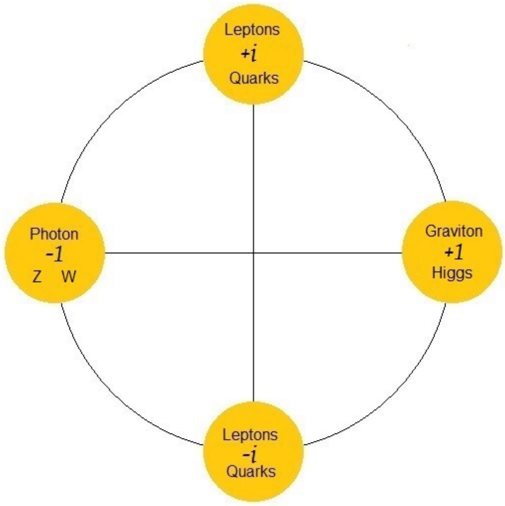
The relation of colour charge to electric charge



Dirac has shown how Einstein's expression for the relation of energy to momentum in Special Relativity can be factored into two linear parts using 4x4 Dirac matrices. *[Dirac, P.A.M., The Principles of Quantum Mechanics, 4th edition (Oxford University Press) ISBN 0-19-852011-5]*

This can also be done using 2x2 Pauli matrices (labelled K,L,M) because two inertial observers agree on the component of momentum Q orthogonal to the component of momentum P in the direction of a Lorentz boost.

$$(E/c)^2 - P^2 - Q^2 - (mc)^2 I = (E/c + rKP + gLQ + mcbM)(E/c - rKP - gLQ - mcbM)$$

This is true for all 3! = 6 permutations of K,L,M where r,g,b equal +1 or -1.
The set {I,K,L,M} forms the basis of a 4-dimensional real vector space.

For leptons r,g,b all equal -1 and for quarks two of r,g,b are equal to +1 and the third equals -1. The signs are all negated for anti-particles as in the equation above.

The 3 cyclic permutations KLM = MKL = LMK count the number of plus signs (say) for r,g,b which is 0 for leptons and 2 for quarks.

The 3 cyclic permutations MLK = LKM = KML count the number of minus signs (say) for r,g,b which is 3 for leptons and 1 for quarks.

For material particles r,g,b all equal -1 which is always true for leptons and true for three distinct quarks with r,g,b equal to -1 separately or a quark and an appropriate anti-quark.

Similarly, for material gauge bosons, the 3 cyclic permutations of KLM squared (which all equal -I) count the number of plus signs (say) for r,g,b which is 0 for the Z boson and the 3 cyclic permutations of MLK squared (which all equal -I) count the number of minus signs (say) for r,g,b which is 3 for the W boson.

The photon, having zero rest mass, carries no electric charge.

The 6 permutations of KLM raised to power zero (which all equal I) present (say) a material particle which carries no electric charge.

The 6 permutations of KLM raised to power four (which all equal I) present (say) a particle with zero rest mass which carries no electric charge.