SEVENTY-EIGHT CONJECTURES ON FERMAT PSEUDOPRIMES

(COLLECTED PAPERS)

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INTRODUCTION

Prime numbers have always fascinated mankind. For mathematicians, they are a kind of “black sheep” of the family of integers by their constant refusal to let themselves to be disciplined, ordered and understood. However, we have at hand a powerful tool, insufficiently investigated yet, which can help us in understanding them: Fermat pseudoprimes. It was a night of Easter, many years ago, when I rediscovered Fermat’s "little" theorem. Excited, I found the first few Fermat absolute pseudoprimes (*i.e.* 561, 1105, 1729, 2465, 2821, 6601, 8911) before I found out that these numbers are already known. Since then, the passion for study these numbers constantly accompanied me.

In this book I gathered together 25 of my articles posted on VIXRA about Poulet numbers (Fermat pseudoprimes to base 2), Carmichael numbers (absolute Fermat pseudoprimes), Fermat pseudoprimes to base 3 and other relative Fermat pseudoprimes and 30 sequences of such numbers posted by me on OEIS.

I titled the book in this way to show how many new and exciting things one can say more about this class of numbers, but, though indeed these collected papers contain 78 conjectures about Fermat pseudoprimes (I will list them at the beginning of this book, not denying it’s title), these collected papers contain also many observations about the properties of Fermat pseudoprimes and generic formulas for many subclasses of such numbers.

Exceptions to the above mentioned theorem, Fermat pseudoprimes seem to be more malleable than prime numbers, more willing to let themselves to be ordered than them, and their depth study will shed light on many properties of the primes, because it seems natural to look for the rule studying it’s exceptions, as a virologist search for a cure for a virus studying the organisms that have immunity to the virus.

The list with the seventy-eight conjectures on Fermat pseudoprimes which are studied in the articles from this book

Conjecture 1: There are infinite many Poulet numbers of the form (4^k – 1)/3, where k is positive integer.

Conjecture 2: Any number of the form (4^k – 1)/3, where k is prime, k ≥ 5, is a Poulet number.

Conjecture 3: The formula (n^(n*k + k + n – 1) – 1)/(n^2 – 1) generates an infinity of Fermat pseudoprimes to base n for any integer n, n > 1.

Conjecture 4: Any 3-Poulet number which has not a prime factor of the form 30k + 23 can be written as p*((n + 1)*p – n*p)*((m + 1)*p – m*p) or as p*((n*p – (n + 1)*p)*(m*p – (m + 1)*p).

Conjecture 5: Any Poulet number with two prime factors can be written as P = (q – 30*n)*(r + 30*n), where q and r are primes or are equal to 1 and n is positive integer, n >= 1.

Conjecture 6: There is an infinity of Poulet numbers of the form p^2 + 81*p + 3*p*q, where p is a prime of the form 30*k + 13 and q is a prime of the form 30*k + 41, where k is an integer, k >= 0.

Conjecture 7: There is an infinity of Poulet numbers of the form p^2 + 81*p + 3*p*q, where p is a prime of the form 30*k + 41 and q is a prime of the form 30*k + 13, where k is an integer, k >= 0.

Conjecture 8: If the number p^2 + 81*p + 3*p*q, where p is a prime of the form 30*k + 13 and q is a prime of the form 30*k + 41, is a Poulet number, then the number p^2 + 81*p + 3*p*q, where p is a prime of the form 30*k + 41 and q is a prime of the form 30*k + 13 is a Poulet number too (k is an integer, k >= 0).

Conjecture 9: For every Wieferich prime p there is an infinity of Poulet numbers which are equal to n*p – n + 1, where n is integer, n > 1.

Conjecture 10: The numbers formed through deconcatanation of Carmichael numbers not divisible by 5 that ends in the digits that form a number of the form 6*k – 1 and the respective number are congruent to 2 (mod 6) or to 5 (mod 6).

Conjecture 11: There is no a Carmichael number with three prime divisors to can be written as (6*x + 1)*(6*y + 1)*(6*z – 1), they are all of the form (6*x + 1)*(6*y + 1)*(6*z + 1), (6*x – 1)*(6*y – 1)*(6*z – 1) or (6*x + 1)*(6*y – 1)*(6*z – 1).

Conjecture 12: For any Carmichael number C that has only prime factors of the form 6*k + 1 is true at least one of the following five relations:
(1) C is a Harshad number;
(2) If we note with s(m) the sum of the digits of the integer m then C is divisible by n*s(C) – n + 1, where n is integer;
(3) C is divisible by s((C + 1)/2);
(4) C is divisible by n*s((C + 1)/2) – n + 1, where n is integer;
(5) s(C) = s((C + 1)/2).

Conjecture 13: The number (30*n + 7)*(60*n + 13)*(150*n + 31) is a Carmichael number if (but not only if) 30*n + 7, 60*n + 13 and 150*n + 31 are all three prime numbers.

Conjecture 14: The number (30*n – 29)*(60*n – 59)*(90*n – 89)*(180*n – 179) is a Carmichael number if (but not only if) 30*n – 29, 60*n – 59, 90*n – 89 and 180*n – 179 are all four prime numbers.

Conjecture 15: The number (330*n + 7)*(660*n + 13)*(990*n + 19)*(1980*n + 37) is a Carmichael number if 330*n + 7, 660*n + 13, 990*n + 19 and 1980*n + 37 are all four prime numbers.

Conjecture 16: The number (30*n – 7)*(90*n – 23)*(300*n – 79) is a Carmichael number if (but not only if) 30*n – 7, 90*n – 23 and 300*n – 79 are all three prime numbers.

Conjecture 17: The number (30*n + 13)*(90*n + 37)*(150*n + 61) is a Carmichael number if (but not only if) 30*n + 13, 90*n + 37 and 150*n + 61 are all three prime numbers.

Conjecture 18: Any Carmichael number can be written as (n^2*p^2 – q^2)/(n^2 – 1), where p and q are primes or power of primes or are equal to 1 and n is positive integer, n > 1.

Conjecture 19: For any Carmichael numbers with three prime factors, C = d_1*d_2*d_3, where d_1 < d_2 < d_3, is true one of the following two statements:
(1) d_2 can be written as d_1*(n + 1) – n and d_3 can be written as d_1*(m + 1) – m;
(2) d_2 can be written as d_1*n – (n + 1) and d_3 can be written as d_1*m – (m + 1),
where m and n are natural numbers.

Conjecture 20: There is an infinity of reversible primes p with the property that the number obtained through concatenation of the digits of p with a number of n digits of 0, where n is equal to one less than the digits of p, and finally with the digit 1 is a prime.

Conjecture 21: If p and 2p + 1 are both primes, then the number n = p*(2p + 1) – 2*k*p is Fermat pseudoprime to base p + 1 for at least one natural value of k.

Conjecture 22: If p and 2p – 1 are both primes, p > 3, then the number n = p*(2p – 1) – 2*k*p is Fermat pseudoprime to base p – 1 for at least one natural value of k.

Conjecture 23: If p and q are primes, where q = sqrt(2*p – 1), then the number p*q is Fermat pseudoprime to base p + 1.

Conjecture 24: If p is prime, p > 3, and k integer, k > 1, then the number n = p*(k*p – k + 1) is Fermat pseudoprime to base k*p – k and to base k*p – k + 2.

Conjecture 25: Any prime number p can be written as p = (C*q – 1)/(q – 1), where C is a Carmichael number and q is a prime.

Conjecture 26: If the number 360*(a*b) + 1, where a and b are primes, is equal to c^2, where c is prime, then exists an infinite series of Carmichael numbers of the form a*b*d, where d is a natural number (obviously odd, but not necessarily prime).

Conjecture 27: The expression n^E mod 544 = n, where n is any natural number, is true if E is an Euler pseudoprime.

Conjecture 28: For any biggest prime factor of a Poulet number p_1 with two prime factors exists a series with infinite many Poulet numbers p_2 formed this way: p_2 mod (p_1 – d) = d, where d is the biggest prime factor of p_1.

Conjecture 29: Any Poulet number p_2 divisible by d can be written as (p_1 – d)*n + d, where n is natural, if exists a smaller Poulet number p_1 with two prime factors divisible by d.

Conjecture 30: For any biggest prime factor of a Poulet number p_1 exists a series with infinite many Poulet numbers p_2 formed this way: p_2 mod (p_1 – d) = d, where d is the biggest prime factor of p_1.

Conjecture 31: Any Poulet number P with three or more prime divisors has at least one prime divisor q for that can be written as P = q*((r + 1)*q – r), where r is a natural number.

Conjecture 32: The only Poulet number divisible by a smaller Poulet number that can't be written as p*((m + 1)*p – m)*((n + 1)*p – n), where m, n, p are natural numbers, are multiples of 5461 and can be written as 5461*(42*k – 13).

Conjecture 33: There are infinite many Poulet numbers of the form 7200*n^2 + 8820*n + 2701.

Conjecture 34: There are infinite many Poulet numbers of the form 144*n^2 + 222*n + 85.

Conjecture 35: If a Poulet number can be written as 8*p*n + p^2, where n is an integer number and p one of it's prime factors, than can be written this way for any of it's prime factors.

Conjecture 36: For any m natural, m > 1, there exist a series with infinite many Fermat pseudoprimes to base 2, P, formed this way: P = (n^m + m*n)/(m + 1).

Conjecture 37: There are infinite many Poulet numbers that can be written as (n + 1)*p^2 – n*p, where n is natural, n > 0, and p is another Poulet number.

Conjecture 38: For any Poulet number p there are infinite many Poulet numbers that can be written as (n + 1)*p^2 – n*p, where n is natural, n > 0.

Conjecture 39: For any Poulet number, p_1, there exist infinite many Poulet numbers, p_2, formed this way: p_2 = (p_1^n + n*p_1)/(n + 1), where n natural, n > 1.

Conjecture 40: For any Carmichael number, C_1, there exist infinite many Carmichael numbers, C_2, formed this way: C_2 = (C_1^n + n*C_1)/(n + 1), where n natural, n > 1.

Conjecture 41: There is no absolute Fermat pseudoprime m for which n = (5*m – 1)/24 is a natural number.

Conjecture 42: Any Carmichael number C divisible by p and 2p – 1 (where p and 2p – 1 are prime numbers) can be written as C = p*(2p – 1)*(n*(2p – 2) + p).

Conjecture 43: For any odd number p we have an infinite number of Carmichael numbers of the form n*(2*n – 1)*(p*n – p + 1)*(2*p*n – 2*p + 1).

Conjecture 45: A Carmichael number C_1 can be written as $C_1 = (C_2 + C_3)/2$, where C_2 and C_3 are also Carmichael numbers, only if both C_1 and C_3 are divisible by C_2.

Conjecture 44: All Carmichael numbers C (not only with three prime divisors) of the form 10*n + 1 that have only prime divisors of the form 10*k + 1 can be written as C = (30*a + 1)*(30*b + 1)*(30*c + 1), C = (30*a + 11)*(30*b + 11)*(30*c + 11), or C = (30*a + 1)*(30*b + 11)*(30*c + 11). In other words, there are no such numbers of the form C = (30*a + 1)*(30*b + 1)*(30*c + 11).

Conjecture 46: If m*126 + n = 1729, m*126 > n, then exists a series with infinite many Carmichael terms of the form C mod m*234 = n.

Conjecture 47: If m*234 + n = 1729, m*234 > n, then exists a series with infinite many Carmichael terms of the form C mod m*234 = n.

Conjecture 48: If m*342 + n = 1729, m*342 > n, then exists a series with infinite many Carmichael terms of the form C mod m*342 = n.

Conjecture 49: For any prime factor of a Carmichael number C_1 exists a series with infinite many Carmichael terms C_2 formed this way: C_2 mod m*18*d = n, where m*18*d + n = C_1, where d is the prime factor of C_1 and m, n are natural numbers, m*18*d < n.

Conjecture 50: There are infinitely many Fermat pseudoprimes to base 3 of the form (3^(4*k + 2) – 1)/8, where k is a natural number.

Conjecture 51: Any Poulet numbers P which have the numbers p = 23 and q = 67 as prime factors can be written as P = p*q*(n*(q – 1) + p) = 3*p^3*(3*n + 1) – p^2*(15*n + 2) + 6*p*n, where n non-null positive integer.

Conjecture 52: Any Poulet numbers P which have the numbers p = 30*k + 23 and q = 90*k + 67, where k non-negative integer, as prime factors can be written as P = 3*p^3*(3*n + 1) – p^2*(15*n + 2) + 6*p*n, where n non-null positive integer.

Conjecture 53: There is an infinity of Poulet numbers which have the numbers p = 30*k + 23 and q = 90*k + 67, where k non-negative integer, as prime factors (implicitly there is an infinity of pairs of primes of the form [30*k + 23, 90*k + 67]).

Conjecture 54: Any Poulet numbers P which have the numbers p = 11 and q = 61 as prime factors can be written as P = p*q*(n*(q – 1) + p) = 6*p^3*(6*n + 1) – p^2*(66*n + 5) + 30*p*n, where n non-null positive integer.

Conjecture 55: Any Poulet numbers P which have the numbers p = 30*k + 11 and q = 180*k + 61, where k non-negative integer, as prime factors can be written as P = 6*p^3*(6*n + 1) – p^2*(66*n + 5) + 30*p*n, where n non-null positive integer.

Conjecture 56: There is an infinity of Poulet numbers which have the numbers p = 30*k + 11 and q = 180*k + 61, where k non-negative integer, as prime factors (implicitly there is an infinity of pairs of primes of the form [30*k + 11, 180*k + 61]).

Conjecture 57: The length of the period of the rational number which is the sum, from n = 1 to n = ∞, of the numbers $1/(C_n - 1)$, where $\{C_1, C_2, ..., C_n\}$ is the ordered set of Carmichael numbers, is always multiple of 66.

Conjecture 58: Any 2-Poulet number P can be written at least in one way as P = (q*2^a*3^b*5^c ± 1)*2^n + 1, where q is a prime, a square of prime or a semiprime, a, b, c are non-negative integers and n is non-null positive integer.

Conjecture 59: If r is equal to the positive rational number $1/(d_1 - 1) + 1/(d_2 - 1) +...+ 1/(d_n - 1)$, where $d_1,..., d_n$ are the prime factors of a Poulet number P, and m is equal to the last denominator obtained applying the Egyptian fraction expansion to r, then the number m + 1 is a prime or a power of prime for an infinity of Poulet numbers.

Conjecture 60: If r is equal to the positive rational number $1/(d_1 - 1) + 1/(d_2 - 1) +...+ 1/(d_n - 1)$, where $d_1,..., d_n$ are the prime factors of a Poulet number P, and r is represented by the irreducible fraction x/y, where x, y positive integers, then the number y + 1 is a prime or a power of prime for an infinity of Poulet numbers.

Conjecture 61: If $d_1,..., d_n$ are the prime factors of a Poulet number P, then the number $lcm((d_1 - 1), (d_2 - 1), ..., (d_n - 1))$ is a prime or a power of prime for an infinity of Poulet numbers.

Conjecture 62: There is an infinity of 2-Poulet numbers which have the set of Smarandache-Coman divisors of order 1 equal to {p, p}, where p is prime.

Conjecture 63: There is an infinity of 2-Poulet numbers which have the set of SC divisors of order 2 equal to {p, p + 20*k}, where p is prime and k is non-null integer.

Conjecture 64: There is an infinity of 2-Poulet numbers which have the set of SC divisors of order 2 equal to {a, b}, where a + b + 1 is prime.

Conjecture 65: There is an infinity of 2-Poulet numbers which have the set of SC divisors of order 2 equal to {a, b}, where a + b – 1 is prime.

Conjecture 66: There is an infinity of 2-Poulet numbers which have the set of SC divisors of order 2 equal to {a, b}, where a + b – 1 and a + b + 1 are twin primes.

Conjecture 67: There is an infinity of pairs of 2-Poulet numbers which have the set of SC divisors of order 2 equal to {a, b}, respectively to {c, d}, where a + b = c + d and a, b, c, d are primes.

Conjecture 68: There is an infinity of pairs of 2-Poulet numbers which have the set of SC divisors of order 2 equal to {a, b}, respectively to {c, d}, where a + b + 1 = c + d – 1.

Conjecture 69: There is an infinity of 2-Poulet numbers which have the set of SC divisors of order 6 equal to {p, q}, where abs{p – q} = 6*k, where p and q are primes and k is non-null positive integer.

Conjecture 70: There is an infinity of 2-Poulet numbers which have the set of SC divisors of order 6 equal to {a, b}, where abs{a – b} = p and p is prime.

Conjecture 71: There is an infinity of 2-Poulet numbers which have the set of SC divisors of order 6 equal to {p, q}, where one from the numbers p and q is prime and the other one is twice a prime.

Conjecture 72: There is an infinity of 3-Poulet numbers which have the set of SC divisors of order 1 equal to {a, b, c}, where a + b + c is prime and a, b, c are primes.

Conjecture 73: There is an infinity of 3-Poulet numbers which have the set of SC divisors of order 1 equal to {a, b, c}, where a + b + c - 1 and a + b + c + 1 are twin primes.

Conjecture 74: There is an infinity of 3-Poulet numbers which have the set of SC divisors of order 1 equal to {n, n, n}.

Conjecture 75: There is an infinity of 3-Poulet numbers which have the set of SC divisors of order 2 equal to {5, p, q}, where p and q are primes and q = p + 6*k, where k is non-null positive integer.

Conjecture 76: There is an infinity of Poulet numbers divisible by 15 which have the set of SC divisors of order 1 equal to {2, 4, 7, n_1, ..., n_i}, where n_1, ..., n_i are non-null positive integers and i > 0.

Conjecture 77: There is an infinity of Poulet numbers divisible by 15 which have the set of SC divisors of order 1 equal to {2, 4, 23, n_1, ..., n_i}, where n_1, ..., n_i are non-null positive integers and i > 0.

Conjecture 78: There is an infinity of Poulet numbers which are multiples of any Poulet number divisible by 15 which has the set of SC divisors of order 1 equal to {2, 4, n_1, ..., n_i}, where n_1 = n_2 =...= n_i = 7 and i > 0.

SUMMARY

Part one. Twenty-five articles on Fermat pseudoprimes

Part two. Thirty sequences of Fermat pseudoprimes

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6. Poulet numbers of the form 7200*n^2 + 8820*n + 2701
7. Poulet numbers of the form 144*n^2 + 222*n + 85
8. Poulet numbers of the form 8*p*n + p^2, where p is prime
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10. Poulet numbers that can be written as 2*p^2 – p, where p is also a Poulet number
11. Poulet numbers of the form m*n^2 + (11*m – 23)*n + 19*m – 49
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17. Carmichael numbers of the form C = (30*n – 7)*(90*n – 23)*(300*n – 79)
18. Carmichael numbers of the form C = (30*n – 17)*(90*n – 53)*(150*n – 89)
19. Carmichael numbers C = (60*k + 13)*(180*k + 37)*(300*k + 61)
20. Carmichael numbers C = (30*n – 29)*(60*n – 59)*(90*n – 89)*(180*n – 179)
21. Carmichael numbers C = (330*k + 7)*(660*k + 13)*(990*k + 19)* (1980*k + 37)
22. Carmichael numbers of the form C = (30*n – p)*(60*n – (2*p + 1))*(90*n – (3*p + 2)), where p, 2*p + 1, 3*p + 2 are all three primes
23. Carmichael numbers of the form C = p*(2*p – 1)*(3*p – 2)*(6*p – 5), where p is prime
24. Carmichael numbers of the form C = p*(2*p – 1)*(n*(2*p – 2) + p), where p and 2*p – 1 are primes
25. Carmichael numbers of the form n*(2*n – 1)*(p*n – p + 1)*(2*p*n – 2*p + 1), where p is odd
26. Carmichael numbers of the form 3*n*(9*n + 2)*(18*n – 1), where n is odd
27. Carmichael numbers that have only prime divisors of the form 10*k + 1
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Part one. Twenty-five articles on Fermat pseudoprimes

1. A formula for generating primes and a possible infinite series of Poulet numbers

Abstract. An amazingly easy to formulate but rich in consequences property of Fermat pseudoprimes to base 2 (Poulet numbers).

A formula for generating primes

I studied Fermat pseudoprimes for quite a while (I posted on OEIS few series and properties of Carmichael numbers and Poulet numbers) and I always believed that in the structure of pseudoprimes resides a key for obtaining primes. Here I expose such a formula that generates primes and products of few primes.

I first noticed that the first Poulet number, 341, can be written as (2^10 – 1)/3 and after that I found other Poulet numbers that can be written as (2^k – 1)/3: 5461, 1398101, 22369621, 5726623061, 91625968981, respectively for k = 14, 22, 26, 34, 38 (I conjecture that there are infinite Poulet numbers of this form).

I then noticed that the third Poulet number, 645, can be written as (2^4*11^2 – 1)/3 and after that I found other Poulet numbers that can be written as (2^k*q^2 – 1)/3, where q is prime: 2465, 2821, 8321, respectively for q = 43, 23, 79 (I conjecture that there are infinite Poulet numbers of this form too).

From the first 23 Poulet numbers, 19 can be written as (2^k*q – 1)/3, where q is prime or square of prime!

So the formula to generate numbers q that are primes, squares of primes and products of few primes or squares of primes is simply q = (3*P + 1)/2^k, where P is a Poulet number and k is the biggest natural number for that q is an integer.

I list below few values of N = 3*P + 1, for 9 consecutive Poulet numbers with 12 digits taken randomly (I note generically with s the squarefree semiprimes and with r the products of 3 distinct prime factors):

for P = 994738556701 we get N = 2^3*s;
for P = 994738580641 we get N = 2^2*746053935481;
for P = 994750702441 we get N = 2^2*r;
for P = 994767925201 we get N = 2^2*746075943901;
for P = 994788345601 we get N = 2^2*746091259201;
for P = 994818048445 we get N = 2^3*s;
for P = 994830588181 we get N = 2^6*46632683821;
for P = 994853432581 we get N = 2^4*29^2*53^2*281^2;
for P = 994868271001 we get N = 2^2*r.

We obtained, from 9 consecutive values of P, four primes, two semiprimes and two products of 3 distinct primes. It can easily be seen the potential of this formula as a generator of primes. I didn't forget the product of 3 squares; here's something interesting; we got through this formula primes, squarefree products of primes, squares of primes and squares of products of primes, but we didn't find a product to contain primes to a bigger power than two or both primes and squares of primes together, therefore we conjecture that there are no such numbers q, where q = (3*P + 1)/2^k (and P is a Poulet number and k is the biggest n natural for that q is an integer).

We know take the four primes randomly generated, *i.e.* 746053935481, 746075943901, 746091259201 and 46632683821, and we see that they have also the property to generate primes; if we put them in a recurrent formula (Cunningham's chain type), we obtain for M = 3*t + 1 the following values:

for t = 746053935481 we get M = 2^2*559540451611;
for t = 746075943901 we get M = 2^3*1381*202591223;
for t = 746091259201 we get M = 2^2*47*11905711583;
for t = 46632683821 we get M = 2^3*174872256433.

We now take a prime newly generated, 559540451611. We have:

3*559540451611 + 1 = 2*839310677417.

I believe these results are encouraging in the study of recurrent sequences of the type $P_n = (3*P_{n-1} + 1)/2^k$, where k is the biggest natural number for that P_n is an integer and P_0 is a Fermat pseudoprime to base 2.

A possible infinite series of Poulet numbers

We saw above that Poulet numbers 341, 5461, 1398101, 22369621, 5726623061, 91625968981 can be written as (4^k – 1)/3 for k = 5, 7, 11, 13, 17, 19. We did'n obtain a Poulet number for any other value of k from 1 to 19 beside those. We calculate now (4^k – 1)/3 for k = 23, 29, 31, 37, 41 and we get respectively:

: 23456248059221 = 47*178481*2796203;
: 96076792050570581 = 59*233*1103*2089*3033169;
: 1537228672809129301 = 715827883*2147483647;
: 6296488643826193618261 = 223*1777*25781083*616318177;
: 1611901092819505566274901 = 83*13367*164511353*8831418697.

Unfortunatelly I have just Mr. Richard Pinch's tables to verify if a number is a Poulet number or not (tables that are just up to 10^12) and there is no such a simple test to verify this as it is the Korselt criterion at Carmichael numbers. But the premises that the numbers we calculated are Poulet numbers are good: they are squarefree products of few primes. I don't have enough data to conjecture that a number of the form (4^k – 1)/3 is a Poulet number *if and only if* k is prime, k ≥ 5 (which would be a tremendously result, to put prime numbers in a bijection with a subset of Poulet numbers!), but I do make two conjectures:

Conjecture 1: There are infinite many Poulet numbers of the form (4^k – 1)/3, where k is positive integer.

Conjecture 2: Any number of the form (4^k – 1)/3, where k is prime, k ≥ 5, is a Poulet number.

The second conjecture, if true, would be, as I know, the first generic formula for an infinite series of Poulet numbers (of type "for any possible value of this we obtain necessarily that", cause formulas that generates Poulet numbers, but not only Poulet numbers I submitted myself a few to OEIS). Besides this, the conjecture has yet another major implication: from the first million natural numbers, about 80 thousand are primes and just about 250 are Poulet numbers, which lead to the conclusion that Poulet numbers are far more rare than prime numbers. The conjecture, if true, would show that, in fact, for the first about 7 consecutive prime numbers, we have 7 corresponding Poulet numbers spread in the first about 40 thousand Poulet numbers and, consequently, the set of prime numbers is so just a mean set beside the set of Poulet numbers!

2. A new class of Fermat pseudoprimes and few remarks about Cipolla pseudoprimes

Abstract. I wrote an article entitled "A formula for generating primes and a possible infinite series of Poulet numbers"; the sequence I was talking about not only that is, indeed, infinite, but is also already known as the sequence of Cipolla pseudoprimes to base 2. Starting from comparing Cipolla pseudoprimes and some of my notes I discovered a new class of pseudoprimes.

Introduction

The article I was talking about in Abstract was my second encounter with Cipolla pseudoprimes. I first submitted a sequence to OEIS (A217853) to define a subset of Fermat pseudoprimes to base 3, *i.e.* numbers of the form (3^(4*k + 2) – 1)/8. I just later saw the note of Mr. Bruno Berselli on this sequence, that for p prime, p = 2*k + 1, is obtained the generating formula for Cipolla pseudoprimes to base 3, namely (9^p – 1)/8, and I made the connection with my further article, in which I was talking about the numbers of the form (4^p – 1)/3, namely Cipolla pseudoprimes to base 2.

The formula (3^(4*k + 2) – 1)/8 generates Fermat pseudoprimes to base 3 not only for k = (p – 1)/2, where p prime (which gives the formula for Cipolla pseudoprimes to base 3), but for other values of k too.

The first few Cipolla pseudoprimes to base 3 are 91, 7381, 597871, 3922632451, 317733228541, 2084647712458321, 168856464709124011 (for more of them, see the sequence A210454 in OEIS).

The first few terms generated by the formula above are 91, 7381, 597871, 48427561, 3922632451, 317733228541, 25736391511831, 2084647712458321, 168856464709124011 (for more of them, see the sequence A217853 in OEIS).

It can be seen that the formula generates until the number 168856464709124011 three more Fermat pseudoprimes to base 3: 48427561, 3922632451 and 25736391511831.

It seemed logic to try to generalize the formula (3^(4*k + 2) – 1)/8, hoping that it can be obtained a class of pseudoprimes that would contain the set of Cipolla pseudoprimes, but instead of this I obtained something even more interesting, an entirely different class of Fermat pseudoprimes (containing pseudoprimes which are not in the Cipolla sequence and, *vice versa*, not containing pseudoprimes that are in Cipolla sequence).

A formula that generates Fermat pseudoprimes

Conjecture: The formula (n^(n*k + k + n – 1) – 1)/(n^2 – 1) generates an infinity of Fermat pseudoprimes to base n for any integer n, n > 1.

Verifying the conjecture

For n = 2 the formula becomes (2^(3*k + 1) – 1)/3 and generates the following Fermat pseudoprimes to base 2, for k = 3, 7, 11: 341, 1398101, 5726623061.

For n = 3 the formula becomes (3^(4*k + 2) – 1)/8 and generates Fermat pseudoprimes to base 3 for 14 values of k from 1 to 20.

For n = 4 the formula becomes (4^(5*k + 3) – 1)/15 and generates the following Fermat pseudoprime to base 4, for k = 1 : 4369.

For n = 5 the formula becomes (5^(6*k + 4) – 1)/24 and generates the following Fermat pseudoprime to base 5, for k = 1 : 406901.

Unfortunatelly the first term of the sequence (corresponding to k = 1) for n = 7 is larger than 10^10 and I do not have the possibility to extend the verifying, but seems there is enough data to justify the conjecture.

Conclusion

It can easily be seen that, for n = 2, the sequence of Cipolla pseudoprimes to base 2 contains until the pseudoprime 5726623061 two more pseudoprimes than the pseudoprimes I defined above (5461 and 22369621 – for the sequence of Cipolla pseudoprimes to base 2 see the sequence A210454 in OEIS) and I have shown above that Cipolla pseudoprimes to base 3 contains until the pseudoprime 168856464709124011 three less pseudoprimes than the pseudoprimes I defined above so it’s no need for a further proof that neither one of the two classes is not a subset of the other.

Reference
Cipolla Pseudoprimes, Y. Hamahata and Y. Kokobun

3. Formulas that generate subsets of 3-Poulet numbers and few types of chains of primes

Abstract. A simple list of sequences of products of three numbers, many of them, if not all of them, having probably an infinity of terms that are Fermat pseudoprimes to base 2 with three prime factors.

Note: I named with "3-Poulet numbers" the Fermat pseudoprimes to base 2 with 3 prime factors, obviously by similarity with the name "3-Carmichael numbers" for absolute Fermat pseudoprimes. For a list with 3-Poulet numbers see the sequence A215672 in OEIS.

I.
Poulet numbers with three prime factors of the form p*((n+1)*p–n*p)*((m+1)*p–m*p), where p prime, m, n natural:

10585 = 5*29*73 = 5*(5*7 – 6)*(5*18 – 17);
13741 = 7*13*151 = 7*(7*2 – 1)*(7*25 – 24);
13981 = 11*31*41 = 11*(11*3 – 2)*(11*4 – 3);
29341 = 13*37*61 = 13*(13*3 – 2)*(13*5 – 4);
137149 = 23*67*89 = 23*(23*3 – 2)*(23*4 – 3).

II.
Poulet numbers with three prime factors of the form p*((n*p – (n + 1)*p)*(m*p – (m + 1)*p),where p prime, m, n natural:

6601 = 7*23*41 = 7*(7*4 – 5)*(7*7 – 8).

Conjecture: Any 3-Poulet number which has not a prime factor of the form 30k + 23 can be written as p*((n + 1)*p – n*p)*((m + 1)*p – m*p) or as p*((n*p – (n + 1)*p)*(m*p – (m + 1)*p).

III.
Poulet numbers with three prime factors of the form p*(p + 2*n)*(p + 2^2*n – 2), where p prime, n natural:

561 = 3*11*17
p = 3; p + 2*4 = 11; p + 2^2*4 – 2 = 17, so [p, n] = [3, 4];

1105 = 5*13*17
p = 5; p + 2*4 = 13; p + 2^2*4 – 2 = 17, so [p, n] = [5, 4].

IV.
Poulet numbers with three prime factors of the form p*(p + 2*n)*(p + 2^k*n), where p prime and n, k natural:

1729 = 7*13*19
p = 7; p + 2*3 = 13; p + 2^2*3 = 19, so [p, n, k] = [7, 3, 2];

2465 = 5*17*29

p = 5; p + 2*6 = 17; p + 2^2*6 = 29, so [p, n, k] = [5, 6, 2];

2821 = 7*13*31
p = 7; p + 2*3 = 17; p + 2^3*3 = 31, so [p, n, k] = [5, 6, 3];

29341 = 13*37*61
p = 13; p + 2*12 = 37; p + 2^2*12 = 61, so [p, n, k] = [13, 12, 2].

V.
Poulet numbers with three prime factors of the form (1 + 2^k*m)*(1 + 2^k*n)*(1 + 2^k*(m + n)), where k, m, n natural:

13981 = 11*31*41
1 + 2^1*5 = 11, 1 + 2^1*15 = 31, 1 + 2^1*(5 + 15) = 41, so [k, m, n] = [1, 5, 15];

252601 = 41*61*101
1 + 2^2*10 = 41, 1 + 2^2*15 = 61, 1 + 2^2*(10 + 15) = 101, so [k, m, n] = [2, 10, 15].

VI.
Poulet numbers with three prime factors of the form (1 + 2^k*m)*(1 + 2^k*n)*(1 + 2^k*(m + n + 2)), where k, m, n natural:

561 = 3*11*17
1 + 2^1*1 = 3, 1 + 2^1*5 = 11, 1 + 2^1*(1 + 5 + 2) = 17, so [k, m, n] = [1, 1, 5].

VII.
Poulet numbers with three prime factors of the form p*(p + 2*n)*(p + 2*n + 2*(n + 1)), where p prime, n natural:

6601 = 7*23*41
p = 7; p + 2*8 = 31; p + 2*8 + 2*9 = 41, so [p, n] = [7, 8].

VIII.
Poulet numbers with three prime factors of the form 3*(3 + 2^k)*(3 + q*2^h), where q prime and k, h natural:

645 = 3*5*43 so [q, h, k] = [5, 1, 3];
1905 = 3*5*127 so [q, h, k] = [31, 1, 2];
8481 = 3*11*257 so [q, h, k] = [127, 3, 1].

Notes

The chains of primes of the form [p, p + 2*n, ..., p + 2^k*n] seems to be a very interesting object of study; such chains are, for instance, [3, 5, 7, 11, 19] for [p, n, k] = [3, 1, 4] and [3, 13, 23, 43, 83, 163] for [p, n, k] = [3, 5, 5].

Also it would be interesting to study the chains of primes formed starting from a prime p and adding 2^k*n, where n is an arbitrarily chosen natural number and k the smallest values for which p + 2^k*n is prime. Such a chain is, for instance, [7, 13, 19, 31, 103, 199, 1543, 3079] for [p, n] = [7, 3] and [k1, k2, k3, k4, k5, k6, k7] = [1, 2, 3, 5, 6, 9, 10].

An interesting triplet of primes is [p + 2*m, p + 2*n, p + 2*(m + n)] where p is prime and m,n natural; such triplets are [11, 13, 17] for [p, m, n] = [7, 2, 3] or [23, 43, 59] for [p, m, n] = [7, 8, 18]. Generalizing, the triplet would be [p + 2^k*m, p + 2^k*n, p + 2^k*(m + n)]; such a triplet is [11, 19, 23] for [p, k, m, n] = [7, 2, 1, 3].

4. A conjecture about 2-Poulet numbers and a question about primes

Abstract. To find generic formulas for Poulet numbers (beside, of course, the formula that defines them) was for long time one of my targets; I maybe found such a formula for Poulet numbers with two prime factors, involving the multiples of the number 30, that also is rising an interesting question about primes.

Conjecture:
Any Poulet number with two prime factors can be written as P = (q – 30*n)*(r + 30*n), where q and r are primes or are equal to 1 and n is positive integer, n >= 1.

Note: For a list of 2-Poulet numbers see the sequence A214305 that I submitted to OEIS.

Verifying the conjecture for the first few 2–Poulet numbers:

: P = 341 = 11*31 = (41 – 30*1)*(1 + 30*1) = (31 – 30*1)*(311 + 30*1);

: P = 1387 = 19*73 = (61 – 30*2)*(1327 + 30*2) = (79 – 30*2)*(13 + 30*2);

: P = 2047 = 23*89 = (31 – 30*1)*(2017 + 30*1) = (53 – 30*1)*(59 + 30*1) = (61 – 30*2)*(1987 + 30*2) = (83 – 30*2)*(29 + 30*2);

: P = 2701 = 37*73 = (31 – 30*1)*(2671 + 30*1) = (67 – 30*1)*(43 + 30*1) = (103 – 30*1)*(7 + 30*1) = (97 – 30*2)*(13 + 30*2) = (151 – 30*5)*(2551 + 30*5);

: P = 3277 = 29*113 = (59 – 30*1)*(83 + 30*1) = (89 – 30*2)*(53 + 30*2) = (211 – 30*7)*(3067 + 30*7) = (241 – 30*8)*(3037 + 30*8) = (421 – 30*14)*(2857 + 30*14) = (571 – 30*19)*(2707 + 30*19) = (601 – 30*20)*(2677 + 30*20) = (631 – 30*21)*(2647 + 30*21).

Note: It is remarkable in how many ways a 2-Poulet number can be written this way.

Note: The conjecture might probably be extended for all Poulet numbers not divisible by 3 or 5, not only with two prime factors.

Verifying the extended conjecture for first few Poulet numbers with more than two prime factors not divisible by 3 or 5:

: P = 1729 = 7*13*19 = (31 – 30*1)*(1699 + 30*1) = (43 – 30*1)*(103 + 30*1);

: P = 2821 = 7*13*31 = (31 – 30*1)*(2791 + 30*1) = (37 – 30*1)*(373 + 30*1) = (61 – 30*1)*(61 + 30*1);

: P = 6601 = 7*23*41 = (31 – 30*1)*(6571 + 30*1) = (53 – 30*1)*(257 + 30*1) = (71 – 30*1)*(131 + 30*1) = (191 – 30*1)*(11 + 30*1).

Note: This conjecture is rising the following question: which pairs of primes (x, y), at least one of them bigger than 30, have the property that can be written as (p – 30*n, q + 30*n), where p and q are primes or are equal to 1 and n is positive integer, n >= 1.

5. A formula that generates a type of pairs of Poulet numbers

Abstract. Starting from the observation that the number 13^2 + 81*13 + 3*13*41 is a Poulet number (2821), and the number 41^2 + 81*41 + 3*13*41 is a Poulet number too (6601), and following my interest for the number 30, I found a formula that generates such pairs of Poulet numbers like (2821, 6601).

Observation: The formula p^2 + 81*p + 3*p*q, where p is a prime of the form 30*k + 13 and q is a prime of the form 30*k + 41 (case I), or, vice versa, p is a prime of the form 30*k + 41 and q is a prime of the form 30*k + 13 (case II), generates Poulet numbers.

Examples:

: for (p, q) = (13, 41), we got 2821, a Poulet number;
: for (p, q) = (41, 13), we got 6601, a Poulet number;

: for (p, q) = (43, 71), we got 14491, a Poulet number;
: for (p, q) = (71, 43), we got 19951, a Poulet number.

Conjecture 1: There is an infinity of Poulet numbers of the form p^2 + 81*p + 3*p*q, where p is a prime of the form 30*k + 13 and q is a prime of the form 30*k + 41, where k is an integer, k >= 0.

Conjecture 2: There is an infinity of Poulet numbers of the form p^2 + 81*p + 3*p*q, where p is a prime of the form 30*k + 41 and q is a prime of the form 30*k + 13, where k is an integer, k >= 0.

Conjecture 3: If the number p^2 + 81*p + 3*p*q, where p is a prime of the form 30*k + 13 and q is a prime of the form 30*k + 41, is a Poulet number, then the number p^2 + 81*p + 3*p*q, where p is a prime of the form 30*k + 41 and q is a prime of the form 30*k + 13 is a Poulet number too (k is an integer, k >= 0).

Note: The differences between the two numbers that form such a pair might also have interesting properties; in the examples above, we have 6601 – 2821 = 3780 and 19951 – 14491 = 5460. Note that 5460 – 3780 = 1680 = 41^2 – 1.

Note: There are many Poulet numbers that can be written as p^2 + 81*p + 3*p*q, where p, q primes, but it's not satisfied the reciprocity from the formula above.

6. A method of finding subsequences of Poulet numbers

Abstract. I was studying the Fermat pseudoprimes in function of the remainder of the division by different numbers, when I noticed that the study of the remainders of the division by 28 seems to be very interesting. Starting from this, I discovered a method to easily find subsequences of Poulet numbers. I understand through "finding subsequences of Poulet numbers" finding such numbers that share a non-trivial property, *i.e.* not a sequence defined like: "Poulet numbers divisible by 7".

Introduction

The way of finding such subsequences is simply to calculate the remainder of the division of a Poulet number P by the number 4*q, where q is a prime which does not divide P; surprisingly, few values of these remainders seems to occur more often than others.

Few subsequences of Poulet numbers

For q = 7, we found out that, from the first 40 Poulet numbers not divisible by 7, 14 numbers can be written as P = 28*n + 1, where n is obviously a natural number; these numbers are:
: 561, 645, 1905, 2465, 3277, 4033, 4369, 5461, 10585, 18705, 25761, 31417, 33153, 34945.

For q = 11, we found out that, from the first 40 Poulet numbers not divisible by 11, 6 numbers can be written as P = 44*n + 1; these numbers are:
: 2465, 6601, 15709, 15841, 30889, 31417.

Also for q = 11 and the first 40 Poulet numbers not divisible by 11, we found out that 6 numbers can be written as P = 44*n + 5; these numbers are:
: 1105, 2821, 4681, 5461, 8321, 18705.

For q = 13, we found out that, from the first 40 Poulet numbers not divisible by 13, 9 numbers can be written as P = 52*n + 1; these numbers are:
: 3277, 4369, 4681, 5461, 7957, 8321, 18721, 30889, 34945.

Also for q = 13 and the first 40 Poulet numbers not divisible by 13, we found out that 5 numbers can be written as P = 52*n + 29; these numbers are:
: 341, 4033, 10585, 23377, 33153.

For q = 17, we found out that, from the first 50 Poulet numbers not divisible by 17, 8 numbers can be written as P = 68*n + 1; these numbers are:
: 341, 1905, 7957, 15709, 31417, 31621, 49981, 52633.

Also for q = 17 and the first 50 Poulet numbers not divisible by 17, we found out that 4 numbers can be written as P = 68*n + 45; these numbers are:
: 10585, 16705, 49141, 60701.

For q = 19, we found out that, from the first 50 Poulet numbers not divisible by 19, 4 numbers can be written as P = 76*n + 5; these numbers are:
: 1905, 4033, 29341, 31621.

Also for q = 19 and the first 50 Poulet numbers not divisible by 19, we found out that 4 numbers can be written as P = 76*n + 37; these numbers are:
: 341, 645, 4369, 8321.

Also for q = 19 and the first 50 Poulet numbers not divisible by 19, we found out that 4 numbers can be written as P = 76*n + 45; these numbers are:
: 4681, 8481, 23377, 49141.

For q = 23, we found out that, from the first 40 Poulet numbers not divisible by 23, 4 numbers can be written as P = 92*n + 1; these numbers are:
: 645, 1105, 23001, 25761.

Also for q = 23 and the first 40 Poulet numbers not divisible by 23, we found out that 4 numbers can be written as P = 92*n + 45; these numbers are:
: 4369, 7957, 18721, 31417.

Note: Yet is interesting to study the quotients n obtained through the method above, *i.e.* the numbers n = (P – r)/4*q, where r is the remainder, *e.g.* the numbers n = (561 – 1)/4*7 = 2^2*5, n = (33153 – 1)/4*7 = 2^5*37, n = (2465 – 1)/4*11 = 2^3*7, n = (2821 – 5)/4*11 = 2^6 and so on.

7. A possible infinite subset of Poulet numbers generated by a formula based on Wieferich primes

Abstract. I was studying the Poulet numbers of the form n*p – n + 1, where p is prime, numbers which appear often related to Fermat pseudoprimes (see the sequence A217835 that I submitted to OEIS) when I discovered a possible infinite subset of Poulet numbers generated by a formula based on Wieferich primes (I pointed out 4 such Poulet numbers).

It is known the following relation between the Fermat pseudoprimes to base 2 (Poulet numbers) and the Wieferich primes: the squares of the two known Wieferich primes, respectively 1194649 = 1093^2 and 12327121 = 3511^2, are Poulet numbers. I discovered yet another relation between these two classes of numbers:

Conjecture 1: For every Wieferich prime p there is an infinity of Poulet numbers which are equal to n*p – n + 1, where n is integer, n > 1.

Note: Because there are just two Wieferich primes known (it's not even known if there are other Wieferich primes beside these two), we verify the conjecture for these two and few values of n (until n < 31).

: 1093*3 – 2 = 3277, a Poulet number;
: 1093*4 – 3 = 4369, a Poulet number;
: 1093*5 – 4 = 5461, a Poulet number;
: 3511*14 – 13 = 49141, a Poulet number.

Observation 1: The formula n*p – n + 1, where p is Wieferich prime and n is integer, n > 1, leads often to semiprimes of the form q*(m*q – m + 1) or of the form q*(m*q + m – 1):

: 1093*11 – 10 = 5*2621 and 2621 = 5*655 – 654;
: 3511*4 – 3 = 19*739 and 739 = 19*41 – 40;
: 3511*9 – 8 = 7*4593 and 4593 = 7*752 – 751;
: 3511*10 – 9 = 11*3191 and 3191 = 11*319 – 318;
: 3511*12 – 11 = 73*577 and 577 = 73*8 – 7;
: 3511*14 – 13 = 157*313 and 313 = 157*2 – 1;
: 3511*21 – 20 = 11*6701 and 6701 = 11*670 – 669;
: 3511*24 – 23 = 61*1381 and 1381 = 61*23 – 22;
: 3511*28 – 27 = 29*3389 and 3389 = 29*121 – 120;

: 1093*11 – 10 = 41*293 and 293 = 41*7 + 6;
: 1093*18 – 17 = 11*1787 and 1787 = 11*149 + 148;
: 1093*29 – 28 = 11*2879 and 2879 = 11*240 + 239;
: 3511*4 – 3 = 19*739 and 739 = 19*37 + 36;
: 3511*19 – 18 = 17*3923 and 3923 = 17*218 + 217;
: 3511*31 – 30 = 233*467 and 467 = 233*2 + 1;
: 3511*28 – 27 = 29*3389 and 3389 = 29*113 + 112.

Note: Every Poulet number obtained so far through the formula above (until n < 31) is semiprime, in other words a 2–Poulet number.

Note: The class of primes p that can be written in both ways, like p = n*q – n + 1 and like m*q + m – 1, where q is prime and m and n are integers larger than 1, seems to be interesting to study. Such primes p are, for instance, 739 = 19*41 – 40 = 19*37 + 36 and 3389 = 29*121 – 120 = 29*113 + 112. Maybe is not a coincidence that both pairs of primes (p, q) are of the form (10*k + 9, 10*h + 9).

Observation 2: Most of the 2-Poulet numbers (for a list with Fermat pseudoprimes to base 2 with two prime factors see the sequence A214305 in OEIS) can be written as d*(d*n – n + 1) or as d*(d*n + n – 1), where d is obviously one of the two prime factors and n is integer, n > 1: for instance 341 = 11*31 = 11*(11*3 – 2) and 1387 = 19*73 = 19*(19*4 – 3). But not all 2-Poulet numbers can be written in one of these two ways: for instance 23377 = 97*241, the 18th 2-Poulet number, can't be written this way.

Observation 3: I also noticed that two semiprimes obtained from the Wieferich primes through the formula above can be written as q*(q*38 + 17):

: 14041 = 19*739 = 19*(19*38 + 17); 52651 = 37*1423 = 37*(37*38 + 17).

Note: That would be also interesting to study the pairs of primes (p, 38*p + 17); such pairs of primes are, for instance, (7, 283), (19, 739), (37, 1423), (73, 2791), (79, 3019), (103, 3931).

8. Four sequences of integers regarding balanced primes and Poulet numbers

Abstract. A simple list of sequences of integers that reveal interesting properties of few subsets of balanced primes.

I.
Balanced primes B that can be written as B = P ± 24, where P is a Fermat pseudoprime to base two (a Poulet number):
1747, 2677, 4657, 41017, 188437, 195997 (...).

Comments:
B that can be written as P + 24: 1747;
B that can be written as P – 24: 2677, 4657, 41017, 188437, 195997.
Note that all these balanced primes are of the form 10*k + 7!

Note: For a list of Poulet numbers see the sequence A001567 in OEIS. For a list of balanced primes see the sequence A006562 in OEIS.

II.
*Balanced primes B2 that can be written as B1 + 330*n – 6, where B1 is also a balanced prime and n is non–negative integer:*
257, 977, 1367, 1511, 1747, 1907, 2417, 2677 (...).

Comments:
B1 corresponding to the least n for that B2 can be written this way and the least n: (263, 0), (653, 1), (53, 4), (1187, 1), (1753, 0), (593, 4), (1103, 4), (373, 7).
Note that 7 from the first 12 balanced primes of the form 10*k + 7 can be written this way!

Note: Seems that the formula p + 330*n produces many primes when p is a balanced prime of the form 10*k + 3 or 10*k + 7; for instance the number 257 + 330*n is prime for n = 0, 1, 5, 6, 8, 10, 12, 13, 14, 17, 18, 20, 21, 22, 26, 28, 31, 35, 39, 40, 43, 45, 47, 48, 49, 52, 53, 54, 59, 62, 64, 66, 67, 68, 69, 70, 71, 74, 77, 78, 81, 83, 85, 88, 94, 95, that means for 46 values of n from the first 99. I also noticed that the same formula produces many primes and squares of primes when p is a square of prime; for instance the number 361 + 330*n is prime or square of prime for n = 0, 1, 2, 4, 5, 6, 7, 8, 9, 13, 16, 18, 20, 22, 23, 26, 28, 29, 33, 37, 42, 43, 46, 51, 53, 54, 58, 60, 64, 68, 69, 74, 75, 77, 79, 81, 83, 84, 85, 88, 90, 91, 93, 96, 97, that means for the first 45 values of n from the first 99.

III.
*Balanced primes B2 that can be written as B1 + 330*n + 6, where B1 is also a balanced prime and n is non–negative integer:*
263, 593, 1753, 2903, 2963, 4013 (...).

Comments:
B1 corresponding to the least n for that B2 can be written this way and the least n: (257, 0), (257, 1), (1747, 0), (257, 8), (977, 6), (1367, 8).
Note that 5 from the first 14 balanced primes of the form 10*k + 3 can be written this way!

IV.
*Balanced primes B2 that can be written as B1 + 1980*n, where B1 is also a balanced prime and n is positive integer:*
3733, 4013, 4657, 6863, 11411, 11807, 11933, 13463, 15193, 15767, 16097, 16787, 16987, 17483, 19463, 19477, 20107, 20123, 22447, 23333, 23893, 24413, 25621, 26177, 26393, 26693, 26723, 27067 (...).

Comments:
The corresponding (B1, n): (1753, 1), (53, 2), (2677, 1), (2903, 2), (1511, 5), (1907, 5), (4013, 4), (7523, 3), (3313, 6), (11807, 2), (257, 8), (947, 8), (5107, 6), (7583, 5), (7583, 6), (3637, 8), (2287, 9), (6263, 7), (12547, 5), (9473, 6), (6073, 9), (653, 12), (21661, 2), (2417, 12), (24413, 1), (10853, 8), (2963, 12), (3307, 12).

Comments:
B2 may sometimes be written this way for more than one set of values of B1 and n (for instance 11933 = 4013 + 4*1980 = 53 + 6*1980); we refered through the corresponding (B1, n) to the least value of n.
Note that 32 from the first 171 balanced primes can be written as B + 1980*n, where B is a smaller balanced prime.

Conjecture: Any balanced prime B beside the first one, 5, generates an infinity of balanced primes of the form B + 1980*n (*e.g.* the second balanced prime, 53, generates for n = 2, 6, 14, 56 the balanced primes 4013, 11933, 27773, 110933).

Conjecture: Any balanced prime B beside the first one, 5, generates through the formula B – 1980*n an infinity of balanced primes in absolute value (*e.g.* 5807 – 6*1980 = – 6073, where 5807 and 6073 are balanced primes).

9. Six polynomials in one and two variables that generate Poulet numbers

Abstract. Fermat pseudoprimes were for me, and they still are, a class of numbers as fascinating as that of prime numbers; over time I discovered few polynomials that generate Poulet numbers (but not only Poulet numbers). I submitted all of them on OEIS; in this paper I get them together.

(1) *Poulet numbers of the form 7200*n^2 + 8820*n + 2701.*

First 8 terms: 2701, 18721, 49141, 93961, 226801, 314821, 534061, 665281 (sequence A214016 in OEIS).

Note: The Poulet numbers above were obtained for the following values of n: 0, 1, 2, 3, 5, 6, 8, 9.

(2) *Poulet numbers of the form 144*n^2 + 222*n + 85.*

First 8 terms: 1105, 2047, 3277, 6601, 13747, 16705, 19951, 31417 (sequence A214017 in OEIS).
Note: The Poulet numbers above were obtained for the following values of n: 2, 3, 4, 6, 9, 10, 11, 14.

(3) *Poulet numbers of the form 3*(2*n + 1)*(18*n + 11)*(36*n + 17).*

First 4 terms: 561, 62745, 656601, 11921001 (sequence A213071 in OEIS).
Note: The Poulet numbers above were obtained for the following values of n: 0, 2, 5, 14.
Note: All 4 terms from above are Carmichael numbers.

(4) *Poulet numbers of the form (6*m – 1)*((6*m – 2)*n + 1).*

First 11 terms: 341, 561, 645, 1105, 1905, 2047, 2465, 3277, 4369, 4371, 6601 (sequence A210993 in OEIS).
Notes:
For m = 1 the formula becomes 20*n + 5 and generates all the Poulet numbers divisible by 5 from the sequence above (beside 645, all of them have another solutions beside n = 1).
For m = 2 the formula becomes 110*n + 11 and generates the Poulet numbers: 341, 561 etc.
For m = 3 the formula becomes 272*n + 17 and generates the Poulet numbers: 561, 1105, 2465, 4369 etc.
For m = 4 the formula becomes 506*n + 23 and generates the Poulet numbers: 2047, 6601 etc.

For n = 1 the formula generates a perfect square.
For n = 2 the formula becomes 3*(6*m – 1)*(4*m – 1) and were found the following Poulet numbers: 561 etc.
For n = 3 the formula becomes (6*m – 1)*(18*m – 5) and were found the following Poulet numbers: 341, 2465 etc.
For n = 4 the formula becomes (6*m – 1)*(24*m – 7) and were found the following Poulet numbers: 1105, 2047, 3277, 6601 etc.

Note: The formula is equivalent to Poulet numbers of the form p*(n*p – n + 1), where p is of the form 6*m – 1. From the first 68 Poulet numbers just 26 of them (1387, 2701, 2821, 4033, 4681, 5461, 7957, 8911, 10261, 13741, 14491, 18721, 23377, 29341, 31609, 31621, 33153, 35333, 42799, 46657, 49141, 49981, 57421, 60787, 63973, 65281) can't be written as p*(n*p – n + 1), where p is of the form 6*m – 1.

(5) *Poulet numbers of the form (6*m + 1)*(6*m*n + 1).*

First 10 terms: 1105, 1387, 1729, 2701, 2821, 4033, 4681, 5461, 6601, 8911 (sequence A214607 in OEIS).
Notes:

For m = 1 the formula becomes 42*n + 7.
For m = 2 the formula becomes 156*n + 13.
For m = 3 the formula becomes 342*n + 19.
For m = 4 the formula becomes 600*n + 25.

For n = 1 the formula generates a perfect square.
For n = 2 the formula becomes (6*m + 1)*(12*m + 1) and were found the following Poulet numbers: 2701, 8911 etc.
For n = 3 the formula becomes (6*m + 1)*(18*m + 1) and were found the following Poulet numbers: 2821, 4033, 5461 etc.
For n = 4 the formula becomes (6*m + 1)*(24*m + 1) and were found the following Poulet numbers: 1387, 83665 etc. (see the sequence A182123 in OEIS).

Note: The formula is equivalent to Poulet numbers of the form p*(n*p – n + 1), where p is of the form 6*m + 1. From the first 68 Poulet numbers just 7 of them (7957, 23377, 33153, 35333, 42799, 49981, 60787) can't be written as p*(n*p – n + 1), where p is of the form 6*m ± 1.

(6) *Poulet numbers of the form m*n^2 + (11*m – 23)*n + 19*m – 49.*

First 10 terms: 341, 645, 1105, 1387, 2047, 2465, 2821, 3277, 4033, 5461 (sequence A215326 in OEIS).
Note: The solutions (m, n) for the Poulet numbers from the sequence above are: (3, 9); (3, 13); (4, 14); (4, 16); (9, 11) and (4, 20); (3, 27); (3, 29); (4, 26); (3, 35); (290, 0).

10. A conjecture about a large subset of Carmichael numbers related to concatenation

Abstract. Though the method of concatenation has it's recognised place in number theory, is rarely leading to the determination of characteristics of an entire class of numbers, which is not defined only through concatenation. We present here a property related to concatenation that appears to be shared by a large subset of Carmichael numbers

Introduction: I was studying the primes of the form 12*k + 5 (*i.e.* the primes 5, 17, 29, 41, 53, 89, 101, 113, 137, 149, 173 and so on) when I noticed that the primes obtained through the concatenation of two of them are easily to find, especially the ones that end in the digits 29: 4129, 6529, 8929, 11329, 13729, 14929 and so on. When I looked on a certain subset of Carmichael numbers I observed an interesting property that appear to be common to the numbers from this subset (Observation) then I saw that the property is in fact common to a much larger subset of Carmichael numbers (Conjecture).

Observation: The numbers obtained through *deconcatenation* (I understand through this word the operation which is the reverse of concatenation) of the digits of the Carmichael numbers that have 29 as the last two digits and the respective two digits appear to be congruent to 5 (mod 6) or to 2 (mod 6).

I checked this property to the first 21 Carmichael numbers of the form 100*k + 29:
: for 1729 we have (17 – 5)/6 = 2;
: for 23382529 we have (233825 – 5)/6 = 38970;
: for 146843929 we have (1468439 – 5)/6 = 244739;
: for 172947529 we have (1729475 – 5)/6 = 288245;
: for 188516329 we have (1885163 – 5)/6 = 314193;
: for 246446929 we have (2464469 – 5)/6 = 410744;
: for 271481329 we have (2714813 – 5)/6 = 452468;
: for 484662529 we have (4846625 – 5)/6 = 807770;
: for 593234929 we have (5932349 – 5)/6 = 988724;
: for 934784929 we have (9347849 – 5)/6 = 1557974;
: for 958762729 we have (9587627 – 5)/6 = 1597937;
: for 1055384929 we have (10553849 – 5)/6 = 1758974;
: for 1688214529 we have (16882145 – 5)/6 = 2813690;
: for 1858395529 we have (18583955 – 5)/6 = 3097325;
: for 1942608529 we have (19426085 – 5)/6 = 3237680;
: for 6218177329 we have (62181773 – 5)/6 = 10363628;
: for 7044493729 we have (70444937 – 5)/6 = 11740822;
: for 10128932929 we have (101289329 – 5)/6 = 101289329;
: for 10387489729 we have (103874897 – 5)/6 = 17312482;
: for 11477658529 we have (114776585 – 5)/6 = 19129430.
: for 12299638429 we have (122996384 – 2)/6 = 20499397.

Note: I expressed this property in the way above so we can see yet another interesting pattern: many of the integers obtained through this operation have the sum of the digits equal to 29: 244739, 288245, 452468, 807770, 2813690, 3097325, 3237680, 10363628, 19129430.

Note: It would be interesting to see what kind of numbers we obtain if we reverse the operations above: let be x a number with the sum of the digits equal to 29, x*6 + 5 = y and z the number obtained through concatenation of y and 29:
: for x = 2999, y = 17999 and z = 1799929 prime;
: for x = 9299, y = 55799 and z = 1553*3593 semiprime;
: for x = 9929, y = 59579 and z = 373*15973 semiprime;
: for x = 9992, y = 59957 and z = 5995729 prime;
: for x = 3899, y = 23399 and z = 2339929 prime;
: for x = 3989, y = 23939 and z = 2393929 prime.
If we take x a number with the sum of the digits equal to another prime of the form 6*k – 1 instead 29, *i.e.* 41, and repeat the same operations from above, we obtain:
: for x = 59999, y = 359999 and z = 35999941 prime;
: for x = 99599, y = 597599 and z = 59759941 semiprime;
: for x = 99959, y = 599759 and z = 59975941 prime;
: for x = 99995, y = 599975 and z = 59997541 prime.
Even more than that, if we take x a number with the sum of the digits equal to 41, but we calculate z as the concatenation of y not cu 41 but with 29, we obtain:
: for x = 59999, y = 359999 and z = 35999929 semiprime;
: for x = 95999, y = 575999 and z = 57599929 prime;
: for x = 99599, y = 597599 and z = 59759929 prime;
: for x = 99959, y = 599759 and z = 59975929 semiprime;
: for x = 99995, y = 599975 and z = 59997599 semiprime.

We saw that, taking randomly 15 numbers with the property that sum of their digits is equal to a prime of the form 6*k – 1 (in fact not entirely random, because 2999 and 59999 are the smaller primes for which the sum of the digits is equal to 29, respectively 41), we obtained 9 primes and 6 semiprimes, so this direction of study seems to be prolific.
It is also interesting to see which are the smaller numbers with the property that the sum of their digits equals a prime p of the form 6*k – 1: these numbers are: 29 (for p = 11), 89 (for p = 17), 599 (for p = 23), 2999 (for p = 29), 59999 (for p = 41), 299999 (for p = 47), 899999 (for p = 53), 5999999 (for p = 59), 89999999 (for p = 71), 2999999999 (for p = 83), 8999999999 (for p = 89), 29999999999 (for p = 101) and so on. If we concatenate, for instance, the number 6*29999999999 + 5 with these numbers we obtain 17999999999929, 17999999999989, 179999999999599 (which are all semiprimes) and so on.

Conjecture: The numbers formed through deconcatanation of Carmichael numbers not divisible by 5 that ends in the digits that form a number of the form 6*k – 1 and the respective number are congruent to 2 (mod 6) or to 5 (mod 6).

I checked this property to the first few Carmichael numbers that ends in digits of this form (beside the cases that I already considered above):

: for 2821, where 821 ≡ 5 (mod 6), we have 2 ≡ 2 (mod 6);
: for 8911, where 11 = 5 (mod 6), we have 89 ≡ 5 (mod 6); but also 911 ≡ 5 (mod 6), and we have 8 ≡ 2 (mod 6);
: for 15841, where 41 ≡ 5 (mod 6), we have 158 ≡ 2 (mod 6);
: for 29341, where 41 ≡ 5 (mod 6), we have 293 ≡ 5 (mod 6); but also 341 ≡ 5 (mod 6), and we have 29 ≡ 5 (mod 6) and also 9341 ≡ 5 (mod 6), and we have 2 ≡ 2 (mod 6);
: for 41041, where 41 ≡ 5 (mod 6), we have 410 ≡ 2 (mod 6);
: for 52633, where 2633 ≡ 5 (mod 6), we have 5 ≡ 5 (mod 6);
: for 101101, where 101 ≡ 5 (mod 6), we have 101 ≡ 5 (mod 6);
: for 115921, where 5921 ≡ 5 (mod 6), we have 11 ≡ 5 (mod 6);
: for 126217, where 17 ≡ 5 (mod 6), we have 1262 ≡ 2 (mod 6);
: for 172081, where 2081 ≡ 5 (mod 6), we have 17 ≡ 2 (mod 6);
: for 188461, where 461 ≡ 5 (mod 6), we have 188 ≡ 2 (mod 6);
: for 252601, where 52601 ≡ 5 (mod 6), we have 2 ≡ 2 (mod 6);
: for 294409, where 4409 ≡ 5 (mod 6), we have 29 ≡ 5 (mod 6); but also 94409 ≡ 5 (mod 6), and we have 2 ≡ 2 (mod 6);
: for 314821, where 821 ≡ 5 (mod 6), we have 314 ≡ 2 (mod 6);
: for 334153, where 53 ≡ 5 (mod 6), we have 3341 ≡ 5 (mod 6);
: for 410041, where 41 ≡ 5 (mod 6), we have 4100 ≡ 2 (mod 6);
: for 488881, where 881 ≡ 5 (mod 6), we have 488 ≡ 2 (mod 6);
: for 512461, where 461 ≡ 5 (mod 6), we have 512 ≡ 2 (mod 6);
: for 530881, where 881 ≡ 5 (mod 6), we have 530 ≡ 2 (mod 6); but also 30881 ≡ 5 (mod 6), and we have 5 ≡ 5 (mod 6);
: for 658801, where 8801 ≡ 5 (mod 6), we have 65 ≡ 2 (mod 6);
: for 748657, where 8657 ≡ 5 (mod 6), we have 74 ≡ 2 (mod 6);
: for 838201, where 8201 ≡ 5 (mod 6), we have 83 ≡ 2 (mod 6);
: for 852841, where 41 ≡ 5 (mod 6), we have 8528 ≡ 2 (mod 6);
: for 1082809, where 809 ≡ 5 (mod 6), we have 1082 ≡ 2 (mod 6);
: for 1152271, where 71 ≡ 5 (mod 6), we have 11522 ≡ 2 (mod 6);
: for 1193221, where 221 ≡ 5 (mod 6), we have 1193 ≡ 5 (mod 6); but also 93221 ≡ 5 (mod 6), and we have 11 ≡ 5 (mod 6);

: for 1461241, where 41 ≡ 5 (mod 6), we have 14612 ≡ 2 (mod 6); but also 1241 ≡ 5 (mod 6), and we have 146 ≡ 2 (mod 6) and 61241 ≡ 5 (mod 6), and we have 14 ≡ 2 (mod 6);
: for 1615681, where 5681 ≡ 5 (mod 6), we have 161 ≡ 5 (mod 6);
: for 1773289, where 89 ≡ 5 (mod 6), we have 17732 ≡ 2 (mod 6); but also 73289 ≡ 5 (mod 6), and we have 17 ≡ 2 (mod 6).

We take now few bigger Carmichael numbers:

: for 998324255809, where 809 ≡ 5 (mod 6), we have 998324255 ≡ 5 (mod 6); but also 255809 ≡ 5 (mod 6), and we have 998324 ≡ 2 (mod 6) and 24255809 ≡ 5 (mod 6), and we have 9983 ≡ 5 (mod 6) and 324255809 ≡ 5 (mod 6), and we have 998 ≡ 2 (mod 6);
: for 998667686017, where 17 ≡ 5 (mod 6), we have 9986676860 ≡ 2 (mod 6); but also 6017 ≡ 5 (mod 6), and we have 99866768 ≡ 2 (mod 6) and 7686017 ≡ 5 (mod 6), and we have 99866 ≡ 2 (mod 6) and 67686017 ≡ 5 (mod 6), and we have 9986 ≡ 2 (mod 6) and 667686017 ≡ 5 (mod 6), and we have 998 ≡ 2 (mod 6);
: for 999607982113, where 113 ≡ 5 (mod 6), we have 999607982 ≡ 2 (mod 6);
: for 999629786233, where 233 ≡ 5 (mod 6), we have 999629786 ≡ 2 (mod 6); but also 6233 ≡ 5 (mod 6), and we have 99962978 ≡ 2 (mod 6) and 786233 ≡ 5 (mod 6), and we have 999629 ≡ 5 (mod 6) and 9786233 ≡ 5 (mod 6), and we have 99962 ≡ 2 (mod 6).

Note: From all the cases which appear until the Carmichael number 1773289 (we saw that for a single Carmichael number we can meet the conditions from hypothesis more than once), I only met one exception: for 162401, where 401 ≡ 5 (mod 6), we have 162 ≡ 0 (mod 6); I didn't change yet the statement from conjecture, waiting for at least one more counterexemple to set a pattern.

Conclusion: The results obtained for Carmichael numbers may have theoretical value, but for a more practical value, for instance to be helpful in a PRP test, let's see if these results can be extended for the class of Fermat pseudoprimes to base 2:
: for 341, where 41 ≡ 5 (mod 6), we have 3 ≡ 3 (mod 6);
: for 2047, where 47 = 5 (mod 6), we have 20 ≡ 2 (mod 6);
: for 2701, where 701 = 5 (mod 6), we have 2 ≡ 2 (mod 6);
: for 3277, where 77 = 5 (mod 6), we have 32 ≡ 2 (mod 6);
: for 4371, where 71 = 5 (mod 6), we have 43 ≡ 1 (mod 6).
Unfortunatelly, from the first 5 cases that we considered it becomes clear that the conjecture can't be extended on Poulet numbers. A resembling pattern seems not to exist in the case of prime numbers also, so this is a feature strictly of absolute Fermat pseudoprimes.

11. A conjecture about primes based on heuristic arguments involving Carmichael numbers

Abstract. The number 30 is important to me because I always believed in the utility of classification of primes in primes of the form 30*k + 1, 30*k + 7, 30*k + 11, 30*k + 13, 30*k + 17, 30*k + 19, 30*k + 23 and 30*k + 29 (which may be interpreted as well as primes of the form 30*h – 29, 30*h – 23, 30*h – 19, 30*h – 17, 30*h – 13, 30*h – 11, 30*h – 7 and 30*h – 1). The following conjecture involves the multiples of the number 30 and is based on the study of Carmichael numbers.

Conjecture: For any three distinct primes p, q, r there exist a positive integer n so that the numbers x = 30*n – p, y = 30*n – q and z = 30*n – r are all three primes.

Comments

I already showed in the article "A list of 13 sequences of Carmichael numbers based on the multiples of the number 30", posted on VIXRA, the importance of the multiples of 30 in the study of Carmichael numbers.

I shall list randomly a number of ways in which a Carmichael number with three prime factors can be written in function of the multiples of the number 30 (we note with C a Carmichael number):

*C = (30*n – p)*(60*n – q)*(90*n – r),* where n is a positive integer and p, q, r are primes. Examples:

C = 8911 = 7*19*67 = (30 – 23)*(60 – 41)*(90 – 23);
C = 15841 = 7*31*73 = (30 – 23)*(60 – 29)*(90 – 17);
C = 29341 = 13*37*61 = (30 – 17)*(60 – 23)*(90 – 29).

*C = (30*n – p)*(90*n – q)*(120*n – r),* where n is a positive integer and p, q, r are primes. Example:

C = 52633 = 7*73*103 = (30 – 23)*(90 – 17)*(120 – 17).

But the most appealing form is the following one: *C = (30*n – p)*(30*n – q)*(30*n – r),* where n is a positive integer and p, q, r are primes.

Examples:
C = 1729 = 7*13*19 = (30*1 – 23)(30*1 – 17)(30*1 – 11);
C = 1729 = 7*13*19 = (30*9 – 263)(30*9 – 257)(30*9 – 251);
C = 2821 = 7*13*31 = (30*9 – 263)(30*9 – 257)(30*9 – 239);
C = 6601 = 7*23*41 = (30*6 – 173)(30*6 – 157)(30*6 – 139);
C = 8911 = 7*19*67 = (30*3 – 83)(30*3 – 71)(30*3 – 23);
C = 15841 = 7*31*73 = (30*3 – 83)(30*3 – 59)(30*3 – 17).

In fact, my initial intention was to conjecture that any Carmichael number can be written in this form, in other words that for any three prime factors p, q, r of a 3-Carmichael number there exist a positive integer n so that the numbers x = 30*n – p, y = 30*n – q and z = 30*n – r are all three primes.

Note: The reason for which I chose 3 primes for the conjecture instead of 2 or 4 is that 3 is the minimum number of prime factors of a Carmichael number but also because I would relate this conjecture with the study of Fermat's last theorem.

Note: The conjecture implies of course that for any pair of twin primes (p, q) there exist a pair of primes (30*n – p, 30*n – q) so that there are infinitely many pairs of primes.

12. A conjecture regarding the relation between Carmichael numbers and the sum of their digits

Abstract. Though they are a fascinating class of numbers, there are very many properties of Carmichael numbers still unstudied enough. I have always thought there is a connection between these numbers and the sum of their digits (few of them are also Harshad numbers). I try here to highlight such a possible connection.

Conjecture: For any Carmichael number C that has only prime factors of the form 6*k + 1 is true at least one of the following five relations:

(1) C is a Harshad number;
(2) If we note with s(m) the sum of the digits of the integer m then C is divisible by n*s(C) – n + 1, where n is integer;
(3) C is divisible by s((C + 1)/2);
(4) C is divisible by n*s((C + 1)/2) – n + 1, where n is integer;
(5) s(C) = s((C + 1)/2).

I verified below the conjecture for the first 23 Carmichael numbers of this type: 1729, 2821, 8911, 15841, 29341, 46657, 52633, 63973, 115921, 126217, 172081, 188461, 294409, 314821, 334153, 399001, 488881, 512461, 530881, 670033, 748657, 838201, 997633.

: 1729 is divisible by 19, where 19 = s(1729); so 1729 satisfies relation (1); also s((1729 + 1)/2) = s(865) = 19 so 1729 satisfies the relations (3) and (5) either;
: 2821 is divisible by 13, where 13 = s(2821); so 2821 satisfies relation (1); also s((2821 + 1)/2) = s(1411) = 7 and 2821 is divisible by 7 so 2821 satisfies the relation (3) either;
: 8911 is divisible by 19, where 19 = s(8911); so 8911 satisfies relation (1); also s((8911 + 1)/2) = s(4456) = 19 so 8911 satisfies the relations (3) and (5) either;
: s(15841) = s((15841 + 1)/2) = 19 and 15841 is divisible by 73 which is equal to 4*19 – 3; so 15841 satisfies relations (2), (4) and (5);
: s(29341) = s((29341 + 1)/2) = 19 and 29341 is divisible by 37 which is equal to 2*19 – 1; so 29341 satisfies relations (2), (4) and (5);
: s((46657 + 1)/2) = 19 and 46657 is divisible by 37, which is equal to 2*19 – 1; so 46657 satisfies relation (4);
: s(52633) = s((52633 + 1)/2) = 19 and 52633 is divisible by 73 which is equal to 4*19 – 3; so 52633 satisfies relations (2), (4) and (5);
: s(63973) = s((63973 + 1)/2) = s(31987) = 28; so 52633 satisfies relation (5);
: s(115921) = 19 and 115921 is divisible by 37 which is equal to 2*19 – 1;
: 126217 is divisible by 19, where 19 = s(126217); so 126217 satisfies relation (1); also s((126217 + 1)/2) = s(63109) = 19 so 126217 satisfies relations (3) and (5) either;
: s(172081) = s((172081 + 1)/2) = s(86041) = 19; so 172081 satisfies relation (5);
: s((188461 + 1)/2) = s(94231) = 19 and 188461 is divisible by 19; so 188461 satisfies relation (3); also s(188461) = 28 and 188461 is divisible by 109 which is equal to 4*28 – 3 so satisfies relation (2) either;
: s(294409) = 28 and 294409 is divisible by 109 which is equal to 4*28 – 3; so 294409 satisfies relation (2); s((294409 + 1)/2) = s(147205) = 19 and 294409 is divisible by 37, 73 and 109 which are equal to 19*2 – 1, 19*4 – 3 and 19*6 – 5 so 294409 satisfies relation (4) either;
: s(314821) = s((314821 + 1)/2) = s(157411) = 19; so 314821 satisfies relation(5);
: 334153 is divisible by 19, where 19 = s(334153); so 334153 satisfies relation (1);

: s(399001) = 22 and 399001 is divisible by 211 which is equal to 22*10 – 9; so 399001 satisfies relation(2);
: 488881 is divisible by 37, where 37 = s(488881); so 488881 satisfies relation (1);
: s(512461) = s((512461 + 1)/2) = s(256231) = 19; so 512461 satisfies relation(5);
: s(530881) = 22 and 530881 is divisible by 421 which is equal to 22*20 – 19; so 530881 satisfies relation(2);
: s(670033) = s((670033 + 1)/2) = s(335017) = 19; so 512461 satisfies relation(5);
: s(748657) = 37 and 748657 is divisible by 433 which is equal to 37*12 – 11; so 748657 satisfies relation(2);
: s((838201 + 1)/2) = s(419101) = 16 and 838201 is divisible by 61 and 151 which are equal to 16*4 – 3 and 16*10 – 5; so 748657 satisfies relation(4);
: s(997633) = s((997633 + 1)/2) = s(498817) = 37; so 997633 satisfies relation(5).

Note: We observed a subset a Carmichael numbers: the numbers 399001 = 31*61*211 and 530881 = 13*97*421 have both the sum of their digits s(C) = 22 and s((C + 1)/2) = 25; also, C is divisible by n*s(C) – n + 1, where n is their greatest prime factor.

Note: Many other Carmichael numbers have resembling properties, the ones that have only prime factors of the form 6*k – 1 for instance, but I didn’t find yet another category of Carmichel numbers that could be set in such a closed form.

Note: For many Carmichael number C that are also Harshad number is true that s(C) = s((C + 1)/2).

Note: For the odd Harshad numbers H that I checked, the first one that satisfy the relation s(H) = s((H + 1)/2) is the number 1387, the fifth Poulet number, which yet again connect this property with Fermat pseudoprimes.

Observation: I also noticed few relations based on the sum of the digits that are satisfied by a Poulet number P that has only two prime factors, both of the form 6*k + 1:

(1) s(P) = s((P + 1)/2);
(2) Both prime factors of P can be written as n*s((P + 1)/2) + 1, where n is integer;
(3) Both prime factors of P can be written as n*s((P + 1)/2) + n + 1, where n is integer;
(4) Both prime factors of P can be written as n*s((P + 1)/2) – n + 1, where n is integer;
(5) Both prime factors of P can be written as n*s(P) – n + 1, where n is integer.

I considered the first 15 Poulet numbers of this type: 1387, 2071, 4033, 4681, 5461, 7957, 10261, 14491, 18721, 23377, 31609, 31621, 42799, 49141, 49981 (for a list of Poulet numbers with two prime factors see the sequence A214305 in OEIS).

: s(1387) = s((1387 + 1)/2) = s(694) = 19, so 1387 satisfies relation (1);
: s(2071) = s((2071 + 1)/2) = s(1351) = 10, so 2071 satisfies relation (1);
: s(4033) = s((4033 + 1)/2) = s(2017) = 10, so 4033 satisfies relation (1);
: s(4681) = 19 and s((4681 + 1)/2) = s(2341) = 10 and 4681 is divisible with 31 which is equal to 3*10 + 1 also with 151 which is equal to 15*10 + 1, so 4681 satisfies relation (2);
: s(5461) = 16 and s((5461 + 1)/2) = s(2731) = 13 and 4681 is divisible with 43 which is equal to 3*13 + 4 also with 127 which is equal to 9*13 + 10, so 1387 satisfies relation (3);
: s(7957) = s((7957 + 1)/2) = s(3979) = 28, so 7957 satisfies relation (1);

: s(10261) = 10 and s((10261 + 1)/2) = s(5131) = 10 and 10261 is divisible with 31 which is equal to 3*10 + 1 also with 331 which is equal to 33*10 + 1, so 10261 satisfies relation (2);
: s(14491) = s((14491 + 1)/2) = s(7246) = 19, so 14491 satisfies relation (1);
: s(18721) = s((18721 + 1)/2) = s(9361) = 19, so 18721 satisfies relation (1);
: s(23377) = 22 and s((23377 + 1)/2) = s(11689) = 25 and 23377 is divisible with 97 which is equal to 4*25 – 3 also with 241 which is equal to 10*25 – 9, so 23377 satisfies relation (4);
: s(31609) = s((31609 + 1)/2) = s(15805) = 19, so 31609 satisfies relation (1);
: s(31621) = 13 and s((31621 + 1)/2) = s(15811) = 16 and 31621 is divisible with 103 which is equal to 6*16 + 7 also with 307 which is equal to 18*16 + 19, so 31621 satisfies relation (3);
: s(42799) = 31 and s((42799 + 1)/2) = s(21400) = 7 and 42799 is divisible with 127 which is equal to 18*7 + 1 also with 337 which is equal to 48*7 + 1, so 42799 satisfies relation (2);
: s(49141) = s((49141 + 1)/2) = s(24571) = 19, so 49141 satisfies relation (1);
: s(49981) = 31 and 49981 is divisible with 151 which is equal to 31*5 – 4 also with 331 which is equal to 31*11 – 10, so 49981 satisfies relation (1).

Conclusion: The relation between the Fermat pseudoprimes and the sum of their digits seems to be obvious even that there are probably better ways to express this relation (I actually only wanted to highlight few such possible ways). The property of a composite odd integer n to be divisible with s((n + 1)/2) deserves further study, also the property of a Harshad odd number n to have s(n) = s((n + 1)/2): we saw that the smallest such number with this property is a Fermat pseudoprime to base 2, the number 1387. It would also be interesting to see what numbers that are products of more than three prime factors of the form 6*k + 1 and are not Carmichael numbers satisfy the relations from the conjecture.

13. A list of 13 sequences of Carmichael numbers based on the multiples of the number 30

Abstract. The applications of the multiples of the number 30 in the study of Fermat pseudoprimes was for a long time one of my favourite subject of study; in this paper I shall list 13 sequences that I discovered, many of them, if not all of them, having probably an infinity of terms that are Carmichael numbers. I posted many of them on OEIS, where I analized more of their attributes; here I’ll just list them, enumerate their first few terms and present few conjectures.

(1) Carmichael numbers of the form C = (30*n + 7)*(60*n + 13)*(150*n + 31).

First 6 terms: 2821, 488881, 288120421, 492559141, 776176261, 1632785701 (sequence A182085 in OEIS).

Conjecture: The number (30*n + 7)*(60*n + 13)*(150*n + 31) is a Carmichael number if (but not only if) 30*n + 7, 60*n + 13 and 150*n + 31 are all three prime numbers.

(2) Carmichael numbers of the form C = (30*n – p)*(60*n – (2*p + 1))*(90*n – (3*p + 2)), where p, 2*p + 1, 3*p + 2 are all three prime numbers.

First 6 terms: 1729, 172081, 294409, 1773289, 4463641, 56052361 (sequence A182087 in OEIS).

Comment: These numbers can be reduced to only two possible forms: C = (30*n – 23)*(60*n – 47)*(90*n – 71) or C = (30*n – 29)*(60*n – 59)*(90*n – 89).

(3) Carmichael numbers of the form C = (30*n – 29)*(60*n – 59)*(90*n –89)*(180*n – 179).

First 4 terms: 31146661, 2414829781, 192739365541, 197531244744661 (sequence A182088 in OEIS).

Conjecture: The number (30*n – 29)*(60*n – 59)*(90*n – 89)*(180*n – 179) is a Carmichael number if (but not only if) 30*n – 29, 60*n – 59, 90*n – 89 and 180*n – 179 are all four prime numbers.

(4) Carmichael numbers of the form C = (330*n + 7)*(660*n + 13)*(990*n + 19)*(1980*n + 37).

First 2 terms: 63973, 461574735553 (sequence A182089 in OEIS).

Conjecture: The number (330*n + 7)*(660*n + 13)*(990*n + 19)*(1980*n + 37) is a Carmichael number if 330*n + 7, 660*n + 13, 990*n + 19 and 1980*n + 37 are all four prime numbers.

(5) Carmichael numbers of the form C = (30*n – 7)*(90*n – 23)*(300*n – 79).

First 5 terms: 340561, 4335241, 153927961, 542497201, 1678569121 (sequence A182132 in OEIS).

Conjecture: The number (30*n – 7)*(90*n – 23)*(300*n – 79) is a Carmichael number if (but not only if) 30*n – 7, 90*n – 23 and 300*n – 79 are all three prime numbers.

(6) Carmichael numbers of the form C = (30*n – 17)*(90*n – 53)*(150*n – 89).

First 5 terms: 29341, 1152271, 34901461, 64377991, 775368901 (sequence A182133 in OEIS).

Conjecture: The number (30*n + 13)*(90*n + 37)*(150*n + 61) is a Carmichael number if (but not only if) 30*n + 13, 90*n + 37 and 150*n + 61 are all three prime numbers.

(7) Carmichael numbers of the form C = (60*n + 13)*(180*n + 37)*(300*n + 61).

First 5 terms: 29341, 34901461, 775368901, 1213619761, 4562359201 (sequence A182416 in OEIS).

Conjecture: The number (60*n + 13)*(180*n + 37)*(300*n + 61) is a Carmichael number if (but not only if) 60*n + 13, 180*n + 37 and 300*n + 61 are all three prime numbers.

(8) Carmichael numbers of the form C = (90*n + 1)*(180*n + 1)*(270*n + 1)*(540*n + 1).

First 2 terms: 2414829781, 192739365541.

Comment:
For n = n/15 the formula becomes (6*n + 1)*(12*n + 1)*(18*n + 1)*(36*n + 1).

(9) Carmichael numbers of the form C = (p + 30)*(q + 60)*(p*q + 90), where p and q are primes.

First 2 terms: 488881, 1033669.

Comment: We obtained Carmichael numbers for [p, q] = [7, 13] and [p, q] = [7, 31].

(10) Carmichael numbers of the form C = (30*p + 1)*(60*p + 1)*(90*p + 1), where p is prime.

First 4 terms: 56052361, 216821881, 798770161, 1976295241.

Comment: We obtained Carmichael numbers for the following values of p: 7, 11, 17, 23.

(11) Carmichael numbers of the form C = 1710*3^m + 60*n + 451.

First 3 terms: 2821, 6601, 15841.

Comment: We obtained Carmichael numbers for the following values of [m, n]: [0, 11], [1, 17], [2, 0].

(12) Carmichael numbers of the form C = 1710*m + 30*n + 1.

First 7 terms: 2821, 6601, 8911, 15841, 29341, 41041, 75361.

Comment: We obtained Carmichael numbers for the following values of [m, n]: [1, 37], [3, 49], [5, 12], [9, 15], [17, 9], [24, 0], [44, 4].

(13) Carmichael numbers of the form C = 60*n + 2281.

First 17 terms: 2821, 6601, 15841, 29341, 41041, 101101, 115921, 172081, 188461, 252601, 314821, 340561, 399001, 410041, 488881, 512461, 530881.

Comment: We obtained Carmichael numbers for the following values of n: 9, 72, 226, 451, 646, 1647, 1894, 2830, 3103, 4172, 5209, 5638, 6612, 6796, 8110, 8503, 8810.

Conjecture: All Carmichael numbers C of the form 10*k + 1 that have digital root equal to 1, 4 or 7 can be written as C = 60*n + 2281.

14. A possible generic formula for Carmichael numbers

Abstract. To find generic formulas for Carmichael numbers (beside, of course, the formula that defines them) was for long time one of my targets; I already found such a formula, based on Korselt's criterion; I possible found now another such a formula.

Conjecture: Any Carmichael number can be written as (n^2*p^2 – q^2)/(n^2 – 1), where p and q are primes or power of primes or are equal to 1 and n is positive integer, n > 1.

The first Carmichael number, 561,
can be written as (4*p^2 – q^2)/3 for [p, q] = [29, 41], [41, 71], [7^2, 89], [421, 29^2]; it can also be written as (16*p^2 – q^2)/15 for [p, q] = [23, 7], [29, 71] etc.

The second Carmichael number, 1105,
can be written as (4*p^2 – q^2)/3 for [p, q] = [29, 7], [31, 23], [53, 89], [59, 103], [67, 11^2], [829, 1657]; it can also be written as (9*p^2 – q^2)/8 for [p, q] = [37, 59], [7^2, 113], [61, 157] etc.

The third Carmichael number, 1729,
can be written as (4*p^2 – q^2)/3 for [p, q] = [37, 17], [43, 47], [67, 113], [73, 127], [103, 193], [433, 863], [1297, 2593]; it can also be written as (9*p^2 – q^2)/8 for [p, q] = [43, 53], [53, 107], [67, 163], [167, 487], [1153, 3457]; it can also be written as (16*p^2 – q^2)/15 for [p, q] = [41, 31], [47, 97], [97, 353], [157, 657], [173, 673], [251, 991]; it can also be written as (25*p^2 – q^2)/24 for [p, q] = [41, 23], [61, 227], [151, 727], [347, 1723] etc. (seems that the famous Hardy–Ramanujan number can set a record for how many ways can be written this way).

Few subsets of Carmichael numbers:
A subset of Carmichael numbers C has the following property: C = (4*p^2 – q^2)/3, where q is the smaller prime that verify the relation q > sqrt (3*C/4), and p is prime or a power of prime; few such numbers are:
1105, 1729, 6601, 41041, 75361, 340561, for corresponding [p, q] = [7, 29], [17, 37], [19, 71], [71, 179], [239, 7^2], [509, 11^4].

Another subset of Carmichael numbers C has the following property: C = (n^2*p^2 – 1)/(n^2 – 1), where p is the smaller prime that verify the relation p > sqrt (3*C/4); few such numbers are:
2465, 8911, 10585, 15841, 162401, for corresponding [n, p] = [2, 43], [3, 89], [3, 97], [2, 109], [2, 349].

Another subset of Carmichael numbers C (but this time only related to the formula above) has the following property: C = (4*p^2 – 7153)/3, where p is prime; such numbers are: 561, 488881, for corresponding p = 47, 607 (interesting that 607 – 47 = 560 and 561 is the first Carmichael number).

Another subset of Carmichael numbers C (this time too only related to the formula above) has the following property: C = (p*q^2 – 1723^2)/(p – 1), where p and q are primes or power of primes; few such numbers are: 1105 for [p, q] = [1249, 59], 1729 for [p, q] = [5^2, 347], 2465 for [p, q] = [7^2, 251].

Note: The formula based on Korselt's criterion that I was talking about in Abstract is: C = p^k + n*p^2 – n*p (if C > p^k) or C = p^k – n*p^2 + n*p (if p^k > C) for any p prime divisor of C and any k natural number. See the sequence A213812 that I submitted to OEIS.

15. An interesting and unexpected property of Carmichael numbers and a question

Abstract. I was researching a kind of generalized Cunningham chains that generate, instead of primes, Fermat pseudoprimes to some base when purely by chance I noticed a property of absolute Fermat pseudoprimes, equally interesting and unexpected. By a childish simple operation, a new class of numbers is obtained from Carmichael numbers.

Like anyone that learned in school that digits are just a way to designate a number and to operate with it, I always looked with reluctance on the arbitrary play with digits. I personally gave credit to the method of concatenation when I saw the relation between it and Fermat pseudoprimes (see my articles, *A conjecture about a large subset of Carmichael numbers related to concatenation* and *Formulas for generating primes involving emirps, Carmichael numbers and concatenation*, posted on viXra).

The property of Carmichael numbers that I discovered now proves the extreme versatility of these numbers: by a childish simple operation, insertion of the digit 0 among the digits of these numbers, we obtain an entirely new class of numbers.

Thus we have the following numbers obtained from Carmichael numbers through the operation that I mentioned:

: 5601 (from 561)
We can see that n^5601 mod 5601 = n^3 for n from 2 to 17 (not for n = 18);

: 28021 (from 2821)
We can see that n^28021 mod 28021 = n^7 for n from 2 to 4 (not for n = 5);

: 24065 (from 2465)
We can see that n^24065 mod 24065 = n^5 for n from 2 to 7 (not for n = 8).

Note: For the number 1729, which is the known Hardy–Ramanujan number, we have p = 10729, p = 17029 and p = 17209 all three primes! (so, of course, n^p mod p = n for any value of n).

Note: For the relative Fermat pseudoprimes, to base 2 and respectivelly to base 3, we don't obtain resembling results through this operation.

Observation:By adding the digit 0 to Carmichael numbers, operation which itself it's not at all special, it's equivalent to a simple formula, the multiplication of a Carmichael number with the number 10, we obtain: n^5610 mod 5610 = n^10 for n = 2 (not for n = 3) and the same result for the numbers 1105 and 1729. Through multiplication of the first Carmichael number, 561, with the number 8, we obtain the number 4488 and also n^4488 mod 4488 = n^8 for n = 2 (not for n =

3). Through multiplication of the first Poulet number, 341, with the number 10, we obtain the number 3410 and also n^3410 mod 3410 = n^10 for n = 2 (not for n = 3). Through multiplication of the first Fermat pseudoprime to base three, 91, with the number 10 we don't obtain resembling results. Seems that this property, that 2^(P*k) mod (P*k) = 2^k, it's a property of Poulet numbers P (it can't be extended for Fermat pseudoprimes to base 3) while the property that I showed above it'a a property of Carmichael numbers (it cant' be extended for relative Fermat pseudoprimes).

Comment: The numbers m that satisfy the relation n^m mod m = n^k, where k > 1, for any consecutive integer value of n from 2 to some larger integer, numbers obtained from Carmichael numbers through this operation or not, seems to deserve further study.

Question: Are there any numbers m to satisfy the relation n^m mod m = n^k, where k > 1, for any value of n?

16. Connections between the three prime factors of 3-Carmichael numbers

Abstract. It was always obvious to me that, beside Korselt's criterion, that gives a relation between any prime factor of a Carmichael number and the number itself, there must be a relation between the prime factors themselves; here I present a conjecture on the Carmichael numbers with three prime factors expressing the larger two prime factors as a function of the smallest one and few particular cases of connections between all three prime factors.

Introduction:

In the sequence A213812 that I posted in OEIS I showed a formula, derived from Korselt's criterion, to express a Carmichael number as a function of any of its prime factors and an integer. In the sequence A215672 that I posted in OEIS I extended this formula for a Poulet number with three or more prime factors, expressing such a number as a function of at least one of its prime factors and an integer. This formula relates a Fermat pseudoprime to one (in the case of Poulet numbers) or to any (in the case of Carmichael numbers) of its prime factors, but says nothing about the relation between the prime factors themselves.

In the sequence A215672 I showed that most of Fermat pseudoprimes to base 2 with three prime factors (so, implicitly, most of Carmichael numbers with three prime factors) can be written in one of the following two ways:

(1) p*((n + 1)*p – n)*((m + 1)*p – m);
(2) p*((n*p – (n + 1))*(m*p – (m + 1)),

where p is the smallest of the three prime factors and n, m are natural numbers.

Exempli gratia for Poulet numbers from first category:
10585 = 5*29*73 = 5*(5*7 – 6)*(5*18 – 17).
Exempli gratia for Poulet numbers from second category:
6601 = 7*23*41 = 7*(7*4 – 5)*(7*7 – 8).

From the first 37 Poulet numbers with three prime factors, just three (30889, 88561 and 91001) can't be written in one of this two ways.

Conjecture: For any Carmichael numbers with three prime factors, $C = d_1*d_2*d_3$, where $d_1 < d_2 < d_3$, is true one of the following two statements:
(1) d_2 can be written as $d_1*(n + 1) – n$ and d_3 can be written as $d_1*(m + 1) – m$;
(2) d_2 can be written as $d_1*n – (n + 1)$ and d_3 can be written as $d_1*m – (m + 1)$,
where m and n are natural numbers.

As I showed, this conjecture holds for the first 13 Carmichael numbers with three prime factors checked. In this article I present few connections that express not the larger two prime factors as a function of the smallest one, as above, but connects all the three prime factors.

Observation: For most of the Carmichael numbers with three prime factors, $C = d_1*d_2*d_3$, where $d_1 < d_2 < d_3$, is true one of the following seventh statements:
(1) d_3 can be written as $d_1*(m + 1) – n$ and as well as $d_2*(n + 1) – m$;
(2) d_3 can be written as $d_1*(m – 1) + n$ and as well as $d_2*(n – 1) + m$;
(3) d_3 can be written as $d_1 + (m + 1)*n$ and as well as $d_2 + m*n$;
(4) d_3 can be written as $d_1*m – 2*n$ and as well as $d_2*n + 2*m$;
(5) d_3 can be written as $d_1*m + 2*n$ and as well as $d_2*n – 2*m$;
(6) d_3 can be written as $d_1*m – 2*n$ and as well as $d_2*n + m$;
(7) d_3 can be written as $d_1*m + n$ and as well as $d_2*n – 2*m$,
where m and n are natural numbers.

Carmichael numbers which verify the first statement:

For C = 561 = 3*11*17 we have [m, n] = [5, 1]:
Indeed, 3*(5 + 1) – 1 = 17 and 11*(1 + 1) – 5 = 17.

For C = 162401 = 17*41*233 we have [m, n] = [13, 5]:
Indeed, 17*(13 + 1) – 5 = 233 and 41*(5 + 1) – 13 = 233.

For C = 314821 = 13*61*397 we have [m, n] = [30, 6]:
Indeed, 13*(30 + 1) – 6 = 397 and 61*(6 + 1) – 30 = 397.

Carmichael numbers which verify the second statement:

For C = 1105 = 5*13*17 we have [m, n] = [4, 2]:
Indeed, 5*(4 – 1) + 2 = 17 and 13*(2 – 1) + 4 = 17.

For C = 2821 = 7*13*31 we have [m, n] = [5, 3]:
Indeed, 7*(5 – 1) + 3 = 31 and 13*(3 – 1) + 5 = 31.

For C = 8911 = 7*19*67 we have [m, n] = [10, 4]:
Indeed, 7*(10 – 1) + 4 = 67 and 19*(4 – 1) + 10 = 67.

For C = 10585 = 5*29*73 we have [m, n] = [15, 3]:
Indeed, 5*(15 – 1) + 3 = 73 and 29*(3 – 1) + 15 = 73.

For C = 15841 = 7*31*73 we have [m, n] = [11, 3]:

Indeed, 7*(11 – 1) + 3 = 73 and 31*(3 – 1) + 11 = 73.

For C = 115921 = 13*37*241 we have [m, n] = [19, 7]:
Indeed, 13*(19 – 1) + 7 = 241 and 37*(7 – 1) + 19 = 241.

For C = 314821 = 13*61*397 we have [m, n] = [31, 7]:
Indeed, 13*(31 – 1) + 7 = 397 and 61*(7 – 1) + 31 = 397.

For C = 334153 = 19*43*409 we have [m, n] = [22, 10]:
Indeed, 19*(22 – 1) + 10 = 409 and 43*(10 – 1) + 22 = 409.

Carmichael numbers which verify the third statement:

For C = 1729 = 7*13*19 we have [m, n] = [1, 6]:
Indeed, 7 + 2*6 = 19 and 13 + 6 = 19.

For C = 2465 = 5*17*29 we have [m, n] = [1, 12]:
Indeed, 5 + 2*12 = 29 and 17 + 12 = 29.

For C = 29341 = 13*37*61 we have [m, n] = [1, 24]:
Indeed, 13 + 2*24 = 61 and 37 + 24 = 61.

For C = 252601 = 41*61*101 we have [m, n] = [2, 32]:
Indeed, 41 + 3*20 = 101 and 61 + 2*20 = 101.

For C = 294409 = 37*73*109 we have [m, n] = [1, 36]:
Indeed, 37 + 2*36 = 109 and 73 + 36 = 109.

For C = 399001 = 31*61*211 we have [m, n] = [5, 36]:
Indeed, 31 + 6*30 = 211 and 61 + 5*30 = 211.

For C = 410041 = 41*73*137 we have [m, n] = [2, 32]:
Indeed, 41 + 3*32 = 137 and 73 + 2*32 = 137.

For C = 488881 = 37*73*181 we have [m, n] = [3, 36]:
Indeed, 37 + 4*36 = 181 and 73 + 3*36 = 181.

For C = 512461 = 31*61*271 we have [m, n] = [7, 30]:
Indeed, 31 + 8*30 = 271 and 61 + 7*30 = 271.

For C = 1152271 = 43*127*211 we have [m, n] = [1, 84]:
Indeed, 43 + 2*84 = 211 and 127 + 84 = 211.

For C = 1152271 = 43*127*211 we have [m, n] = [1, 84]:
Indeed, 43 + 2*84 = 211 and 127 + 84 = 211.

For C = 1857241 = 31*181*331 we have [m, n] = [1, 150]:
Indeed, 31 + 2*150 = 331 and 181 + 150 = 331.

Carmichael numbers which verify the fourth statement:

For C = 52633 = 7*73*103 we have [m, n] = [15, 1]:
Indeed, 7*15 – 2*1 = 103 and 73*1 + 2*15 = 103.

For C = 1461241 = 37*73*541 we have [m, n] = [15, 7]:
Indeed, 37*15 – 2*7 = 541 and 73*7 + 2*15 = 541.

Carmichael numbers which verify the fifth statement:

For C = 46657 = 13*37*97 we have [m, n] = [7, 3]:
Indeed, 13*7 + 2*3 = 97 and 37*3 – 2*7 = 97.

Carmichael numbers which verify the sixth statement:

For C = 1193221 = 31*61*631 we have [m, n] = [21, 10]:
Indeed, 31*21 – 2*10 = 631 and 61*10 + 21 = 631.

Carmichael numbers which verify the seventh statement:

For C = 530881 = 13*97*421 we have [m, n] = [32, 5]:
Indeed, 13*32 + 5 = 421 and 97*5 – 2*32 = 421.

Note: From the first 31 Carmichael numbers with three prime factors checked, only four of them (6601 = 7*23*41, 1024651 = 19*199*271, 1615681 = 23*199*353 and 1909001 = 41*101*461) don't satisfy any of the seventh statements.

Note: Obviously the prime factors of Chernick's Carmichael numbers satisfy the third statement.

Note: There are Carmichael numbers, like 314821 = 13*61*397, that satisfy both the first and the second statement. The triplets of primes like [p_1, p_2, p_3] = [13, 61, 397], for which p_3 = p_1*(m + 1) – n = p_2*(n + 1) – m = p_1*m + n + 1 = p_2*n + m + 1, deserve further study, also the question if and when the products p_1*p_2*p_3 are Carmichael numbers.

Note: The Carmichael number 252601 = 41*61*101 can be written as p*(p*n – m)*(p*(n + 1) – (m + 1)), where p is prime and m, n natural numbers (because 61 = 41*2 – 21 and 101 = 41*3 – 22). Also the triplets of primes of the form [p, p*n – m, p*(n + 1) – (m + 1)] deserve further study as well as the question if and when the products of the primes that form such a triplet are Carmichael numbers.

Note: For Carmichael numbers with three prime factors, see the sequence A087788 in OEIS.

17. Formulas for generating primes involving emirps, Carmichael numbers and concatenation

Abstract. Observations on generating primes or products of very few primes from reversible primes and Carmichael numbers using the method of concatenation.

I. *On the numbers obtained through concatenation from emirps and Carmichael numbers using only the digits of the number itself and the digits of its square*

Note: First we notice that, if p is a reversible prime and the number q is the number obtained through concatenation of the digits of p^2 with the digits of p, then the number q/p is often the product of very few primes (for a list of emirps see the sequence A006567 in OEIS).

Observation: If p is a reversible prime and the number q obtained through concatenation of the digits of p^2 with the digits of p has the sum of digits equal to 29, then the number q/p is often a prime or a semiprime.

16913/13 = 1301 is prime;
136937/37 = 3701 is prime;
624179/79 = 7901 is prime;
564001751/751 = 751001 is prime;
10180811009/1009 = 101*99901 is semiprime;
17450411321/1321 = 7*1887143 is semiprime.

Note that the first digits of the resulted primes are the same with the digits of p. The pairs of primes [13, 1301], [37, 3701], [79, 7901], [751, 751001] and so on deserve further study.

Conjecture: There is an infinity of reversible primes p with the property that the number obtained through concatenation of the digits of p with a number of n digits of 0, where n is equal to one less than the digits of p, and finally with the digit 1 is a prime.

Note: We also notice that, if C is a Carmichael number and the number s is the number obtained through concatenation of the digits of C^2 with the digits of C, then the number C/s is often the product of very few primes (for a list of Carmichael numbers see the sequence A002997 in OEIS):

Few examples:

314721561/561 = 7^2*107^2;
79405921/8911 = 59*1510339;
79580412821/2821 = 28210001;
435732016601/6601 = 2593*25457;
7115017141014721843 50561/84350561 = 3*2811685366666667.

Note the interesting value of C/s for C = 2821.

II. *On the numbers obtained through concatenation from emirps and Carmichael numbers using the digits of the number itself, the digits of its square and the digits 0001*

Observation: If C is a Carmichael number then the number obtained through the concatenation of the digits of C with the digits 0001 is often a product of very few primes.

Few examples:
5610001 = 1129*4969; 17290001 = 1051*16451; 28210001 is prime; 66010001 = 2593*25457; 89110001 = 59*1510339; 105850001 = 911*116191; 158410001 is prime.

Note the values obtained for 2821 and 15841, both divisible with 31.

Observation: If C is a Carmichael number divisible by 31 then the number obtained through the concatenation of the digits of C with the digits 0001 is often a product of very few primes.

28210001 is prime; 158410001 is prime; 753610001 is prime; 1720810001 is prime; 21009010001 = 7*3001287143; 9912830875210001 is prime.

Observation: If C is a Carmichael number having 561 (a Carmichael number, also) as the last digits then the following numbers are often a product of very few primes:

: M, obtained through the concatenation of the digits of C with the digits 0001;
: N, obtained through the concatenation of the digits of C^2 with the digits 0001.

Few examples:

C = 340561; C^2 = 115981794721

M = 3405610001 is semiprime; N = 1159817947210001 is semiprime;

C = 8134561; C^2 = 66171082662721

M = 81345610001 is prime; N = 661710826627210001 is prime;

C = 10024561; C^2 = 100491823242721

M = 100245610001 is prime; N = 1004918232427210001 is semiprime;

C = 10402561; C^2 = 108213275358721

M = 1104025610001 is semiprime; N = 1082132753587210001 is semiprime;

C = 45318561; C^2 = 2053771971110721

M = 453185610001 is semiprime; N = 20537719711107210001 is semiprime.

C = 84350561; C^2 = 7115017141014721

M = 843505610001 is semiprime; N = 71150171410147210001 is semiprime.

Note: Probably the formulas could be extrapolated for Carmichael numbers having as the last digits not 561 but another Carmichael number but the results that we obtained, *exempli gratia*, with Carmichael numbers 1729 and 6601 were not encouraging.

III. *On the numbers obtained through concatenation from emirps using only the digits of the number itself*

Observation: We noticed that, through succesive concatenation of the digits of a reversible prime with the digits of its reversal, is obtained an interesting sequence of primes.

Primes obtained through concatenation of the digits of the numbers p, q, p, q and p, where p is an emirp and q is its reversal (this formula also conducts to products of very few primes):

1331133113, 9779977997, 769967769967769, 15111151151111511511

Observation: There is an infinity of primes formed this way.

IV. *On the extension of few of these observations from the set of emirps to set of all primes*

Note: We observed three interesting series of primes.

(1) Primes q of the form n/p, where p is prime and n is formed through concatenation this way: first digits of n are the digits of the square of p and last digits of n are the digits of p itself:

First few such primes:

31, 71, 1301, 1901, 3701, 6101, 6701, 7901, 103001, 109001, 181001 (...).

(the corresponding p: 3, 7, 13, 19, 37, 61, 67, 79, 103, 109, 181)

(2) Primes formed through succesive concatenation of the digits of the prime p with the digits of its reversal, not necessarily prime, q (this formula also conducts to products of very few primes).

First few such primes:

1331133113, 2992299229, 4334433443, 9779977997, 127721127721127 (...).

Note that, from the primes obtained this way, we can also obtain interesting primes from adding numbers of the form 18*10^k.

Few examples:

: 1331133113 + 18*10^8 = 3131133113 prime(which is the concatenation of q, q, p, q, p);
: 9779977997 + 18*10^7 = 9959977997 prime;
: 9779977997 + 18*10^9 = 27779977997 prime;
: 769967769967769 + 18*10^9 = 769985769967769 prime;
: 769967769967769 + 18*10^11 = 771767769967769 prime.

(3) Primes q formed through concatenation of the digits of the squares of the primes p with the digits 0001.

First few such primes:

90001, 490001, 2890001, 8410001, 18490001, 22090001

(the corresponding p: 3, 7, 17, 29, 43, 47).

18. Four conjectures regarding Fermat pseudoprimes and few known types of pairs of primes

Abstract. There are already known some relations between Fermat pseudoprimes and the pairs of primes [p, 2*p – 1]. We will here show few relations between Fermat pseudoprimes and the pairs of primes of the type [p, 2*p – 1], [p, 2*p + 1], [p, sqrt(2*p – 1)], respectivelly [p, k*p – k + 1].

Introduction

Due to mathematician Farideh Firoozbakht, we have in OEIS few interesting observations about the relation between Fermat pseudoprimes and the pairs of primes [p, 2*p – 1]. We will list only few of them (see the sequences A005935 - A005937):
: if p and 2*p – 1 are both primes, and p > 3, then p*(2*p – 1) is pseudoprime to base 3;
: if p and 2p – 1 are both primes, then p*(2*p – 1) is pseudoprime to base 5 iff p is of the form 10*k + 1;
: if p and 2*p – 1 are both primes, then p*(2*p – 1) is pseudoprime to base 6 iff p is of the form 12*k + 1.

Now that the relation between Fermat pseudoprimes and the pairs of primes [p, 2*p – 1] appears to be clear, we will make four conjectures regarding the relation between Fermat pseudoprimes and the pairs of primes of the type [p, 2*p + 1], [p, 2*p – 1], [p, sqrt(2*p – 1)], respectivelly [p, k*p – k + 1].

Conjecture 1: If p and 2*p + 1 are both primes, then the number n = p*(2*p + 1) – 2*k*p is Fermat pseudoprime to base p + 1 for at least one natural value of k.

Verifying the conjecture:
(for the first 8 such pairs of primes)

For [p, 2*p + 1] = [3, 7] we have, for k = 1, n = 15, which is, indeed, pseudoprime to base p + 1 = 4.
For [p, 2*p + 1] = [5, 11] we have, for k = 2, n = 35, which is, indeed, pseudoprime to base p + 1 = 6.
For [p, 2*p + 1] = [11, 23] we have, for k = 5, n = 143, which is, indeed, pseudoprime to base p + 1 = 12.
For [p, 2*p + 1] = [23, 47] we have, for k = 6, n = 805, which is, indeed, pseudoprime to base p + 1 = 24.

For [p, 2*p + 1] = [29, 59] we have, for k = 3, n = 1537, which is, indeed, pseudoprime to base p + 1 = 30.
For [p, 2*p + 1] = [41, 83] we have, for k = 9, n = 2665, which is, indeed, pseudoprime to base p + 1 = 42.
For [p, 2*p + 1] = [53, 107] we have, for k = 4, n = 5247, which is, indeed, pseudoprime to base p + 1 = 54.
For [p, 2*p + 1] = [83, 167] we have, for k = 24, n = 9877, which is, indeed, pseudoprime to base p + 1 = 84.

Note: For the list of Sophie Germain primes, see the sequence A005384 in OEIS.

Conjecture 2: If p and 2*p – 1 are both primes, p > 3, then the number n = p*(2*p – 1) – 2*k*p is Fermat pseudoprime to base p – 1 for at least one natural value of k.

Verifying the conjecture:
(for the first 6 such pairs of primes)

For [p, 2*p – 1] = [7, 13] we have, for k = 4, n = 21, which is, indeed, pseudoprime to base p – 1 = 6.
For [p, 2*p – 1] = [19, 37] we have, for k = 10, n = 323, which is, indeed, pseudoprime to base p – 1 = 18.
For [p, 2*p – 1] = [31, 61] we have, for k = 5, n = 1581, which is, indeed, pseudoprime to base p – 1 = 30.
For [p, 2*p – 1] = [37, 73] we have, for k = 2, n = 2553, which is, indeed, pseudoprime to base p – 1 = 36.
For [p, 2*p – 1] = [79, 157] we have, for k = 7, n = 11297, which is, indeed, pseudoprime to base p – 1 = 78.
For [p, 2*p – 1] = [97, 193] we have, for k = 8, n = 17169, which is, indeed, pseudoprime to base p – 1 = 96.

Note: For the list of primes p for which 2*p – 1 is also prime, see the sequence A005382 in OEIS.

Conjecture 3: If p and q are primes, where q = sqrt(2*p – 1), then the number p*q is Fermat pseudoprime to base p + 1.

Verifying the conjecture:
(for the first 8 such pairs of primes)

For [p, q] = [13, 5] we have p*q = 65 which is, indeed, pseudoprime to base 14.
For [p, q] = [61, 11] we have p*q = 671 which is, indeed, pseudoprime to base 62.
For [p, q] = [181, 19] we have p*q = 3439 which is, indeed, pseudoprime to base 182.
For [p, q] = [421, 29] we have p*q = 12209 which is, indeed, pseudoprime to base 422.
For [p, q] = [1741, 59] we have p*q = 102719 which is, indeed, pseudoprime to base 1742.
For [p, q] = [1861, 61] we have p*q = 113521 which is, indeed, pseudoprime to base 1862.
For [p, q] = [2521, 71] we have p*q = 178991 which is, indeed, pseudoprime to base 2522.
For [p, q] = [3121, 79] we have p*q = 246559 which is, indeed, pseudoprime to base 3122.

Note: For the list of primes p for wich sqrt(2*p – 1) is also prime, see the sequence A067756 in OEIS.

Conjecture 4: If p is prime, p > 3, and k integer, k > 1, then the number n = p*(k*p – k + 1) is Fermat pseudoprime to base k*p – k and to base k*p – k + 2.

Verifying the conjecture:

For the first 4 such pairs of primes, when p = 5:

For [p, 2*p – 1] = [5, 9] we have p*(2*p – 1) = 45 which is, indeed, pseudoprime to bases 8 and 10.
For [p, 3*p – 2] = [5, 13] we have p*(3*p – 2) = 65 which is, indeed, pseudoprime to bases 12 and 14.
For [p, 4*p – 3] = [5, 17] we have p*(4*p – 3) = 85 which is, indeed, pseudoprime to bases 16 and 18.
For [p, 5*p – 4] = [5, 21] we have p*(5*p – 4) = 105 which is, indeed, pseudoprime to bases 20 and 22.

For the first 4 such pairs of primes, when p = 7:

For [p, 2*p – 1] = [7, 13] we have p*(2*p – 1) = 91 which is, indeed, pseudoprime to bases 12 and 14.
For [p, 3*p – 2] = [7, 19] we have p*(3*p – 2) = 133 which is, indeed, pseudoprime to bases 18 and 20.
For [p, 4*p – 3] = [7, 25] we have p*(4*p – 3) = 175 which is, indeed, pseudoprime to bases 26 and 28.
For [p, 5*p – 4] = [7, 31] we have p*(5p – 4) = 217 which is, indeed, pseudoprime to bases 30 and 32.

For the next 4 such pairs of primes, when k = 3:

For [p, 3*p – 2] = [11, 31] we have p*(3*p – 2) = 341 which is, indeed, pseudoprime to bases 30 and 32.
For [p, 3*p – 2] = [13, 37] we have p*(3*p – 2) = 481 which is, indeed, pseudoprime to bases 36 and 38.
For [p, 3*p – 2] = [23, 67] we have p*(3*p – 2) = 1541 which is, indeed, pseudoprime to bases 66 and 68.
For [p, 3*p – 2] = [37, 109] we have p*(3*p – 2) = 4033 which is, indeed, pseudoprime to bases 108 and 110.

Note: The formula p*(k*p – k + 1), where p is prime and k integer, seems to appear often related to Fermat pseudoprimes (see the sequence A217835 that I submitted to OEIS).

19. Special properties of the first absolute Fermat pseudoprime, the number 561

Abstract. Though is the first Carmichael number, the number 561 doesn't have the same fame as the third absolute Fermat pseudoprime, the Hardy–Ramanujan number, 1729. I try here to repair this injustice showing few special properties of the number 561.

I will just list (not in the order that I value them, because there is not such an order, I value them all equally as a result of my more or less inspired work, though they may or not "open a path") the interesting properties that I found regarding the number 561, in relation with other Carmichael numbers, other Fermat pseudoprimes to base 2, with primes or other integers.

1. The number 2*(3 + 1)*(11 + 1)*(17 + 1) + 1, where 3, 11 and 17 are the prime factors of the number 561, is equal to 1729. On the other side, the number 2*lcm((7 + 1), (13 + 1), (19 + 1)) + 1, where 7, 13 and 19 are the prime factors of the number 1729, is equal to 561. We have so a function on the prime factors of 561 from which we obtain 1729 and a function on the prime factors of 1729 from which we obtain 561.

***Note**: The formula N = 2*(d_1 + 1)*...*(d_n + 1) + 1, where d_1, d_2, ...,d_n are the prime divisors of a Carmichael number, leads to interesting results (see the sequence A216646 in OEIS); the formula M = 2*lcm((d_1 + 1), ...,(d_n + 1)) + 1 also leads to interesting results (see the sequence A216404 in OEIS). But we didn't obtained anymore through one of these two formulas a Carmichael number from another, so this bivalent realtion might only exist between the numbers 561 and 1729.*

2. The number 561 can be expressed as C = a*b + b – a, where b is prime and a can be any prime factor of the number 1729: 561 = 7*71 + 71 – 7 = 13*41 + 41 – 13 = 19*29 + 29 = 19 (even more than that, for those that consider that 1 is a prime number, so a prime factor of 1729, 561 = 1*281 + 281 – 1).

***Note**: The formula (a + 1)*(b + 1)*(b – a + 1) + 1 seems to lead to interesting results: for instance, (19 + 1)*(29 + 1)*(29 – 19 + 1) = 6601, also a Carmichael number and for the pairs [a, b] = [7, 71] and [a, b] = [13, 41] we obtain through this formula primes, which make us think that this formula deserves further study. Also the triplets [a, b, a*b + b – a], where a, b and a*b + b – a are all three primes might lead to interesting results.*

***Note**: I can't, unfortunately, to state that 561 is the first integer that can be written in three (or even four, if we consider that 1 is prime) distinct ways as a*b + b – a, where a and b are primes, because there is a smaller number that has this property: 505 = 3*127 + 124 = 11*43 + 32 = 13*37 + 24 = 17*29 + 12. I yet assert that Carmichael numbers (probably the Fermat pseudoprimes to base 2 also) and the squares of primes can be written in many ways as such.*

3. Another interesting formula inspired by the number 561: we have the expression (2*3 + 3)*(2*11 + 3)*(2*17 + 3) – 4, where 3, 11 and 17 are the prime factors of 561, egual to 8321, a Fermat pseudoprime to base 2.

***Note**: If we apply this formula to the prime factors of another Carmichael number, 2821 = 7*13*31, we obtain 32041 = 179^2, an interesting result.*

4. We consider the triplets of primes of the form [p, p + 560, p + 1728]. The first triplet of such primes, [59, 619, 1787], we notice that has the following property: 59 + 619 + 1787 = 2465, a Carmichael number.

***Note**: For the next two such triplets, [83, 643, 1811] and [149, 709, 1877] we didn't obtain convincing results.*

5. The number 561 is the first term in the sequence of Fermat pseudoprimes to base 2 of the form 3*(4*n – 1)*(6*n – 2), where n is integer different from 0.

***Note**: See the sequence A210993 in OEIS.*

6. The number 561 is the first term in the sequence of Fermat pseudoprimes to base 2 of the form 3*n*(9*n + 2)*(18*n – 1), where n is an odd number.

***Note**: See the sequence A213071 in OEIS.*

7. The number 561 is the first term in the sequence of Fermat pseudoprimes to base 2 of the form 8*p*n + p^2, where p is prime and n is integer (for n = 0 we include in this sequence the squares of the only two Wieferich primes known).

***Note**: See the sequence A218483 in OEIS.*

8. The number 561 is the first term in the sequence of Fermat pseudoprimes to base 2 of the form 5*p^2 – 4*p, where p is prime.

***Note**: See the sequence A213812 in OEIS.*

9. The numbers obtained through the method of concatenation from reversible primes and the number 561 are often primes.

***Note**: We obtain 11 primes from the first 20 reversible primes concatenated with the number 561; these primes are: 37561, 73561, 79561, 97561, 149561, 157561, 311561, 337561, 347561, 359561, 389561.*

10. The numbers obtained through the method of concatenation from palindromic primes and the number 561 are often primes.

***Note**: We obtain 9 primes from the first 20 palindromic primes concatenated with the number 561; these primes are: 101561, 131561, 151561, 191561, 313561, 373561, 727561, 797561, 929561.*

11. The numbers obtained through the method of concatenation from the powers of 2 and the number 561 are often primes or products of few primes.

***Note**: The numbers 4561, 16561, 32561, 256561 are primes.*

12. Yet another relation between the numbers 561 and 1729: the numbers obtained through the method of concatenation from the prime factors of 1729 raised to the third power and the number 561 are primes.

***Note**: These are the numbers: 343561 (where 7^3 = 343); 2197561 (where 13^3 = 2197) and 6859561 (where 19^3 = 6859).*

13. The number (561*n – 1)/(n – 1), where n is integer different from 1, is often integer; more than that, is often prime.

***Note**: We obtained the following primes (in the brackets is the corresponding value of n): 701(5), 673(6), 641(8), 631(9), 617(11), 601(14), 421(–3), 449(–4), 491(–7), 521(–13) etc. I assert that for a Carmichael number C the number (C*n – 1)/(n – 1), where n is integer different from 1, is often an integer (comparing to other integers beside C). In fact the primes appear so often that I will risk a conjecture.*

***Conjecture**: Any prime number p can be written as p = (C*q – 1)/(q – 1), where C is a Carmichael number and q is a prime.*

14. The number 561 is the first term in the sequence of Fermat pseudoprimes to base 2 of the form (n*109^2 – n)/360, where n is integer (561 is obtained for n = 17).

***Note**: Another term of this sequence, obtained for n = 19897, is the Carmichael number 656601.*

***Note**: The number 1729 is the first term in the sequence of Fermat pseudoprimes to base 2 of the form (n*181^2 – n)/360, where n is integer (1729 is obtained for n = 19). The next terms of the sequence, obtained for n = 31, is the Carmichael number 2821.*

***Note**: Because the numbers 561 and 1729 have both three prime factors, the sequences from above can be eventually translated into the property of the numbers of the form 360*(a*b) + 1, where a and b are primes, to generate squares of primes. Corresponding to the sequences above, for [a, b] = [3, 11] we obtain 109^2 and for [a, b] = [7, 13] we obtain 181^2.*

***Conjecture**: If the number 360*(a*b) + 1, where a and b are primes, is equal to c^2, where c is prime, then exists an infinite series of Carmichael numbers of the form a*b*d, where d is a natural number (obviously odd, but not necessarily prime).*

***Note**: The numbers of the form 360*(a*b) + c, where a, b and c are primes, seems to have also the property to generate primes. Indeed, if we take for instance [a, c] = [3, 7], we obtain primes for b = 5, 11, 17, 23, 29, 31, 43, 47, 59, 67 etc. (note the chain of 5 consecutive primes of the form 6*k – 1).*

15. The number 561 is the first term of the sequence of Carmichael numbers that can be written as 2^m + n^2, where m and n are integers (561 is obtained for m = 5 and n = 23).

***Note**: The next few terms of this sequence are: 1105 = sqrt(2^4 + 33^2), 2465 = sqrt(2^6 + 49^2) etc.*

16. Some Carmichael numbers are also Harshad numbers but the most of them aren't. The number 561 has yet another interesting related property; if we note with s(n) the iterated sum of the digits of a number n that not goes until the digital root but stops to the last odd prime obtained before this, than 561 is divisible by s((561 + 1)/2) equivalent to s(281) equivalent to 11. Also other Carmichael numbers have this property: 1105 is divisible by s(1105) = 13 and 6601 is divisible by s(6601) = 7.

17. For the randomly chosen, but consecutive, 7 primes (129689, 1299709, 1299721, 1299743, 1299763, 1299791 and 1299811) we obtained 3 primes and 3 semiprimes when introduced them in the formula 2*561 + p^2 – 360.

18. Another relation between 561 and Hardy Ramanujan number: (62745 + 24) mod 1728 = 561 (where 24 is, *e.g.*, the sum of the digits of the Carmichael number 62745 or a constant and 1728 is, obviously, one less than Hardy–Ramanujan number).

19. Yet another relation between 561 and Hardy Ramanujan number: 561 mod 73 = 1729 mod 73 = 50. The formula 73*n + 50, from which we obtain 561 and 1729 for n = 7 and n = 23, leads to other interesting results for n of the form 7 + 16*k: we obtain primes for n = 39, 71, 119, 167 etc.

20. A formula that generating primes: 561^2 – 561 – 1 = 314159 is prime; 561^4 – 561^3 – 561^2 – 561 – 1 = 98872434077 is prime. Also for other Carmichael number the formula C^2 – C – 1 conducts to: 1105^2 – 1105 – 1 = 1219919 prime, 6601^2 – 6601 – 1 = 43566599 prime (semiprimes were obtained for the numbers 1729, 2465, 2821 and so on). Yet the number 2465^4 – 2465^3 – 1 = 36905532355999 is prime and the number 15841^4 – 15841^3 – 1 = 62965543898204639 is prime.

21. The formula N = d_1^2 + d_2^2 + d_3^2 – 560, where d_1, d_2 and d_3 are the only prime factors of a Carmichael number, and they are all three of the form 6*k + 1, seems to generate an interesting class of primes:

: for C = 1729 = 7*13*19 we have N = 19 prime;
: for C = 2821 = 7*13*31 we have N = 619 prime;
: for C = 8911 = 7*19*67 we have N = 4339 prime;
: for C = 15841 = 7*31*73 we have N = 5779 prime.

22. The number 544, obtained as the difference between the first two Carmichael numbers, 1105 and 561, has also a notable property: the relation n^C mod 544 = n seems to be verified for a lot of natural numbers n and a lot of Carmichael numbers C, especially when C is also an Euler pseudoprime.

Conjecture*: The expression n^E mod 544 = n, where n is any natural number, is true if E is an Euler pseudoprime.*

23. The difference between the squares of the first two Carmichael numbers, 1105 and 561, has also the notable property that results in a square of an integer: 952^2 = 1105^2 – 561^2.

Conclusion: I am aware of the excessive use of the word "interesting" in this article, but this was the purpose of it: to show how many "interesting" paths can be opened just studying the number 561, not to follow until the last consequences one of these paths. I didn't succed to show that the properties of the number 561 eclipses the ones of the number 1729 (very present in this article) but hopefully I succeded to show that they are both a pair of extraordinary numbers (and that the number 561 deserves his place on the license plate of a taxi-cab).

20. Six conjectures and the generic formulas for two subsets of Poulet numbers

Abstract. I was following an interesting "track", *i.e.* the pairs of primes [p, q] that apparently can form strictly Carmichael numbers of the form p*q*(n*(q – 1) + p), like for instance [23, 67] and [41, 241], when I observed that also all the Poulet numbers P which have the numbers p = 30*k + 23 and q = 90*k + 67 respectively p = 30*k + 11 and q = 180*k + 61 as prime factors can be written as P = p*q*(n*(q – 1) + p) and I made few conjectures.

I. The generic formula for Poulet numbers which have two prime factors of the form 30*k + 23 and 90*k + 67

Conjecture 1:

Any Poulet numbers P which have the numbers p = 23 and q = 67 as prime factors can be written as P = p*q*(n*(q – 1) + p) = 3*p^3*(3*n + 1) – p^2*(15*n + 2) + 6*p*n, where n non-null positive integer (we took q = 3*p – 2).

Verifying the conjecture (for the first few such Poulet numbers):

: For n = 1 we have the Poulet number P = 137149 = 23*67*89;
: For n = 3 we have the Poulet number (also Carmichael number) P = 340561 = 13*17*23*67;
: For n = 9 we have the Poulet number P = 950797 = 23*67*617;
: For n = 10 we have the Poulet number P = 1052503 = 23*67*683;
: For n = 13 we have the Poulet number P = 1357621 = 23*67*881.

Comment:

This formula is important for determining sequences of Poulet numbers; in their case there is not an instrument for obtaining such formulas as there is the Korselt's criterion in the case of Carmichael numbers. See also the sequence A182515 that I submitted to OEIS.

Note:

The formula P = p*q*(n*(q – 1) + p) is not a pattern for any Poulet numbers which have two prime factors of the form p and q = 3*p – 2; for instance, for [p, q] = [7, 19] and Carmichael numbers 1729 and 63973 the formula doesn't apply.

Conjecture 2:

Any Poulet numbers P which have the numbers p = 30*k + 23 and q = 90*k + 67, where k non-negative integer, as prime factors can be written as P = 3*p^3*(3*n + 1) – p^2*(15*n + 2) + 6*p*n, where n non-null positive integer.

Note:

As it can be seen, the formula from above it is not anymore derived from and equivalent to the formula P = p*q*(n*(q – 1) + p), equivalence that exists only in the case of the Conjecture 1.

Verifying the conjecture for p = 53 and q = 157 (for the first few such Poulet numbers):

: For n = 3 we have the Poulet number (also Carmichael number) P = 4335241 = 53*157*521;
: For n = 10 we have the Poulet number P = 13421773 = 53*157*1613;
: For n = 13 we have the Poulet number (also Carmichael number) P = 17316001 = 53*157*2081.

Verifying the conjecture for p = 113 and q = 337 (for the first few such Poulet numbers):

: For n = 1 we have the Poulet number (also Carmichael number) P = 17098369 = 113*337*449;
: For n = 7 we have the Poulet number (also Carmichael number) P = 93869665 = 5*17*29*113*337;
: For n = 13 we have the Poulet number P = 170640961 = 113*337*4481.

Note:

It is notable how easily we found Poulet numbers with this formula, for at least three values of n from n = 1 to n = 13, for any of the three pairs of primes considered: [23, 67], [53, 157], [113, 337].

Conjecture 3:

There is an infinity of Poulet numbers which have the numbers p = 30*k + 23 and q = 90*k + 67, where k non-negative integer, as prime factors (implicitly there is an infinity of pairs of primes of the form [30*k + 23, 90*k + 67]).

II. The generic formula for Poulet numbers which have two prime factors of the form 30*k + 11 and 180*k + 61

Conjecture 4:

Any Poulet numbers P which have the numbers p = 11 and q = 61 as prime factors can be written as P = p*q*(n*(q – 1) + p) = 6*p^3*(6*n + 1) – p^2*(66*n + 5) + 30*p*n, where n non-null positive integer (we took q = 6*p – 5).

Verifying the conjecture (for the first such Poulet number):

: For n = 21 we have the Poulet number (also Carmichael number) P = 852841 = 11*31*41*61.

Note:

The formula P = p*q*(n*(q – 1) + p) is not a pattern for any Poulet numbers which have two prime factors of the form p and q = 6*p – 5; for instance, for [p,q] = [7,37] and Carmichael number 63973 = 7*13*19*37 the formula doesn’t apply.

Conjecture 5:

Any Poulet numbers P which have the numbers p = 30*k + 11 and q = 180*k + 61, where k non-negative integer, as prime factors can be written as P = 6*p^3*(6*n + 1) – p^2*(66*n + 5) + 30*p*n, where n non-null positive integer.

Verifying the conjecture for p = 41 and q = 241 (for the first few such Poulet numbers):

: For n = 2 we have the Poulet number (also Carmichael number) P = 5148001 = 41*241*521;
: For n = 3 we have the Poulet number (also Carmichael number) P = 7519441 = 41*241*761;
: For n = 4 we have the Poulet number (also Carmichael number) P = 9890881 = 7*11*13*41*241;
: For n = 5 we have the Poulet number (also Carmichael number) P = 12262321 = 17*41*73*241.

Conjecture 6:

There is an infinity of Poulet numbers which have the numbers p = 30*k + 11 and q = 180*k + 61, where k non-negative integer, as prime factors (implicitly there is an infinity of pairs of primes of the form [30*k + 11, 180*k + 61]).

21. A pattern that relates Carmichael numbers to the number 66

Abstract. The length of the period of the rational number which is the sum, from n = 1 to n = ∞, of the numbers $1/(C_n – 1)$, where $\{C_1, C_2, ..., C_n\}$ is the ordered set of Carmichael numbers, i.e. {561, 1105, 1729, 2465, ...}, seems to be always multiple of 66. This property doesn't apply always when $C_1, C_2, ..., C_n$ are not consecutive, so this pattern could be a way to determinate if between two known Carmichael numbers there exist other unknown Carmichael numbers.

Conjecture:

The length of the period of the rational number which is the sum, from n = 1 to n = ∞, of the numbers $1/(C_n – 1)$, where $\{C_1, C_2, ..., C_n\}$ is the ordered set of Carmichael numbers, is always multiple of 66.

Verifying the conjecture (for n ≤ 12)**:**

: the sum 1/560 + 1/1104 is equal to a rational number with the length of the period 66;
: the sum 1/560 + 1/1104 + 1/1728 is equal to a rational number with the length of the period 66;
: the sum 1/560 + 1/1104 + 1/1728 + 1/2464 is equal to a rational number with the length of the period 66;
: the sum 1/560 +...+ 1/2464 + 1/2820 is equal to a rational number with the length of the period 1518 = 66*23;
: the sum 1/560 +...+ 1/2820 + 1/6600 is equal to a rational number with the length of the period 1518 = 66*23;
: the sum 1/560 +...+ 1/6600 + 1/8910 is equal to a rational number with the length of the period 4554 = 66*69;

: the sum 1/560 +...+ 1/8910 + 1/10584 is equal to a rational number with the length of the period 31878 = 66*483;

: the sum 1/560 +...+ 1/10584 + 1/15840 is equal to a rational number with the length of the period 31878 = 66*483;

: the sum 1/560 +...+ 1/15840 + 1/29340 is equal to a rational number with the length of the period 286902 = 66*4347;

: the sum 1/560 +...+ 1/29340 + 1/41040 is equal to a rational number with the length of the period 286902 = 66*4347;

: the sum 1/560 +...+ 1/41040 + 1/46656 is equal to a rational number with the length of the period 286902 = 66*4347.

Note:

This is a characteristc only of absolute Fermat pseudoprimes; in the case of relative Fermat pseudoprimes, Poulet numbers for instance, this pattern doesn't apply: for the first two Poulet numbers, 341 and 561, we have the sum 1/340 + 1/560 equal to a rational number with the length of the period 48.

Note:

This is a characteristc only of the sum of ordered Carmichael numbers, for instance, for the first and the third Carmichael numbers, 561 and 1729, we have the sum 1/560 + 1/1728 equal to a rational number with the length of the period 6.

Comment:

This property could be a way to determinate if between two known Carmichael numbers there exist other unknown Carmichael numbers.

22. A generic formula of 2-Poulet numbers and also a method to obtain sequences of n-Poulet numbers

Abstract. In this paper I present a formula based on 2-Poulet numbers which seems to conduct always to a prime, a square of prime or a semiprime, a conjecture that this formula is generic for 2-Poulet numbers, and, in case that the conjecture doesn't hold, I present another utility for this formula, namely to generate sequences of n-Poulet numbers.

Conjecture:

Any 2-Poulet number P can be written at least in one way as P = (q*2^a*3^b*5^c ± 1)*2^n + 1, where q is a prime, a square of prime or a semiprime, a, b, c are non-negative integers and n is non-null positive integer.

In other words, there always exist a number q = ((P – 1)/2^n ± 1)/(2^a*3*b*5^c), where P is a 2-Poulet number, a, b, c are non-negative integers and n is non-null positive integer, such that q is a prime, a square of prime or a semiprime.

Note: In this paper I will consider the number 1 to be a prime (not to repeat the formulation: q is prime, square of prime, semiprime or is equal to number 1).

Verifying the conjecture:
[for the first ten 2-Poulet numbers, only for a restrictive version of the conjecture, considering just the formula P = (q*2^a*3^b*5^c + 1)*2^n + 1]

For P = 341, we have:
    q = ((341 – 1)/2^1 – 1)/(2^0*3^0*5^0) = 13^2;
    q = ((341 – 1)/2^2 – 1)/(2^2*3^1*5^0) = 7.
For P = 1387, we have:
    q = ((1387 – 1)/2^1 – 1)/(2^2*3^0*5^0) = 173.
For P = 2047, we have:
    q = ((2047 – 1)/2^1 – 1)/(2^1*3^0*5^0) = 7*73.
For P = 2701, we have:
    q = ((2701 – 1)/2^1 – 1)/(2^0*3^0*5^0) = 19*71;
    q = ((2701 – 1)/2^2 – 1)/(2^1*3^0*5^0) = 337.
For P = 3277, we have:
    q = ((3277 – 1)/2^1 – 1)/(2^0*3^0*5^0) = 1637;
    q = ((3277 – 1)/2^2 – 1)/(2^1*3^0*5^0) = 409.
For P = 4033, we have:
    q = ((4033 – 1)/2^1 – 1)/(2^0*3^0*5^1) = 13*31;
    q = ((4033 – 1)/2^2 – 1)/(2^0*3^0*5^0) = 19*53;
    q = ((4033 – 1)/2^3 – 1)/(2^0*3^0*5^0) = 503;
    q = ((4033 – 1)/2^4 – 1)/(2^0*3^0*5^0) = 251;
    q = ((4033 – 1)/2^5 – 1)/(2^0*3^0*5^3) = 1;
    q = ((4033 – 1)/2^6 – 1)/(2^1*3^0*5^0) = 31.
For P = 4369, we have:
    q = ((4369 – 1)/2^1 – 1)/(2^0*3^0*5^0) = 37*59;
    q = ((4369 – 1)/2^2 – 1)/(2^0*3^0*5^0) = 1091;
    q = ((4369 – 1)/2^3 – 1)/(2^0*3^0*5^1) = 109;
    ((4369 – 1)/2^4 – 1)/(2^0*3^0*5^0) = 251;
For P = 4681, we have:
    q = ((4681 – 1)/2^1 – 1)/(2^0*3^0*5^0) = 2339;
    q = ((4681 – 1)/2^2 – 1)/(2^0*3^0*5^0) = 7*167;
    q = ((4681 – 1)/2^3 – 1)/(2^3*3^0*5^0) = 73;
For P = 5461, we have:
    q = ((2701 – 1)/2^1 – 1)/(2^0*3^0*5^0) = 2729;
    q = ((2701 – 1)/2^2 – 1)/(2^2*3^0*5^0) = 11*31.
For P = 7957, we have:
    q = ((7957 – 1)/2^1 – 1)/(2^0*3^0*5^1) = 37*43;
    q = ((7957 – 1)/2^2 – 1)/(2^2*3^0*5^0) = 7*71.

Verifying the conjecture:
(for seven greater consecutive 2-Poulet numbers)

For P = 27657600833, we have:

q = ((27657600833 – 1)/2^4 – 1)/(2^0*3^1*5^0) = 653*882389;
q = ((27657600833 – 1)/2^1 + 1)/(2^0*3^1*5^0) = 22433*205483;
q = ((27657600833 – 1)/2^2 + 1)/(2^0*3^0*5^0) = 6914400209.
q = ((27657600833 – 1)/2^4 + 1)/(2^0*3^0*5^0) = 6911*250123.
q = ((27657600833 – 1)/2^6 + 1)/(2^1*3^0*5^0) = 8093*26699.

For P = 27667059281, we have:

q = ((27667059281 – 1)/2^1 – 1)/(2^0*3^0*5^0) = 103*134306113;
q = ((27667059281 – 1)/2^4 + 1)/(2^1*3^0*5^0) = 864595603.

For P = 27675991081, we have:

q = ((27675991081 – 1)/2^1 – 1)/(2^0*3^0*5^0) = 10169*680401;
q = ((27675991081 – 1)/2^2 – 1)/(2^2*3^0*5^0) = 1109*779869.

For P = 27681232903, we have:

q = ((27681232903 – 1)/2^1 – 1)/(2^1*3^0*5^2) = 276812329;
q = ((27681232903 – 1)/2^2 + 1)/(2^4*3^0*5^0) = 67*807083.

For P = 27685810639, we have:

q = ((27685810639 – 1)/2^1 + 1)/(2^3*3^0*5^1) = 4740721.

For P = 27686175193, we have:

q = ((27686175193 – 1)/2^1 - 1)/(2^0*3^0*5^1) = 20208887;
q = ((27686175193 – 1)/2^1 + 1)/(2^0*3^0*5^0) = 2837*4879481;
q = ((27686175193 – 1)/2^3 + 1)/(2^2*3^0*5^2) = 113*306263.

For P = 27702689701, we have:

q = ((27702689701 – 1)/2^2 - 1)/(2^3*3^0*5^0) = 11*78700823;
q = ((27702689701 – 1)/2^1 + 1)/(2^0*3^0*5^0) = 15971*867281;
q = ((27702689701 – 1)/2^2 + 1)/(2^1*3^0*5^0) = 199*17401187.

Comment:

If the Conjecture doesn’t hold, it may be considered a more premissive version: Any 2-Poulet number P can be written at least in one way as P = (q*2^a*3^b*5^c ± 1)*2^n + 1, where q is a prime, a square of prime or a semiprime and a, b, c, n are non-negative integers.
In this case we have, for instance for P = 27686175193, q = ((27686175193 – 1)/2^0 - 1)/(2^0*3^0*5^0) = 27686175191 which is prime.

Comment:

If the Conjecture doesn’t hold, it has anyhow at least one utility: it’s a method for finding sequences of Poulet numbers (not only 2-Poulet numbers).

Taking, for instance, q = 223*r, where r is prime, we have the sequence of Poulet numbers P defined as P = (223*r + 1)*2^n + 1, with the first three terms {41041, 10261, 52633}, obtained for the following values of (r, n): {23, 1), (23, 3), (59, 2)}.
Taking, for instance, q = 29*r, where r is prime, we have the sequence of Poulet numbers P defined as P = (29*r + 1)*3^n + 1, with the first term 2701, obtained for the following value of (r, n): (31, 1).
Taking, for instance, q = 37*r, where r is prime, we have the sequence of Poulet numbers P defined as P = (37*r + 1)*5^n + 1, with the first term 561, obtained for the following value of (r, n): (3, 5).
Taking, for instance, q = 13^2, we have the sequence of Poulet numbers P defined as P = (13^2 + 1)*2^n + 1, with the first term 341, obtained for the following value of n: 1.

23. Few interesting results regarding Poulet numbers and Egyptian fraction expansion

Abstract. Considering r being equal to the positive rational number 1/(d1 – 1) + 1/(d2 – 1) +...+ 1/(dn – 1), where d1,..., dn are the prime factors of a Poulet number, the Egyptian fraction expansion applied to r leads to interesting results.

Note:

An Egyptian fraction is a sum of distinct unit fractions, such as 1/a + 1/b + 1/c +...+/m, where the denominators a, b, c, ..., m are positive, distinct, integers. Every positive rational number can be represented by an Egyptian fraction.

The Egyptian fraction expansion is an algorithm due to Fibonacci for computing Egyptian fractions: the number x/y, where x, y are positive, distinct, integers, is written as follows:

x/y = 1/ceiling(y/x) + ((-1)mod x)/y*ceiling(y/x), where the function ceiling(z) represents the smaller integer equal to or greater than z.

This algorithm is repetead to the second term of the summation above and so on until is obtained an egyptian fraction.

Conjecture 1:

If r is equal to the positive rational number $1/(d_1 – 1) + 1/(d_2 – 1) +...+ 1/(d_n – 1)$, where $d_1,..., d_n$ are the prime factors of a Poulet number P, and m is equal to the last denominator obtained applying the Egyptian fraction expansion to r, then the number m + 1 is a prime or a power of prime for an infinity of Poulet numbers.

Examples:

: For P = 341 = 11*31, we have r = 1/10 + 1/30 = 2/15 = 1/8 + 1/120; the number m + 1 = 120 + 1 = 121 = 11^2, a square of prime.
: For P = 561 = 3*11*17, we have r = 1/2 + 1/10 + 1/16 = 53/80 = 1/2 + 1/7 + 1/51 + 1/28560; the number m + 1 = 28560 + 1 = 28561 = 13^4, a power of prime.
: For P = 645 = 3*5*43, we have r = 1/2 + 1/4 + 1/42 = 65/84 = 1/2 + 1/4 + 1/42; the number m + 1 = 42 + 1 = 43, a prime number.
: For P = 1105 = 5*13*17, we have r = 1/4 + 1/12 + 1/16 = 19/48 = 1/3 + 1/16; the number m + 1 = 16 + 1 = 17, a prime number.
: For P = 1387 = 19*73, we have r = 1/18 + 1/72 = 5/72 = 1/15 + 1/360; the number m + 1 = 360 + 1 = 361 = 19^2, a square of prime.
: For P = 1729 = 7*13*19, we have r = 1/6 + 1/12 + 1/18 = 11/36 = 1/4 + 1/18; the number m + 1 = 18 + 1 = 19, a prime number.
: For P = 1905 = 3*5*127, we have r = 1/2 + 1/4 + 1/126 = 191/252 = 1/2 + 1/4 + 1/126; the number m + 1 = 126 + 1 = 127, a prime number.
: For P = 6601 = 7*23*41, we have r = 1/6 + 1/22 + 1/40 = 313/1320 = 1/5 + 1/27 + 1/11880; the number m + 1 = 11880 + 1 = 11881 = 109^2, a square of prime.

: For P = 8911 = 7*19*67, we have r = 1/6 + 1/18 + 1/66 = 47/198 = 1/5 + 1/27 + 1/2970; the number m + 1 = 2970 + 1 = 2971, a prime number.

: For P = 52633 = 7*73*103, we have r = 1/6 + 1/72 + 1/102 = 233/1224 = 1/6 + 1/43 + 1/2289 + 1/8031644 + 1/80634123646776; the number m + 1 = 80634123646776 + 1 = 80634123646777, a prime number.

Note:
For the first ten Carmichael numbers C divisible by 7 and 19 (we don't have a comprehensive list of Poulet numbers indexed together with their prime factors) we always obtain for the number m + 1 a prime or a square of prime; we have the following values for (C, m + 1): (1729, 19), (8911, 2971), (63973, 2^2), (126217, 19^2), (188461, 433), (748657, 433), (825265, 1009), (997633, 577), (1050985, 23), (1773289, 1321).

Conjecture 2:

If r is equal to the positive rational number $1/(d_1 - 1) + 1/(d_2 - 1) +...+ 1/(d_n - 1)$, where $d_1,..., d_n$ are the prime factors of a Poulet number P, and r is represented by the irreducible fraction x/y, where x, y positive integers, then the number y + 1 is a prime or a power of prime for an infinity of Poulet numbers.

Examples:
(as it can be seen above)

: For P = 341, we have r = x/y = 2/15; the number y + 1 = 15 + 1 = 16 = 2^4, a power of prime.

: For P = 561, we have r = x/y = 53/80; the number y + 1 = 80 + 1 = 81 = 3^4, a power of prime.

: For P = 1105, we have r = x/y = 19/48; the number y + 1 = 48 + 1 = 49, a square of prime.

: For P = 1387, we have r = x/y = 5/72; the number y + 1 = 72 + 1 = 73, a prime number.

: For P = 1729, we have r = x/y = 11/36; the number y + 1 = 36 + 1 = 37, a prime number.

: For P = 6601, we have r = x/y = 313/1320; the number y + 1 = 1320 + 1 = 1321, a prime number.

: For P = 8911, we have r = x/y = 47/198; the number y + 1 = 198 + 1 = 199, a prime number.

Note:
As it can be seen above, the number y is sometimes equal to $lcm((d_1 - 1), (d_2 - 1), ..., (d_n - 1))$, which is, for instance, the case of the Poulet number 1387 = 19*73, where y = 72 = lcm(18,72), but this is not always true: this is, for instance, the case of Poulet number 341, where y = 15 and lcm(10,30) = 30.

Conjecture 3:

If $d_1,..., d_n$ are the prime factors of a Poulet number P, then the number $lcm((d_1 - 1), (d_2 - 1), ..., (d_n - 1))$ is a prime or a power of prime for an infinity of Poulet numbers.

24. The Smarandache-Coman divisors of order k of a composite integer n with m prime factors

Abstract. We will define in this paper the Smarandache-Coman divisors of order k of a composite integer n with m prime factors, a notion that seems to have promising applications, at a first glance at least in the study of absolute and relative Fermat pseudoprimes, Carmichael numbers and Poulet numbers.

Definition 1:

We call *the set of Smarandache-Coman divisors of order 1 of a composite positive integer n with m prime factors*, $n = d_1*d_2*...*d_m$, where the least prime factor of n, d_1, is greater than or equal to 2, the set of numbers defined in the following way:
$SCD_1(n) = \{S(d_1 – 1), S(d_2 – 1), ..., S(d_m – 1)\}$, where S is the Smarandache function.

Examples:

1. The set of SC divisors of order 1 of the number 6 is {S(2 – 1), S(3 – 1)} = {S(1), S(2)} = {1, 2}, because 6 = 2*3;
2. $SCD_1(429)$ = {S(3 – 1), S(11 – 1), S(13 – 1)} = {S(2), S(10), S(12)} = {2, 5, 4}, because 429 = 3*11*13.

Definition 2:

We call *the set of Smarandache-Coman divisors of order 2 of a composite positive integer n with m prime factors*, $n = d_1*d_2*...*d_m$, where the least prime factor of n, d_1, is greater than or equal to 3, the set of numbers defined in the following way:
$SCD_2(n) = \{S(d_1 – 2), S(d_2 – 2), ..., S(d_m – 2)\}$, where S is the Smarandache function.

Examples:

1. The set of SC divisors of order 2 of the number 21 is {S(3 – 2), S(7 – 2)} = {S(1), S(5)} = {1, 5}, because 21 = 3*7;
2. $SCD_2(2429)$ = {S(7 – 2), S(347 – 2)} = {S(5), S(345)} = {5, 23}, because 2429 = 7*347.

Definition 3:

We call *the set of Smarandache-Coman divisors of order k of a composite positive integer n with m prime factors*, $n = d_1*d_2*…*d_m$, where the least prime factor of n, d_1, is greater than or equal to k + 1, the set of numbers defined in the following way:
$SCD_k(n) = \{S(d_1 – k), S(d_2 – k), …, S(d_m – k)\}$, where S is the Smarandache function.

Examples:

1. The set of SC divisors of order 5 of the number 539 is {S(7 – 5), S(11 – 5)} = {S(2), S(6)} = {2, 3}, because 539 = 7^2*11;
2. $SCD_6(221)$ = {S(13 – 6), S(17 – 6)} = {S(7), S(11)} = {7, 11}, because 221 = 13*17.

Comment:

We obviously defined the sets of numbers above because we believe that they can have interesting applications, in fact we believe that they can even make us re-think and re-consider the Smarandache function as an instrument to operate in the world of number theory: while at the beginning its value was considered to consist essentially in that to be a criterion for primality, afterwards the Smarandache function crossed a normal process of substantiation, so it was constrained to evolve in a relatively closed (even large) circle of equalities, inequalities, conjectures and theorems concerning, most of them, more or less related concepts. We strongly believe that some of the most important applications of the Smarandache function are still undiscovered. We were inspired in defining the Smarandache-Coman divisors by the passion for Fermat pseudoprimes, especially for Carmichael numbers and Poulet numbers, by the Korselt's criterion, one of the very few (and the most important from them) instruments that allow us to comprehend Carmichael numbers, and by the encouraging results we easily obtained, even from the first attempts to relate these two types of numbers, Fermat pseudoprimes and Smarandache numbers.

Smarandache-Coman divisors of order 1 of the 2-Poulet numbers:

(See the sequence A214305 in OEIS, posted by us, for a list with Poulet numbers with two prime factors)

$SCD_1(341) = \{S(11 – 1), S(31 – 1)\} = \{S(10), S(30)\} = \{5, 5\}$;
$SCD_1(1387) = \{S(19 – 1), S(73 – 1)\} = \{S(18), S(72)\} = \{6, 6\}$;
$SCD_1(2047) = \{S(23 – 1), S(89 – 1)\} = \{S(22), S(88)\} =\{11, 11\}$;
$SCD_1(2701) = \{S(37 – 1), S(73 – 1)\} = \{S(36), S(72)\} = \{6, 6\}$;
$SCD_1(3277) = \{S(29 – 1), S(113 – 1)\} = \{S(28), S(112)\} = \{7, 7\}$;
$SCD_1(4033) = \{S(37 – 1), S(109 – 1)\} = \{S(36), S(108)\} = \{6, 9\}$;
$SCD_1(4369) = \{S(17 – 1), S(257 – 1)\} = \{S(16), S(256)\} = \{6, 10\}$;
$SCD_1(4681) = \{S(31 – 1), S(151 – 1)\} = \{S(30), S(150)\} = \{5, 10\}$;
$SCD_1(5461) = \{S(43 – 1), S(127 – 1)\} = \{S(42), S(126)\} = \{7, 7\}$;
$SCD_1(7957) = \{S(73 – 1), S(109 – 1)\} = \{S(72), S(108)\} = \{6, 9\}$;
$SCD_1(8321) = \{S(53 – 1), S(157 – 1)\} = \{S(52), S(156)\} = \{13, 13\}$.

Comment:

It is notable how easily are obtained interesting results: from the first 11 terms of the 2-Poulet numbers sequence checked there are already foreseen few patterns.

Open problems:

1. Is for the majority of the 2-Poulet numbers the case that the two Smarandache-Coman divisors of order 1 are equal, as for the seven from the eleven numbers checked above?
2. Is there an infinity of 2-Poulet numbers for which the set of SCD of order 1 is equal to {6, 6}, the case of Poulet numbers 1387 and 2701, or with {6, 9}, the case of Poulet numbers 4033 and 7957?

Smarandache-Coman divisors of order 2 of the 2-Poulet numbers:

$SCD_2(341) = \{S(11 – 2), S(31 – 2)\} = \{S(9), S(29)\} = \{6, 29\}$;
$SCD_2(1387) = \{S(19 – 2), S(73 – 2)\} = \{S(17), S(71)\} =\{17, 71\}$;
$SCD_2(2047) = \{S(23 – 2), S(89 – 2)\} = \{S(21), S(87)\} = \{7, 29\}$;
$SCD_2(2701) = \{S(37 – 2), S(73 – 2)\} = \{S(35), S(71)\} = \{7, 71\}$;
$SCD_2(3277) = \{S(29 – 2), S(113 – 2)\} = \{S(27), S(111)\} = \{9, 37\}$;
$SCD_2(4033) = \{S(37 – 2), S(109 – 2)\} = \{S(35), S(107)\} = \{7, 107\}$;
$SCD_2(4369) = \{S(17 – 2), S(257 – 2)\} = \{S(15), S(255)\} = \{5, 17\}$;
$SCD_2(4681) = \{S(31 – 2), S(151 – 2)\} =\{S(29), S(149)\} = \{29, 149\}$;

$SCD_2(5461)$ = {S(43 – 2), S(127 – 2)} = {S(41), S(125)} = {41, 15};
$SCD_2(7957)$ = {S(73 – 2), S(109 – 2)} ={S(71), S(107)} = {71, 107};
$SCD_2(8321)$ = {S(53 – 2), S(157 – 2)} = {S(52), S(156)} = {17, 31}.

Comment:
In the case of SCD of order 2 of the 2-Poulet numbers there are too foreseen few patterns.

Open problems:

1. Is for the majority of the 2-Poulet numbers the case that the two Smarandache-Coman divisors of order 2 are both primes, as for the eight from the eleven numbers checked above?
2. Is there an infinity of 2-Poulet numbers for which the set of SCD of order 2 is equal to {p, p + 20*k}, where p prime and k positive integer, the case of Poulet numbers 4033 and 4681?

Smarandache-Coman divisors of order 1 of the 3-Poulet numbers:

(See the sequence A215672 in OEIS, posted by us, for a list with Poulet numbers with two prime factors)

$SCD_1(561)$ = $SCD_1(3*11*17)$ = {S(2), S(10), S(16)} = {2, 5, 6};
$SCD_1(645)$ = $SCD_1(3*5*43)$ = {S(2), S(4), S(42)} = {2, 4, 7};
$SCD_1(1105)$ = $SCD_1(5*13*17)$ = {S(4), S(12), S(16)} = {4, 4, 6};
$SCD_1(1729)$ = $SCD_1(7*13*19)$ = {S(6), S(12), S(18)} = {3, 4, 6};
$SCD_1(1905)$ = $SCD_1(3*5*127)$ = {S(2), S(4), S(126)} = {2, 4, 7};
$SCD_1(2465)$ = $SCD_1(5*17*29)$ = {S(4), S(16), S(28)} = {4, 6, 7};
$SCD_1(2821)$ = $SCD_1(7*13*31)$ = {S(6), S(12), S(30)} = {3, 4, 5};
$SCD_1(4371)$ = $SCD_1(3*31*47)$ = {S(2), S(30), S(46)} = {2, 5, 23};
$SCD_1(6601)$ = $SCD_1(7*23*41)$ = {S(6), S(22), S(40)} = {3, 11, 5};
$SCD_1(8481)$ = $SCD_1(3*11*257)$ = {S(2), S(10), S(256)} = {2, 5, 10};
$SCD_1(8911)$ = $SCD_1(7*19*67)$ = {S(6), S(18), S(66)} = {3, 19, 67}.

Open problems:

1. Is there an infinity of 3-Poulet numbers for which the set of SCD of order 1 is equal to {2, 4, 7}, the case of Poulet numbers 645 and 1905?
2. Is there an infinity of 3-Poulet numbers for which the sum of SCD of order 1 is equal to 13, the case of Poulet numbers 561 (2 + 5 + 6 = 13), 645 (2 + 4 + 7 = 13), 1729 (3 + 4 + 6 = 13), 1905 (2 + 4 + 7 = 13) or is equal to 17, the case of Poulet numbers 2465 (4 + 6 + 7 = 17) and 8481 (2 + 5 + 10 = 17)?
3. Is there an infinity of Poulet numbers for which the sum of SCD of order 1 is prime, which is the case of the eight from the eleven numbers checked above? What about the sum of SCD of order 1 plus 1, the case of Poulet numbers 2821 (3 + 4 + 5 + 1 = 13) and 4371 (2 + 5 + 23 + 1 = 31) or the sum of SCD of order 1 minus 1, the case of Poulet numbers 1105 (4 + 4 + 6 – 1 = 13), 2821 (3 + 4 + 5 – 1 = 11) and 4371 (2 + 5 + 23 – 1 = 29)?

25. Seventeen sequences of Poulet numbers characterized by a certain set of Smarandache-Coman divisors

Abstract. In a previous article I defined the Smarandache-Coman divisors of order k of a composite integer n with m prime factors and I sketched some possible applications of this concept in the study of Fermat pseudoprimes. In this paper I make few conjectures about few possible infinite sequences of Poulet numbers, characterized by a certain set of Smarandache-Coman divisors.

Conjecture 1:

There is an infinity of 2-Poulet numbers which have the set of SC divisors of order 1 equal to {p, p}, where p is prime.

The sequence of this 2-Poulet numbers is: 341, 2047, 3277, 5461, 8321, 13747, 14491, 19951, 31417, ... (see the lists below).

Conjecture 2:

There is an infinity of 2-Poulet numbers which have the set of SC divisors of order 2 equal to {p, p + 20*k}, where p is prime and k is non-null integer.

The sequence of this 2-Poulet numbers is: 4033, 4681, 10261, 15709, 23377, 31609, ... (see the lists below).

Conjecture 3:

There is an infinity of 2-Poulet numbers which have the set of SC divisors of order 2 equal to {a, b}, where a + b + 1 is prime.

The sequence of this 2-Poulet numbers is: 1387, 2047, 2701, 3277, 4369, 4681, 8321, 13747, 14491, 18721, 31417, 31609, ... (see the lists below).
Note: This is the case of twelve from the first twenty 2-Poulet numbers.

Conjecture 4:

There is an infinity of 2-Poulet numbers which have the set of SC divisors of order 2 equal to {a, b}, where a + b – 1 is prime.

The sequence of this 2-Poulet numbers is: 4033, 8321, 10261, 13747, 14491, 15709, 19951, 23377, 31417, ... (see the lists below).

Conjecture 5:

There is an infinity of 2-Poulet numbers which have the set of SC divisors of order 2 equal to {a, b}, where a + b – 1 and a + b + 1 are twin primes.

The sequence of this 2-Poulet numbers is: 13747, 14491, 23377, 31417, ... (see the lists below).

Conjecture 6:

There is an infinity of pairs of 2-Poulet numbers which have the set of SC divisors of order 2 equal to {a, b}, respectively to {c, d}, where a + b = c + d and a, b, c, d are primes.

Such pair of 2-Poulet numbers is: (4681, 7957), because 29 + 149 = 71 + 107 = 178.

Conjecture 7:

There is an infinity of pairs of 2-Poulet numbers which have the set of SC divisors of order 2 equal to {a, b}, respectively to {c, d}, where a + b + 1 = c + d – 1.

Such pairs of 2-Poulet numbers are:
(3277, 8321), because 9 + 37 + 1 = 17 + 31 - 1 = 47;
(19951, 5461), because 23 + 31 + 1 = 41 + 15 – 1 = 55.

Conjecture 8:

There is an infinity of 2-Poulet numbers which have the set of SC divisors of order 6 equal to {p, q}, where abs{p – q} = 6*k, where p and q are primes and k is non-null positive integer.

The sequence of this 2-Poulet numbers is:
1387, 2047, 2701, 3277, 4033, 4369, 7957, 13747, 14491, 15709, 23377, 31417, 31609, ... (see the lists below).

Note: This is the case of thirteen from the first twenty 2-Poulet numbers.

Conjecture 9:

There is an infinity of 2-Poulet numbers which have the set of SC divisors of order 6 equal to {a, b}, where abs{a – b} = p and p is prime.

The sequence of this 2-Poulet numbers is: 341, 4681, 10261, ... (see the lists below).

Conjecture 10:

There is an infinity of 2-Poulet numbers which have the set of SC divisors of order 6 equal to {p, q}, where one from the numbers p and q is prime and the other one is twice a prime.

The sequence of this 2-Poulet numbers is: 341, 4681, 5461, 10261, ... (see the lists below).

Conjecture 11:

There is an infinity of 3-Poulet numbers which have the set of SC divisors of order 1 equal to {a, b, c}, where a + b + c is prime and a, b, c are primes.

The sequence of this 2-Poulet numbers is: 561, 645, 1729, 1905, 2465, 6601, 8481, 8911, 10585, 12801, 13741, ... (see the lists below).

Note: This is the case of eleven from the first twenty 2-Poulet numbers.

Conjecture 12:

There is an infinity of 3-Poulet numbers which have the set of SC divisors of order 1 equal to {a, b, c}, where a + b + c - 1 and a + b + c + 1 are twin primes.

The sequence of this 3-Poulet numbers is: 2821, 4371, 16705, 25761, 30121, ... (see the lists below)

Conjecture 13:

There is an infinity of 3-Poulet numbers which have the set of SC divisors of order 1 equal to {n, n, n}.

Such 3-Poulet number is 13981.

Conjecture 14:

There is an infinity of 3-Poulet numbers which have the set of SC divisors of order 2 equal to {5, p, q}, where p and q are primes and q = p + 6*k, where k is non-null positive integer.

Such 3-Poulet numbers are:
1729, because $SCD_2(1729)$ = {5, 11, 17} and 17 = 11 + 6*1;
2821, because $SCD_2(2821)$ = {5, 11, 29} and 29 = 11 + 6*3;
6601, because $SCD_2(6601)$ = {5, 7, 13} and 13 = 7 + 6*1;
13741, because $SCD_2(13741)$ = {5, 11, 149} and 149 = 11 + 6*23;
15841, because $SCD_2(15841)$ = {5, 29, 71} and 71 = 29 + 6*7;
30121, because $SCD_2(30121)$ = {5, 11, 329} and 329 = 11 + 6*53.

Conjecture 15:

There is an infinity of Poulet numbers divisible by 15 which have the set of SC divisors of order 1 equal to {2, 4, 7, n_1, ..., n_i}, where n_1, ..., n_i are non-null positive integers and i > 0.

The sequence of this 3-Poulet numbers is: 18705, 55245, 72855, 215265, 831405, 1246785, ... (see the lists below)

Conjecture 16:

There is an infinity of Poulet numbers divisible by 15 which have the set of SC divisors of order 1 equal to {2, 4, 23, n_1, ..., n_i}, where n_1, ..., n_i are non-null positive integers and i > 0.

The sequence of this 3-Poulet numbers is: 62745, 451905, ... (see the lists below)

Conjecture 17:

There is an infinity of Poulet numbers which are multiples of any Poulet number divisible by 15 which has the set of SC divisors of order 1 equal to {2, 4, n_1, ..., n_i}, where $n_1 = n_2 =...= n_i = 7$ and i > 0.

Examples:

The Poulet number 645 = 3*5*43, having $SCD_1(645)$ = {2, 4, 7}, has the multiples the Poulet numbers 18705, 72885, which have SCD_1 = {2, 4, 7, 7}.

The Poulet number 1905 = 3*5*127, having $SCD_1(1905)$ = {2, 4, 7}, has the multiples 55245, 215265 which have SCD_1 = {2, 4, 7, 7}.

(see the sequence A215150 in OEIS for a list of Poulet numbers divisible by smaller Poulet numbers)

List of SC divisors of order 1 of the first twenty 2-Poulet numbers:

(see the sequence A214305 that I submitted to OEIS for a list of 2-Poulet numbers)

$SCD_1(341)$ = {S(11 – 1), S(31 – 1)} = {S(10), S(30)} = {5, 5};
$SCD_1(1387)$ = {S(19 – 1), S(73 – 1)} = {S(18), S(72)} = {6, 6};
$SCD_1(2047)$ = {S(23 – 1), S(89 – 1)} = {S(22), S(88)} = {11, 11};
$SCD_1(2701)$ = {S(37 – 1), S(73 – 1)} = {S(36), S(72)} = {6, 6};
$SCD_1(3277)$ = {S(29 – 1), S(113 – 1)} = {S(28), S(112)} = {7, 7};
$SCD_1(4033)$ = {S(37 – 1), S(109 – 1)} = {S(36), S(108)} = {6, 9};
$SCD_1(4369)$ = {S(17 – 1), S(257 – 1)} = {S(16), S(256)} = {6, 10};
$SCD_1(4681)$ = {S(31 – 1), S(151 – 1)} = {S(30), S(150)} = {5, 10};
$SCD_1(5461)$ = {S(43 – 1), S(127 – 1)} = {S(42), S(126)} = {7, 7};
$SCD_1(7957)$ = {S(73 – 1), S(109 – 1)} = {S(72), S(108)} = {6, 9};
$SCD_1(8321)$ = {S(53 – 1), S(157 – 1)} = {S(52), S(156)} = {13, 13};
$SCD_1(10261)$ = {S(31 – 1), S(331 – 1)} = {S(30), S(330)} = {5, 11};
$SCD_1(13747)$ = {S(59 – 1), S(233 – 1)} = {S(58), S(232)} = {29, 29};
$SCD_1(14491)$ = {S(43 – 1), S(337 – 1)} = {S(42), S(336)} = {7, 7};
$SCD_1(15709)$ = {S(23 – 1), S(683 – 1)} = {S(22), S(682)} = {11, 31};
$SCD_1(18721)$ = {S(97 – 1), S(193 – 1)} = {S(96), S(192)} = {8, 8};
$SCD_1(19951)$ = {S(71 – 1), S(281 – 1)} = {S(70), S(280)} = {7, 7};
$SCD_1(23377)$ = {S(97 – 1), S(241 – 1)} = {S(96), S(240)} = {8, 6};
$SCD_1(31417)$ = {S(89 – 1), S(353 – 1)} = {S(88), S(352)} = {11, 11};
$SCD_1(31609)$ = {S(73 – 1), S(433 – 1)} = {S(72), S(432)} = {6, 9}.

List of SC divisors of order 2 of the first twenty 2-Poulet numbers:

(see the sequence A214305 that I submitted to OEIS for a list of 2-Poulet numbers)

$SCD_2(341)$ = {S(11 – 2), S(31 – 2)} = {S(9), S(29)} = {6, 29};
$SCD_2(1387)$ = {S(19 – 2), S(73 – 2)} = {S(17), S(71)} = {17, 71};
$SCD_2(2047)$ = {S(23 – 2), S(89 – 2)} = {S(21), S(87)} = {7, 29};
$SCD_2(2701)$ = {S(37 – 2), S(73 – 2)} = {S(35), S(71)} = {7, 71};
$SCD_2(3277)$ = {S(29 – 2), S(113 – 2)} = {S(27), S(111)} = {9, 37};
$SCD_2(4033)$ = {S(37 – 2), S(109 – 2)} = {S(35), S(107)} = {7, 107};
$SCD_2(4369)$ = {S(17 – 2), S(257 – 2)} = {S(15), S(255)} = {5, 17};
$SCD_2(4681)$ = {S(31 – 2), S(151 – 2)} = {S(29), S(149)} = {29, 149};
$SCD_2(5461)$ = {S(43 – 2), S(127 – 2)} = {S(41), S(125)} = {41, 15};
$SCD_2(7957)$ = {S(73 – 2), S(109 – 2)} = {S(71), S(107)} = {71, 107};
$SCD_2(8321)$ = {S(53 – 2), S(157 – 2)} = {S(51), S(155)} = {17, 31};
$SCD_2(10261)$ = {S(31 – 2), S(331 – 2)} = {S(29), S(329)} = {29, 47};

$SCD_2(13747)$ = {S(59 – 2), S(233 – 2)} = {S(57), S(231)} = {19, 11};
$SCD_2(14491)$ = {S(43 – 2), S(337 – 2)} = {S(41), S(335)} = {41, 67};
$SCD_2(15709)$ = {S(23 – 2), S(683 – 2)} = {S(21), S(681)} = {7, 227};
$SCD_2(18721)$ = {S(97 – 2), S(193 – 2)} = {S(95), S(191)} = {19, 191};
$SCD_2(19951)$ = {S(71 – 2), S(281 – 2)} = {S(69), S(279)} = {23, 31};
$SCD_2(23377)$ = {S(97 – 2), S(241 – 2)} = {S(95), S(239)} = {19, 239};
$SCD_2(31417)$ = {S(89 – 2), S(353 – 2)} = {S(87), S(351)} = {29, 13};
$SCD_2(31609)$ = {S(73 – 2), S(433 – 2)} = {S(71), S(431)} = {71, 431}.

List of SC divisors of order 6 of the first twenty 2-Poulet numbers:
(see the sequence A214305 that I submitted to OEIS for a list of 2-Poulet numbers)

$SCD_6(341)$ = {S(11 – 6), S(31 – 6)} = {S(5), S(25)} = {5, 10};
$SCD_6(1387)$ = {S(19 – 6), S(73 – 6)} = {S(13), S(67)} = {13, 67};
$SCD_6(2047)$ = {S(23 – 6), S(89 – 6)} = {S(17), S(83)} = {17, 83};
$SCD_6(2701)$ = {S(37 – 6), S(73 – 6)} = {S(31), S(67)} = {31, 67};
$SCD_6(3277)$ = {S(29 – 6), S(113 – 6)} = {S(23), S(107)} = {23, 107};
$SCD_6(4033)$ = {S(37 – 6), S(109 – 6)} = {S(31), S(103)} = {31, 103};
$SCD_6(4369)$ = {S(17 – 6), S(257 – 6)} = {S(11), S(251)} = {11, 251};
$SCD_6(4681)$ = {S(31 – 6), S(151 – 6)} = {S(25), S(145)} = {10, 29};
$SCD_6(5461)$ = {S(43 – 6), S(127 – 6)} = {S(37), S(121)} = {37, 22};
$SCD_6(7957)$ = {S(73 – 6), S(109 – 6)} = {S(67), S(103)} = {67, 103};
$SCD_6(8321)$ = {S(53 – 6), S(157 – 6)} = {S(47), S(151)} = {47, 151};
$SCD_6(10261)$ = {S(31 – 6), S(331 – 6)} = {S(25), S(325)} = {10, 13};
$SCD_6(13747)$ = {S(59 – 6), S(233 – 6)} = {S(53), S(227)} = {53, 227};
$SCD_6(14491)$ = {S(43 – 6), S(337 – 6)} = {S(37), S(331)} = {37, 331};
$SCD_6(15709)$ = {S(23 – 6), S(683 – 6)} = {S(17), S(677)} = {17, 677};
$SCD_6(18721)$ = {S(97 – 6), S(193 – 6)} = {S(91), S(187)} = {13, 17};
$SCD_6(19951)$ = {S(71 – 6), S(281 – 6)} = {S(65), S(275)} = {13, 11};
$SCD_6(23377)$ = {S(97 – 6), S(241 – 6)} = {S(91), S(235)} = {13, 47};
$SCD_6(31417)$ = {S(89 – 6), S(353 – 6)} = {S(83), S(347)} = {83, 347};
$SCD_6(31609)$ = {S(73 – 6), S(433 – 6)} = {S(67), S(427)} = {67, 61}.

List of SC divisors of order 1 of the first twenty 3-Poulet numbers:
(see the sequence A215672 that I submitted to OEIS for a list of 3-Poulet numbers)

$SCD_1(561)$ = $SCD_1(3*11*17)$ = {S(2), S(10), S(16)} = {2, 5, 6};
$SCD_1(645)$ = $SCD_1(3*5*43)$ = {S(2), S(4), S(42)} = {2, 4, 7};
$SCD_1(1105)$ = $SCD_1(5*13*17)$ = {S(4), S(12), S(16)} = {4, 4, 6};
$SCD_1(1729)$ = $SCD_1(7*13*19)$ = {S(6), S(12), S(18)} = {3, 4, 6};
$SCD_1(1905)$ = $SCD_1(3*5*127)$ = {S(2), S(4), S(126)} = {2, 4, 7};
$SCD_1(2465)$ = $SCD_1(5*17*29)$ = {S(4), S(16), S(28)} = {4, 6, 7};
$SCD_1(2821)$ = $SCD_1(7*13*31)$ = {S(6), S(12), S(30)} = {3, 4, 5};
$SCD_1(4371)$ = $SCD_1(3*31*47)$ = {S(2), S(30), S(46)} = {2, 5, 23};
$SCD_1(6601)$ = $SCD_1(7*23*41)$ = {S(6), S(22), S(40)} = {3, 11, 5};
$SCD_1(8481)$ = $SCD_1(3*11*257)$ = {S(2), S(10), S(256)} = {2, 5, 10};
$SCD_1(8911)$ = $SCD_1(7*19*67)$ = {S(6), S(18), S(66)} = {3, 19, 67};
$SCD_1(10585)$ = $SCD_1(5*29*73)$ = {S(4), S(28), S(72)} = {4, 7, 6};

SCD_1(12801) = SCD_1(3*17*251) = {S(2), S(16), S(250)} = {2, 6, 15};
SCD_1(13741) = SCD_1(7*13*151) = {S(6), S(12), S(150)} = {3, 4, 10};
SCD_1(13981) = SCD_1(11*31*41) = {S(10), S(30), S(40)} = {5, 5, 5};
SCD_1(15841) = SCD_1(7*31*73) = {S(6), S(30), S(72)} = {3, 5, 6};
SCD_1(16705) = SCD_1(5*13*257) = {S(4), S(12), S(256)} = {4, 4, 10};
SCD_1(25761) = SCD_1(3*31*277) = {S(2), S(30), S(276)} = {2, 5, 23};
SCD_1(29341) = SCD_1(13*37*61) = {S(12), S(36), S(60)} = {4, 6, 5};
SCD_1(30121) = SCD_1(7*13*331) = {S(6), S(12), S(330)} = {3, 4, 11}.

List of SC divisors of order 2 of the first twenty 3-Poulet numbers:
(see the sequence A215672 that I submitted to OEIS for a list of 3-Poulet numbers)

SCD_2(561) = SCD_1(3*11*17) = {S(1), S(9), S(15)} = {1, 6, 5};
SCD_2(645) = SCD_1(3*5*43) = {S(1), S(3), S(41)} = {1, 3, 41};
SCD_2(1105) = SCD_1(5*13*17) = {S(3), S(11), S(15)} = {3, 11, 5};
SCD_2(1729) = SCD_1(7*13*19) = {S(5), S(11), S(17)} = {5, 11, 17};
SCD_2(1905) = SCD_1(3*5*127) = {S(1), S(3), S(125)} = {1, 3, 15};
SCD_2(2465) = SCD_1(5*17*29) = {S(3), S(15), S(27)} = {3, 5, 9};
SCD_2(2821) = SCD_1(7*13*31) = {S(5), S(11), S(29)} = {5, 11, 29};
SCD_2(4371) = SCD_1(3*31*47) = {S(1), S(29), S(45)} = {1, 29, 6};
SCD_2(6601) = SCD_1(7*23*41) = {S(5), S(21), S(29)} = {5, 7, 13};
SCD_2(8481) = SCD_1(3*11*257) = {S(1), S(9), S(255)} = {1, 6, 17};
SCD_2(8911) = SCD_1(7*19*67) = {S(5), S(17), S(65)} = {5, 17, 13};
SCD_2(10585) = SCD_1(5*29*73) = {S(3), S(27), S(71)} = {3, 9, 71};
SCD_2(12801) = SCD_1(3*17*251) = {S(1), S(15), S(249)} = {1, 5, 83};
SCD_2(13741) = SCD_1(7*13*151) = {S(5), S(11), S(149)} = {5, 11, 149};
SCD_2(13981) = SCD_1(11*31*41) = {S(9), S(29), S(39)} = {6, 29, 13};
SCD_2(15841) = SCD_1(7*31*73) = {S(5), S(29), S(71)} = {5, 29, 71};
SCD_2(16705) = SCD_1(5*13*257) = {S(3), S(111), S(255)} = {3, 11, 17};
SCD_2(25761) = SCD_1(3*31*277) = {S(1), S(29), S(275)} = {1, 29, 11};
SCD_2(29341) = SCD_1(13*37*61) = {S(11), S(35), S(59)} = {11, 7, 59};
SCD_2(30121) = SCD_1(7*13*331) = {S(5), S(11), S(329)} = {5, 11, 329}.

List of SC divisors of order 1 of the first ten Poulet numbers divisible by 3 and 5:
(see the sequence A216364 that I submitted to OEIS for a list of Poulet numbers divisible by 15)

SCD_1(645) = SCD_1(3*5*43) = {2, 4, 7};
SCD_1(1905) = SCD_1(3*5*127) = {2, 4, 7};
SCD_1(18705) = SCD_1(3*5*29*43) = {2, 4, 7, 7};
SCD_1(55245) = SCD_1(3*5*29*127) = {2, 4, 7, 7};
SCD_1(62745) = SCD_1(3*5*47*89) = {2, 4, 23, 11};
SCD_1(72855) = SCD_1(3*5*43*113) = {2, 4, 7, 7};
SCD_1(215265) = SCD_1(3*5*113*127) = {2, 4, 7, 7};
SCD_1(451905) = SCD_1(3*5*47*641) = {2, 4, 23, 8};
SCD_1(831405) = SCD_1(3*5*43*1289) = {2, 4, 7, 23};
SCD_1(1246785) = SCD_1(3*5*43*1933) = {2, 4, 7, 23}.

Part two. Thirty sequences of Fermat pseudoprimes

1. Poulet numbers with two prime factors

First 38 terms of the sequence (A214305 in OEIS): 341, 1387, 2047, 2701, 3277, 4033, 4369, 4681, 5461, 7957, 8321, 10261, 13747, 14491, 15709, 18721, 19951, 23377, 31417, 31609, 31621, 35333, 42799, 49141, 49981, 60701, 60787, 65077, 65281, 80581, 83333, 85489, 88357, 90751, 104653, 123251, 129889, 130561.

Conjecture 1:

For any biggest prime factor of a Poulet number P_1 with two prime factors exists a series with infinite many Poulet numbers P_2 formed this way: P_2 mod $(P_1 – d) = d$, where d is the biggest prime factor of P_1.

Note: It can be seen that the Poulet numbers divisible by 73 bigger than 2701 (7957, 10585, 15841, 31609 etc.) can be written as 1314*n + 73 as well as 2628*m + 73.

Conjecture 2:

Any Poulet number P_2 divisible by d can be written as $(P_1 – d)*n + d$, where n is natural, if exists a smaller Poulet number P_1 with two prime factors divisible by d.

Note: This conjecture can't be extrapolated for Poulet numbers P_1 with more than two prime factors; for instance, if is taken 561 = 3*11*17 as p1, are indeed found bigger Poulet numbers divisible by 17 as 1105 and 4369 that can be written as 544*n + 17 but exists also such numbers that can’t be written this way, like 2465. But can be extrapolated the first conjecture.

Conjecture 3:

For any biggest prime factor of a Poulet number P_1 exists a series with infinite many Poulet numbers p2 formed this way: P_2 mod $(P_1 – d) = d$, where d is the biggest prime factor of P_1.

Examples:

For P_1 = 341 = 11*31 were obtained the following Poulet numbers P_2 for which P_2 mod 310 = 31: 2821, 4371, 4681, 10261 etc.
For P_1 = 1387 = 19*73 were obtained the following Poulet numbers P_2 for which P_2 mod 1314 = 73: 2701, 7957, 10585, 15841 etc.
For P_1 = 2047 = 23*89 were obtained the following Poulet numbers P_2 for which P_2 mod 1958 = 89: 31417, 35333, 60787, 62745 etc.
For p1 = 2701 = 37*73 were obtained the following Poulet numbers P_2 for which P_2 mod 2628 = 73: 7957, 10585, 15841 etc.

2. Poulet numbers with three prime factors

First 37 terms of the sequence (A215672 in OEIS): 561, 645, 1105, 1729, 1905, 2465, 2821, 4371, 6601, 8481, 8911, 10585, 12801, 13741, 13981, 15841, 16705, 25761, 29341, 30121, 30889, 33153, 34945, 41665, 52633, 57421, 68101, 74665, 83665, 87249, 88561, 91001, 93961, 113201, 115921, 121465, 137149.

Comments:

The most of the terms shown can be written in one of the following two ways:

(1) p*((n + 1)*p – n*p)*((m + 1)*p – m*p);

(2) p*((n*p – (n + 1)*p)*(m*p – (m + 1)*p),

where p is the smallest of the three prime factors and n, m natural numbers.

Exempli gratia for Poulet numbers from first category:

10585 = 5*29*73 = 5*(5*7 – 6)*(5*18 – 17);
13741 = 7*13*151 = 7*(7*2 – 1)*(7*25 – 24);
13981 = 11*31*41 = 11*(11*3 – 2)*(11*4 – 3);
29341 = 13*37*61 = 13*(13*3 – 2)*(13*5 – 4);
137149 = 23*67*89 = 23*(23*3 – 2)*(23*4 – 3).

Exempli gratia for Poulet numbers from second category:

6601 = 7*23*41 = 7*(7*4 – 5)*(7*7 – 8).

Note: From the numbers from the sequence above, just the numbers 30889, 88561 and 91001 can't be written in one of the two ways. What these three numbers have in common: they all have a prime divisor q of the form 30*k + 23 (*i.e.* 23, 53, 83) and can be written as q*((r + 1)*q – r), where r is a natural number.

Conjecture:

Any Poulet number P with three or more prime divisors has at least one prime divisor q for that can be written as P = q*((r + 1)*q – r), where r is a natural number.

Note: It can be proved that a Carmichael number can be written this way for any of its prime divisors – see the sequence A213812 in OEIS.

Note: There are also a lot of Poulet numbers with two prime divisors that can be written this way, but here are few exceptions: 7957, 23377, 42799, 49981, 60787.

3. Poulet numbers with three prime factors divisible by a smaller Poulet number

First 30 terms of the sequence (A215944 in OEIS): 13981, 137149, 158369, 176149, 276013, 285541, 294409, 348161, 387731, 423793, 488881, 493697, 617093, 625921, 847261, 1052503, 1052929, 1104349, 1128121, 1152271, 1398101, 1461241, 1472353, 1507561, 1534541, 1549411, 1746289, 1840357, 1857241, 2299081.

Comments:

Almost all the numbers from the sequence above can be written as p*((m + 1)*p – m)*((n + 1)*p – n), where m, n, p are natural numbers (in the brackets is written the Poulet number which every one of them is divisible by):
(1) n*(2*n – 1)*(3*n – 2): the number 294409 (2701);
(2) n*(2*n – 1)*(5*n – 4): the numbers 285541 (4681), 488881 (2701);
(3) n*(2*n – 1)*(11*n – 10): the number 625921 (10261);
(4) n*(2*n – 1)*(15*n – 14): the number 1461241 (2701);
(5) n*(3*n – 2)*(4*n – 3): the numbers 13981 (341), 137149 (2047);
(6) n*(3*n – 2)*(5*n – 4): the number 1152271 (5461);
(7) n*(3*n – 2)*(8*n – 7): the number 1840357 (5461);
(8) n*(3*n – 2)*(10*n – 9): the number 2299081 (5461);
(9) n*(3*n – 2)*(12*n – 11): the number 1746289 (4033);
(10) n*(3*n – 2)*(31*n – 30): the number 1052503 (15709);
(11) n*(3*n – 2)*(102*n – 101): the number 348161 (341);
(12) n*(3*n – 2)*(442*n – 441): the number 1507561 (341);
(13) n*(4*n – 3)*(7*n – 6): the number 176149 (1387);
(14) n*(4*n – 3)*(11*n – 10): the number 276013 (1387);
(15) n*(4*n – 3)*(12*n – 11): the number 1104349 (3277);
(16) n*(4*n – 3)*(31*n – 30): the number 1398101 (15709);
(17) n*(5*n – 4)*(6*n – 5): the number 847261 (4681);
(18) n*(5*n – 4)*(8*n – 7): the number 1128121 (4681);
(19) n*(5*n – 4)*(11*n – 10): the number 1549411 (4681);
(20) n*(6*n – 5)*(11*n – 10): the number 1857241 (10261);
(21) n*(6*n – 5)*(16*n – 15): the number 423793 (4369);
(22) n*(7*n – 6)*(16*n – 15): the number 493697 (4369);
(23) n*(15*n – 14)*(16*n – 15): the number 1052929 (4369);
(24) n*(16*n – 15)*(21*n – 20): the number 1472353 (4369).

Note: The only few numbers from the sequence above that can't be written this way are multiples of the Poulet number 5461 and can be, instead, written as 5461*(42*k – 13): 158369 = 5461*29, 387731 = 5461*71, 617093 = 5461*113 and 1534541 = 5461*281.

Conjecture:

The only Fermat pseudoprimes to base 2 divisible by a smaller Fermat pseudoprime to base 2 that can't be written as p*((m + 1)*p – m)*((n + 1)*p – n), where m, n, p are natural numbers, are multiples of 5461 and can be written as 5461*(42*k – 13).

Note: Conjecture is checked for the numbers from the sequence above and for the first 15 Poulet numbers with four prime factors.

Note: There are Fermat pseudoprimes to base 2 divisible with 5461 that can be written as p*((m + 1)*p – m)*((n + 1)*p – n); these ones can be written as 5461*(42*k + 43)). Numbers from this category are: 1152271 = 5461*211, 1840357 = 5461*337, 2299081 = 5461*421.

4. Poulet numbers of the form (6*k + 1)*(6*k*n + 1), where k, n are integers different from 0

First 30 terms of the sequence (A214607 in OEIS): 1105, 1387, 1729, 2701, 2821, 4033, 4681, 5461, 6601, 8911, 10261, 10585, 11305, 13741, 13981, 14491, 15841, 16705, 18721, 29341, 30121, 30889, 31609, 31621, 39865, 41041, 41665, 46657, 49141, 52633, 57421, 63973, 65281, 68101, 75361.

Comments:

A few examples of how the formula looks like for k and n from 1 to 4:
For k = 1 the formula becomes 42*n + 7.
For k = 2 the formula becomes 156*n + 13.
For k = 3 the formula becomes 342*n + 19.
For k = 4 the formula becomes 600*n + 25.
For n = 1 the formula generates a perfect square.
For n = 2 the formula becomes (6*k + 1)*(12*k + 1) and were found the following Poulet numbers: 2701, 8911, 10585, 18721, 49141 etc.
For n = 3 the formula becomes (6*k + 1)*(18*k + 1) and were found the following Poulet numbers: 2821, 4033, 5461, 15841, 31621, 68101 etc.
For n = 4 the formula becomes (6*k + 1)*(24*k + 1). See the sequence A182123 in OEIS.

Note: The formula is equivalent to Poulet numbers of the form p*(n*p – n + 1), where p is of the form 6*k + 1. From the first 68 Poulet numbers just 7 of them (7957, 23377, 33153, 35333, 42799, 49981, 60787) can't be written as p*(n*p – n + 1), where p is of the form 6*k ± 1 and k, n are integers different from 0.

5. Poulet numbers of the form (6*k – 1)*((6*k – 2)*n + 1), where k, n are integers different from 0

First 37 terms of the sequence (A210993 in OEIS): 341, 561, 645, 1105, 1905, 2047, 2465, 3277, 4369, 4371, 6601, 8321, 8481, 10585, 11305, 12801, 13747, 13981, 15709, 16705, 18705, 19951, 23001, 30889, 31417, 34945, 39865, 41041, 41665, 55245, 60701, 62745, 65077, 68101, 72885, 74665, 75361.

Comments:

A Poulet number can be written in more than one way in this form: e.g. 561 = (6*2 – 1)*((6*2 – 2)*5 + 1) = (6*3 – 1)*((6*3 – 2)*2 + 1).

Few examples of how the formula looks like for k and n from 1 to 4:
For k = 1 the formula becomes 20*n + 5 and generates all the Poulet numbers divisible by 5 from the sequence above (beside 645, all of them have another solutions beside k = 1).

For k = 2 the formula becomes 110*n + 11 and generates the Poulet numbers: 341, 561 etc.
For k = 3 the formula becomes 272*n + 17 and generates the Poulet numbers: 561, 1105, 2465, 4369 etc.
For k = 4 the formula becomes 506*n + 23 and generates the Poulet numbers: 2047, 6601 etc.
For n = 1 the formula generates a perfect square.
For n = 2 the formula becomes 3*(6*k – 1)*(4*k – 1) and were found the following Poulet numbers: 561 etc.
For n = 3 the formula becomes (6*k – 1)*(18*k – 5) and were found the following Poulet numbers: 341, 2465, 8321, 83333 etc.
For n = 4 the formula becomes (6*k – 1)*(24*k – 7) and were found the following Poulet numbers: 1105, 2047, 3277, 6601, 13747, 16705, 19951, 31417, 74665, 88357 etc.

Note: The formula is equivalent to Poulet numbers of the form p*(n*p – n + 1), where p is of the form 6*k – 1. From the first 68 Poulet numbers just 26 of them (1387, 2701, 2821, 4033, 4681, 5461, 7957, 8911, 10261, 13741, 14491, 18721, 23377, 29341, 31609, 31621, 33153, 35333, 42799, 46657, 49141, 49981, 57421, 60787, 63973, 65281) can't be written as p*(n*p – n + 1), where p is of the form 6*k – 1 and k, n are integers different from 0.

6. Poulet numbers of the form 7200*n^2 + 8820*n + 2701

First 29 terms of the sequence (A214016 in OEIS): 2701, 18721, 49141, 93961, 226801, 314821, 534061, 665281, 1537381, 1755001, 1987021, 2233441, 3059101, 3363121, 4014361, 5489641, 6313681, 8134561, 9131401, 10185841, 13073941, 13694761, 18443701, 21474181, 27331921, 30058381, 30996001, 32914441, 34890481.

Comments:
Poulet numbers were obtained for the following values of n: 0, 1, 2, 3, 5, 6, 8, 9, 14, 15, 16, 17, 20, 21, 23, 27, 29, 33, 35, 37, 42, 43, 50, 54, 61, 64, 65, 67, 69.

Conjecture:
There are infinite many Poulet numbers of the form 7200*n^2 + 8820*n + 2701.

7. Poulet numbers of the form 144*n^2 + 222*n + 85

First 29 terms of the sequence (A214017 in OEIS): 1105, 2047, 3277, 6601, 13747, 16705, 19951, 31417, 74665, 88357, 275887, 514447, 604117, 642001, 741751, 916327, 1293337, 1433407, 1520905, 2205967, 2387797, 2976487, 2976487, 3316951, 3539101, 4005001, 4101637, 4863127, 5575501, 8209657.

Comments:
Poulet numbers were obtained for the following values of n: 2, 3, 4, 6, 9, 10, 11, 14, 22, 43, 59, 64, 66, 71, 79, 94, 99, 102, 123, 128, 143, 151, 156, 166, 168, 183, 196, 238.

Conjecture:

There are infinite many Poulet numbers of the form 144*n^2 + 222*n + 85.

8. Poulet numbers of the form 8*p*n + p^2, where p is prime

First 29 terms of the sequence (A218483 in OEIS): 561, 1105, 1729, 1905, 2465, 4033, 4369, 4681, 6601, 8321, 8481, 10585, 11305, 12801, 15841, 16705, 18705, 18721, 23001, 23377, 25761, 30121, 30889, 31417, 31609, 33153, 34945, 35333, 39865, 41041, 41665, 46657, 52633, 62745, 65281, 74665, 75361, 83665, 85489.

Comments:

For p = 5 the formula becomes 40*n + 25. From the first 15 pseudoprimes divisible by 5, 12 are of the form 40*n + 25 (beside 3 of them which are of the form 40*n + 5). Conjecture: there are no pseudoprimes to base 2 of the form 40*n + 15.

Note: it can be seen that a pseudoprime can be written in this formula in more than one way: e.g. 561 = 8*3*23 + 3^2 = 8*11*5 + 11^2 = 8*17*2 + 17^2 or 1905 = 8*3*79 + 3^2 = 8*5*47 + 5^2.

Conjecture 1:

If a Fermat pseudoprime to base 2 can be written as 8*p*n + p^2, where n is an integer number and p one of it’s prime factors, than can be written this way for any of it’s prime factors. Checked for all pseudoprimes from the sequence above.

Conjecture 2:

If a Fermat pseudoprime to base 2 with two prime factors can be written as 8*p1*n + p1^2, where n is a natural number and p1 one of it’s two prime factors, than can be written too as 8*p2*(–n) + p2^2, where p2 is the other prime factor. Checked for 4033 = 37*109(n = 9), 4369 = 17*257(n = 30), 4681 = 31*151(n = 15), 8321 = 53*157(n = 13), 18721 = 97*193(n = 12), 23377 = 97*241(n = 18), 31417 = 89*353(n = 33), 31609 = 73*433 (n = 45), 65281 = 97*673(n = 72), 85489 = 53*1613 (n = 195).

Conjecture 3:

If a Fermat pseudoprime to base 2 can’t be written as 8*p*n + p^2, where n is an integer number and p one of it’s prime factors, than can’t be written this way for any of it’s prime factors. Checked for the following pseudoprimes: 341, 645, 1387, 2047, 2701, 2821, 3277, 4371, 5461, 7957, 10261, 13741, 13747, 13981, 14491, 15709, 19951, 29341, 31621, 42799, 49141, 49981, 55245, 60701, 60787, 63973, 65077, 68101, 72885, 80581, 83333.

Note: from the first 72 pseudoprimes, 39 can be written this way.

9. Poulet numbers of the form (n^2 + 2*n)/3

First 33 terms of the sequence (A216170 in OEIS): 341, 645, 2465, 2821, 4033, 5461, 8321, 15841, 25761, 31621, 68101, 83333, 162401, 219781, 282133, 348161, 530881, 587861, 653333, 710533, 722261, 997633, 1053761, 1082401, 1193221, 1246785, 1333333, 1357441, 1398101, 1489665, 1584133, 1690501, 1735841.

Comments:

The corresponding values of n: 31, 43, 85, 91, 109, 127, 157, 217, 277, 307, 451, 499, 697, 811, 919, 1021, 1261, 1327, 1399, 1459, 1471, 1729, 1777, 1801, 1891, 1933, 1999, 2017, 2047, 2113, 2177, 2251.

The formula can be generalised this way: Fermat pseudoprimes to base 2 of the form (n^m + m*n)/(m + 1).

For m = 3 the formula becomes (n^3 + 3*n)/4 and were obtained the following Poulet numbers: 341, 1729, 188461, 228241, 1082809 (for n = 11, 19, 91, 97, 163).

Conjecture:

For any m natural, m > 1, there exist a series with infinite many Fermat pseudoprimes to base 2, P, formed this way: P = (n^m + m*n)/(m+1).

10. Poulet numbers that can be written as 2*p^2 – p, where p is also a Poulet number

First 22 terms of the sequence (A215343 in OEIS): 831405, 5977153, 15913261, 21474181, 38171953, 126619741, 210565981, 224073865, 327718401, 377616421, 390922741, 558097345, 699735345, 1327232481, 1999743661, 4996150741, 8523152641, 11358485281, 13999580785, 15613830541, 17657245081, 20442723301.

Comments:

The correspondent p for the numbers from the sequence above: 645, 1729, 2821, 3277, 4369, 7957, 10261, 10585, 12801, 13741, 13981, 16705, 18705, 25761, 31621, 49981, 65281, 75361, 83665, 88357, 93961, 101101.

Note that for 22 from the first 80 Poulet numbers we obtained thru this formula another Poulet numbers!

The formula could be generalised this way: Poulet numbers that can be written as (n + 1)*p^2 – n*p, where n is natural, n > 0, and p is another Poulet number.

For n = 1 that formula becomes the formula set out for the sequence above.

For n = 2 that formula becomes 3*p^2 – 2*p and were obtained the following Poulet numbers: 348161, 1246785 (for p = 341, 645) etc.

For n = 3 that formula becomes 4*p^2 – 3*p and were obtained the following Poulet numbers: 119273701 (for p = 5461) etc.

For n = 4 that formula becomes 5*p^2 – 4*p and were obtained the following Poulet numbers: 2077545, 9613297 (for p = 645, 1387) etc.

Conjecture:

There are infinite many Poulet numbers that can be written as (n + 1)*p^2 – n*p, where n is natural, n > 0, and p is another Poulet number.

Note: Finally, considering, *e.g.*, that for the Poulet number 645 were obtained Poulet numbers for n = 1, 2, 4 (i.e. 831405, 1246785, 2077545), yet another conjecture.

Conjecture: For any Poulet number p there are infinite many Poulet numbers that can be written as (n + 1)*p^2 – n*p, where n is natural, n > 0.

11. Poulet numbers of the form m*n^2 + (11*m – 23)*n + 19*m – 49

First 37 terms of the sequence (A215326 in OEIS): 341, 645, 1105, 1387, 2047, 2465, 2821, 3277, 4033, 5461, 6601, 7957, 8321, 11305, 13747, 15841, 16705, 19951, 23001, 25761, 30889, 31417, 31621, 39865, 41665, 49981, 65077, 68101, 74665, 83333, 83665, 85489, 88357, 90751, 107185, 137149, 158369.

Comments:

The solutions (m,n) for the Poulet numbers from the sequence above are: (3, 9); (3, 13); (4, 14); (4, 16); (9, 11) and (4, 20); (3, 27); (3, 29); (4, 26); (3, 35); (290, 0) and (3, 41); (350, 0) and (4, 38); (259, 1); (3, 51); (367, 1); (4, 56); (94, 8) and (3, 71); (4, 62); (329, 3) and (4, 68); (379, 3); (3, 91); (182, 8); (319, 5) and (4, 86); (3, 101); (888, 2); (928, 2) and (66, 20); (43, 29); (659, 5); (3, 149); (438, 8) and (4, 134); (3, 165); (4406, 0) and (4, 142); (4502, 0); (4, 146); (4, 148); (2384, 2) and (38, 48); (1387, 5); (5111, 1).

Few examples of how the formula looks like for m from 3 to 4.

For m = 3 the formula becomes 3*n^2 + 10*n + 8 and were found the following Poulet numbers: 341, 645, 2465, 2821, 4033, 5461, 8321, 15841, 25761, 31621, 68101, 83333 etc. (12 from the first 100 Poulet numbers can be written this way!).

For m = 4 the formula becomes 4*n^2 + 21*n + 27 and were found the following Poulet numbers: 1105, 1387, 2047, 3277, 6601, 13747, 16705, 19951, 31417, 83665, 88357, 90751 etc. (12 from the first 100 Poulet numbers can be written this way!).

Note: For n = –2 the formula becomes (m – 3) and for n = –9 becomes (m + 158) so all the Poulet numbers have at least these integer solutions to this formula.

Note: For n = –1 the formula becomes (9*m – 26) and 37 from the first 100 Poulet numbers can be written this way! That means that for more than a third from Poulet numbers P checked is true that (P + 8) is divizible by 9 (for comparison, this relation is true for just 14 from the first 100 primes).

12. Poulet numbers that can be written as (p^2 + 2*p)/3, where p is also a Poulet number

First 22 terms of the sequence (A216276 in OEIS): 997633, 1398101, 3581761, 26474581, 37354465, 63002501, 70006021, 82268033, 93030145, 561481921, 804978721, 1231726981, 2602378721, 12817618945, 15516020833, 16627811905, 22016333333, 25862624705, 53707855201, 67220090785, 95074073281, 144278347201.

Comments:

The corresponding values of the Fermat pseudoprime p: 1729, 2047, 3277, 8911, 10585, 13747, 14491, 15709, 16705, 41041, 49141, 60787, 88357, 196093, 215749, 223345, 256999, 278545, 401401, 449065, 657901.

Note that for 22 from the first 200 Fermat pseudoprimes to base 2 were obtained also Fermat pseudoprimes to base 2 through this formula!

Conjecture 1:

For any Fermat pseudoprime to base 2, p1, there exist infinite many Fermat pseudoprimes to base 2, p2, formed this way: p2 = (p1^n + n*p1)/(n + 1), where n natural, n > 1.

Conjecture 2:

For any Carmichael number, c1, there exist infinite many Carmichael numbers, c2, formed this way: c2 = (c1^n + n*c1)/(n + 1), where n natural, n > 1. Note that, in the sequence above, from Fermat pseudoprimes to base 2 that are also Carmichael numbers (1729, 8911, 10585, 41041, 278545, 449065) were obtained too Carmichael numbers.

13. Poulet numbers that can be written as p^2*n – p*n + p, where p is also a Poulet number

First 22 terms of the sequence (A217835 in OEIS): 348161, 831405, 1246785, 1275681, 2077545, 2513841, 5977153, 9613297, 13333441, 13823601, 18137505, 19523505, 21474181, 21880801, 37695505, 38171953, 44521301, 47734141, 54448153, 72887585, 75151441, 95423329.

Comments:

The numbers from sequence are the all numbers of this type up to 10^8.

The corresponding (p, n): (341, 3), (645, 2), (645, 3), (341, 11), (645, 5), (561, 8), (1729, 2), (1387, 5), (341, 120), (561, 44), (1905, 5), (645, 47), (3277, 2), (2701, 3), (2047, 9), (4369, 2), (341, 384), (2821, 6), (2047, 13), (2465, 12), (3277, 7), (4369, 5).

Conjecture 1:

For any Fermat pseudoprime p to base 2 there are infinitely many Fermat psudoprimes to base 2 equal to p^2*n – p*n + p, where n is natural.

Note: See the sequence A215343: the generalised formula from there is p^2*n – p*n + p^2, which suggests an extrapolated formula for obtaining some Fermat pseudoprime to base 2 from other: p^2*n – p*n + p^k.

Conjecture 2:

For any Fermat pseudoprime p to base 2 and any k natural, k > 0, there are infinitely many Fermat psudoprimes to base 2 equal to p^2*n – p*n + p^k, where n is natural.

14. Primes of the form (24*p + 1)/5, where p is a Poulet number

First 22 terms of the sequence (A218010 in OEIS): 1637, 2693, 20981, 22469, 40709, 42773, 49253, 65957, 69557, 123653, 140837, 235877, 451013, 623621, 626693, 716549, 1095557, 1370597, 1634693, 2108597, 2459813, 2548229, 2554421, 2563493, 2869781, 3534197, 3669557, 3755237, 4093637, 4337429, 4567109.

Comments:

The corresponding values of p: 341, 561, 4371, 4681, 8481, 8911, 10261, 13741, 14491, 25761, 29341, 49141, 93961, 129921, 130561, 149281, 228241, 285541, 340561, 439291, 512461, 530881, 532171, 534061, 597871, 736291, 764491, 782341, 852841, 903631, 951481.

It is notable that, from the first 128 natural solutions of this equation ((24*p + 1)/5, where p is Fermat pseudoprime to base 2), 31 are primes (the ones from the sequence above), 51 are products (not necessary squarefree) of two prime factors and 41 are products of three prime factors; only 5 of them are products of four prime factors.

It is notable yet another relation between numbers of the form (24*n + 1)/5, where n natural, and Fermat pseudoprimes:

Conjecture:

There is no absolute Fermat pseudoprime m for which n = (5*m – 1)/24 is a natural number (checked for the first 300 Carmichael numbers; if true, then the formula is a criterion to separate pseudoprimes at least from a subset of primes, because there are 37 primes m from the first 300 primes for which n = (5*m – 1)/24 is a natural number).

15. The smallest m for which the n-th Carmichael number can be written as p^2*(m + 1) – p*m

First 60 terms of the sequence (A213812 in OEIS): 1, 3, 4, 2, 2, 3, 1, 1, 2, 7, 24, 4, 4, 7, 47, 80, 9, 1, 23, 2, 46, 15, 24, 21, 24, 1, 1, 76, 8, 21, 16, 14, 6, 2, 150, 16, 8, 16, 3, 156, 36, 232, 2, 13, 10, 788, 40, 25, 2, 4, 123, 12, 44, 16, 8, 207, 226, 462, 92, 6.

Comments:

The corresponding values of p are (we write the Carmichael number in brackets): 17(561), 17(1105), 19(1729), 29(2465), 31(2821), 41(6601), 67(8911), 73(10585), 73(15841), 61(29341), 41(41041), 97(46657), 103(52633), 89(62745), 37(63973), 31(75361), 101(101101), 241(115921), 73(126217), 233(162401), 61(172081), 109(188461), 101(252601), 113(278545), 109(294409), 397(314821), 409(334153), 67(340561), 211(399001), 137(410041), 163(449065), 181(488881), 271(512461), 421(530881), 61(552721), 197(656601), 271(658801), 199(670033), 433(748657), 73(825265), 151(838201), 61(852841), 577(997633), 271(1024651), 307(1033669), 37(1050985), 163(1082809), 211(1152271), 631(1193221), 541(1461241), 113(1569457), 353(1615681), 199(1773289), 331(1857241), 461(1909001), 101(2100901), 97(2113921), 73(2433601), 163(2455921), 599(2508013).

Note: Any Carmichael number C can be written as C = p^2*(n + 1) – p*n, where p is any prime divisor of C (it can be seen that the smallest n is obtained for the biggest prime divisor). The formula C = p^2*(n + 1) – p*n is equivalent to C = p^2*m – p*(m – 1) = p^2*m – p*m + p, equivalent to p^2 – p divides C – p, which is a direct consequence of Korselt's criterion. It can be shown from p – 1 divides C – 1 not that just p^2 – p divides C – p but even that p^2 – p divides C – p^k (if C > p^k) or p^k – C (if p^k > C) which leads to the generic formula for a Carmichael number: C = p^k + n*p^2 – n*p (if C > p^k) or C = p^k – n*p^2 + n*p (if p^k > C) for any p prime divisor of C and any k natural number.

Note: The formulas generated giving values of k seems to be very useful in the study of Fermat pseudoprimes; also, the composite numbers C for which the equation C = p^k – n*p^2 + n*p gives, over the integers, as solutions, all their prime divisors, for a certain k, deserve further study.

16. Carmichael numbers of the form (30*k + 7)*(60*k + 13)*(150*k + 31)

First 18 terms of the sequence (A182085 in OEIS): 2821, 488881, 288120421, 492559141, 776176261, 1632785701, 3835537861, 6735266161, 9030158341, 21796387201, 167098039921, 288374745541, 351768558961, 381558955141, 505121232001, 582561482161, 915245066821, 2199733160881.

Conjecture:

The number C = (30*k + 7)*(60*k + 13)*(150*k + 31) is a Carmichael number if (but not only if) 30*k + 7, 60*k + 13 and 150*k + 31 are all three prime numbers.

Note: We got Carmichael numbers with three prime divisors for k = 0, 1, 10, 12, 18, 24, 32, 43, 85, 102, 123, 129, 150, 201, 207, 256.
We got Carmichael numbers with more than three prime divisors for n = 14, 29, 109, 112.

Note: All these numbers can be written as well as N = (n + 1)*(2*n + 1)*(5*n + 1), where n = 30*k + 6.

17. Carmichael numbers of the form C = (30*n – 7)*(90*n – 23)*(300*n – 79)

First 16 terms of the sequence (A182132 in OEIS): 340561, 4335241, 153927961, 542497201, 1678569121, 2598933481, 25923026641, 63280622521, 88183003921, 155999871721, 209850699601, 240893092441, 274855097881, 380692027321, 733547013841, 1688729866321.

Conjecture:

The number C = (30*n – 7)*(90*n – 23)*(300*n – 79) is a Carmichael number if (but not only if) 30*n – 7, 90*n – 23 and 300*n – 79 are all three prime numbers.

Note: We got Carmichael numbers with three prime divisors for n = 2, 9, 15, 32, 43, 48, 58, 64, 67, 78, 97, 128.
We got Carmichael numbers with more than three prime divisors for n = 1, 6, 13, 70.

18. Carmichael numbers of the form C = (30*n – 17)*(90*n – 53)*(150*n – 89)

First 17 terms of the sequence (A182133 in OEIS): 29341, 1152271, 34901461, 64377991, 775368901, 1213619761, 4562359201, 8346731851, 9293756581, 48874811311, 68926289491, 72725349421, 147523256371, 235081952731, 672508205281, 707161856941, 779999961061.

Conjecture:

The number C = (30*n + 13)*(90*n + 37)*(150*n + 61) is a Carmichael number if (but not only if) 30*n + 13, 90*n + 37 and 150*n + 61 are all three prime numbers.

Note: We got Carmichael numbers with three prime divisors for n = 0, 1, 5, 12, 14, 12, 27, 28, 49, 55, 56, 71, 83, 121, 125.
We got Carmichael numbers with more than three prime divisors for n = 4 and n = 119.

19. Carmichael numbers C = (60*k + 13)*(180*k + 37)*(300*k + 61)

First 16 terms of the sequence (A182416 in OEIS): 29341, 34901461, 775368901, 1213619761, 4562359201, 9293756581, 72725349421, 672508205281, 707161856941, 779999961061, 983598759361, 1671885346141, 1800095194261, 3459443867461, 6513448976101, 8369282635561.

Conjecture:

N = (60*k + 13)*(180*k + 37)*(300*k + 61) is a Carmichael number if (but not only if) 60*k + 13, 180*k + 37 and 300*k + 61 are all three prime numbers.

Note: We got Carmichael numbers with three prime divisors for k = 0, 6, 7, 11, 14, 28, 60, 62, 80, 102, 126, 137, 139, 157, 171.
We got Carmichael numbers with more than three prime divisors for k = 2, 59, 67, 82.

Note: We can see that 13 = 7*2 – 1, 37 = 7*6 – 5 and 61 = 7*10 – 9, while 60 = 30*2, 180 = 30*6 and 300 = 30*10; we also have Carmichael numbers that can be written as (30*n – 11)*(60*n – 23)*(150*n – 59), for instance 63973, or as (30*n – 7)*(90*n – 23)*(300*n – 79), for instance 340561; we can see that, this time, 23 = 11*2 + 1, 59 = 11*5 + 4, 23 = 7*3 + 2 and 79 = 7*10 + 9, while 60 = 30*2, 150 = 30*5, 90 = 30*3 and 300 = 30*10.

Observation:
Many Carmichael numbers, not only with three prime divisors, can be written in one of the following two forms: C = ((30*a*m – (a*p + a – 1))*((30*b*m – (b*p + b – 1))*((30*c*m – (c*p + c – 1)) or C = ((30*a*m + (a*p – a + 1))*((30*b*m + (b*p – b + 1))*((30*c*m + (c*p – c + 1)), where p, a*p + a – 1, b*p + b – 1 and c*p + c – 1 are all (four or three, if a = 1)primes (in the first case) or p, a*p – a + 1, b*p – b + 1 and c*p – c + 1 are all primes (in the second case).

20. Carmichael numbers C = (30*n – 29)*(60*n – 59)*(90*n – 89)*(180*n – 179)

First 9 terms of the sequence (A182088 in OEIS): 31146661, 2414829781, 192739365541, 197531244744661, 741700610203861, 973694665856161, 2001111155103061, 2278278996452641, 4271903575869601.

Conjecture:
The number C = (30*n – 29)*(60*n – 59)*(90*n – 89)*(180*n – 179) is a Carmichael number if (but not only if) 30*n – 29, 60*n – 59, 90*n – 89 and 180*n – 179 are all four prime numbers.

Note: We got Carmichael numbers with three prime divisors for n = 10, 52, 77, 143.
We got Carmichael numbers with more than three prime divisors for n = 2, 4, 72, 92, 95, 111.

21. Carmichael numbers C = (330*k + 7)*(660*k + 13)*(990*k + 19)*(1980*k + 37)

First 11 terms of the sequence (A182089 in OEIS): 63973, 461574735553, 7103999557333, 35498632881313, 111463190499493, 271061745643873, 560604728986453, 1036648928639233, 1765997490154213, 2825699916523393, 4303052068178773.

Conjecture:

The number C = (330k + 7)*(660k + 13)*(990k + 19)*(1980k + 37) is a Carmichael number if 330k + 7, 660k + 13, 990k + 19 and 1980k + 37 are all four prime numbers.

22. Carmichael numbers of the form C = (30*n – p)*(60*n – (2*p + 1))*(90*n – (3*p + 2)), where p, 2*p + 1, 3*p + 2 are all three primes

First 17 terms of the sequence (A182087 in OEIS): 1729, 172081, 294409, 1773289, 4463641, 56052361, 118901521, 172947529, 216821881, 228842209, 295643089, 798770161, 1150270849, 1299963601, 1504651681, 1976295241, 2301745249.

Comments:

These numbers can be reduced to only two possible forms: C = (30*n – 23)*(60*n – 47)*(90*n – 71) or C = (30*n – 29)*(60*n – 59)*(90*n – 89). In the first form, for the particular case when 30*n – 23, 60n – 47 and 90n – 71 are all three prime numbers, we obtain the Chernick numbers of the form 10*m + 1 (for k = 5*n – 4 we have C = (6*k + 1)*(12*k + 1)*(18*k + 1)). In the second form, for the particular case when 30*n – 29, 60*n – 59 and 90*n – 89 are all three prime numbers, we obtain the Chernick numbers of the form 10*m + 9 (for k = 5*n – 5 we have C = (6*k + 1)*(12*k + 1)*(18*k + 1)).

So the Chernick numbers can be divided into two categories: Chernick numbers of the form (30*n + 7)*(60*n + 13)*(90*n + 19) and Chernick numbers of the form (30*n + 1)*(60*n + 1)*(90*n + 1).

23. Carmichael numbers of the form C = p*(2*p – 1)*(3*p – 2)*(6*p – 5), where p is prime

First 15 terms of the sequence (A182518 in OEIS): 63973, 31146661, 703995733, 21595159873, 192739365541, 461574735553, 3976486324993, 10028704049893, 84154807001953, 197531244744661, 741700610203861, 973694665856161, 2001111155103061, 3060522900274753, 3183276534603733.

Comments:

We get Carmichael numbers with four prime divisors for p = 7, 271, 337, 727, 1237, 1531, 2281, 3037, 3067.

We get Carmichael numbers with more than four prime divisors for p = 31, 67, 157, 577, 2131, 2731, 3301.

Note: We can see that p, 2*p – 1, 3*p – 2 and 6*p – 5 can all four be primes only for p = 6*k + 1 (for p = 6*k + 5, we get 2*p – 1 divisible by 3), so in that case the formula is equivalent to C = (6*k + 1)*(12*k + 1)*(18*k + 1)*(36*k + 1).

24. Carmichael numbers of the form C = p*(2*p – 1)*(n*(2*p – 2) + p), where p and 2*p–1 are primes

First 29 terms of the sequence (A182207 in OEIS): 1729, 2821, 41041, 63973, 101101, 126217, 172081, 188461, 294409, 399001, 488881, 512461, 670033, 748657, 838201, 852841, 997633, 1033669, 1050985, 1082809, 1461241, 2100901, 2113921, 2628073, 4463641, 4909177, 7995169, 8341201, 8719309.

Conjecture:

Any Carmichael number C divisible by p and 2*p – 1 (where p and 2*p – 1 are prime numbers) can be written as C = p*(2*p – 1)*(n*(2*p – 2) + p).

Checked for the first 30 Carmichael numbers divisible by p and 2*p – 1.

Note: We can see how easy is to obtain Carmichael numbers with this formula:

For n = 1 we get p*(2*p – 1)*(3*p – 2) and Carmichael numbers 1729, 172081, 294409 etc.

For n = 2 we get p*(2*p – 1)*(5*p – 4) and Carmichael numbers 2821, 63973, 488881 etc.

For n = 3 we get p*(2*p – 1)*(7*p – 6) and Carmichael numbers 399001, 53711113 etc.

25. Carmichael numbers of the form n*(2*n – 1)*(p*n – p + 1)* (2*p*n – 2*p + 1), where p is odd

First 17 terms of the sequence (A212882 in OEIS): 63973, 172081, 31146661, 167979421, 277241401, 703995733, 1504651681, 2414829781, 117765525241, 192739365541, 461574735553, 881936608681, 2732745608209, 3145699746793, 3307287048121, 3976486324993, 7066238244481.

Comments:

The following Carmichael numbers are of the form n*(2*n – 1)*(3*n – 2)*(6*n – 5): 63973, 31146661, 703995733, 2414829781, 192739365541, 461574735553, 3976486324993.

The following Carmichael numbers are of the form n*(2*n – 1)*(5*n – 4)*(10*n–9): 172081, 881936608681, 3307287048121, 8916642713161.

The following Carmichael number is of the form n*(2*n – 1)*(7*n – 6)*(14*n – 13): 167979421.

The following Carmichael number is of the form n*(2*n – 1)*(9*n – 8)*(18*n – 17): 277241401.

The following Carmichael number is of the form n*(2*n – 1)*(11*n – 10)*(22*n – 21): 9924090391909.

The following Carmichael number is of the form n*(2*n – 1)*(15*n – 14)*(30*n – 29): 7932245192461.

The following Carmichael number is of the form n*(2*n – 1)*(17*n – 16)*(34*n – 33): 3145699746793.

The following Carmichael numbers are of the form n*(2*n – 1)*(21*n – 20)*(42*n – 41): 1504651681, 117765525241, 2732745608209.
The following Carmichael number is of the form n*(2*n – 1)*(23*n – 22)*(46*n – 45): 7066238244481.
For p = 13 and p = 19, there is no Carmichael number up to 10^13.
There is not any other Carmichael number of this form, for p from 3 to 23, up to 10^13.

Conjecture:
For any odd number p we have an infinite number of Carmichael numbers of the form n*(2*n – 1)*(p*n – p + 1)*(2*p*n – 2*p + 1).

Note: Many numbers of the form n*(2*n – 1)*(p*n – p + 1)*(2*p*n – 2*p + 1), not divisible by 2, 3 or 5, where p is odd or even, are squarefree and respects the Korselt's criterion for many of their prime divisors or are not squarefree but respects the Korselt's criterion sometimes even for all their divisors (but we didn't find Carmichael numbers when p is even).

26. Carmichael numbers of the form 3*n*(9*n + 2)*(18*n – 1), where n is odd

First 29 terms of the sequence (A213071 in OEIS): 561, 13833, 62745, 170625, 360801, 656601, 1081353, 1658385, 2411025, 3362601, 4536441, 5955873, 7644225, 9624825, 11921001, 14556081, 17553393, 20936265, 24728025, 28952001, 33631521, 38789913, 44450505, 50636625, 57371601, 64678761, 72581433, 81102945, 90266625.

Comments:
Carmichael numbers (561, 62745, 656601, 11921001, 174352641) were obtained for the following values of n: 1, 5, 11, 29, 71.

Note: The sequence can be generalized this way: C = p*n*(3*p*n + 2)*(6*p*n – 1), where p is prime.
Few examples for p from 5 to 23:
For p = 5 the formula becomes 5*n*(15*n + 2)*(30*n – 1) and were obtained the following Carmichael numbers: 2465, 62745, 11119105, 3249390145 (for n = 1, 3, 17, 113);
For p = 7 the formula becomes 7*n*(21*n + 2)*(42*n – 1) and were obtained the following Carmichael numbers: 6601 (for n = 1);
For p = 11 the formula becomes 11*n*(33*n + 2)*(66*n – 1) and were obtained the following Carmichael numbers: 656601 (for n = 3);
For p = 13 the formula becomes 13*n*(39*n + 2)*(78*n – 1) and were obtained the following Carmichael numbers: 41041, 271794601 (for n = 1, 21);
For p = 17 the formula becomes 17*n*(51*n + 2)*(102*n – 1) and were obtained the following Carmichael numbers: 11119105, 2159003281 (for n = 5);
For p = 19 the formula becomes 19*n*(57*n + 2)*(114*n – 1) and were obtained the following Carmichael numbers: 271794601 (for n = 13);

For p = 23 the formula becomes 23*n*(69*n + 2)*(138*n – 1) and were obtained the following Carmichael numbers: 5345340001 (for n = 29).

27. Carmichael numbers that have only prime divisors of the form 10k+1

First 28 terms of the sequence (A212843 in OEIS): 252601, 399001, 512461, 852841, 1193221, 1857241, 1909001, 2100901, 3828001, 5049001, 5148001, 5481451, 6189121, 7519441, 8341201, 9439201, 10024561, 10837321, 14676481, 15247621, 17236801, 27062101, 29111881, 31405501, 33302401, 34657141, 40430401, 42490801.

Conjecture:

All Carmichael numbers C (not only with three prime divisors) of the form 10*n + 1 that have only prime divisors of the form 10*k+1 can be written as C = (30*a + 1)*(30*b + 1)*(30*c + 1), C = (30*a + 11)*(30*b + 11)*(30*c + 11), or C = (30*a + 1)*(30*b + 11)*(30*c + 11). In other words, there are no numbers of the form C = (30*a + 1)*(30*b + 1)*(30*c + 11).

28. Carmichael numbers divisible by a smaller Carmichael number

First 29 terms of the sequence (A214758 in OEIS): 63973, 126217, 172081, 188461, 278545, 748657, 997633, 1773289, 5310721, 8719921, 8830801, 9890881, 15888313, 18162001, 26474581, 26921089, 31146661, 36121345, 37354465, 41471521, 93614521, 93869665, 101957401, 120981601, 151813201.

Comments:

Carmichael numbers by which the numbers from sequence are divisible: 1729, 1729, 2821, 1729, 2465, 1729, 1729, 8911, 29341, 6601, 8911, 41041, 8911, 75361, 8911, 46657, 2821 and 172081, 1105, 10585, 2821 and 172081, 41041, 2465, 1729 and 188461, 46657, 252601.

Note: A Carmichael number can be divisible by more than one Carmichael number: *e.g.* 31146661, 41471521, 101957401.

A subsequence of this sequence contains the numbers C_1 (and another subsequence the numbers C_3) that can be written as $C_1 = (C_2 + C_3)/2$, where C_1, C_2 and C_3 are Carmichael numbers and C_1 and C_3 are both divisible by C_2 (e.g. 63973 = (1729 + 126217)/2; 93614521 = (41041 + 187188001)/2).

Conjecture:

A Carmichael number C_1 can be written as $C_1 = (C_2 + C_3)/2$, where C_2 and C_3 are also Carmichael numbers, only if both C_1 and C_3 are divisible by C_2.

29. Carmichael numbers divisible by 1729

First 29 terms of the sequence (A212920 in OEIS): 1729, 63973, 126217, 188461, 748657, 997633, 101957401, 509033161, 705101761, 1150270849, 1854001513, 2833846561, 7103660473, 8039934721, 9164559313, 10298458261, 14530739041, 23597511301, 41420147041, 49923611101, 50621055121, 55677010753, 65039877721.

Conjecture:

If m*126 + n = 1729, m*126 > n, then exists a series with infinite many Carmichael terms of the form C mod m*234 = n.

Verifying the conjecture:

(1) For m < 7 we have m*126 < n;
(2) For m = 7 the formula becomes C mod 882 = 847 and were obtained the Carmichael numbers: 1729, 15841, 1033669 etc.;
(3) For m = 8 the formula becomes C mod 1008 = 721 and were obtained the Carmichael numbers: 1729, 15841, 41041, 172081, 670033, 748657, 825265, 997633 etc.;
(4) For m = 9 the formula becomes C mod 1134 = 595 and were obtained the Carmichael numbers: 1729, 1033669 etc.;
(5) For m = 10 the formula becomes C mod 1260 = 469 and were obtained the Carmichael numbers: 1729, 1033669 etc.;
(6) For m = 11 the formula becomes C mod 1386 = 343 and were obtained the Carmichael numbers: 1729, 1082809 etc.;
(7) For m = 12 the formula becomes C mod 1512 = 217 and were obtained the Carmichael numbers: 1729, 41041 etc.;
(8) For m = 13 the formula becomes C mod 1638 = 91 and were obtained the Carmichael numbers: 1729, 41041, 63973, 670033, 997633 etc.

Conjecture:

If m*234 + n = 1729, m*234 > n, then exists a series with infinite many Carmichael terms of the form C mod m*234 = n.

Verifying the conjecture:

(1) For m < 4 we have m*234 < n;
(2) For m = 4 the formula becomes C mod 936 = 793 and were obtained the Carmichael numbers: 1729, 41041, 46657, 126217, 748657, 4909177, 65037817, 193910977, 311388337, 633639097 etc.;
(3) For m = 5 the formula becomes C mod 1170 = 559 and were obtained the Carmichael numbers: 1729, 1033669, 1082809, 7995169, 26921089 etc.;
(4) For m = 6 the formula becomes C mod 1404 = 325 and were obtained the Carmichael numbers: 1729, 41041, 46657, 188461, 314821 etc.;
(5) For m = 7 the formula becomes C mod 1638 = 91 and the case is similar with one from precedent conjecture.

Conjecture:

If m*342 + n = 1729, m*342 > n, then exists a series with infinite many Carmichael terms of the form C mod m*342 = n.

Verifying the conjecture:

(1) For m < 2 we have m*342 < n;
(2) For m = 3 the formula becomes C mod 1026 = 703 and were obtained the Carmichael numbers: 1729, 8911 etc.;
(3) For m = 4 the formula becomes C mod 1368 = 361 and were obtained the Carmichael numbers: 1729, 126217 etc.;
(4) For m = 5 the formula becomes C mod 1710 = 19 and were obtained the Carmichael numbers: 1729, 1773289 etc.

Conclusion:

We can see that 126 = 18*7, 234 = 18*13 and 342 = 18*19 and 7, 13, and 19 are the prime factors of 1729, so the three conjectures could be expressed all in one. Even more than that, taking randomly another Carmichael number, 8911 = 7*19*67, taking randomly m = 7 in the formula m*18*67, we obtain the formula C mod 8442 = 469, which, indeed, leeds to a series of Carmichael numbers: 8911, 1773289, 8830801 etc., which means that the conjecture could be generalised:

Conjecture:

For any prime factor of a Carmichael number C1 exists a series with infinite many Carmichael terms C2 formed this way: C2 mod m*18*d = n, where m*18*d + n = C1, where d is the prime factor of C1 and m, n are natural numbers, m*18*d < n.

Note: Finally, if we have a Carmichael number divisible by 1729 (i.e. 63973, see the sequence above), we can see that the formula C mod 62244 = 1729 (it can see that 62244 + 1729 = 63973) leeds too to a series of Carmichael numbers: 126217 etc. which means that 1729 can be treated like a prime factor. This can be probably generalised to the Carmichael numbers that are divisible with other Carmichael numbers or probably even for a randomly chosen product of prime factors.

30. Fermat pseudoprimes n to base 3 of the form n = (3^(4*k + 2) – 1)/8

First 9 terms of the sequence (A217853 in OEIS): 91, 7381, 597871, 48427561, 3922632451, 317733228541, 25736391511831, 2084647712458321, 168856464709124011.

Comments:

These numbers were obtained for values of k from 1 to 20, with the following exceptions: k = 10, 12, 13, 16, 17, 19, for which were obtained 3^n mod n = 3^7, 3^31, 3^37, 3^25, 3^31, 3^13.

Conjecture:

There are infinitely many Fermat pseudoprimes to base 3 of the form (3^(4*k + 2) – 1)/8, where k is a natural number.