v_f$$

Now given Newton:

$$E = \frac{1}{2}mv^2$$

We find our factor:

$$\Delta E = E_2 - E_1$$

$$\Delta E = \frac{1}{2}m{v_2}^2 - \frac{1}{2}m{v_1}^2$$

$$\Delta E = \frac{1}{2}m({v_2}^2 - {v_1}^2)$$

$$\Delta E = \frac{1}{2}m((v_1 v_f)^2 - {v_1}^2) \ \ where \ \ v_2 = v_1 v_f$$

$$\Delta E = \frac{1}{2}m({v_1}^2 {v_f}^2 - {v_1}^2)$$

$$\Delta E = \frac{1}{2}m{v_1}^2({v_f}^2 - 1)$$

$$\Delta E = E_1({v_f}^2 - 1) \ \ where \ \ E_1 = \frac{1}{2}m{v_1}^2$$

$$and from \ (4)...$$

$$E_f = \frac{\Delta E + E_1}{E_1}$$

$$E_f = \frac{\Delta E}{E_1} + 1$$

$$E_f = \frac{E_1({v_f}^2 - 1)}{E_1} + 1 \ \ where \ \ \Delta E = E_1({v_f}^2 - 1)$$

$$E_f = ({v_f}^2 - 1) + 1$$

$$E_f = {v_f}^2$$

This relationship for the factor between energy and velocity is for non relativistic movement. When speeds are moderately high, the energy is affected by relativity. It turns out that the energy of a moving body is tied perfectly to the proper velocity. Given this, we can show that $E_f = {v_{af}}^2$ where v_{af} is the change in *actual* velocity (aka proper velocity).
Classical momentum:

$$p = mv$$
$$v = \frac{p}{m}$$

Classical energy in terms of momentum:

$$E = \frac{1}{2}mv^2$$
$$E = \frac{1}{2}m\left(\frac{p}{m}\right)^2$$
$$E = \frac{1}{2}m\left(\frac{p^2}{m^2}\right)$$
$$E = \frac{1}{2}\left(\frac{p^2}{m}\right)$$
$$E = \frac{p^2}{2m}$$

Relativistic momentum:

$$Einstein: \; p = mv\gamma$$

Relativistic energy:

$$E = \frac{p^2}{2m}$$
$$E = \frac{(mv\gamma)^2}{2m}$$
$$E = \frac{(m^2v^2\gamma^2)}{2m}$$
$$E = \frac{1}{2}mv^2\gamma^2$$
$$E = \frac{1}{2}m(v\gamma)^2$$
$$E = \frac{1}{2}m{v_a}^2 \; where \; v_a = v\gamma \tag{9}$$

Relativistic energy change factor:

$$E_f = {v_{af}}^2 \tag{10}$$

This relativistic factor is true because we already established that $E_f = {v_f}^2$ (8) earlier and the relativistic energy formula above is the same formula, except that we use v_a instead of v.

In the next section, we find the connection. To begin, please refer back to the "Rate of Time" graph shown earlier. We mentioned that rate of time is the *mirror* of velocity. This "mirror" suggests that the vertical side of this graph is a sort of reciprocal to the horizontal side. On the horizontal side, however, we have *two* variables: velocity (v) and *actual* velocity (v_a aka Proper velocity where $v_a = v\gamma$). We now hypothesize that there must also be another variable on the time side of the graph, called "*actual* rate of time" (r_{ta}). Whether or not this variable actually means anything in the real universe is irrelevant. We can calculate it anyhow and it is our key to solving the puzzle. It will be calculated in a comparable way to how our *actual* velocity is calculated.

Actual velocity (v_a) is calculated by multiplying perceived velocity (v_p) by gamma. Gamma as we have seen earlier (2) is the ratio between two rates of time:

$$\gamma = \frac{r_{to}}{r_{tt}}$$

Where the observer's speed is always zero:

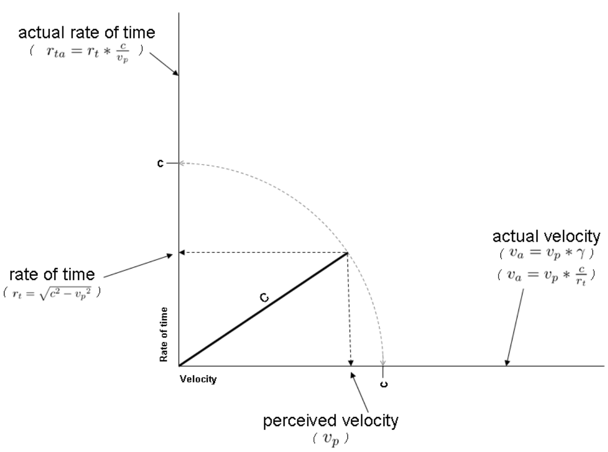
$$r_{to} = \sqrt{c^2 - 0^2} = c$$
$$\gamma = \frac{c}{r_{tt}}$$

So that actual velocity ($v_a = v_p\gamma$) can be written as:

$$v_a = v_p * \frac{c}{r_t}$$

Therefore, the mirror of this on the time axis would be:

$$r_{ta} = r_t * \frac{c}{v_p} \tag{11}$$



The next section derives the relationship between *actual velocity* and this new *actual rate of time* variable. Finding this is just a matter of using algebra to traverse the rate of time graph above. The formula that is derived from this is:

$$v_a = \frac{c^2}{r_{ta}}$$

To do this, we will first define two new functions that we'll use extensively throughout the rest of this paper. One is to

convert from perceived speed into actual, and the other is to covert back again, from actual into perceived.
To convert into actual, it is simply (9):

$$v_a = f_a(v_p) = v_p\gamma \tag{12}$$

To convert back again to perceived from actual, the formula is:

$$v_p = f_p(v_a) = f_a{}^{-1}(v_a) = \sqrt{\frac{c^2 v_a{}^2}{c^2 + v_a{}^2}} \tag{13}$$

Because:

$$Since:$$

$$v_a = v_p\gamma$$

$$And from\ (3):$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{c}{\sqrt{c^2 - v_p{}^2}}$$

$$Then:$$

$$v_a = \frac{v_p c}{\sqrt{c^2 - v_p{}^2}}$$

$$v_a{}^2 = \frac{c^2 v_p{}^2}{(c^2 - v_p{}^2)}$$

$$v_a{}^2(c^2 - v_p{}^2) = c^2 v_p{}^2$$

$$v_a{}^2 c^2 - v_a{}^2 v_p{}^2 = c^2 v_p{}^2$$

$$v_a{}^2 c^2 = c^2 v_p{}^2 + v_a{}^2 v_p{}^2$$

$$v_a{}^2 c^2 = v_p{}^2(c^2 + v_a{}^2)$$

$$\frac{v_a{}^2 c^2}{c^2 + v_a{}^2} = v_p{}^2$$

$$v_p = \sqrt{\frac{c^2 v_a{}^2}{c^2 + v_a{}^2}}$$

Then one more preparation. We can mirror a couple more formulas because they all work the same on the time axis; except that the variables are reversed. First, mirror our $f_p()$ function (13) that we just derived above.

$$since\ v_p = \sqrt{\frac{c^2 v_a{}^2}{c^2 + v_a{}^2}}\ \ then\ \ r_t = \sqrt{\frac{c^2 r_{ta}{}^2}{c^2 + r_{ta}{}^2}}$$

$$since\ r_t = \sqrt{c^2 - v_p{}^2}\ \ then\ \ v_p = \sqrt{c^2 - r_t{}^2}$$

Finally derive the relationship between actual rate of time and actual velocity:

$$v_a = \frac{c^2}{r_{ta}} \tag{14}$$

Because:

$$v_p = \sqrt{c^2 - r_t{}^2}$$

$$v_p = \sqrt{c^2 - \left(\sqrt{\frac{c^2 r_{ta}{}^2}{c^2 + r_{ta}{}^2}}\right)^2}$$

$$v_p = \sqrt{c^2 - \left(\frac{c^2 r_{ta}{}^2}{c^2 + r_{ta}{}^2}\right)}$$

$$v_p{}^2 = c^2 - \frac{c^2 r_{ta}{}^2}{c^2 + r_{ta}{}^2}$$

$$(c^2 + r_{ta}{}^2)(v_p{}^2 - c^2) = -c^2 r_{ta}{}^2$$

$$c^2 v_p{}^2 - c^4 + r_{ta}{}^2 v_p{}^2 - c^2 r_{ta}{}^2 = -c^2 r_{ta}{}^2$$

$$c^2 v_p{}^2 - c^4 + r_{ta}{}^2 v_p{}^2 = 0$$

$$v_p{}^2(c^2 + r_{ta}{}^2) = c^4$$

$$v_p = \sqrt{\frac{c^4}{c^2 + r_{ta}{}^2}}$$

$$v_p = \sqrt{\frac{\frac{c^4}{c^2}}{\frac{c^2}{c^2} + \frac{r_{ta}{}^2}{c^2}}}$$

$$v_p = \sqrt{\frac{c^2}{1 + \frac{r_{ta}{}^2}{c^2}}} \tag{15}$$

Now include our proper velocity (aka *actual* velocity) (9):

$$v_a = v_p\gamma$$

$$And\ from\ (3):$$

$$v_a = \frac{v_p c}{\sqrt{c^2 - v_p{}^2}}$$

$$From\ above\ (15):$$

$$v_a = \frac{c * \sqrt{\frac{c^2}{1+\frac{r_{ta}{}^2}{c^2}}}}{\sqrt{c^2 - \left(\sqrt{\frac{c^2}{1+\frac{r_{ta}{}^2}{c^2}}}\right)^2}}$$

$$v_a{}^2 = \frac{c^2 * \left(\frac{c^2}{1+\frac{r_{ta}{}^2}{c^2}}\right)}{c^2 - \left(\frac{c^2}{1+\frac{r_{ta}{}^2}{c^2}}\right)}$$

$$v_a{}^2 c^2 - v_a{}^2\left(\frac{c^2}{1 + \frac{r_{ta}{}^2}{c^2}}\right) = \frac{c^4}{1 + \frac{r_{ta}{}^2}{c^2}}$$

$$v_a{}^2 c^2 - v_a{}^2\left(\frac{c^2}{1 + \frac{r_{ta}{}^2}{c^2}}\right) = \frac{c^4}{1 + \frac{r_{ta}{}^2}{c^2}}$$

$$v_a^2 c^2\left(1 + \frac{r_{ta}{}^2}{c^2}\right) - v_a^2\left(\frac{c^2}{1 + \frac{r_{ta}{}^2}{c^2}}\right)\left(1 + \frac{r_{ta}{}^2}{c^2}\right) = \frac{c^4}{1 + \frac{r_{ta}{}^2}{c^2}}\left(1 + \frac{r_{ta}{}^2}{c^2}\right)$$

$$v_a{}^2 c^2\left(1 + \frac{r_{ta}{}^2}{c^2}\right) - v_a{}^2 c^2 = c^4$$

$$v_a{}^2 c^2 + \frac{v_a{}^2 c^2 r_{ta}{}^2}{c^2} = c^4 + v_a{}^2 c^2$$

$$\frac{{v_a}^2 c^2 {r_{ta}}^2}{c^2} = c^4$$
$${v_a}^2 {r_{ta}}^2 = c^4$$
$$v_a r_{ta} = c^2$$
$$v_a = \frac{c^2}{r_{ta}}$$
$$r_{ta} = \frac{c^2}{v_a}$$
$$v_a = \frac{c^2}{r_{ta}}$$

Now that we have this relationship, we can turn that into a factor - to see how much *actual rate of time* changes for a change in *actual velocity*.

$$from\ (5):$$
$$r_{taf} = \frac{r_{ta_{total}}}{r_{ta_{initial}}}$$
$$\frac{1}{r_{taf}} = \frac{r_{ta_{initial}}}{r_{ta_{total}}}$$
$$from\ above\ (14):$$
$$v_a = \frac{c^2}{r_{ta}}$$
$$and\ from\ (5): \quad v_{af} = \frac{v_{a_{total}}}{v_{a_{initial}}}$$
$$v_{af} = \frac{\frac{c^2}{r_{ta_{total}}}}{\frac{c^2}{r_{ta_{initial}}}}$$
$$v_{af} = \frac{c^2\ r_{ta_{initial}}}{c^2\ r_{ta_{total}}}$$
$$v_{af} = \frac{1}{r_{taf}} \quad where\ from\ above \quad \frac{1}{r_{taf}} = \frac{r_{ta_{initial}}}{r_{ta_{total}}} \tag{16}$$

With this, we will find that the connection between frequency and velocity is this:

$$f = \frac{1}{{r_{ta}}^2}$$

Because, we know that (10):

$$E_f = {v_{af}}^2$$

And we know that (7):

$$E_f = f_f$$

Therefore:

$$f_f = {v_{af}}^2$$

And since we also know that (16):

$$v_{af} = \frac{1}{r_{taf}}$$

We can see that:

$$f_f = \frac{1}{{r_{taf}}^2}$$

Given this, we now know the relationship between a *change in actual rate of time* and *change in frequency*. Now here's the trick. We make a small leap and hypothesize that this means the following formula is also true:

$$f = \frac{1}{{r_{ta}}^2} \tag{17}$$

At this point, there is no guarantee that this formula is correct. We only know how one variable *changes* in relation to the other, not how they relate statically. We make this hypothesis and then confirm it by simply putting it to the test. It works.

With this connection, it is easy to derive the rest of the velocity-frequency formula. The final formula converting between frequency and velocity is this:

$$v = \sqrt{\frac{c^2}{\left(\frac{1}{fc^2} + 1\right)}}$$

Because:
From earlier we saw that our new variable *actual rate of time* was (11):

$$r_{ta} = r_t * \frac{c}{v_p}$$

And since we also know that (1):

$$r_t = \sqrt{c^2 - {v_p}^2}$$

We can see that:

$$r_{ta} = \frac{c}{v_p}\left(\sqrt{c^2 - {v_p}^2}\right)$$

Now by taking our hypothesis from above (17):

$$f = \frac{1}{{r_{ta}}^2}$$
$$f = \frac{1}{\left(\frac{c}{v}\sqrt{c^2 - v^2}\right)^2}$$
$$f = \frac{1}{\frac{c^2}{v^2}(c^2 - v^2)}$$
$$f = \frac{v^2}{c^2(c^2 - v^2)}$$
$$fc^2 = \frac{v^2}{c^2 - v^2}$$
$$\frac{1}{fc^2} = \frac{c^2 - v^2}{v^2}$$
$$\frac{1}{fc^2} = \frac{c^2}{v^2} - 1$$

$$\frac{1}{fc^2} + 1 = \frac{c^2}{v^2}$$
$$\frac{1}{v^2} = \frac{\left(\frac{1}{fc^2} + 1\right)}{c^2}$$
$$v^2 = \frac{c^2}{\left(\frac{1}{fc^2} + 1\right)}$$
$$v = \sqrt{\frac{c^2}{\left(\frac{1}{fc^2} + 1\right)}}$$

And the reverse of this to solve for frequency is:

$$v = \sqrt{\frac{c^2}{\left(\frac{1}{fc^2} + 1\right)}}$$
$$v^2 = \frac{c^2}{\left(\frac{1}{fc^2} + 1\right)}$$
$$\frac{1}{fc^2} + 1 = \frac{c^2}{v^2}$$
$$\frac{1}{fc^2} = \frac{c^2}{v^2} - 1$$
$$\frac{1}{f} = c^2\left(\frac{c^2}{v^2} - 1\right)$$
$$f = \frac{1}{\left(\frac{c^4}{v^2} - c^2\right)}$$

These two formulas will be denoted with the functions:

$$f_v(f) = \sqrt{\frac{c^2}{\left(\frac{1}{fc^2} + 1\right)}} = v_p \tag{18}$$

And:

$$f_f(v) = f_v^{-1}(v) = \frac{1}{\left(\frac{c^4}{v^2} - c^2\right)} = f \tag{19}$$

3.1.3 Moderate Speed Additive Velocities

Everything works better when we use *proper velocity*. All the math becomes simple; the explanations become comprehensible and new solutions become possible. Proper velocity is the equivalent but alternate variable that can be used instead of the shrunken distance involved in Lorentz's length-contraction hypothesis. Time dilation can be interpreted in either of *two* ways. Lorentz chose the length contraction approach. But, when less time passes for a particle than what passes for us as it travels at a fast speed between two points, it can be interpreted in the following two mathematical ways: 1) that there must be some sort of special shorter distance in the particle's world that I cannot see, or 2) the particle is travelling faster at some special *speed* that I cannot see. Either of these two interpretations work mathematically. One is the length-contraction hypothesis; the other is the velocity-view of time dilation. There is no argument up front that can lean towards one or the other of these two views - they are *equivalent*. Only by following the tangent of each can we see which view works better. The length-contraction hypothesis is a nightmare. It carries all sorts of theoretical side effects, such as the idea that mass must increase with speed, and that the universe of a body in motion *actually* distorts and becomes different than the world we live in. The length-contraction math to describe relativity is complex and unintuitive. The velocity-view, however, works gracefully. The math is much simpler. And the explanation does not carry the nasty side effects we see with the length-contraction hypothesis. For instance, the speed that I "perceive" a body travelling, simply, is not the speed that it is *actually* travelling. This idea is immediately more plausible - because it suggests that the "distortion" of relativity (i.e. the odd size of the traveller's world) is in my *perception* - rather than in the *actual* world of the traveller. Secondly, the velocity-view shows that the reason why more and more energy is required to accelerate a particle as it gains speed, is because we do not *see* it fully accelerate. It actually *does* accelerate according to the energy that is applied to it - maintaining a constant mass; we just don't see that actual speed change. Our *perception* is distorted.

Take for instance, Einstein's *additive velocity* formula. It is based on on the math of the length-contraction hypothesis. The math is pretty complex, as it struggles to work with *speeds* when given only distance as a variable. With the velocity-view, the math is simple. The following formula is equivalent to Einstein's additive velocity formula. The math behind this one, however, is intuitive. Basically, just convert the light speed to its *actual* velocity, add it to the second velocity and then convert back to perceived speed. That's all.
First, let us define the method of *adding* that will be used here and in later formulas. When adding two variables, $v + u$, we can use the *factor* approach and end up with this:

$$total = v + u$$
$$total = \frac{vu}{u} + u$$
$$total = \left(\frac{v}{u} + 1\right)u \tag{20}$$

which is a *factor* that is useful because:

$$from\ earlier\ (6):$$
$$total = (u_f)u$$
$$\left(\frac{v}{u} + 1\right)u = (u_f)u$$
$$\therefore u_f = \left(\frac{v}{u} + 1\right) \tag{21}$$

We will use this method of addition for adding velocities in the following formula but will multiply the factor $(\frac{v}{u} + 1)$

times the *actual* speed of u ($u\gamma$), then convert back to perceived speed by dividing again by gamma. Here is the new velocity-view formula for additive velocities. Note: from this point onward we make use of the earlier functions from (12) and (13) $f_a(v_p)$ and $f_p(v_a)$ for converting back and forth between actual and perceived speeds since it is a much more workable notation.
We begin by increasing *actual* velocity by the factor:

$$u_{a_{total}} = u_{a_{initial}} \left(1 + \frac{v}{u}\right) \tag{22}$$

But add in conversions to and from actual speed:

$$\frac{u\gamma(1 + \frac{v}{u})}{\gamma} \quad or \quad f_p\left(fa(u) * \left(1 + \frac{v}{u}\right)\right)$$

And see that it is equivalent to Einstein:

$$s = \frac{v + u}{1 + \frac{vu}{c^2}} \quad \approx \quad f_p\left(fa(u) * \left(1 + \frac{v}{u}\right)\right)$$

For the above formula, there are no other steps to derive it. We simply write it as we explained it; according to what is intuitive, and then it just works. It's simple. The velocity view works far, far better than the length-contraction hypothesis. We will use this symbol to denote this addition formula:

$$\diamondplus(u, v) = f_p\left(fa(u) * \left(1 + \frac{v}{u}\right)\right) \tag{23}$$

Which can be used like this for adding v and u:

$$s = u \diamondplus v$$

An important observation to note here, is that the above two formulas are not *perfectly* the same. In a formula below, we will add an adjustment to *make* these perfectly the same; however, it is more likely this new formula above (23) is the more correct of the two. The formula above, is not *symmetrical*. Reversing the two variables passed to it results in a different answer. For reasons that are beyond the scope of this paper to fully discuss, it is more likely that real addition of velocities is not symmetrical because in the real world we are dealing with two bodies that are not symmetrical. All calculations in this formula are *relative to the observer* only. This formula cannot be reversed to calculate speeds relative to other points of view. Here is a brief explanation. Wherever there is velocity "addition," there are always *three* frames of reference involved since with *two* frames you can only have *one* speed. Let's name them for simplicity's sake: "you", the "other guy," and the "photon." There are also always three *speeds* involved: the speed between the other guy and the *photon* (u); between the other guy and *you* (v); between *you and the photon* (s). The speed between you and the other guy is also the speed that corresponds to the "speed change" because that is the speed that that is *affecting* the photon. This speed is normally a reasonably small change (i.e., well below c). If we were to reverse the variables u and v, we would be implying that a small speed is being changed *drastically* to become a very high speed, requiring an entirely different amount of energy than the other way around. It also implies that the change in speed spans from our "moderate speed calculation" to the "high speed calculation" as the particle accelerates, requiring a much more complex formula (we will discuss this transition shortly). In the real world, the variables are not swappable.

For the sake of completeness, we will make a minor adjustment to this formula to make it symmetrical. When symmetrical, it is exactly equivalent to Einstein's additive velocity formula. In this example, instead of converting the perceived speed to actual by using our $f_a()$ function as we did above, this time we will do it manually by multiplying the perceived speed by gamma (which is the same thing: $f_a(u) = u\gamma$); however, we *also* multiply it by the gamma of the other speed as well - making the formula symmetrical, even though the gamma for v is generally insignificant ($\gamma \approx 1$). This version suggests that the total amount of "relativity" involved in the addition is from *both* speeds combined. To make this more readable, we add another function for the Lorentz factor calculation (γ) where:

$$f_\gamma(v) = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma$$

So that the above *asymmetrical* formula (23) is written:

$$\begin{gathered} from\ earlier\ (12): \\ u f_\gamma(u) = f_a(u) = u\gamma \\ \therefore s = \frac{v + u}{1 + \frac{vu}{c^2}} \quad \approx \quad f_p\left(u f_\gamma(u)\left(1 + \frac{v}{u}\right)\right) \end{gathered}$$

And the *symmetrical* version of this is:

$$including\ a\ gamma\ for\ v: f_\gamma(v) \approx 1\ since\ v << c$$

$$s = \frac{v + u}{1 + \frac{vu}{c^2}} \quad = \quad f_p\left(u f_\gamma(u) f_\gamma(v)\left(1 + \frac{v}{u}\right)\right)$$

We see that this formula is exactly the same as Einstein's:

$$\begin{aligned} s &= f_p\left(u f_\gamma(u) f_\gamma(v)\left(1 + \frac{v}{u}\right)\right) \\ Let\ \ v_a &= u f_\gamma(u) f_\gamma(v)\left(1 + \frac{v}{u}\right) \\ \therefore s &= f_p(v_a) \end{aligned} \tag{24}$$

$$v_a = u * \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}} * \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} * \left(1 + \frac{v}{u}\right)$$

$$v_a{}^2 = u^2 * \frac{1}{\left(1-\frac{u^2}{c^2}\right)} * \frac{1}{\left(1-\frac{v^2}{c^2}\right)} * \left(1+\frac{v}{u}\right)^2$$

$$v_a{}^2 = \frac{u^2\left(1+\frac{v}{u}\right)^2}{\left(1-\frac{u^2}{c^2}\right)\left(1-\frac{v^2}{c^2}\right)}$$

$$v_a{}^2 = \frac{u^2\left(1+\frac{v}{u}\right)^2}{1-\frac{v^2}{c^2}-\frac{u^2}{c^2}+\frac{u^2v^2}{c^4}}$$

$$v_a{}^2 = \frac{u^2\left(1+\frac{v}{u}\right)^2}{\frac{c^4}{c^4}-\frac{c^2v^2}{c^4}-\frac{c^2u^2}{c^4}+\frac{u^2v^2}{c^4}}$$

$$v_a{}^2 = \frac{u^2\left(1+\frac{v}{u}\right)^2}{\frac{c^4-c^2v^2+u^2(v^2-c^2)}{c^4}}$$

$$v_a{}^2 = \frac{c^4u^2\left(1+\frac{v}{u}\right)^2}{c^4-c^2v^2+u^2(v^2-c^2)}$$

$$v_a{}^2 = \frac{c^4u^2\left(1+2\frac{v}{u}+\frac{v^2}{u^2}\right)}{c^4-c^2v^2+u^2(v^2-c^2)}$$

$$v_a{}^2 = \frac{c^4u^2\left(\frac{u^2+2vu+v^2}{u^2}\right)}{c^4-c^2v^2+u^2(v^2-c^2)}$$

$$v_a{}^2 = \frac{c^4u^2\left(\frac{(v+u)^2}{u^2}\right)}{c^4-c^2v^2+u^2(v^2-c^2)}$$

$$v_a{}^2 = \frac{c^4(v+u)^2}{c^4-c^2v^2+u^2(v^2-c^2)}$$

$And\ since\ from\ above$ (24) and (13) :

$$s = f_p(v_a) = v_a\gamma^{-1} = \sqrt{\frac{c^2v_a{}^2}{c^2+v_a{}^2}}$$

$Then$:

$$s = f_p(v_a) = \sqrt{\frac{c^2\left(\frac{c^4(v+u)^2}{c^4-c^2v^2+u^2(v^2-c^2)}\right)}{c^2+\left(\frac{c^4(v+u)^2}{c^4-c^2v^2+u^2(v^2-c^2)}\right)}}$$

$$s = \sqrt{\frac{\left(\frac{c^6(v+u)^2}{c^4-c^2v^2+u^2(v^2-c^2)}\right)}{\left(\frac{c^2\left(c^4-c^2v^2+u^2(v^2-c^2)\right)}{c^4-c^2v^2+u^2(v^2-c^2)}\right)+\left(\frac{c^4(v+u)^2}{c^4-c^2v^2+u^2(v^2-c^2)}\right)}}$$

$$s = \sqrt{\frac{\left(\frac{c^6(v+u)^2}{c^4-c^2v^2+u^2(v^2-c^2)}\right)}{\left(\frac{c^2\left(c^4-c^2v^2+u^2(v^2-c^2)\right)\ +\ c^4(v+u)^2}{c^4-c^2v^2+u^2(v^2-c^2)}\right)}}$$

$$s = \sqrt{\frac{(c^6(v+u)^2)\,(c^4-c^2v^2+u^2(v^2-c^2))}{(c^4-c^2v^2+u^2(v^2-c^2))\,(c^2\,(c^4-c^2v^2+u^2(v^2-c^2))+c^4(v+u)^2)}}$$

$$s = \sqrt{\frac{c^6(v+u)^2}{c^2\,(c^4-c^2v^2+u^2(v^2-c^2))+c^4(v+u)^2}}$$

$$s = \sqrt{\frac{c^4(v+u)^2}{(c^4-c^2v^2+u^2(v^2-c^2))+c^2(v+u)^2}}$$

$$s = \sqrt{\frac{c^4(v+u)^2}{(c^4-c^2v^2+u^2v^2-u^2c^2)+c^2(v+u)^2}}$$

$$s = \sqrt{\frac{c^4(v+u)^2}{c^2\left(c^2-v^2+\frac{u^2v^2}{c^2}-u^2\right)+c^2(v+u)^2}}$$

$$s = \sqrt{\frac{c^2(v+u)^2}{\left(c^2-v^2+\frac{u^2v^2}{c^2}-u^2\right)+(v+u)^2}}$$

$$s = \sqrt{\frac{c^2(v+u)^2}{\left(c^2-v^2+\frac{u^2v^2}{c^2}-u^2\right)+(v^2+2vu+u^2)}}$$

$$s = \sqrt{\frac{c^2(v+u)^2}{c^2+\frac{u^2v^2}{c^2}+2vu}}$$

$$s = \sqrt{\frac{(v+u)^2}{\left(\frac{c^2+\frac{u^2v^2}{c^2}+2vu}{c^2}\right)}}$$

$$s = \sqrt{\frac{(v+u)^2}{\frac{c^2}{c^2}+\frac{2vu}{c^2}+\frac{u^2v^2}{c^4}}}$$

$$s = \sqrt{\frac{(v+u)^2}{\left(1+\frac{vu}{c^2}\right)^2}}$$

$$s = \frac{v+u}{1+\frac{vu}{c^2}}$$

3.1.4 High Speed Additive Velocities

Einstein's additive velocity formula is for the addition of two *moderate* speeds; speeds well beyond those which we experience from day to day but well under that of light. Einstein's formula is a modern version of an equation that Fizeau derived many years earlier. It calculates the combined speed of water and of light that is travelling through it. What this formula *doesn't* calculate, however, are changes in speeds of light that are travelling in vacuum.

We know from earlier on that Planck's energy formula changes linearly with changes in frequency ($E_f = f_f$). We also know that Newton's energy of a moving body formula will change *exponentially* with velocity ($E_f = v_f{}^2$). If light has mass, then these two formulas must be describing the same thing; however, they have two very different behaviours: linear versus nonlinear. We could reconcile these two equations if somehow it turned out that when two fast speeds are added together, the formula looked like this:

$$E_f = \left(\sqrt{v_f}\right)^2 \qquad (25)$$

Where the speed change is first reduced to its *square-root* before being squared into its kinetic energy. This would result in a linear relationship between v_f and E_f, just like we see with Planck (7). This behaviour is in fact exactly how nature

works. If we implement the refraction of index (n) into Einstein's formula and set it to 1 for vacuum, then actual speeds increase by precisely the square root of the classical change in perceived speeds. Have a look:
Start with Einstein's formula and then swap out the light's speed with refraction of index:

$$s = \frac{v+u}{1+(\frac{vu}{c^2})}$$
$$where \;\; u = \frac{c}{n}$$
$$s = \frac{v+\frac{c}{n}}{1+\left(\frac{v\frac{c}{n}}{c^2}\right)}$$
$$s = \frac{v+\frac{c}{n}}{1+\left(\frac{v}{c}\right)\left(\frac{\frac{c}{n}}{c}\right)}$$
$$s = \frac{v+\frac{c}{n}}{1+\left(\frac{v}{c}\right)\left(\frac{1}{n}\right)}$$
$$s = \frac{v+\frac{c}{n}}{1+\frac{v}{cn}}$$

We can now specify the speed of the light (u) by instead using the refraction of index n. We will set $n = 1$ for vacuum; however, if $n = 1$ then u must equal c since $u = \frac{c}{n}$. This cannot be. By the rules of relativity anything with mass must always be less than c. We must fix this formula to account for the fact that light travels slightly less than c. We will swap out the c and replace it with L where $L \approx c$ and add a restriction that it can never equal c such that: $L < c$. L and c are effectively the same thing and the formula does not change, except that we have implemented a restriction on L. Our velocity of frequency formula from earlier (18) tells us that the velocity of green light is extremely close to c: $L = .9999999999999999999999999999999990127 * c$.
Swap L for c:

$$s = \frac{v+\frac{L}{n}}{1+\frac{v}{cn}}$$
$$where\ L \approx c\ and\ L < c$$

This formula will be denoted with a $\oplus$ symbol. For subtraction, the $\ominus$ symbol will be used:

$$\oplus(L,v) = \frac{v+\frac{L}{n}}{1+\frac{v}{cn}} \tag{26}$$
$$\ominus(L,v) = \frac{-v+\frac{L}{n}}{1+\frac{-v}{cn}} \tag{27}$$

So that:

$$s = u \oplus v$$

Watch what happens when we compare the factor of velocity change between perceived and actual speeds. Take two speeds, v and u (water and light) and add them together:

$$v = 30$$
$$u = .9999999999999999999999999999999990127 * c$$
$$u = 299,792,457.999,999,999,999,999,999,999,997,040,35\boldsymbol{0,850,897}$$
$$s = u \oplus v$$
$$s = 299,792,457.999,999,999,999,999,999,999,997,040,35\boldsymbol{1,147,067}$$

Now, have a look at the factors. Using equations from earlier, we calculate factors for both the *perceived* speed change (u_f) and *actual* speed change (u_{af}):

$$u_f = \frac{u_{total}}{u_{initial}} \quad from\ (5)$$
$$\therefore u_f = \frac{v}{u}+1 \quad from\ (21)$$
$$u_{af} = \frac{u_{a_{total}}}{u_{a_{initial}}} \quad from\ (5)$$
$$\therefore u_{af} = \frac{f_a(s)}{f_a(u)} \quad from\ (12)$$

And we see that the change in actual speed is precisely the *square root* of the change in perceived:

$$\sqrt{u_f} = 1.0000000500346130279915570059052847215 65$$
$$u_{af} = 1.0000000500346130279915570059052847215 65$$

In other words, the speed that the light *actually* changes is the square-root of the change we would see according to classical physics. For example, when light is shone forward from a moving vehicle, where that vehicle is moving at 10% of the speed of that light, the *actual* velocity increases by about 4.9% (i.e. $\sqrt{1.10}$). We can also describe this by saying that high velocity relativistic speed change is the square root of classical speed change. Also, we might say, the speed change in the frame of the traveller, when in vacuum, is the square root of the speed change that is perceived from the frame of the observer. This is demonstrated simply by comparing the perceived speed change to the actual. The result is this relationship:

$$v_{af} = \sqrt{v_{pf}} \tag{28}$$

And since we know (10):

$$E_f = {v_{af}}^2$$

Then what he hoped for in (25) is true:

$$E_f = \left(\sqrt{v_{pf}}\right)^2$$
$$E_f = v_{pf}$$

Which is for high speeds close to c in vacuum. It tells us that energy changes *linearly* with changes in the speeds we see when dealing with light particles in vacuum. It means that we have reconciliation of behaviour between Newton and Planck.

We now have a formula for adding high speed velocities of light particles. We also have a formula for calculating the velocity for any given frequency. If light has mass then it is just a particle like any other, and we should be able to use Einstein's additive velocity to do the same job as Doppler's. Changes in velocity and changes in frequency are the same thing, and we already have partial reconciliation between the behaviour (i.e. both formulas are linear in regards to the energy changes involved).

3.1.5 Energy of a Fast Moving Particle

The previous section demonstrated that the addition of high speed velocities results in changes in actual speed that are the square root of changes in perceived speed. There is also an interesting mathematical pattern where taking the square root of any small *factor* is the same as dividing the decimal portion by 2 (ex. $\sqrt{1.0000000000000008} \approx 1.0000000000000004$). This is demonstrated here with some basic calculus but what it tells us is that for speed changes, the *actual* speed change is *half* the perceived change. It means it takes twice as much energy to accelerate a high speed particle than it takes to accelerate a low speed one. For this reason, there is twice as much energy in a high speed particle than there ought to be given Newton's kinetic energy formula, and so the formula for high speed kinetic energy close to c is:

$$E = m{v_a}^2$$

Because if we start with our speed change factor (21):

$$u_f = \frac{v}{u} + 1$$

But since for high speeds, the resulting actual change is (28):

$$u_{af} = \sqrt{\frac{v}{u} + 1}$$

Then the linear approximation for changes in $\frac{v}{u}$ around zero shows that:

$$\begin{aligned}
Let \;\; f(x) = \sqrt{x+1} \;\; where \; x = \frac{v}{u} \\
f(x) \approx f'(a)(x-a) + f(a) \\
f(x) \approx f'(0)(x-0) + f(0) \\
f(x) \approx \frac{1}{2\sqrt{0+1}}(x-0) + \sqrt{0+1} \\
f(x) \approx \frac{x}{2} + 1 \\
f(x) \approx \frac{\left(\frac{v}{u}\right)}{2} + 1
\end{aligned}$$

Which is exactly half of a normal low speed energy change:

$$f(x) = x + 1$$

$$f(x) = \frac{v}{u} + 1$$

Leading to our *high-speed* kinetic energy formula that requires twice as much energy to accelerate a particle (where $v \approx c$):

$$\begin{aligned}
&from\ (9) \\
&E = (2)\frac{1}{2}mv^2\gamma^2 \\
&E = mv^2\gamma^2 \qquad (29) \\
&or \\
&E = m(v\gamma)^2 \\
&E = m{v_a}^2 \qquad (30)
\end{aligned}$$

3.1.6 Mass of the Photon

From the formulas we have described thus far, we are now able to calculate the precise mass of a photon. We make use of the following formulas to accomplish this:

Newton:	$E = mv^2\gamma^2$	$from$ (29)
Planck:	$E = hf$	
Lorentz:	$\frac{c}{\sqrt{c^2 - v^2}}$	
Velocity:	$v = \sqrt{\frac{c^2}{\left(\frac{1}{fc^2} + 1\right)}}$	$from$ (18)

Calculate the mass of a photon:

$$\begin{aligned}
E &= mv^2\gamma^2 \\
m &= \frac{E}{v^2\gamma^2} \\
m &= \frac{hf}{\left(\sqrt{\frac{c^2}{\frac{1}{fc^2}+1}}\right)^2 * \left(\frac{c}{\sqrt{c^2-v^2}}\right)^2} \\
m &= \frac{hf}{\left(\frac{c^2}{\frac{1}{fc^2}+1}\right) * \left(\frac{c^2}{c^2-v^2}\right)} \\
m &= \frac{hf}{\frac{c^4}{\frac{c^2}{fc^2} - \frac{v^2}{fc^2} + c^2 - v^2}} \\
mc^4 &= hf\left(\frac{c^2 - v^2}{fc^2} + c^2 - v^2\right) \\
mc^4 &= \frac{hf(c^2 - v^2)}{fc^2} + hfc^2 - hfv^2 \\
mc^4 &= \frac{hfc^2}{fc^2} - \frac{hfv^2}{fc^2} + hfc^2 - hfv^2 \\
mc^4 &= h - \frac{hv^2}{c^2} + hfc^2 - hfv^2
\end{aligned}$$

$$substituting\ldots\ v^2 = \left(\sqrt{\frac{c^2}{\frac{1}{fc^2}+1}}\right)^2 = \left(\frac{c^2}{\frac{1}{fc^2}+1}\right)$$

$$mc^4 = h - \frac{h}{c^2}\left(\frac{c^2}{\frac{1}{fc^2}+1}\right) + hfc^2 - hf\left(\frac{c^2}{\frac{1}{fc^2}+1}\right)$$

$$mc^4 = h - \frac{h}{\frac{1}{fc^2}+1} + hfc^2 - \frac{hfc^2}{\frac{1}{fc^2}+1}$$

$$mc^4 - h - hfc^2 = \frac{-h - hfc^2}{\frac{1}{fc^2}+1}$$

$$(mc^4 - h - hfc^2)(\frac{1}{fc^2}+1) = -h - hfc^2$$

$$\frac{mc^4}{fc^2} - \frac{h}{fc^2} - \frac{hfc^2}{fc^2} + mc^4 - h - hfc^2 = -h - hfc^2$$

$$\frac{mc^4 - h - hfc^2}{fc^2} = -mc^4$$

$$\frac{mc^4 - h}{fc^2} - h = -mc^4$$

$$mc^4 - h = fc^2(-mc^4 + h)$$

$$mc^4 - h = -fc^2mc^4 + hfc^2$$

$$mc^4 + fc^2mc^4 = h + hfc^2$$

$$mc^4(fc^2+1) = h(fc^2+1)$$

$$m = \frac{h(fc^2+1)}{c^4(fc^2+1)}$$

Giving us the precise mass of a photon:

$$m = \frac{h}{c^4} \tag{31}$$

Where m is constant for all velocities and frequencies, and equals about $8.2 * 10^{-68} Kg$ which is right in the neighbourhood of where it ought to be given current conjecture of what its maximum size could be. Note again that the velocity-view of time dilation does not have the side effect where mass increases with speed; mass is constant.

3.2 The Proofs and Unifications

3.2.1 Unification 1: Planck with Newton

Since a photon has mass, it is nothing more than a small particle of matter that is travelling very fast. Its structure relative to a stationary observer transitions more and more into a wave the faster it moves (de Broglie). We can now use Planck's ($E = hf$) formula for energy of frequencies interchangeably with Newton's formula for kinetic energy. We must always use the *actual* velocity of the particle in this formula and also double the energy as we saw earlier due to the square root effect of velocity addition ($E = mv_a{}^2$).

Newton (29): $E = mv^2\gamma^2 \quad = mv_a{}^2$
Planck: $E = hf$
Therefore: $hf = mv^2\gamma^2$

We can find the energy of a photon with Newton using (31) and (18):

$$m = \frac{h}{c^4} \qquad and \qquad v = \sqrt{\frac{c^2}{\left(\frac{1}{fc^2}+1\right)}}$$

For Newton's formula ($E = mv^2\gamma^2$) where:

$$E = \left(\frac{h}{c^4}\right)\left(\sqrt{\frac{c^2}{\left(\frac{1}{fc^2}+1\right)}}\right)^2 \gamma^2$$

And the kinetic energy of a particle with Planck using (31) and (19):

$$m = \frac{h}{c^4} \quad\rightarrow\quad h = mc^4 \quad and \quad f = \frac{1}{\left(\frac{c^4}{v^2} - c^2\right)}$$

For Planck's formula ($E = hf$) where:

$$E = (mc^4)\left(\frac{1}{\left(\frac{c^4}{v^2} - c^2\right)}\right)$$

3.2.2 Unification 2: Everything with de Broglie

Earlier we figured out an alternative to Lorentz's factor using *rate of time* and used this information to derive a formula for calculating the velocity of any frequency. We then figured out a perfectly equivalent and simple alternative to Einstein's additive velocity formula, after which we altered Einstein's formula to work with particles in vacuum, illustrating a *square root effect* that we then used to alter Newton's kinetic energy formula for high speeds. Using these formulas, we derived the mass of a photon using simple algebra. We will now take all these formulas and verify their accuracy by showing how they perfectly combine to produce de Broglie's formula for the waveform of matter - the basis for quantum physics. This is an extremely significant proof of the accuracy of these preceding formulas. Our velocity-of-frequency formula *must* be the same formula as de Broglie's wavelength of matter formula because they are measuring the same thing: matter that is moving.

Using the mass of a photon and the velocity formula we derived above, we arrive at de Broglie's Nobel Prize winning formula. Start with the following formulas.

Frequency of speed from above (19):

$$f = \frac{1}{\left(\frac{c^4}{v^2} - c^2\right)}$$

And mass of the photon (31):

$$h = mc^4$$

And standard conversion from frequency to wavelength:

$$\lambda = \frac{v}{f}$$

We end up with de Broglie:

$$\lambda = \frac{h}{p} = \frac{h}{mv\gamma}$$

Because:

$$f = \frac{1}{\left(\frac{c^4}{v^2} - c^2\right)}$$

$$f = \frac{(v^2)1}{(v^2)\left(\frac{c^4}{v^2} - c^2\right)}$$

$$f = \frac{v^2}{v^2c^2\left(\frac{c^2}{v^2} - 1\right)}$$

$$f = \frac{v^2}{c^2\left(\frac{c^2v^2}{v^2} - v^2\right)}$$

$$f = \frac{v^2}{c^2\left(c^2 - v^2\right)}$$

$$f = \frac{(\gamma)v^2}{(\gamma)c^2\left(c^2 - v^2\right)}$$

$$f = \frac{\gamma v^2}{\left(\frac{c}{\sqrt{c^2-v^2}}\right)c^2\left(c^2 - v^2\right)}$$

$$f = \frac{\gamma v^2}{c^3\frac{(c^2-v^2)}{\sqrt{c^2-v^2}}}$$

$$f = \frac{\gamma v^2}{c^3\sqrt{c^2 - v^2}}$$

$$\textit{But since}: \sqrt{c^2-v^2} \approx c \quad \textit{when } v << c \ (\textit{see footnote}^*):$$

$$f = \frac{\gamma v^2}{c^4}$$

$$f = \frac{v(v\gamma)}{c^4}$$

$$f = \frac{v}{\left(\frac{c^4}{v\gamma}\right)}$$

$$f = \frac{v}{\left(\frac{(m)c^4}{(m)v\gamma}\right)}$$

$$f = \frac{v}{\left(\frac{h}{mv\gamma}\right)} \quad where\ mc^4 = h$$

$$\lambda = \frac{v}{\left(\frac{v}{\left(\frac{h}{mv\gamma}\right)}\right)} \quad where\ \lambda = \frac{v}{f}$$

$$\lambda = \frac{h}{mv\gamma}$$

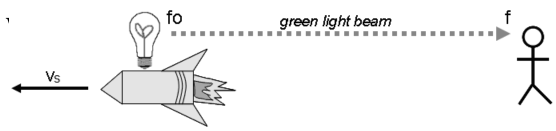
3.2.3 Unification 3: Einstein with Doppler

Given that matter and energy are the same thing and frequency is equivalent to velocity, we should be able to use Doppler's formula for adding frequency interchangeably with Einstein's formula for adding velocities. Here we see the *Champagne* formula:

$$f_f(\ f_v(fo) \ominus v_s\ ) = \left(1 - \frac{v_s}{c}\right) f_o$$

We start with the Doppler formula for a body that is moving away from the observer:

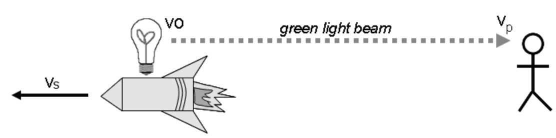
$$f = \left(1 - \frac{v_s}{c}\right) f_o$$



Where we see green light shining back from a moving body towards a stationary person. This light will be red-shifted when observed by the person on the right. The initial frequency of the light is fo while the observed frequency is reduced to f. Given that we are able to convert between frequency and velocity, we can describe this same scenario in terms of speeds. Here we have vo representing the velocity of the light relative to the spaceship, and v_p as the perceived speed. Starting with our formula for converting frequency to velocity (18):

$$v_o = f_v(f_o)$$

We can redraw our diagram in terms of velocity:



And we can state the following about this diagram:

$$v_p = vo \ominus v_s$$

So that v_p is the resulting velocity of the perceived light (f). If we convert that resulting total velocity (v_p) back into frequency (19):

$$f = f_f(v_p)$$

Then we find that this is the same value from Doppler:

$$f = \left(1 - \frac{v_s}{c}\right) f_o$$

*An electron's speed (v) is about $6 * 10^6 m/s$ which is only 2% of light speed resulting in an approximation ($\sqrt{c^2 - v^2} \approx c$) that is 99.98% accurate.

Giving us the complete Champagne formula:

$$f_f(\ f_v(fo) \ominus v_s\ ) = \left(1 - \frac{v_s}{c}\right) f_o \tag{32}$$

These two formulas are equal to the following accuracy, for a brief range of values of fo and v_s (note: in the next section, we add a *third* equivalent formula to this list):

$fo = 1Hz$ and $v_s = 30m/s$	
$f_f(f_v(fo) \ominus v_s)$	0.99999989999307714405543842
$\left(1 - \frac{v_s}{c}\right) f_o$	0.99999989999307714405543851

$fo = 3000Hz$ and $v_s = 3000m/s$	
$f_f(f_v(fo) \ominus v_s)$	2999.96997923143216631553811
$\left(1 - \frac{v_s}{c}\right) f_o$	2999.96997923143216631553819

$fo = 563519657894736Hz$ and $v_s = 296794532.42m/s$	
$f_f(f_v(fo) \ominus v_s)$	5635198458646.608120297942918889
$\left(1 - \frac{v_s}{c}\right) f_o$	5635198458646.608120297942918897

The higher the speeds are, the more accurate the calculation because these formulas are for *high speeds only*. Both the Doppler formula as well as the additive-velocity-in-vacuum formula contain the square root effect. *Low speeds* do not have this effect. There is a transition from the low-speed addition calculation to the high-speed addition calculation as velocities increase, which we will discuss soon.

This *Champagne* formula significantly unites the laws of matter with those for energy. It shows us that the Doppler formula, as is it stands, is *already* relativistic and that there is no need for an additional "relativistic" version. It reaffirms that both our velocity-of-frequency formula is accurate as well as the additive velocity formula in vacuum. It demonstrates that matter and energy are the same thing and that the Doppler is tied via these formulas directly to *rate of time* (1). Even more notable, however, is that this tells us that time dilation is *not* affected by gravity. If gravity affected time, then the Doppler formula would be gravity-dependent - but it is not. The Doppler formula is incredibly consistent for all light travelling all across the universe, under all types of gravitational influences. It works the same here on Earth as it does on the moon and in all directions. It means that rate of time (*aka time dilation*) is **strictly** tied to velocity. Velocity and rate of time are the perfect mirror of each other and gravity has nothing to do with relativity. This is a good time to note also that the path of a massful photon does not require *space to bend* in order for it to curve by a star (*Einstein/Eddington*). The path of light bends near a star - because gravity pulls on it like it pulls on *all other matter*. There is no longer any reason to suggest that "space bends." It should also be pointed out that the formula $E = mc^2$ incorporates math for mass that increases with speed, but as we have seen, mass is constant across all velocities. The accurate formula is $E = m{v_a}^2$ (30) which, aside from the square-root effect for high speed velocity addition, is pretty much dead on to what Newton gave us - except we now recognize that we must use the *actual* speed that an object is travelling, not the velocity we perceive (i.e. use the velocity from the *traveller's* frame of reference, not its speed relative to ours).

3.2.4 Unification 4: Grand Unifying Formula: Matter and Energy

The *grand unifying formula* is essentially another version of the additive velocity in vacuum formula we used earlier. In a previous step, we replaced Einstein's *moderate* additive speed formula with another that is based on the velocity-view of time dilation (23). Now, we replace his *high* velocity (in vacuum) formula with a new one that is also based on the velocity-view. We will now have *five* addition formulas in total: two different *types* of relativity-addition (fast speeds versus low/moderate speeds), each with two formulas (length contraction versus velocity-view) - plus the Doppler formula as a third way of adding high speeds. This new *grand unifying* high speed formula, however, is intuitive and simple. We will add two high speeds together simply by adding their energies together. The resulting speed is the same result returned by **both** Einstein's high speed additive velocity (in vacuum) formula as well as the Doppler formula. This formula makes use of the earlier velocity-view addition formula technique as well as the velocity of frequency formula and makes it very clear just how matter and energy are related, as well as identifies exactly how the square-root effect takes place. This formula brings everything together. This formula works like this. For high speeds, particles are primarily in the form of a wave. They are energy. Adding two energies together is a bit different than adding the energies of two moving bodies together. For energy, $E_t = E_1 + E_2$, except that we will use a factor to add the two speeds together like we did earlier with the moderate velocity addition formula. To calculate the resulting speed of two high velocities, simply, convert the light's velocity into its actual speed ($v_a = v\gamma$), then convert into energy ($E = m{v_a}^2$), then increase the energy in the same sort of way we did earlier based on the factor of change between the two perceived speeds $E_t = (\frac{v_1}{v_2} + 1)E$, and finally convert back into actual and then perceived speed. For the following calculations, we will assume a mass of 1. This simplifies our math since mass has no relevance on the speed changes.
Add two speeds:

$$v \oplus u$$

Convert to actual speed (12):

$$v_a = f_a(u)$$

Convert to energy (30) (assume $m = 1$):

$$E = m{v_a}^2$$

$$E = (1){v_a}^2$$
$$E = {v_a}^2 \qquad (33)$$

Increase the energy by the factor of speed change (21):

$$E_t = E\left(\frac{v}{u} + 1\right) \qquad (34)$$

Convert back into actual speed (33):

$$E_t = {v_{at}}^2$$
$$v_{at} = \sqrt{E_t}$$

And finally convert back into perceived speed to get the total of the two speeds added together (13):

$$s = f_p(v_{at})$$

All together:

$$s = f_p\left(\sqrt{\left(\frac{v}{u} + 1\right) f_a(u)^2}\right)$$

Or, in other equivalent more familiar notation:

$$s = \frac{\sqrt{\left(\frac{v}{u} + 1\right)(u\gamma)^2}}{\gamma}$$

We use these symbols for addition and subtraction using this new formula ($\oplus$ *and* $\ominus$):

$$\oplus(u,v) = f_p\left(\sqrt{\left(\frac{v}{u} + 1\right) f_a(u)^2}\right) \qquad (35)$$
$$\ominus(u,v) = f_p\left(\sqrt{\left(\frac{-v}{u} + 1\right) f_a(u)^2}\right)$$

So that with this *new* addition formula ($\oplus$ *and* $\ominus$), we again reconcile with Doppler:

$$f_f(\ f_v(fo) \ominus v_s) = \left(1 - \frac{v_s}{c}\right) f_o$$

Where this formula reconciles with *both* the Doppler as well as the Einstein additive-velocity-in-vacuum formulas for a range of values of v_s and f_o:

$fo = 1Hz$ and $v_s = 30m/s$	
$f_f(f_v(fo) \ominus v_s)$	0.99999989999307714405543842
$\left(1 - \frac{v_s}{c}\right) f_o$	0.99999989999307714405543851
$f_f(f_v(fo) \ominus v_s)$	**0.99999989999307714405543845**

$fo = 3000Hz$ and $v_s = 3000m/s$	
$f_f(f_v(fo) \ominus v_s)$	2999.96997923143216631553811
$\left(1 - \frac{v_s}{c}\right) f_o$	2999.96997923143216631553819
$f_f(f_v(fo) \ominus v_s)$	**2999.96997923143216631553814**

$fo = 563519657894736Hz$ and $v_s = 296794532.42m/s$	
$f_f(f_v(fo) \ominus v_s)$	5635198458646.608120297942918889
$\left(1 - \frac{v_s}{c}\right) f_o$	5635198458646.608120297942918897
$f_f(f_v(fo) \ominus v_s)$	**5635198458646.608120297942918892**

3.2.5 Doppler Formula Derived from Particle Velocity Addition

Now that we have this velocity-view version of high speed addition, notice here how it is actually the same formula as the Doppler formula. Taking our formula from above (35):

$$\oplus(u,v) = f_p\left(\sqrt{\left(\frac{v}{u} + 1\right) f_a(u)^2}\right)$$

If we use use E for the energy involved (30) and (12) where m=1:

$$E = m(u\gamma)^2 = 1 * f_a(u)^2$$

And cut back this calculation to leave the results as energy without converting back into actual and perceived speeds (i.e. the square-root above converts energy into speed and the $f_p()$ converts it into perceived speed - remove these steps. See earlier (34)):

$$E_t = \left(1 + \frac{v}{u}\right) E_u$$

Now have a look at Doppler. We will mix Planck's formula into Doppler:

$$E = hf$$
$$f = \frac{E}{h}$$
$$f = (1 + \frac{v}{c}) f_o$$
$$\frac{E_t}{h} = (1 + \frac{v}{c})\frac{E_{f_o}}{h}$$
$$E_t = (1 + \frac{v}{c}) E_{f_o}$$

Now, if we consider that the speed of the light in Doppler's formula (c) is actually always less than c for a photon with mass, then this variable becomes the speed of f_o where $u = f_v(f_o)$. And we have this:

$$E_t = (1 + \frac{v}{u}) E_{f_o}$$

Which is the same as our additive velocity formula.

Unification is really quite simple. Matter transitions into energy with speed; therefore, to add two high speeds together - add them as energy. Where this becomes a bit more complicated is for particles travelling at speeds somewhere in the middle; below *high* speeds but above *moderate* speeds. These particles are both part matter and part energy. Both our moderate and high speed formulas need to work together, each contributing its own share of the resulting total speed change based on each's relevance to the speed involved. In the next section, we discuss this and provide an equation for approximating this transition between matter and energy.

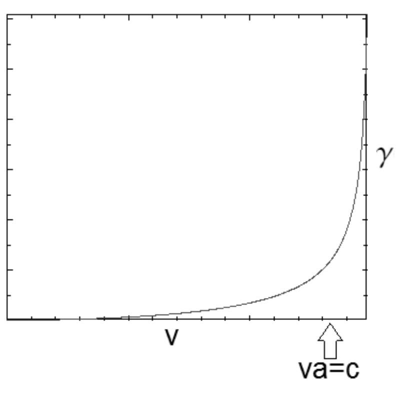
3.2.6 Transitioning from Matter to Energy

We have two types of formulas: addition of *moderate* speed velocities versus addition of *high* speed velocities (energy). As a particle increases in speed, it must transition between these two equations. There is essentially a playoff between these two formulas (22) and (34); one for change in actual speed (for moderate speeds) while another for change in energy (for high speeds):

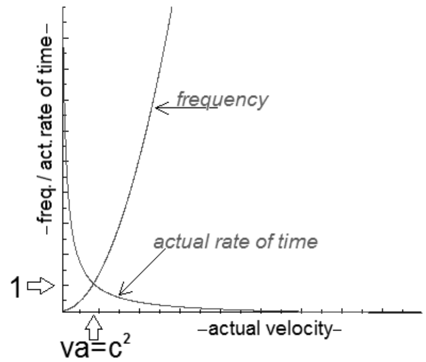
$$u_{at} = \left(1 + \frac{v}{u}\right) u_a$$
$$versus: \qquad E_t = \left(1 + \frac{v}{u}\right) E_u$$

Earlier, we saw how to derive the formula for converting between frequency and velocity. As part of that, we discovered the relationship between actual velocity and actual rate of time (14) and then between frequency and actual rate of time (17). From these formulas, we now propose that the midpoint point where matter transitions into energy is at an *actual* speed of c^2 (which is 299792457.9999999983321 in *perceived* speed).

The graph of the Lorentz factor already has a well known transition point where the line gets abruptly steep. There is a drastic change to the slope right at about 212,000km/s. This is the point where rate of time (1) equals velocity $r_t = v = \sqrt{c^2 - v^2} = cos(45°)c$. This point also happens to be where the *actual* velocity equals exactly "c" (i.e. $v\gamma = c$). This is a very significant velocity where relativity starts to have a noticeable effect. Here is the Lorentz factor - notice that it gets steep when $v_a = c$:



There is an another significant *transition point* at an actual speed of c^2. In this case, both *actual rate of time* and *frequency* equal "one" ($f = r_{ta} = 1$). This is the point where frequency and actual rate of time flip and cross each other at a value of 1 and we again see a drastic change in the slope of the graph but in this case it's for actual-rate-of-time:



From (14) and (17):

$$r_{ta} = \frac{c^2}{v_a}$$
$$\therefore r_{ta} = 1 \; when \; v_a = c^2$$
$$f = \frac{1}{{r_{ta}}^2}$$
$$\therefore f = 1 \; when \; r_{ta} = 1$$

And this is where the frequency of matter gains a foothold and changes from less than one hertz to above it. It is proposed here in this paper that this is the *transition point* between matter and energy. Above this speed, a particle is mostly energy and in wave form. Below this speed, a particle is mostly matter and in solid form.

Given this central point, we will now propose an *approximate* formula for the gradual transition between matter and energy. At this stage in time, there are no real observed data points available to accurately determine how the moderate-speed-additive-velocity formula transitions to the high-speed-additive-formula. Eventually time will tell but for now we will begin with this formula which combines both formulas to produce the sum of two velocities by weighting each formula according to the speed's (u) position relative to the transition point, where ($v_a = c^2$ or $f, r_{ta} = 1$). This formula is *only* an approximation and only works for speed changes that are not so large that they require a significant variation in the weight of each formula as a particle accelerates. It is primarily useful for speed changes somewhere between moderate and high that don't span a large change.

We start with the "weighting" formula and use frequency as the main variable. We will first create two *factors* to represent the portion of matter versus the portion of energy that a speed represents (f_m *and* f_e). The "transition" point is when $f = 1$ so we obtain our factors for matter and energy by comparing each to 1 (where $f = f_f(u)$ (19)):

$$f_m = \frac{1}{f}$$
$$f_e = \frac{f}{1}$$

Now figure out the percentage that each comprises of the whole. We will use p_m for the percent matter and p_e for the percent energy:

$$p_m = \frac{f_m}{f_e + f_m}$$
$$p_e = \frac{f_e}{f_e + f_m}$$

And finally apply each of the percentage *weights* to each of the corresponding formulas for moderate-speed "✦" (23) and high-speed "⊕" (35) velocity addition:

$$s = (u \mathbin{\text{✦}} v) * p_m + (u \oplus v) * p_e$$

4 Conclusion

This paper began by proposing that a photon has mass. It ends with a complete unification between classical physics, relativity and quantum mechanics.

Let us summarize our findings. Proposing that a photon has mass carries the following logical side effects that *must* also be true:

- **If a photon has mass then a photon is a particle.**
  - It has already been established that a *photon* exhibits dual wave/particle behaviour.
  - It has already been established that a *particle* exhibits dual wave/particle behaviour.
- **If a photon is a particle then it must travel *less than* c.**
  - By the rules of relativity, nothing can reach the speed of c.
  - Our velocity-view tells us that c would represent an *infinite* proper speed.
- **If a photon is a particle then it must obey the same laws as for matter.**
  - One set of laws must apply to all, particularly when dealing with the same phenomenon.
- **If a photon obeys laws for matter then Newton's nonlinear energy formula must reconcile with Planck's linear formula.**
  - Even though the laws for light are very different than for bodies, they *must* be describing the same thing.
- **If Newton's formula reconciles with Planck's then there *must* exist a "square root effect" for changes in speed.**
  - Since Newton's formula $E = 1/2mv^2$ squares the velocity while Planck's does not $E = hf$ then changes in speed must nullify that squaring of the velocity or else these two formulas could never produce the same results.
- **A photon's frequency must relate to its speed.**
  - De Broglie demonstrated that all moving matter has a wavelength that relates to its speed; therefore, a photon with mass must also have a frequency that relates to its speed.
- **If we have a formula to calculate a photon's speed, it must reconcile with de Broglie's formula that calculates wavelengths of matter.**

In addition to proposing that a photon has mass, we interjected a mathematical equivalent of length contraction: the "velocity-view of time dilation" which carries these logical effects. From the idea of velocity-expansion instead of length contraction:

- **There are *two speeds* rather than two distances; one for each frame of reference.**
  - A discrepancy in time can be explained by either distance or velocity ($t = d/v$).
  - Ex: If you know that a distance is 120km and the speed limit is 60km/h, but your friend drives the distance in *one* hour, then you can conclude that either: the distance was shorter or that they drove faster.
  - In length contraction, there are two variations of the distance: one for the observer's frame and another for the traveller. In the velocity-view, there are two *speeds* instead.
- **One of these two speeds we see, the other we do not.**
  - Like the distances, one of these two speeds is the one we actually perceive from our frame while the other is not visible since it is from another frame's point of view.
- **The unseen speed of the traveller's frame obeys the laws of classical physics.**
  - This fact has long been established that "proper" velocity ($v * \gamma$) conveniently obeys laws for momentum and energy ($E = 1/2m(v\gamma)^2$ and $p = mv\gamma$).
  - In other words, the unseen speed from the *traveller's* frame of reference is a **key** speed in the frame of the observer. It is not simply a matter of "one speed to each's frame" but instead a matter of *two* relevant speeds per frame - one that is accurate and one that is not, where the one that is accurate is the one that is unseen.
- **Mass does not increase with velocity when gauging by the "proper" velocity.**
  - If we regard the proper velocity as the "*actual*" velocity while the speed we *see* from our frame as the "*perceived*" speed (since the proper velocity is the one that obeys classical laws), then mass is constant since there is no limit of c for proper velocity, and the "missing" energy of an accelerating particle can be attributed to a distortion in perception.
- **Relativity is a distortion in the *perception* of the observer rather than an actual distortion of the traveller's universe.**
  - There are two speeds: actual and perceived. All classical laws abide by the actual speed; therefore, the speed we perceive is *not* the speed to which physical laws apply and so, the speed we *see* is not the "*real*" speed - *as far as all physical laws are concerned*.
  - We can describe this discrepancy in the perception of all speeds that we see as a "distorted perception" of the observer.
  - The size of the frame of the traveller is unaffected by their speed difference from the observer.

From all the points above, we were able to achieve the following:

- **Derived a new equivalent formula to Einstein's additive velocity - based on the *velocity-view* of time dilation.**
  - This new formula is derived very simply, strongly reinforcing the velocity-view point of view.
  - Effectively, convert speeds to their "actual" velocity; add; convert back to "perceived." Simple.
- **Derived a new equivalent for the Lorentz factor.**
  - This is based on the concept that each frame has its own "rate of time." The Lorentz factor can be derived by taking the ratio of two frames' rates of time.
- **Derived a formula that converts between frequency and speed.**
  - By expanding on the "rate of time" concept, we find the formula to find a photon's speed from its frequency.
  - This formula applies to light or any other particle.
- **Reconciled this "velocity of frequency" formula with de Broglie's waveform of matter.**
  - Since "waveform of moving matter" and "frequency of photon/particle speed" are the same thing, these two formulas reconcile.
- **Adjusted Einstein's "additive velocities" formula to work in vacuum and observed the required square root effect that reconciles Newton with Planck.**
  - If a photon is a particle then the energy formulas of Newton and Planck must work for either light or bodies ($E = 1/2mv^2$ and $E = hf$)
- **Derived the high speed kinetic energy formula based on the square-root effect ($E = m(v\gamma)^2$).**
  - The square-root effect causes the $1/2$ to drop out of Newton's energy formula for high speeds.
- **Derived the mass of the photon.**
  - Using the "velocity of frequency" formula and the high speed kinetic energy formula, we deduce the mass of a photon (via simple algebra).
- **Reconciled the "additive velocities in vacuum" and "velocity of frequency" formulas with the Doppler formula.**
  - Adding high speed particle velocities is the same thing as adding speeds to frequencies of light if light is a particle.
- **Derived an equivalent of the "additive velocities in vacuum" formula but using the velocity view.**
  - This greatly simplified the formula. With this we see exactly how matter is related to energy.
  - Effectively, convert speeds to their "actual" velocity; convert to kinetic energy; add; convert back to actual speed; convert back to "perceived." Simple.
  - This formula reconciles with both Doppler and Einstein's in vacuum formula.
- **Observed that the Doppler formula is strictly related to the Lorentz factor and time dilation (aka "rate of time").**
  - This tells us that since the Doppler formula does not have a variable for gravity (G), gravity has nothing to do with relativity.
  - In other words, we derived two different formulas that were based on relativity to reconcile with the regular Doppler formula. Since measurements of changes in light speeds (i.e. by using the Doppler formula) is entirely unaffected by gravity, relativity is therefore unaffected by gravity.
- **Observed that mass is constant for all speeds.**
  - The mass of the photon does not have a variable for v and is constant under all circumstances in formulas that reconcile with Doppler and de Broglie.
  - The velocity-view of time dilation does not carry the bad logical side effect of a mass that increases with speed, and this view clearly demonstrates itself to describe relativity better than the length-contraction view.

We have unified Newton with relativity and with quantum mechanics and demonstrated that the same laws apply to the astronomically large as they do to the microscopically small, as they do to the fast as well as the slow, and as they do for matter and as they do for energy. This paper summarizes one of five chapters from the book, "The Theory of Infinity."

References

1. Nelson L. W. Theory of Infinity. Printorium Bookworks, Canada, 2012