

# A CIRCLE WITHOUT " $\pi$ "

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## Abstract

This paper provides the proof of invalidity of the most fundamental constant known to mankind. Imagining a circle without " $\pi$ " is simply unthinkable but it's going to be a reality very soon. " $\pi$ " is not a true circle constant. This paper explores this idea and proposes a new constant in the process which gives the correct measure of a circle. It is given by " $\tau$ ". As a result, it redefines the area of the circle. The circle area currently accounted is wrong and therefore needs correction. This has serious implications for science. I have also discovered the fundamental geometrical ratio b/w a circle and a square in which it's inscribed and have also discovered a new circle formula. This paper makes this strong case with less ambiguity.

KEYWORDS: Algebra, Fundamental Geometry, Number Theory

## 1. INTRODUCTION

The geometrical figure "*Circle*" has been in the imagination of many ordinary and the extraordinary people from the time immemorial. And the value of the most fundamental constant " $\pi$ " associated with it has been known since many centuries. Great mathematicians of our times have looked at it with great fascination. Our science has been developed with a central focus on this fundamental constant. It finds its applications in mathematics and numerous other branches of science. Imagining mathematics without " $\pi$ " is unbelievable. But this paper presented here proves its invalidity <sup>[2]</sup>. There exists no doubts about the numerical value of " $\pi$ " but it's not the true constant which defines a circle? And this paper explores this idea deeply and proposes a new constant in the process. It also gives the correct area of a circle. The area of the circle is not properly accounted. *In this context*, I would also like to state that all mathematical expressions which are associated with " $\pi$ " are beautiful but only when such expressions define completeness they look elegant and moreover they depict the true picture of our physical world <sup>[4, 5]</sup>. This paper presented here is one such serious attempt made in this direction. I am absolutely sure that this paper will truly enrich our science and will lead us to greater heights of knowledge and understanding and will be an effective tool in our continuous efforts to push the boundaries of limits to unlock the hidden mysteries of this world. " $\pi$ " has indeed made a long journey in man's scientific quest up until this paper.

## 2. NEW FORMULA DISCOVERED

$$1) \mathbf{A} = r \mathbf{C} ; \quad \text{Where, } A = \text{Area of the given circle}$$

$$C = \text{Circumference}$$

$$r = \text{Radius}$$

$$2) \mathbf{A} = \mathbf{C} = \tau \text{ ( when } r = \text{unity; newly defined as a unit circular block)}$$

Where  $\tau = 6.28\dots$ ; the *fundamental circle constant*

$$3) \left( \frac{a}{r} \right) = 2; \text{ where 'a' = side of any square and 'r' = radius of the circle}$$

inscribed in it. I call it the *fundamental geometrical ratio*.

## 3. GENERAL DISCUSSION

I would like to start this discussion with a basic question. Is " $\pi$ " the constant which truly defines a circle? *To know the answer*, we have to start up from scratch. Therefore, a basic understanding of circles is mentioned here to clarify this situation. A circle, as we all know is basically a geometrical figure which is drawn with the help of a compass with a central point and radius. Circumference, Area and Radius are the basic properties of it. *Importantly, it comprises of six equilateral triangles of radius length*. Since the circle is drawn with radius and therefore it's ideal to state that the other two components of circle, *namely*, circumference and area is directly related to its radius. And " $\pi$ " is the constant which relates them. And this constant is determined presently by dividing the circumference of a circle with its diameter<sup>[1]</sup>. This logically seems to be incorrect but for the time being we would not like to disturb this notion. Also, I will not delve too much into the properties of this constant but will focus my attention on the constant and its effect on the circumference and area of the circle. It clearly highlights the error in current science. It also highlights the geometrical inconsistency. EXP: *Pythagoras theorem is an inequality*. Also, the "*fundamental geometrical ratio*" along with the new "*Circle Formula*" discovered in this manuscript gives new insights in to our science. *In this paper*, I am considering a unit circle to prove my mathematical concept on circles. The complete analysis is done keeping this unit circle in mind. The value of the constant thus obtained in our analysis would ultimately test the validity of the existing constant " $\pi$ ". It would also test the validity of the circle area. Therefore, let's study and analyze this interesting situation. I was in pursuit of finding a simple solution to this end, and *when simplicity is the order*, where to find it other than the time tested *Algebra and Geometry*.

### 3.1 PROBLEM STATEMENT DETAILED

a) The crux of the problem associated with circle constant is, whether circumference over diameter is the correct measure of a circle? *The answer is a strict no.* We know that a circle is drawn with radius, therefore the natural and logical way of determining this constant would have been by taking circumference over the radius. Failure of this had led to the current problem which is being highlighted.

b) *In the general discussion section above*, I had mentioned that our analysis is done by considering a Unit Circle, therefore its radius is ( $r = 1$  unit), specialty of the unit circle is discussed in later sections. But, let's quickly apply this value to the existing relations which define a circle. *We know the following :-*

$$\{\text{Area of Circle} = f_1(r) = \pi \times r^2 = \pi \times 1 = \pi \text{ unit}\};$$

$$\{\text{Circumference - Circle} = f_2(r) = 2 \times \pi \times r = 2 \times \pi \times 1 = 2\pi \text{ unit}\};$$

If one takes a careful look at the result, one will be stunned to absolute silence, the **Circumference > Area**. *In other words ( $f_2(r) > f_1(r)$ )*, How on earth is that possible? *Firstly*, one must establish math equality before one makes the choice of measuring units. Therefore, there is something which is clearly wrong with the existing laws that define a circle. Therefore, the problem is finally identified above. It seems the problem is with the constant value which is associated with circle. It's " $\pi$ " the fundamental circle constant which is in question? *This is simply established above.*

c) Also, please kindly (*ref section 7*) in this manuscript highlighting the flaw in "*Archimedes Technique*" used for measuring the Area of a Circle.

d) '*Circle*' & '*Square*' scale needs to be congruent. "*Fundamental Geometrical Ratio*" discovered is clear evidence showing the measuring scale In-compatibility (*ref 5.1 s*)

e) Current science made a fatal mistake of inscribing a square in a unit circle, *where as it actually had to inscribe a unit circle in a square*, thereby undervaluing the true dimensions of a square by 50% and as a result our current science laws are undervalued by fifty percent (*ref 5.1 sec*). A simple analogy would be to say, that instead of counting two apples, *one counts it as one apple*, and keeps that ratio as the standard through out science, thereby not properly accounting 50% of reality.

## 4. ALGEBRAIC PROOF 1

Before we proceed with our theory, a few things needs to be addressed firsthand. *For our analysis*, as mentioned earlier, let's take a unit circle into our consideration. A unit circle is one whose radius = 1 unit. The idea of considering unit circle is because the area of the circle equals the circle constant. It is newly defined as a unit circular block, one on the same lines as used to define a unit square block. It is purely done for equivalence of the measuring scales in the “*square*” and “*circle*” dimensions.

We initially assume the circle constant to be “K” and it needs to be determined. We use simple algebra and fundamental geometry to carry out our study and analysis. *In this paper*, a circle which is derived by constant “ $\pi$ ” is called as “*Pie – Circle*” and similarly, a circle which is derived by another constant “ $\tau$ ” is called as “*Tau – Circle*”. *For math simplicity*, the value of the circle constant is rounded to two decimal places.

I have developed a new method of analysis called SC – Analysis. In this analysis, we consider a geometrical combination of a “square” and “circle” figures. Here, direct substitution of values into the current formulas is prohibited because of the unique situation in the form of “*fundamental geometrical ratio*” discovered in this paper highlighting the in-compatibility b/w a square and a circle scales. With this analysis one can negate the in-equality existing b/w them. Therefore, the math operation needs to be performed only on the “side” & “radius” of the “*Square – Circle*” geometrical combination and hence the “*Constant & Area*” needs to be interpreted in this context to better understand the algebraic & geometrical analysis done. Meaning: A division operation performed on ‘r’ also applies to ‘a’. **EX**: ‘r/2’ also applies to ‘a/2’ thereby yielding a resultant “*Square – Circle*” combination ( $r_c$ ,  $a_c$ ). This situation is because current science has considered the geometrical figures “*Circle*” and “*Square*” to be in (1:1) and not (1:2) ratio which is the reality. *Discussed in detail in later sections*.

Now, let's analyze the situation in the new context defined above. As discussed above, “*Area*” and “*Circumference*” of a circle is directly proportional to its “*Radius*”.

*Mathematically*, it can be expressed as  $Area \propto (Radius)^2$

$$Area = K * (radius)^2 \text{ ----- (1);} \quad \text{Where 'K' is the Circle Constant}$$

*Circumference*  $\propto$  *Radius*

(OR)

$$\text{Circumference} = K * \text{radius} \text{ ----- (2)}$$

**Re-arranging** (1) and (2), we get

$$K = \left( \frac{\text{Area}}{(\text{radius})^2} \right) \text{----- (3)}$$

$$K = \left( \frac{\text{Circumference}}{\text{radius}} \right) \text{----- (4)}$$

**Equating** (3) and (4), we get

$$\left( \frac{\text{Area}}{(\text{radius})^2} \right) = \left( \frac{\text{Circumference}}{\text{radius}} \right)$$

$$\left( \frac{\text{Area}}{\text{Circumference}} \right) = \left( \frac{(\text{radius})^2}{\text{radius}} \right)$$

$$\left( \frac{\text{Area}}{\text{Circumference}} \right) = \text{Radius} \text{ ----- (5)}$$

Thus, we have obtained a new formula relating all the three basic properties of a circle.

We use this formula to define a “Unit – Circle” by equating it to the numerical value “1” unit.

$$\left( \frac{\text{Area}}{\text{Circumference}} \right) = \text{Radius} = 1$$

Mathematically, it can be written as follows

$$\left(\frac{K * r^2}{K * r}\right) = 1 \text{ ----- (6)}$$

If we apply the conventional wisdom of circle in equation (5), we get as follows

$$\left(\frac{\text{Area}}{\text{Circumference}}\right) = \left(\frac{\pi * r^2}{2\pi * r}\right) = 1$$

$$\left(\frac{r}{2}\right) \neq 1 \text{ ----- (7)}$$

**NOTE 1:** The area of the circle where "π" constant was used is actually defining a circle whose radius = *half of the unit radius*. The “area of Circle” law breaks down above;

To uphold the validity of the unit circle, one has to multiply the LHS, *in particular the numerator*, of equation (7), which is nothing but the area of the circle by a factor of **two**

Hence, it takes the below form

$$\text{True Area of a circle} = 2 * (\pi * r^2) \text{ ----- (8)}$$

The simple conclusion from equation (7) is that the usage of constant "π" was actually not defining a unit circle area (**please kindly take note of this situation**) instead it was defining a circle whose radius is half the unity and given by (r = ½ = 0.5 unit, from *SC – Analysis* perspective (ref fig 2)) and hence its area is half the unit circle area. *Therefore, it's wrong and needs to be corrected*. This correction is done above. (refer equation (8))

Finally, the true circle constant “K” is obtained by equating (3) and (8) and a new notation is given {Greek letter (*tau*) "τ"} <sup>[2, 3]</sup>. It defines one complete cycle for a circle.

$$\mathbf{K = \{ \tau \} = 2 * \mathbf{Pie} = 6.28} \text{ (proved) ----- (9)}$$

From equation (4), the circumference of the circle

$$\text{Circumference} = \tau * r$$

$$\text{Circumference} = 6.28 * r \text{ (Proved) ----- (10)}$$

Hence, the fundamental circle constant " $\tau$ " is obtained by dividing the circumference by its radius.

From equation (3), the true area of the circle is

$$\text{Area} = \tau * r * r$$

$$\text{Area} = 6.28 * r * r \text{ (Proved) ----- (11)}$$

**(NOTE 2: this area is twice the current circle area)**

## 5. GEOMETRICAL ANALYSIS OF THE TWO CIRCLES

The new circle defined by constant " $\tau$ " described geometrically is depicted in Fig (1),

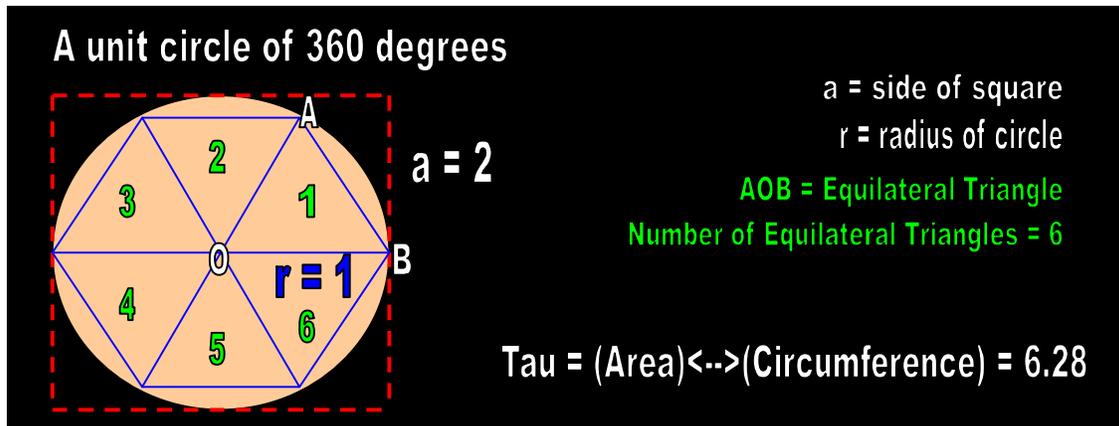


Fig (1) a geometrical depiction of unit circle formed by using " $\tau$ " constant

The current circle defined by constant " $\pi$ " described geometrically is depicted in the Fig (2).

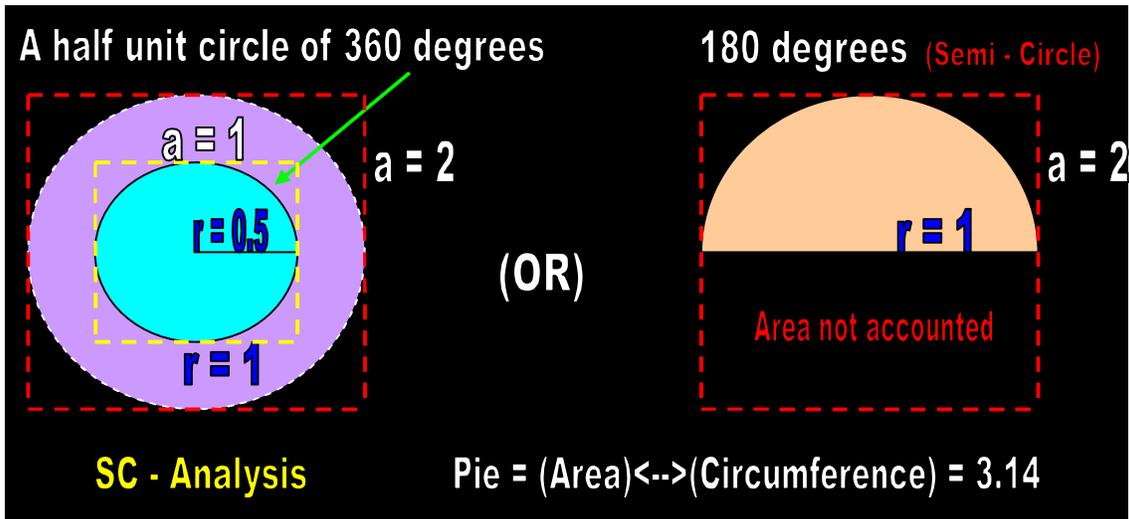


Fig (2) incorrectness of " $\pi$ " circle depicted geometrically for a unit circle

Geometrical mismatch of the area of the " $\pi$ " circle is depicted in Fig (3). A clear proof

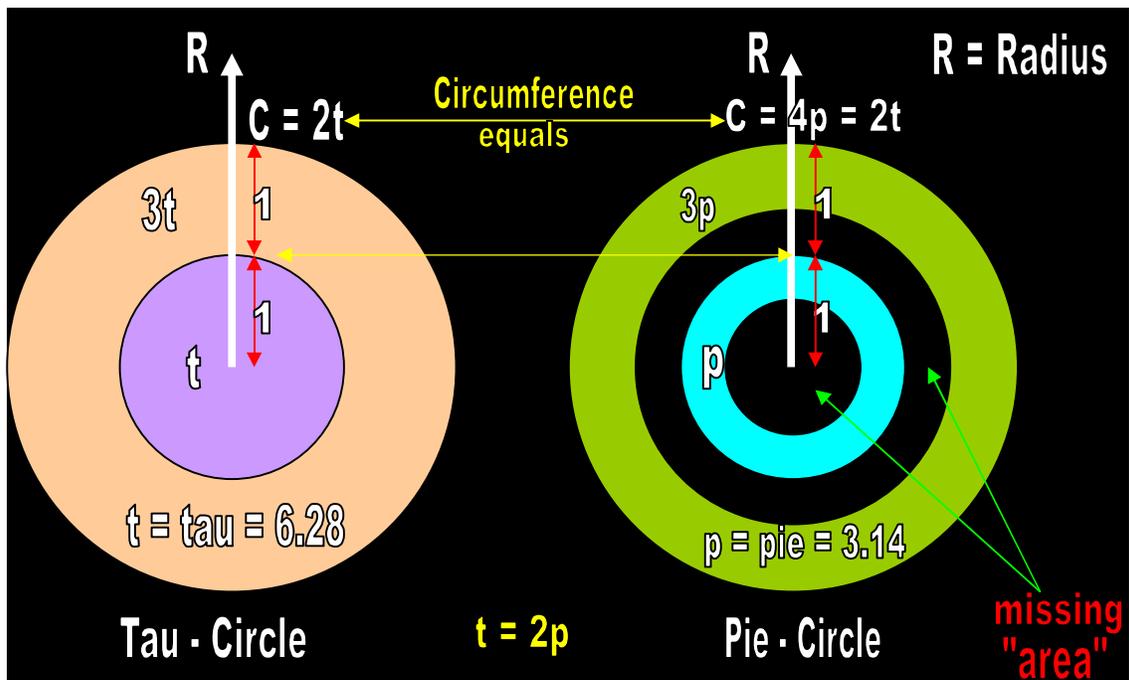


Fig (3) geometrical mismatch in area of " $\pi$ " circle at radius "1" and "2" depicted

Now, let's do the geometrical comparison of areas of circles formed by the current constant " $\pi$ " and the new constant " $\tau$ " in our new context (SC – Analysis perspective) and is as shown in the Fig (4).

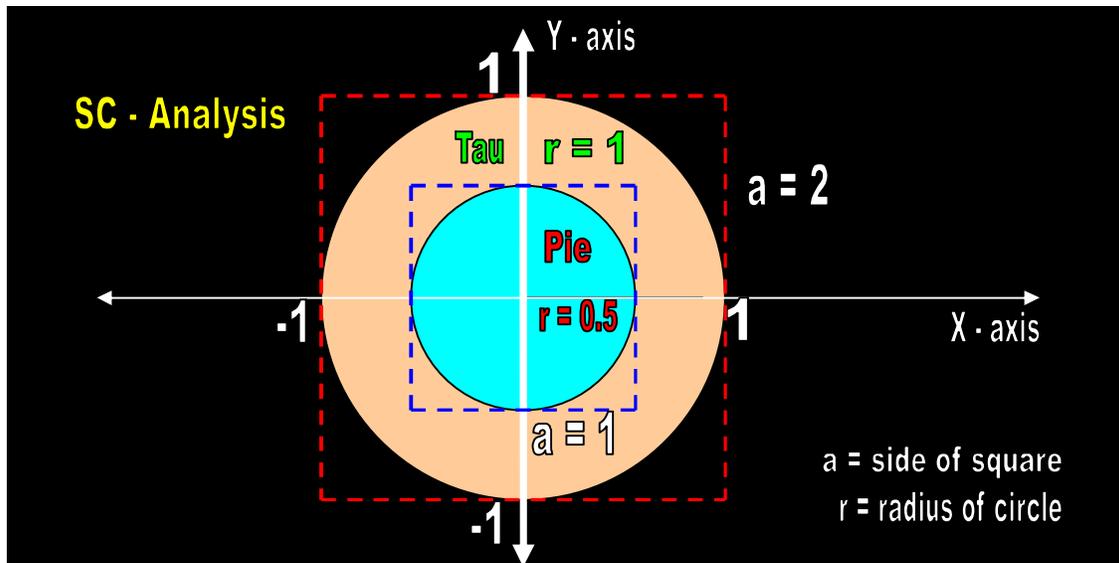


Fig (4) geometrical comparison of area of circles formed by " $\pi$ " and the new " $\tau$ "

From the Fig {1, 2, 3}, it's absolutely clear that the " $\pi$ " constant was not defining a unit circle and therefore is incorrect and hence the correct description of a unit circle is defined by " $\tau$ " constant as depicted in Fig (1). *Theoretically, the relationship which current science established was that of a circle enclosed in a unit square.* As a result of this, the existing area of the circle doubles and this is what is depicted in the Fig (4). *Convincing geometrical proof.*

## 5.1 FUNDAMENTAL GEOMETRICAL RATIO

Now, let us analyze the "*Pie – Circle*" and "*Tau – Circle*" constructed within the perfect squares. This is achieved by first drawing a unit – circle and then constructing a bigger square around it and therefore, this also gives room for constructing a circle of smaller dimensions. The smaller dimension circle is the "*Pie – Circle*" and larger dimension circle is the "*Tau – Circle*" which is same as the unit – circle used for our analysis. The "*Pie – Circle*" can be constructed any where. I have chosen the first quadrant. Their dimensions are as depicted in the Fig (5). *Please kindly take note the fundamental geometrical ratio of radius of the inscribed circle to the side of the square.* The value of "1" of a "*Square*" side is two times the value of "1" of a "*Circle*" radius. Leaving as it is produces mathematical inconsistency. Therefore, this incompatibility needs to be nullified before doing any analysis. Therefore, the math operation needs to be performed on the "*Square – Circle*" combination and not individually to show consistency in the math operations being performed and also is a clear evidence for why the factor "**2**" needs to be properly accounted through out current science.

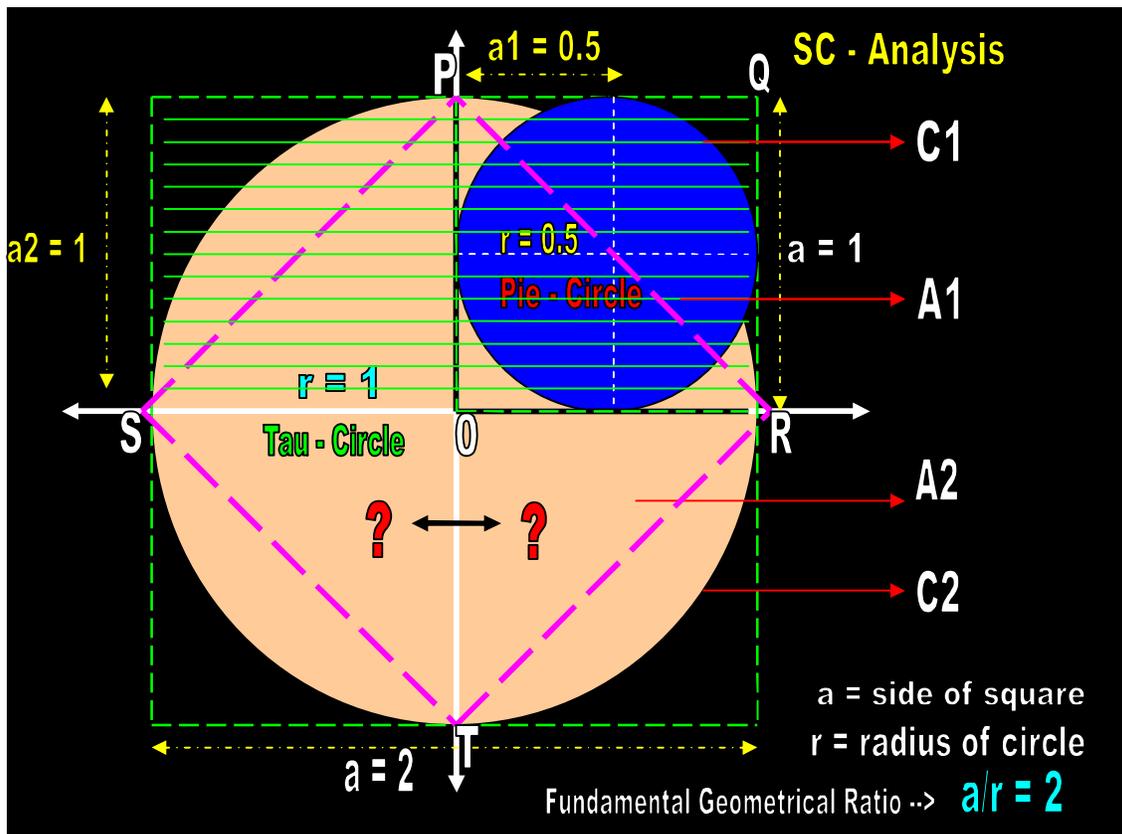


Fig (5) a unit circle and half unit circle enclosed in their respective squares

In the Fig (5), kindly note that the total area enclosed by the “Square” {PQRO} of side length of numerical value “1” is less than the area enclosed by the “Circle” of radius of the same numerical value “1”. Area of PQRO = Area  $\Delta^{le}$  POR + Area  $\Delta^{le}$  PQR. We know, Area  $\Delta^{le}$  SOP = Area  $\Delta^{le}$  PQR. Therefore, the current area of the unit square covers only 50 % of the area of the unit circle geometrically. In the fig (5), Area  $\Delta^{le}$  STR = Area  $\Delta^{le}$  (SOT + ROT) is not accounted. This is inconsistent. Therefore, this inconsistency b/w a normal & curved “areas” needs to be removed. Also, from our mathematical analysis in this manuscript, we have established this discrepancy to be (deficient by 50% or by a factor of “2”). Hence, the case being made for correcting our science laws. From the fig (5); the following is deduced

Pie – Circle Values

- C1 = Circumference
- A1 = Area
- r = Radius = 0.5 unit;
- Radius correction (r = 0.5 \* 2 = 1 unit);

Tau – Circle Values

- C2 = Circumference
- A2 = Area
- r = Radius = 1 unit

**PROOF 2:** From the circumference perspective and using the simple method of substitution

*Pie – Circle Analysis*

From equation (4), we know

$$C_1 = K * r$$

$$C_1 = \pi * r$$

By making the radius correction and equating it to the unit circle, one can write as follows

$$\pi * 2r = 1 \text{ ----- (12)}$$

*Tau – Circle Analysis*

From equation (4), we know

$$C_2 = K * r$$

$$C_2 = \tau * r$$

By equating it to the unit circle, one can write as follows

$$\tau * r = 1 \text{ ----- (13)}$$

Equating (12) and (13), we get

$$\mathbf{\tau = 2 * \pi (Proved)}$$

**PROOF 3:** From the area of the circle perspective and using the same method of substitution

*Pie – Circle Analysis*

From equation (3), we know

$$A_1 = K * r * r$$

$$A_1 = \pi * r * r$$

By making the radius correction and equating it to the unit circle, one can write as follows. (Note: *Radius correction is applied to one 'r' which is sufficient*)

$$\pi * 2r * r = 1 \text{ ----- (14)}$$

### Tau – Circle Analysis

From equation (3), we know

$$A_2 = K * r * r$$

$$A_2 = \tau * r * r$$

By equating it to the unit circle, one can write as follows

$$\tau * r * r = 1 \text{ ----- (15)}$$

Equating (14) and (15), we get

$$\mathbf{\tau = 2 * \pi (Proved)}$$

**PROOF 4:** From the same area of the circle perspective but by adopting a different method which uses the squares produced by these two circles to derive their respective constants.

*In this case*, both the circles have their respective squares associated with them and is depicted in the Fig (5).

From equation (3), we know

$$\text{Area of a Circle} = K * r * r$$

We can write it is as follows,

$$\text{Area of a Circle} = K * (\text{Current Area of the Square}) \text{ ----- (16)}$$

We interchange the variables for avoiding confusion,

Here onwards the variable ‘r’ is associated with the square and two new variables ‘a<sub>1</sub>’ and ‘a<sub>2</sub>’ define the radius of the “Pie – Circle” and “Tau – Circle” respectively. This is purely done by considering the square term on the RHS of the above equation. The following analysis is done to remove the inequalities as a result of our new findings.

Let us construct the following table for this purpose

Square = (Radius) <sup>2</sup>	Square = (Radius) = r	Pi – Circle = ‘a <sub>1</sub> ’	Tau – Circle = ‘a <sub>2</sub> ’
0	0	0	0
1	1	0.5	0
4	2	0	1

**Table 1:** Values obtained for square of varying side

For our consideration, it’s enough to know the values corresponding to r = 1 and r = 2. *In the above table*, the unit circle is given by r = 2.

Put (r<sup>2</sup> = 1, CRV a<sub>1</sub> = 0.5) in equation (16); where CRV = Corresponding Value

$$\left(\frac{A1}{1}\right) = \pi$$

$$\left(\pi * a_1^2\right) = \pi \text{ ----- (17)}$$

Put (r<sup>2</sup> = 4, CRV a<sub>2</sub> = 1) in equation (16)

$$\left(\frac{A2}{4}\right) = \tau$$

$$\left(\frac{\pi * a_2^2}{4}\right) = \tau \text{ ----- (18)}$$

Divide (18) by (17)

$$\left(\frac{\left(\frac{\pi * a_2^2}{4}\right)}{\left(\frac{\pi * a_1^2}{1}\right)}\right) = \frac{\tau}{\pi}$$

$$\pi * \left(\frac{\pi * a_2^2}{4}\right) = \tau * \left(\frac{\pi * a_1^2}{1}\right)$$

$$\pi * (\pi * a_2^2) = 4 * \tau * (\pi * a_1^2)$$

*Hence forth*, please kindly consider the RHS term only, because LHS is already in Unit Circle dimension

(Since,  $r^2 = 4$  in Sq D =  $a_2$  in CD of “*Tau – Circle*”)

$$\pi * (\pi * a_2^2) = a_2 * \tau * (\pi * a_1^2)$$

Converting “*Tau – Circle*” CD = “*Pie – Circle*” CD

$$\pi * (\pi * a_2^2) = 2a_1 * \tau * (\pi * a_1^2) \quad (\text{Since } a_2 = 2a_1)$$

Now, converting the CD of “*Pie – Circle*” into its value

$$\pi * (\pi * a_2^2) = 2 * 0.5 * \tau * (\pi * a_1^2)$$

$$\pi * (\pi * a_2^2) = 1 * \tau * (\pi * a_1^2)$$

Thus far we have made the corrections w.r.t to “*Pie – Circle*”

The RHS term is not the unit circle; therefore we have to get back to the original position by repeating the steps in the reverse direction.

The first step involved is to convert the value ‘1’ of CD “*Pie – Circle*” to its equivalent value in the CD of “*Tau – Circle*” which is nothing but ‘2’.

$$\pi * (\pi * a_2^2) = 2 * \tau * (\pi * a_1^2)$$

Convert the CD “*Pie – Circle*” to CD “*Tau – Circle*” which is a unit circle

$$\pi * (\pi * a_2^2) = 2 * \tau * \left( \pi * \frac{a_2^2}{4} \right) \quad \text{Since } (a_2 = 2a_1)$$

Now, both of them are in the unit circle dimensions and can be equated

$$\pi * (\pi * a_2^2) = \tau * \left( \pi * \frac{a_2^2}{2} \right)$$

$$\pi = \tau * \left( \frac{1}{2} \right)$$

$$\tau = 2 * \pi \quad (\text{Proved})$$

Where, Sq D = Measuring scale of a *Square* Dimension

CD = Measuring scale of a *Circle* Dimension

**Let us see some geometrical aberration of circle formed by usage of "π" constant,**

< Refer the “*annexure*” document provided >

**NOTE 3:** I would like to re-iterate the point that by under – estimating the fundamental circle constant by 50 % one was over – estimating the unit circle by the same amount. This in mathematical terms means that one was accounting 50 % more of unit circle unnecessarily.

6. PROOF BY METHOD OF INTEGRATION

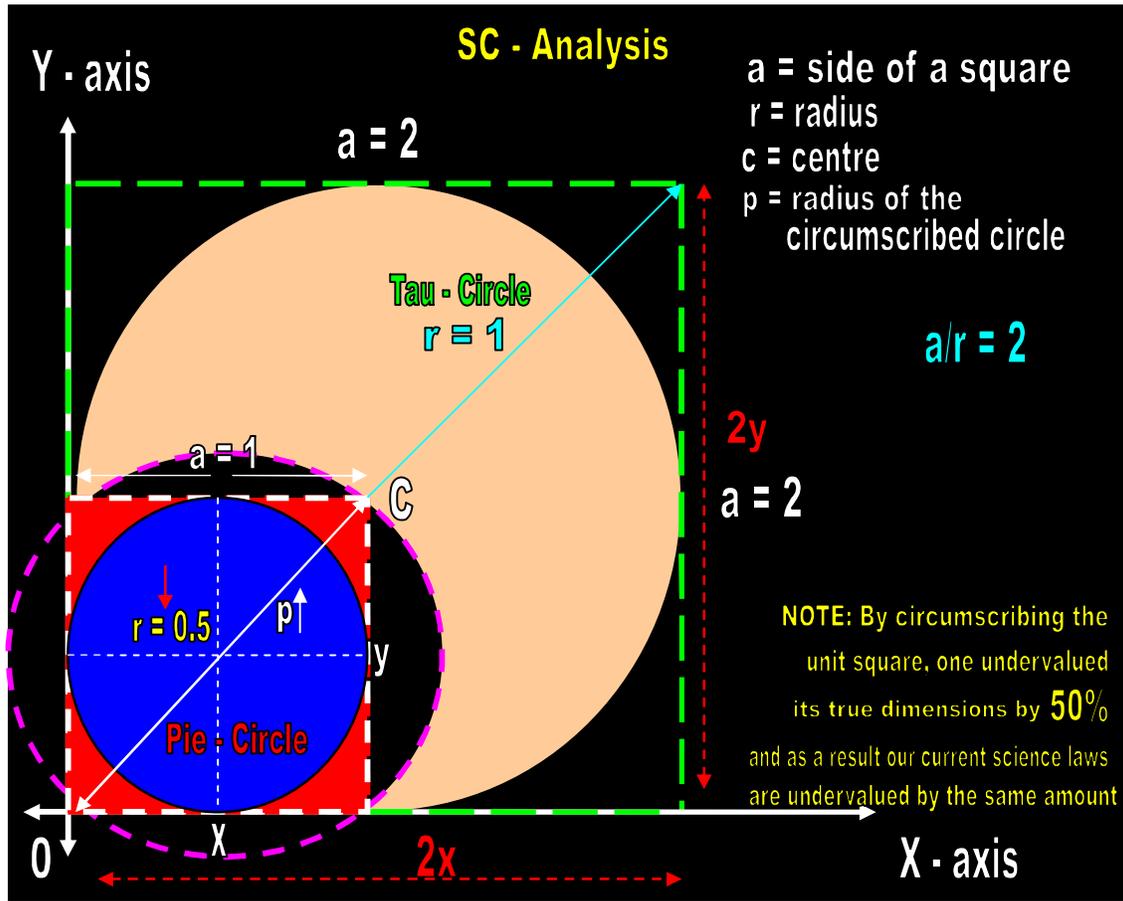


Fig (6) Distinction b/w the two circles described in a normal plane

Consider Fig (5) for our analysis, please kindly note that the circle enclosed in a unit square is that of a circle whose radius = *half the unit circle* and the actual unit circle is that which is enclosed in a square whose side is given by the value '2'. This is what is depicted in Fig (6). The diagram is presented in this fashion to convey the idea in simple and clear terms. Hence, we have to multiply the existing area of a circle by a factor of two. This clarifies the given situation and would aid the mathematical analysis which follows.

According to current mathematics,

Equation of a circle in a Cartesian plane is given by

$$x^2 + y^2 = r^2 \text{ ----- (19)}$$

True equation of a circle in a Cartesian plane according to me is given by

$$2 * (x^2 + y^2 = r^2) \text{----- (20)} \quad (\text{since a factor of '2'})$$

From (19), we can write as follows

$$x = \pm \sqrt{r^2 - y^2}$$

We can integrate this function to find the area of the given circle.

$$A = \int_0^r \sqrt{r^2 - y^2} dy$$

$$y = r \sin \phi \text{ ----- (21)}$$

$$dy = r \cos \phi$$

Trigonometric functions known,

$$\sin^2 \phi + \cos^2 \phi = 1$$

$$\cos 2\phi = 2 \cos^2 \phi - 1$$

Put (y=0) in equation (21), we get

$$0 = \sin \phi \text{ (This implies } \phi = \frac{\pi}{2} \text{)}$$

Put (y=r) in equation (21), we get

$$1 = \sin \phi \text{ (This implies } \phi = 0 \text{)}$$

The above represents the first quadrant and therefore multiplying it four times gives the total area of the circle.

Substituting the corresponding values and changing the limits to ‘ $\phi$ ’

$$A = 4 * \int_0^{\pi/2} \sqrt{r^2 - (r \sin \phi)^2} (r \cos \phi) d\phi$$

$$A = 4 * \int_0^{\pi/2} \sqrt{r^2 (1 - \sin^2 \phi)} (r \cos \phi) d\phi$$

$$A = 4r \int_0^{\pi/2} \sqrt{r^2 (1 - \sin^2 \phi)} (\cos \phi) d\phi$$

$$A = 4r^2 \int_0^{\pi/2} \sqrt{(1 - \sin^2 \phi)} (\cos \phi) d\phi$$

$$A = 4r^2 \int_0^{\pi/2} \cos \phi (\cos \phi) d\phi$$

$$A = 4r^2 \int_0^{\pi/2} (\cos^2 \phi) d\phi$$

$$A = 4r^2 \int_0^{\pi/2} \frac{1}{2} (\cos 2\phi + 1) d\phi$$

$$A = 2r^2 \int_0^{\pi/2} (\cos 2\phi + 1) d\phi$$

$$A = 2r^2 * \left( [\sin 2\phi]_0^{\pi/2} - [\phi]_0^{\pi/2} \right)$$

$$A = 2r^2 \left[ \sin \pi - \sin 0 + \frac{\pi}{2} - 0 \right]$$

$$A = 2r^2 \left\{ 0 + \frac{\pi}{2} \right\} = \pi * r^2$$

This is the area of the circle currently accounted, but this needs to be multiplied by a factor of two as per our analysis, *therefore the true area of a circle is twice this value.*

$$A = 2\pi * r^2 = \tau * r^2 \text{ (Proved) ----- (22)}$$

## 7. UNDERSTANDING THE FALLACY IN ARCHIMEDES PROOF OF AREA OF A CIRCLE

I reconstruct the method which was used by Archimedes to derive the formula for the area of a circle <sup>[7,8]</sup>. Archimedes constructed a number of polygons in the given circle and to compute the area of the polygon, Archimedes divided it into triangles, one triangle for each pair of sides whose height is the radius of the circle and whose base is its circumference, then the circle was split up and re-arranged to form a rectangular shape and thereby calculated its area as depicted in the Figures {a, b, c}. *For our study, we take  $n = "8"$  polygons only.* This is done purely for simplicity and keeping the space constraints in mind. We have to look into this aspect from the *SC – Analysis* perspective for the obvious reasons mentioned earlier.

### **STEP 1:**

*A circle with polygons inscribed.*

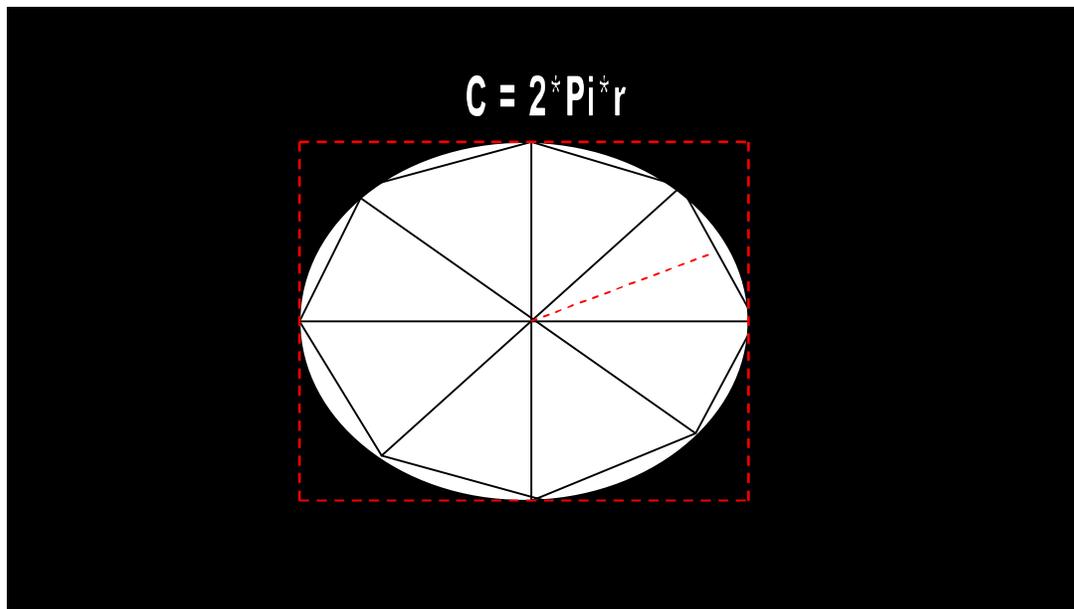


Fig (a) a circle is divided into 'n' polygons

**NOTE 4:** the split shape when re – arranged looks like a parallelogram in this document but when a large number of sides are taken into consideration it turns out be a rectangle.

**STEP 2:** *Splitting of the circle is as shown below*

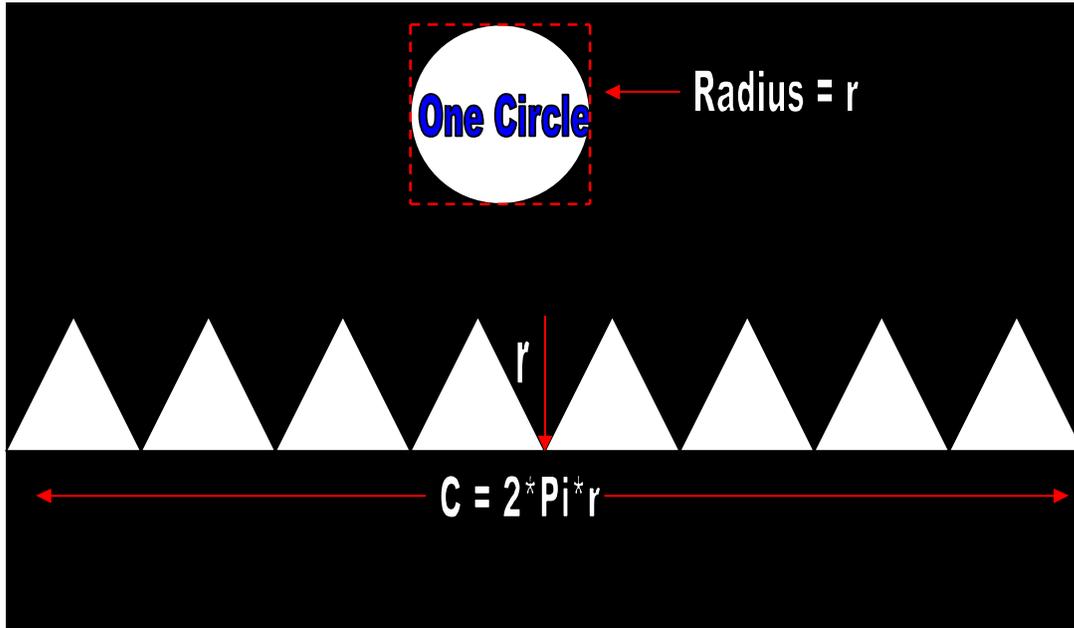


Fig (b) Polygons split open to form the above shape

**STEP 3:** *Re-arranging them to form a rectangle*

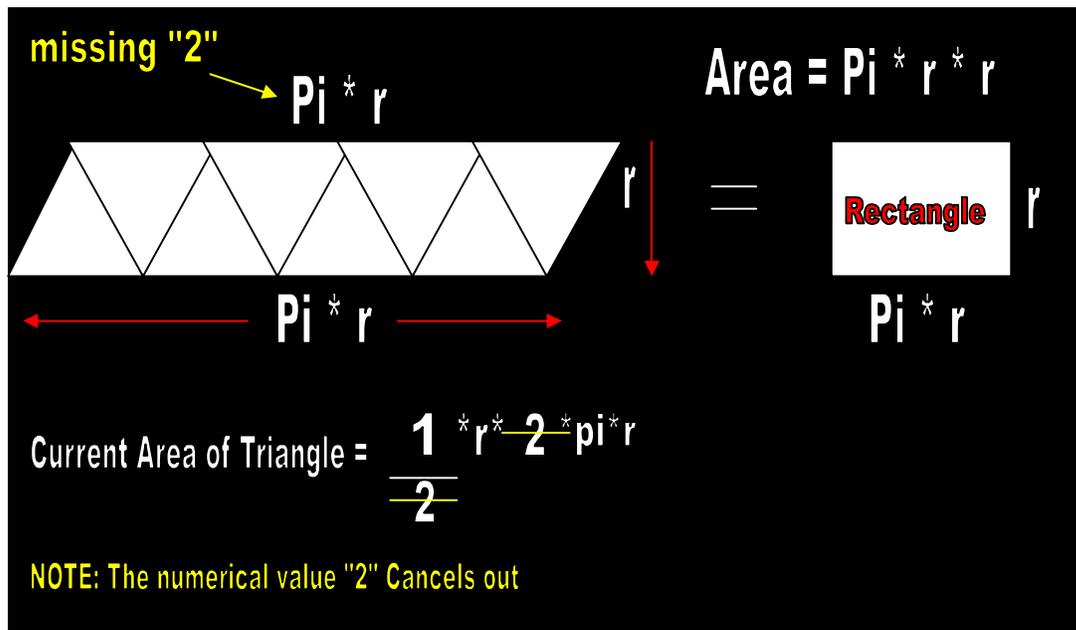


Fig © polygons re-arranged to form the shape of a rectangle

And the area was thus calculated to be the area of the rectangle which is nothing but the current area of the circle. *But there is a terrible flaw here which needs to be pointed out.*

Now, let's understand this flaw geometrically, I am reproducing the “**Step 2**” for this purpose here.

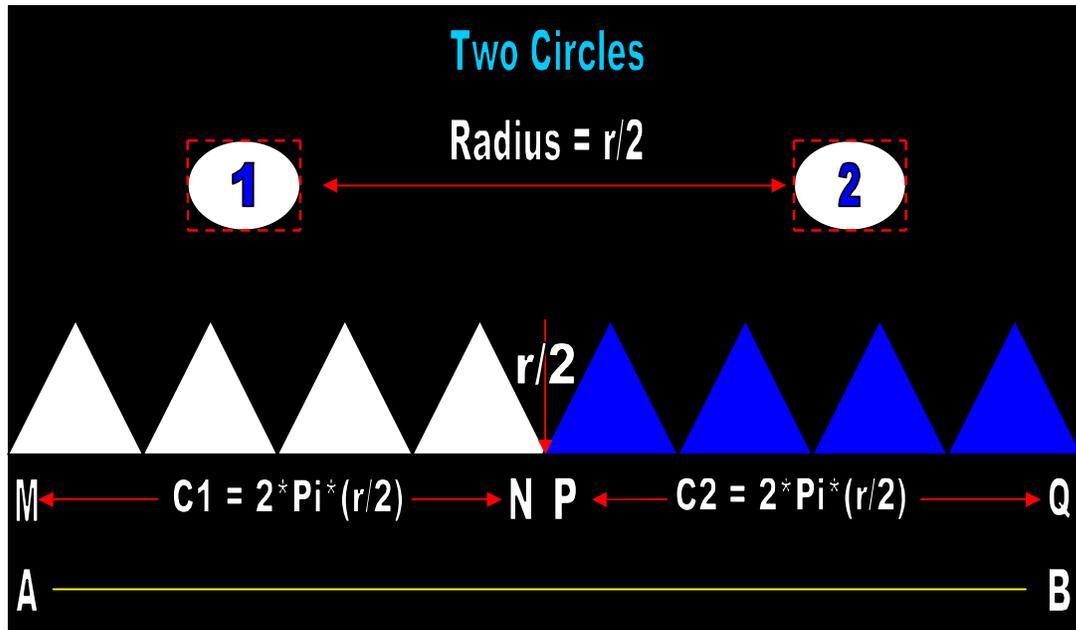


Fig (d) splitting here essentially means two different circles

In the Fig (d), when Archimedes split the circumference into two parts, Archimedes failed to take into account that it represented two separate circles of lower radius (and also Archimedes was unaware of the *fundamental geometrical ratio (a new discovery in science)*, a relationship between circle and a square in geometry). This is what is depicted in the figure Fig (d). In this figure, AB represents a circle whose radius is “r” and MN and PQ represent the circumference of two new circles whose radius is halved by the process of splitting it into two. The problem with splitting is that it changes the original geometry and by doing that one needs to take the changes into consideration. This is the key to understanding the flaw in the technique used.

*Therefore*, its new circumference and radius is as given in Fig (d).

Kindly note that;  $MN = PQ = \frac{1}{2} * (AB) = \frac{1}{2} * \text{radius}$

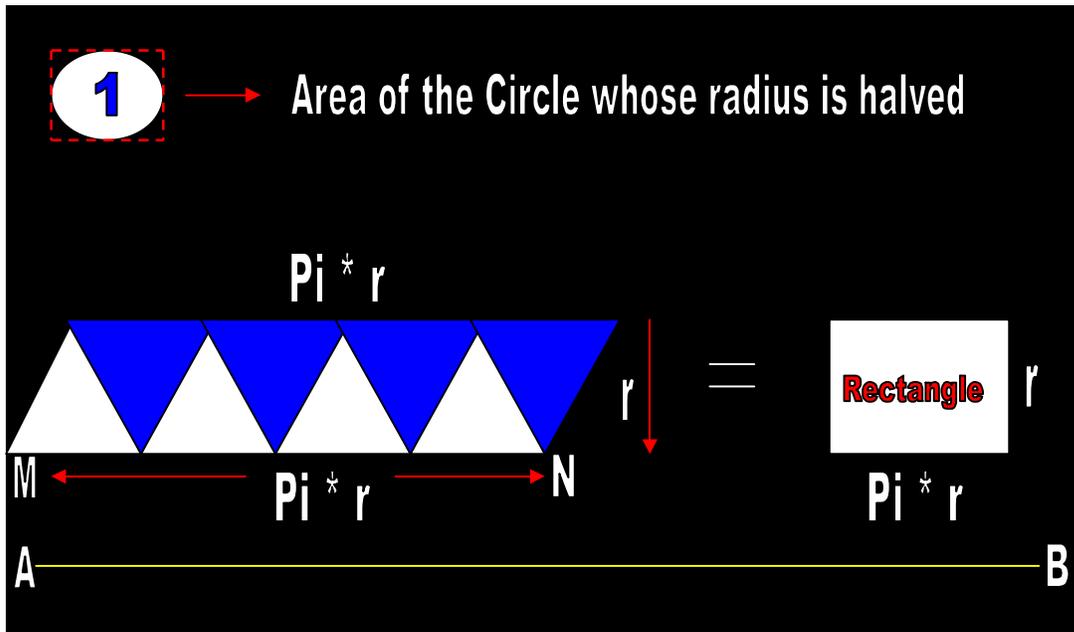


Fig (e) Archimedes Area of the circle

From the Fig (d) and Fig (e), it's clear that the area which Archimedes was calculating was that of a circle whose radius is one half the actual radius of the circle. *Therefore*, one has to multiply the final result by a factor of “2” to get the correct area of the circle.

By doing that, one gets the below value

$$\text{True area of the circle} = 2\pi * r^2 = \tau * r^2 \text{----- (23)}$$

Also, please kindly note that the ‘r’ formed in Fig (e) has to be treated as one of the sides of the rectangle got by adding ( $\frac{1}{2} * r + \frac{1}{2} * r = r$ ) and not the actual radius of the circle. In other words, we should interpret this geometric analysis as, two inscribed circles formed in squares of side length ‘2’ and ‘1’ respectively. The *SC – Analysis* perspective given because of the fundamental geometrical ratio. *This clarifies the situation*. This analysis is a solid proof of how the radius of a circle and the side of any square in which it is inscribed or otherwise is in the ratio ( **1 : 2** ) and is termed as fundamental geometrical ratio and is geometrically depicted in Fig (5) & Fig (6). Therefore, this difference needs to be nullified first and hence the factor “2” needs to be properly accounted in our science to get geometrical completeness.

*The flaw was in the technique used by Archimedes*. This solves the issue once and for all.

### 7.1 CALCULATING AREA OF A CIRCLE USING SIMILAR METHOD

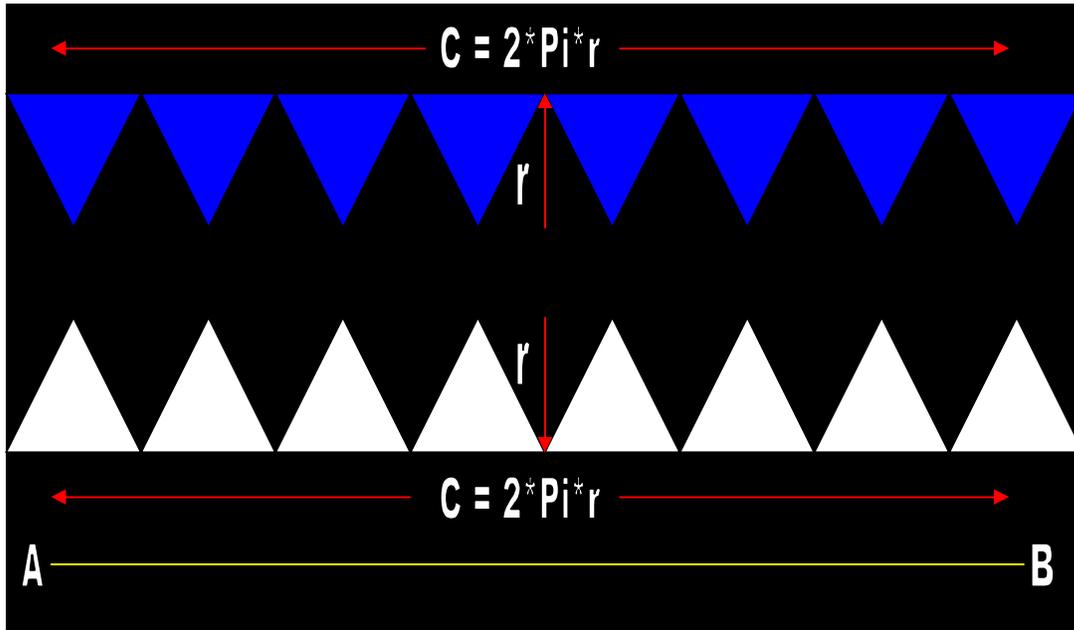


Fig (f) adding one more circle of the same dimension at the top

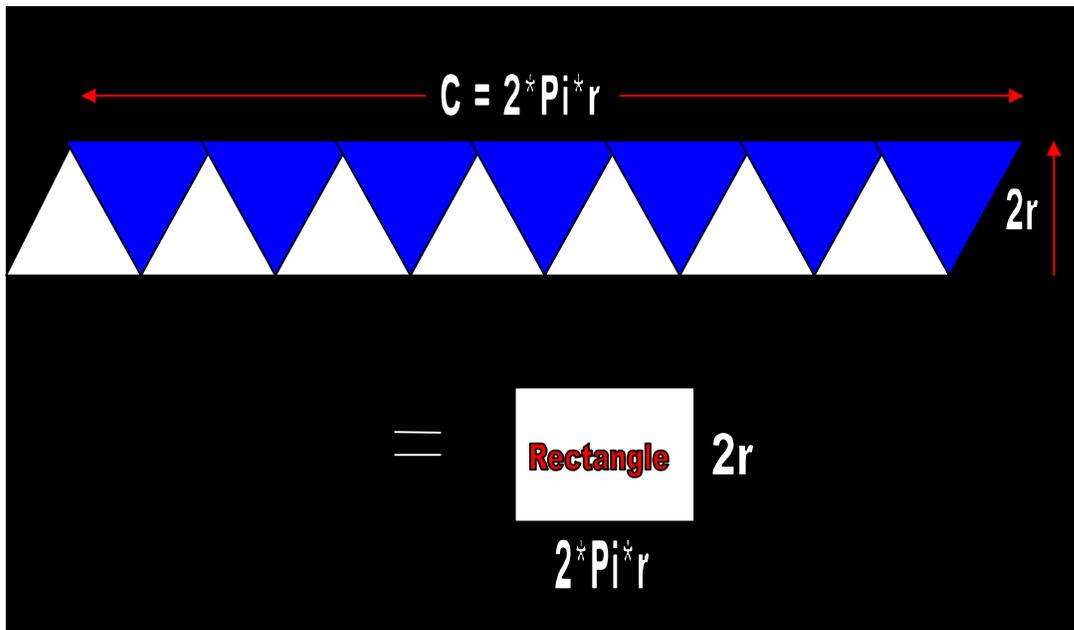


Fig (g) clubbing them together to form a rectangle

Now,

$$\text{The area of this circle} = 2\pi * r * 2r = 4\pi * r^2$$

We know that, this area is the “*area of the circle*” which is double of the original circle; *therefore*, to get the correct area of the given circle, one needs to half this quantity which is nothing but the actual area which one is calculating,

$$\text{Area of the circle} = 2\pi * r^2 = \tau * r^2 \text{ ----- (24)}$$

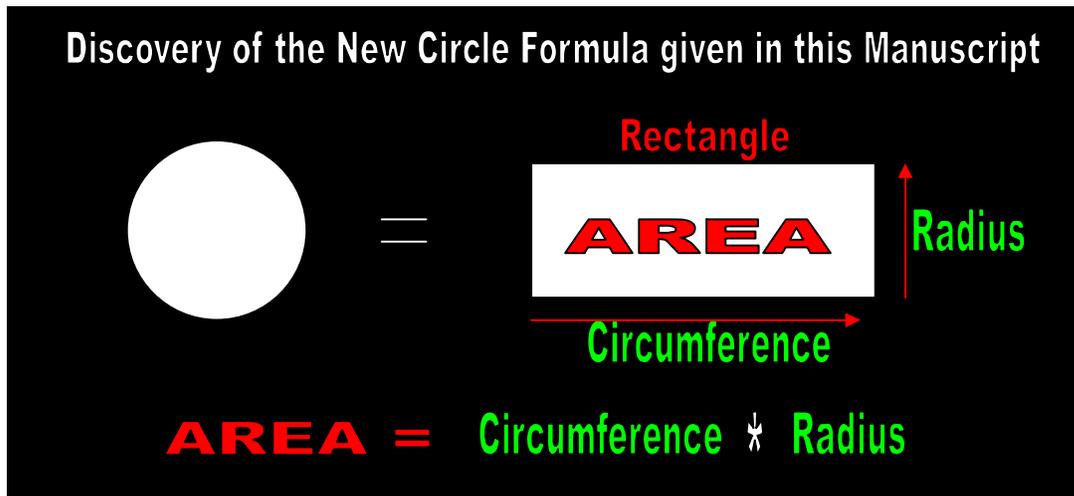


Fig (h) Fundamental formula relating the 3 basic properties of a circle

By replacing the values of the rectangle with the respective components of the circle, we obtain the new circle formula as depicted in Fig (h) and also given in this manuscript. *This concludes my geometrical analysis of Archimedes Method. Matter solved beautifully.*

**7.2 THEOREM:** - *The ratio of any circle’s circumference to its diameter is a constant (proof already known)<sup>[6]</sup> and so is its radius*

**PROOF:** We already know,  $\frac{C1}{D1} = \frac{C2}{D2} = \pi$

(Where C1 = C2 = Circumference of two Concentric circles, D1 and D2 its diameters, and R1 and R2 its radius respectively)

Therefore,  $\frac{C1}{2 * R1} = \frac{C2}{2 * R2} = \pi$

$$\frac{C1}{R1} = \frac{C2}{R2} = 2\pi = \tau$$

**Hence, the simple proof**

**7.3 THEOREM:** - *The area of any circle is equal to a right-angled triangle in which one of the sides about the right triangle is equal to the radius and the other to the circumference of the circle. (Proof already known) [7, 8]*

**PROOF:** We already know the proof given by Archimedes in this connection. We have to see this proof in the new context defined in this manuscript; the new area of the right – triangle of equal side lengths is proved to be  $a^2$ , (ref 8.1 & 8.2 sec). In general terms, it can be expressed as (base \* height) which is double the current expression for the area of the triangle. *Kindly note this.*

Therefore, new Area of right  $\Delta^{le} = \text{base} * \text{height}$

Area of right  $\Delta^{le} = \text{Radius} * \text{Circumference}$  (substitution of circle components)

Area of right  $\Delta^{le} = \text{Area of the Circle} = r * 2\pi * r = 2\pi * r^2 = \tau * r^2$ .

This is also confirmed from the equations (8), (22), (23), (24) & (25).

**Hence, the proof**

**7.4 THEOREM:** - *A circle comprises of six kites whose area is equal to area of the right triangle of radius length.*

**PROOF:** Before we proceed with our proof, It's essential to give a thought on equilateral triangles [9] which are of great significance in our geometry and also a circle consists of six equilateral triangles of radius length as shown in Fig (i).

Consider  $\Delta^{le} POA$ , we extend P and A outwards to meet at point 'X' on the Circumference of the circle such that if one draws a line to the origin 'O' then it would bisect the line PA at midpoint of it. We get an isosceles  $\Delta^{le} PAX$ . Hence, the combination makes a Kite [10] POAX with diagonals OX & PA. The same process is repeated for all other equilateral triangles and is as depicted in the Fig (i). Now, let's analyze this situation in the context of the unit circle.

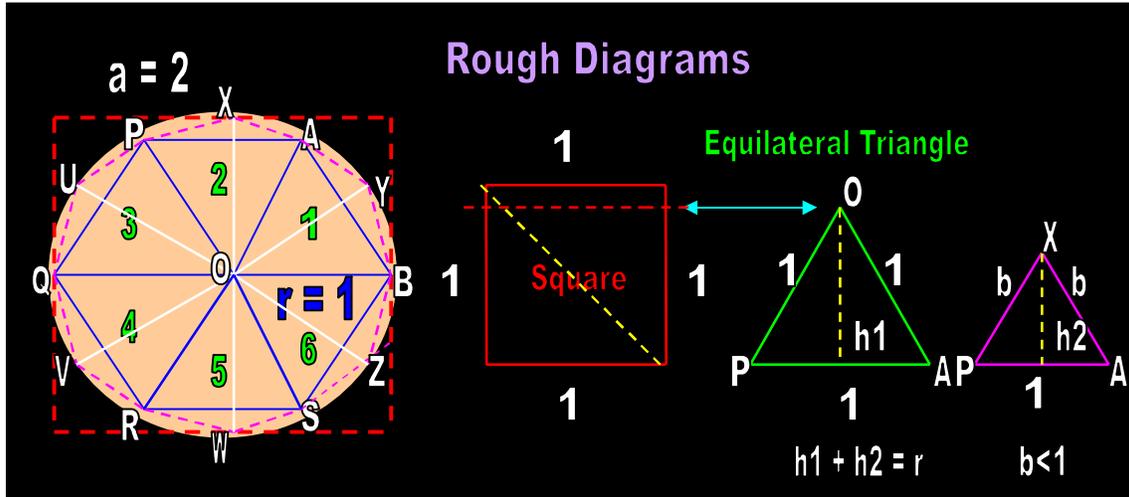


Fig (i) Description of equilateral triangles and kites in a unit circle

We have to look at our analysis from a fresh perspective. From the fig (i), we calculate

$$\text{New Area of the Equilateral } \Delta^{le} \text{ PAO} = \text{base} * \text{height} = r * h_1 \text{ (ref 8.1 \& 8.2 sec)}$$

$$\text{New Area of the Isosceles } \Delta^{le} \text{ PAX} = \text{base} * \text{height} = r * h_2$$

$$\text{Area of the Kite POAX} = \text{Area } \Delta^{le} \text{ PAO} + \text{Area } \Delta^{le} \text{ PAX}$$

$$\text{Area of the Kite POAX} = r * h_1 + r * h_2 = r * (h_1 + h_2) = r * r = r^2 \text{ (Right } \Delta^{le} \text{)}$$

Therefore, the conjunction of Equilateral & Isosceles  $\Delta^{le}$  forms Kite, a special quadrilateral and in this instance, its area is calculated to be the area of the “right triangle of radius length”.

$$\text{Six such combinations have an area (K)} = 6r^2$$

$$\text{And, (Area of the circle - K)} = \tau * r^2 - 6r^2 \approx (0.28 * r^2) < r^2$$

Therefore, a maximum of Six Kites can only be constructed in a circle. Therefore, a circle is made up of **6 “Kites”** and *not* **4 “Squares”**. **Hence, the proof**

## 8. UNDERSTANDING THE FALLACY OF AREA OF A SQUARE

It's widely believed that in classical times, the second power was described in terms of a square, *as is the current expression for a square*, but there is incompleteness in this belief as it gives significance only to the “*power*” value and completely ignores the “*co-efficient*” value associated with a variable which is as important as the latter. *This notion is without geometrical basis*. Therefore, I question the rationality of the area of the square expression. *Currently*, it's expressed as  $a^2$  and according to my new findings after accounting the factor ‘2’ needs to be  $2a^2$ . Here is the justification for the case being presented beautifully.

*For our study and analysis*, we consider a unit square and also we know that a square is formed by the joining of ‘4’ equal lengths and also importantly two right triangles. *Let's keep these important things in mind*. We know that areas of all geometrical figures are expressed in terms of square units. There is nothing wrong in it, but what current science failed to question was the fact as to what the geometrical figure “*Square*” is expressed of in terms of its own area?

*No doubts*, it has to be expressed in terms of the square itself since all other geometrical figures are expressed in terms of it. *Pure logic and reasoning is enough to come to this conclusion*. So, let's test the validity of the unit square based on these mathematical grounds.

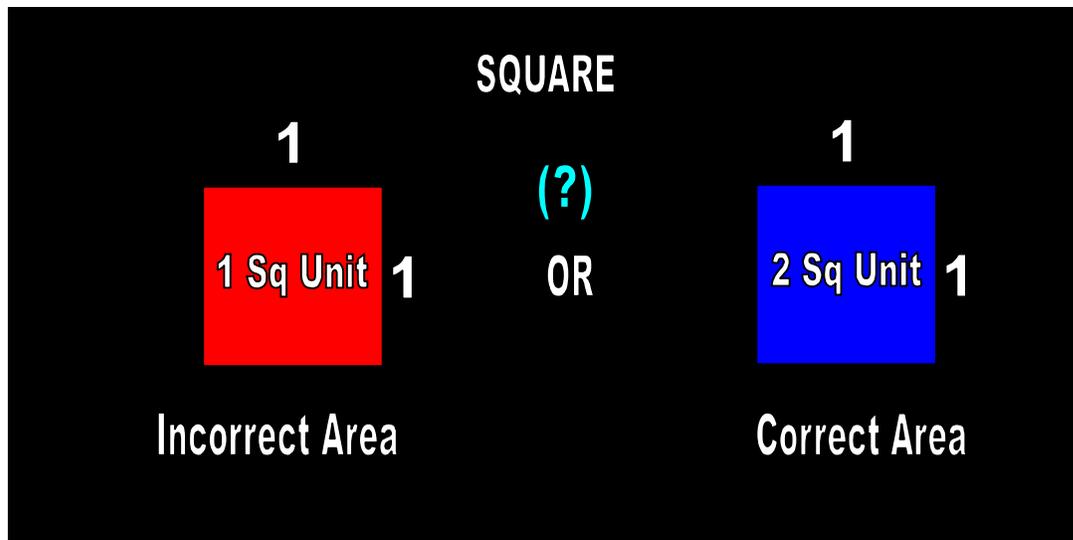


Fig (g1) Area of Square (*current science*) and as per my new research findings

### 8.1 GEOMETRICAL PROOF OF AREA OF A SQUARE

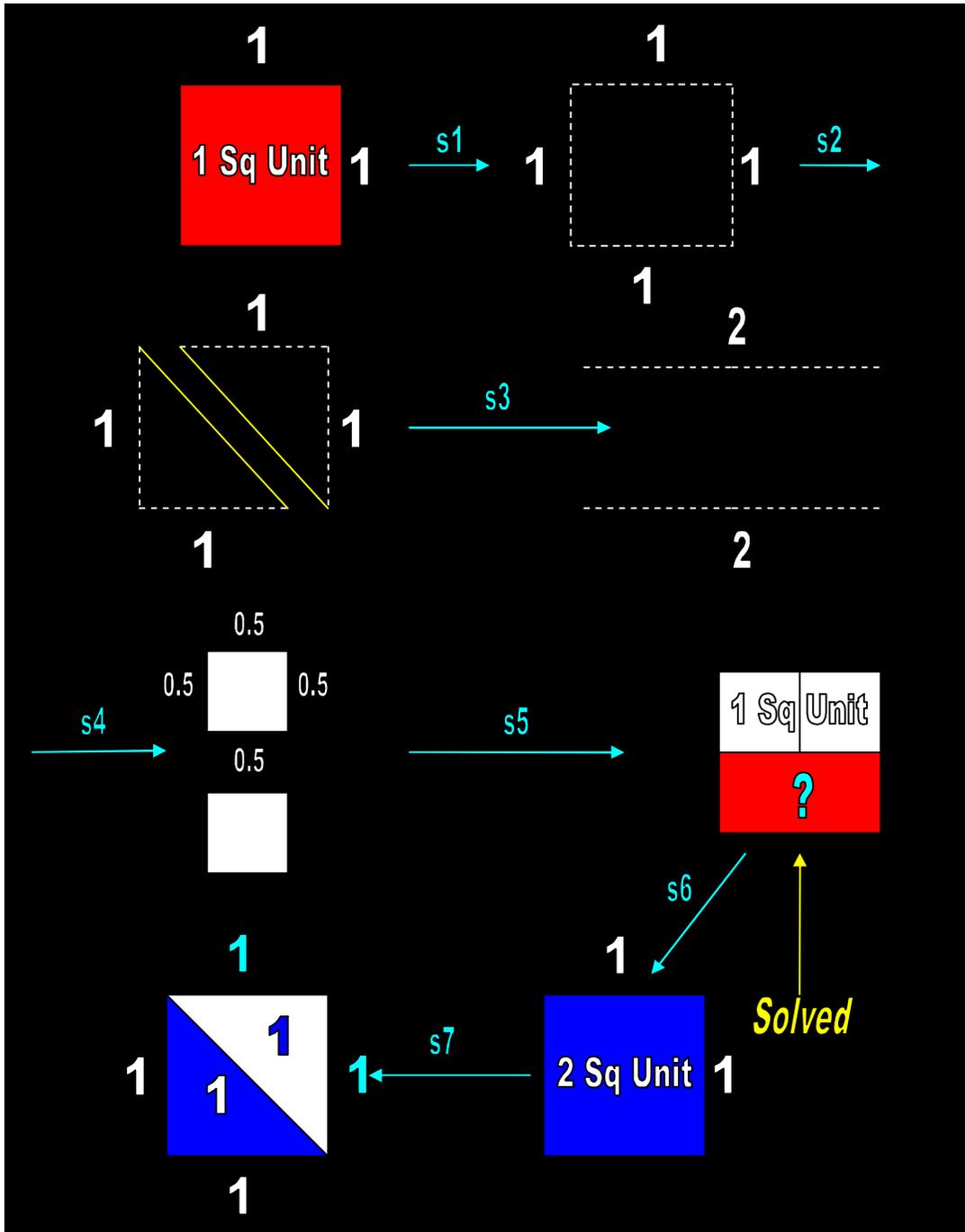


Fig (g2) Geometrical proof of Area of a Square is  $2a^2$  and that of a right triangle is  $a^2$

Quick Comparison b/w a Triangle and a Square Unit:-

$$\begin{aligned} \text{Triangle} &= a^2 = (1 \text{ unit})^2 = \underline{1} \text{ unit}^2 \text{ (geometrically in-complete)} \\ \text{Square} &= 2 a^2 = 2 * (1 \text{ unit})^2 = \underline{2} \text{ unit}^2 \text{ (Geometrically Complete)} \end{aligned}$$

Please kindly note the *co-efficient* value and that of the *power* value.

## 8.2 DISPROOF OF PYTHAGORAS THEOREM

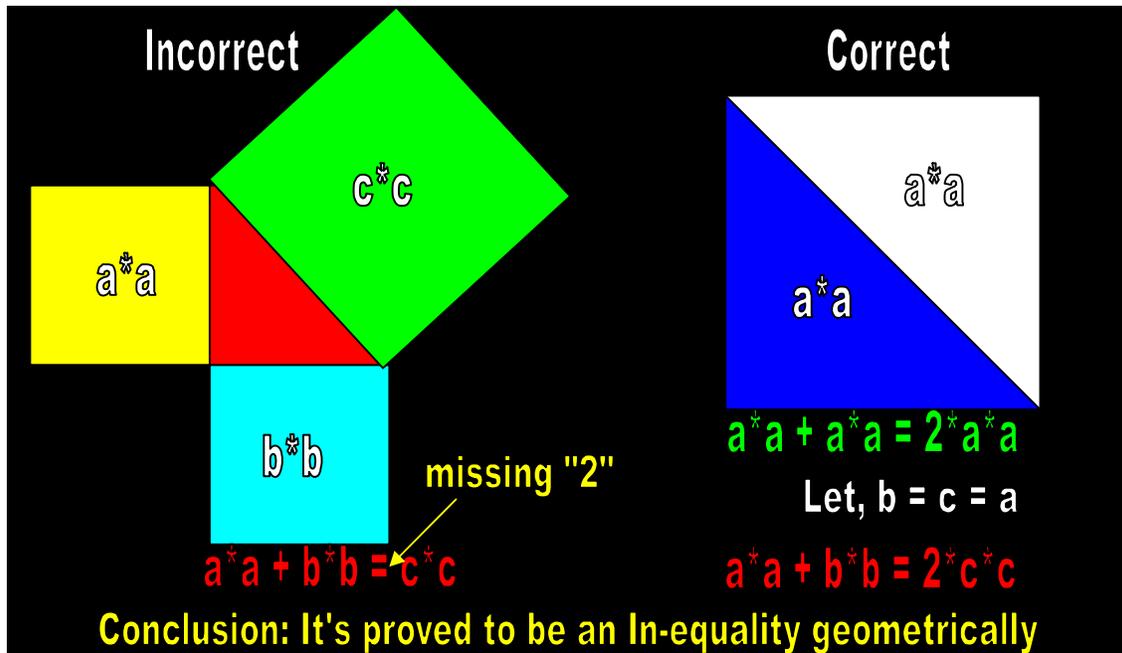


Fig (g3) disproof of Pythagoras theorem done geometrically

Please kindly follow the sequence  $\{s1 \rightarrow s2 \rightarrow s3 \rightarrow s4 \rightarrow s5 \rightarrow s6 \rightarrow s7\}$  in the Fig (g2). *The above geometrical proof given is self explanatory.* Therefore, this invalidates the current expression for the area of a square and upholds my new expression for the area of a square. The area which current science was accounting for a “**Square**” is actually that of a “**Triangle**”. *Currently, science was accounting the areas of all geometrical figures in (triangle units or semi-square units) and not in square units. In otherwords, one-half of square units. It's a clear misrepresentation of geometry.* This clarifies the existing situation. From Fig (g3), it's proved that the area of square can be split up into areas of two right triangles of equal side.

**Definition 1:** Area of a right triangle of equal side lengths when added to its symmetrical pair forms the area of a **Square**. Mathematically, it's expressed as  $a^2 + a^2 = 2a^2$ . *This is the actual basis for the geometrically incomplete Pythagoras theorem currently in use (fig (g3)).* The presence of the factor “**2**” in the above expression convincingly proves that Pythagoras theorem is an inequality. Hence “*Fermat's last theorem*”, “*Beal's conjecture*” are inequalities.

**Definition 2:** Area of a right triangle of varied side lengths when added to its symmetrical pair forms the area of a **Rectangle**. Mathematically, it's expressed as  $ab + ab = 2ab$ .

### 8.3 DERIVATION OF AREA OF CIRCLE IN THE CONTEXT OF NEW AREA OF THE SQUARE PROVED ABOVE

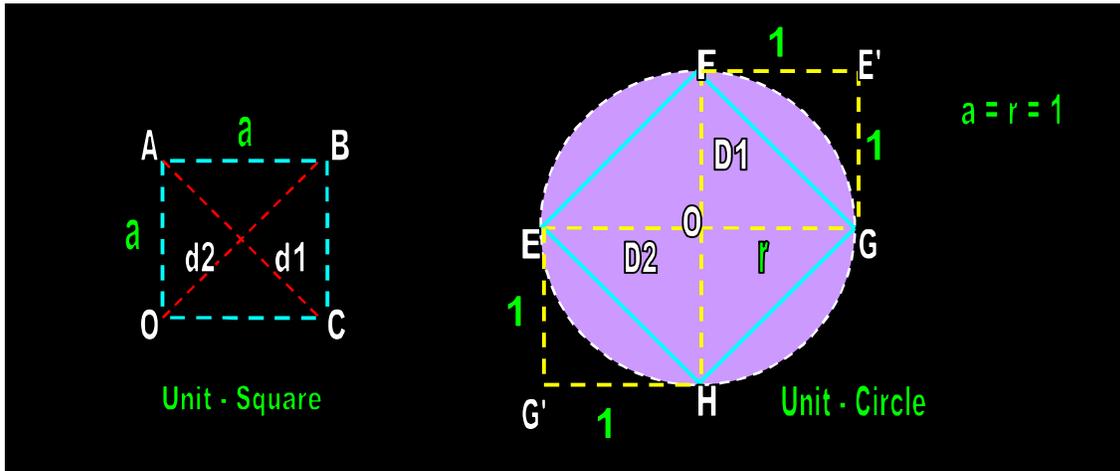


Fig (g4) Relationship b/w Unit Square and Unit Circle

#### Unit – Square Analysis:

Consider the “Unit Square” ABCO in the fig (g4); it has two diagonals AC and OB represented by ‘ $d_1$ ’ and ‘ $d_2$ ’. It consists of two right triangles of equal sides  $\Delta^{le} AOC$  and  $\Delta^{le} ABC$ . Area of ABCO = Area  $\Delta^{le} AOC$  + Area  $\Delta^{le} ABC$ . Now, we can apply the Pythagoras theorem in this instance as the side lengths are equal

Therefore, for  $\Delta^{le} AOC$ ,

$$AC^2 = AO^2 + OC^2$$

$$AC^2 = a^2 + a^2$$

$$d_1^2 = a^2 + a^2 = 2a^2$$

$$d_1 = \sqrt{2} * a$$

Similarly,

For  $\Delta^{le} BCO$ ,

$$OB^2 = OC^2 + BC^2$$

$$OB^2 = a^2 + a^2$$

$$d_2^2 = a^2 + a^2 = 2a^2$$

$$d_2 = \sqrt{2} * a$$

The product of the two diagonals gives us the following result

$$d_1 \times d_2 = \sqrt{2} * a * \sqrt{2} * a = 2a^2$$

But, we know that this is the new area of the “Square” derived (ref 8.1 & 8.2 sec); therefore, we have derived a new relation for the area of the square which is given as follows,

$$\text{Area of the Square} = d_1 \times d_2 \text{ (Proved)}$$

Unit – Circle Analysis:

Consider the “Unit Circle” in fig (g4); it encloses one square EFGH, this square consists of 4 right  $\Delta^e$  EOF,  $\Delta^e$  FOG,  $\Delta^e$  EOH,  $\Delta^e$  GOH. (This is the preface for the “4 – Squares” theory currently involving a circle). Importantly, it consists of two diagonals FH, EG for our consideration. Kindly note that the “Circle” and “Right triangles” inside it are in unit dimensions but the “Square” is not. So, we cannot establish a direct relationship with the square in its current form and therefore we have to bring it in unit dimensions, let’s analyze.

We know, CA of Unit Circle =  $\pi$  \* CA of Unit Square (where CA = Current Area)

Theoretically, if we assume that the “area of the circle” and “area of the square” to be two independent properties then one can establish a direct relationship between them as follows

$$\begin{aligned} &\text{CA of Unit Circle } (\propto) \text{ CA of Unit Square} \\ &\text{CA of Unit Circle} = \pi * \text{CA of Unit Square (where } \pi = \text{Current Constant)} \end{aligned}$$

But, the area of the “Square” EFGH is not in unit dimension; therefore it has to be brought to it, so let’s see how this is achieved.

$$\begin{aligned} \text{CA of Unit Circle} &= \pi * \text{CA of Square “EFGH”} \\ \text{CA of Unit Circle} &= \pi * (\text{FH} * \text{EG}) = \pi * D_1 \times D_2 \text{ ( } \because \text{ proved above )} \\ \text{CA of Unit Circle} &= \pi * 2r \times 2r = \pi * 4r^2 \text{ (since it’s not the unit square)} \\ \text{CA of Unit Circle} &= \pi * 2 * 2r^2 \text{ (new unit square)} \end{aligned}$$

$$\text{New Area of Unit Circle} = 2\pi * \text{CA of Unit Square} = \tau * r^2 \text{ ----- (25)}$$

**Hence, the final proof.**

## 8.4 A SIMPLE EXPERIMENT WITH WATER

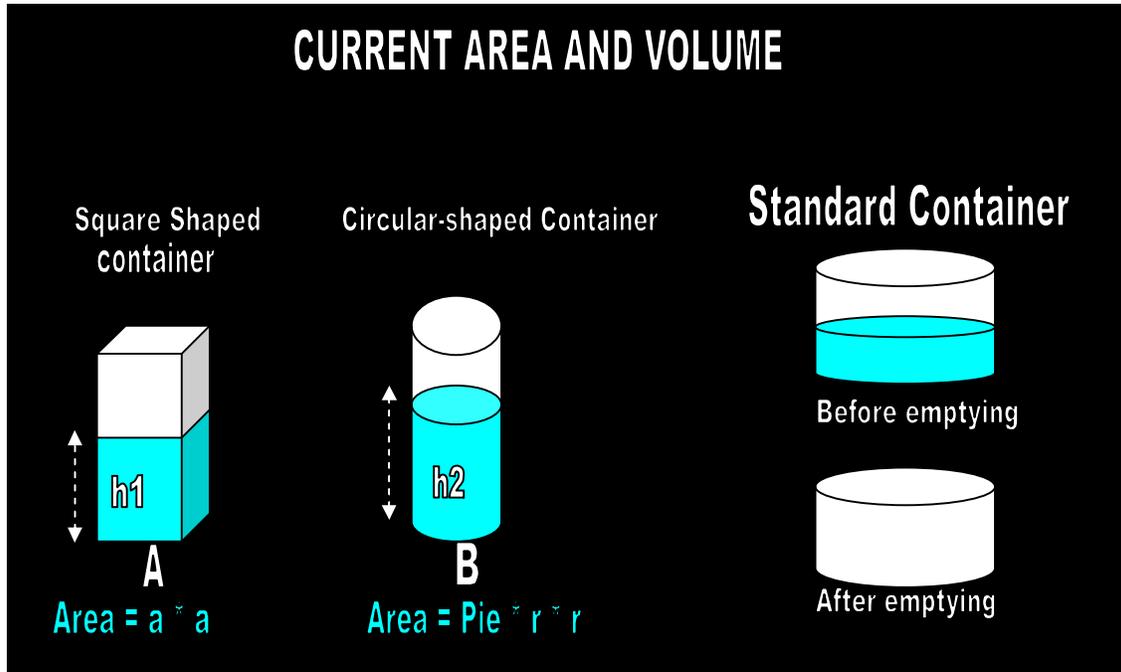


Fig (g5) Volume analysis b/w a Square & Circular shaped containers currently

Let us consider the simple setup described above, where in we test the validity of the existing geometrical laws by measuring the volume of water contained in each of it. We have considered a standard container for volume measurement. We have taken a fixed quantity of water in this container. We have also considered two other specially made containers, one is “*square shaped*” and the other is “*circular shaped*” as shown in the fig (g5). Now, let us transfer the water from the standard container into the circular shaped container first and thus emptying it, *now*, the water in the circular shaped container attains some height ‘ $h_2$ ’ as shown in the fig (g5). We note down the readings. Now, again we transfer this water into the square shaped container, it attains some height ‘ $h_1$ ’ as shown in the fig (g5). There are two situations which need to be studied here. So, *let’s analyze these situations*.

CURRENT SITUATION: (*Situation according to current science*)

Case ‘A’: Volume of the water contained in it is given by the following relation,

$$V_s = \text{Square\_area} * \text{Square\_height}$$

$$V_s = a^2 * h_1$$

Case 'B': Volume of the water contained in it is given by the following relation

$$V_c = \text{Circle\_area} * \text{Circle\_height}$$

$$V_c = \pi * r^2 * h_2$$

Therefore, one can establish a simple relationship b/w them as follows,

$$\text{Square\_area} * \text{Square\_height} = \text{Circle\_area} * \text{Circle\_height}$$

$$a^2 * h_1 = \pi * r^2 * h_2$$

Since, we have considered unit dimension for our study, let's take an example to prove this,

Let's say  $h_2 = 1$  unit, be the height attained;

Therefore, by substituting in the above relation, one gets as follows,

$$h_1 = \frac{3.14 \times 1 \times 1}{1} = \mathbf{3.14 \text{ units}}$$

NEW SITUATION: (My theory)

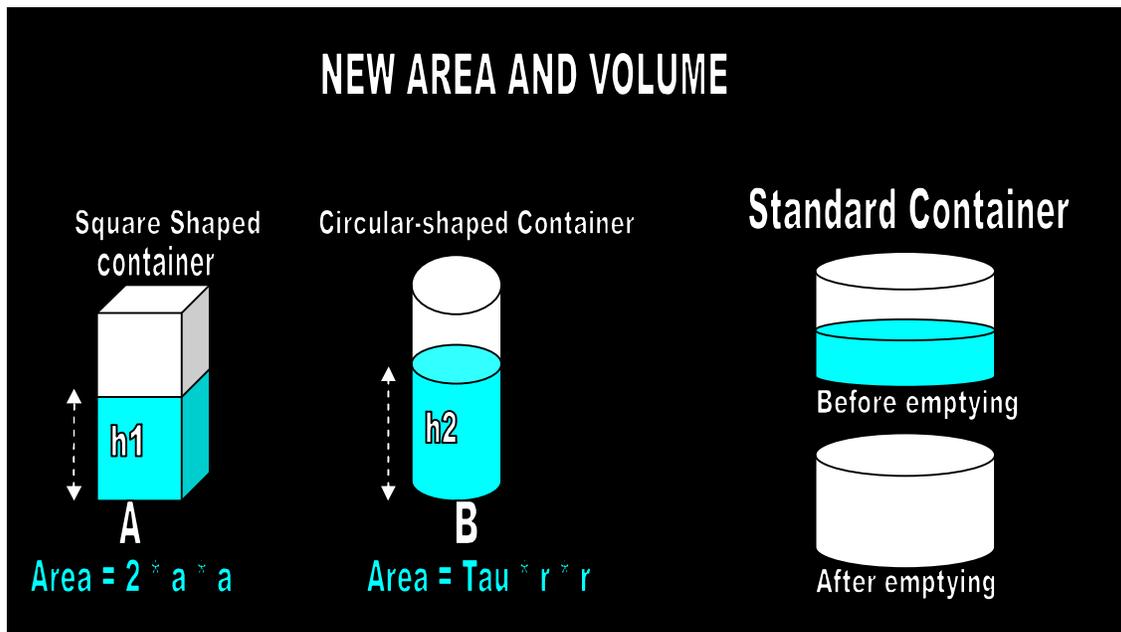


Fig (g6) Volume analysis b/w a Square & Circular shaped containers in new situation

Case 'A': Volume of the water contained in it is given by the following relation,

$$V_s = \text{New Square\_area} * \text{Square\_height}$$

$$V_s = 2a^2 * h_1$$

Case 'B': Volume of the water contained in it is given by the following relation

$$V_c = \text{New Circle\_area} * \text{Circle\_height}$$

$$V_c = \tau * r^2 * h_2$$

Similarly, one can establish a simple relationship b/w them as follows,

$$\text{New Square\_area} * \text{Square\_height} = \text{New Circle\_area} * \text{Circle\_height}$$

$$2a^2 * h_1 = \tau * r^2 * h_2$$

Since, we have considered unit dimension for our study, let's take an example to prove this,

Lets say  $h_2 = 1$  unit, be the height attained;

Therefore, by substituting in the above relation, we get,

$$h_1 = \frac{6.28 \times 1 \times 1}{2} = \mathbf{3.14 \text{ units}}$$

Circle_Area	Square_Area	Circle_Height (h2)	Square_Height (h1)	Remarks
$\pi * r^2$	$a^2$	1	3.14	<i>False</i> , its correct height needs to be either double of the existing height or it fills only ½ the quantity of water
$\tau * r^2$	$2a^2$	1	3.14	<i>True</i>

**Table 2:** Volume analysis of the two situations tabulated

It seems that we have a paradox, *because both attain the same height*. But, *this is impossible*. We already know that the new areas of the geometrical object being studied, to be double of their existing areas. Therefore, the simple conclusion which one can arrive at is that the current situation in science is absolutely wrong; it's a wrong perception of volume analysis and according to me, it's an improper representation of geometry. The actual situation as per my assessment is that, with the current laws one should attain double the height which is not obtained above or only half the quantity of water can be filled. This is the correct volume analysis. This is clearly illustrated in the fig (g7). Please kindly note that we at present are accounting only 50% of the “**actual areas & volumes**” of our geometrical figures. *It's completely a distorted geometry in use and is as shown in the fig (g7)*. Hence, the clear proof of incorrectness in the measurement of the areas & volumes of geometrical figures in our science is established beyond doubts. Hence, the urgent need for rectifying our science laws.

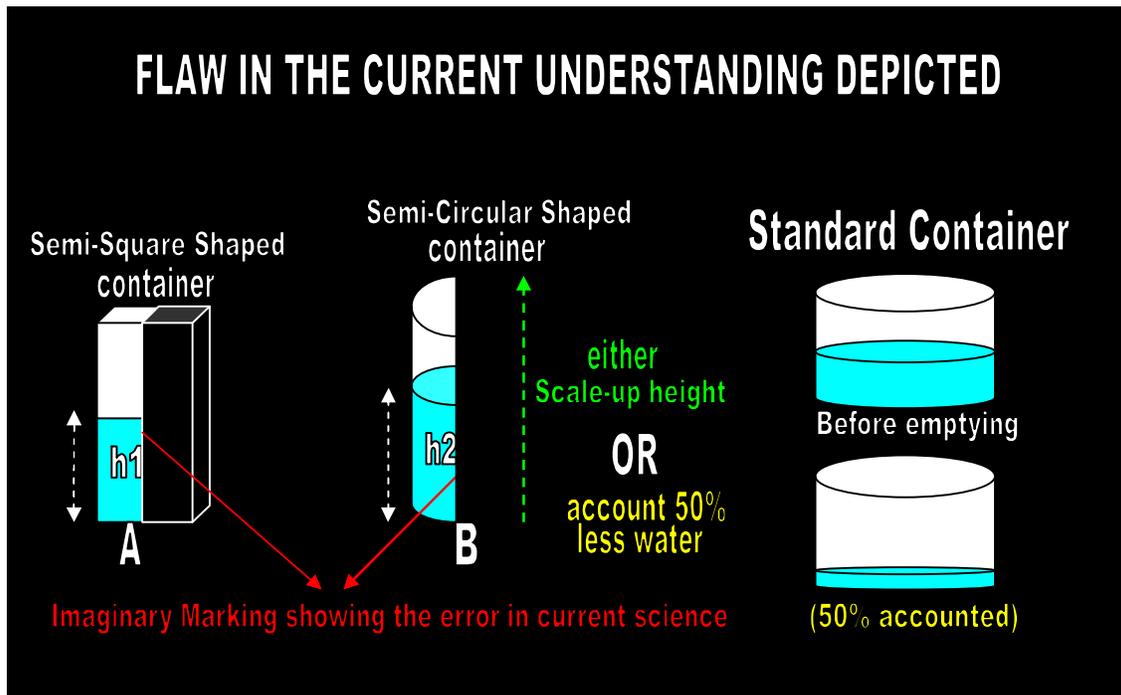
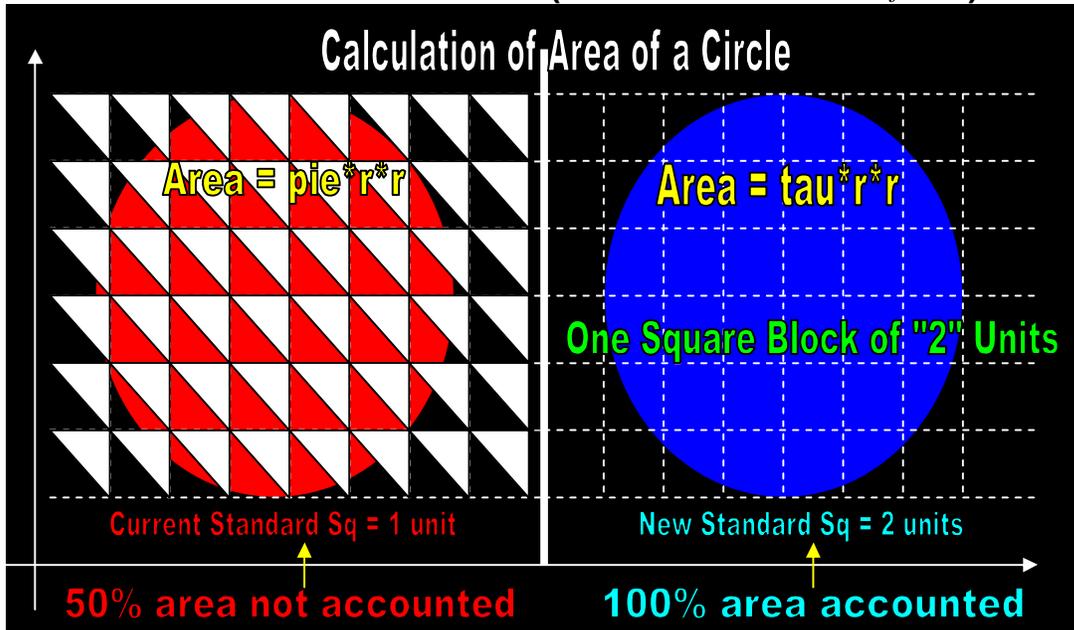


Fig (g7) Distorted geometry currently in use is shown diagrammatically

Please kindly note that the rough diagrams are used for our study as is depicted above and are thus given to demonstrate the invalidity of the current laws in geometry and to uphold the validity of my new theory on circle. This study convincingly proves the necessity for a change in our science philosophy. My theory gives the complete picture of our physical world. And also it sheds light on the missing aspect in our science which has been largely unexplored.

<This concludes my theory on circle>.

**SPECIMEN PRODUCED BELOW (Current Vs New Standard of units)**



**EXAMPLE for PYTHAGORAS IN-EQUALITY:-**

Let's consider the simplest equation,  $4^2 + 3^2 = 5^2$  (but this is Incorrect)

From the new definition given for "Square" and "Rectangle", we can say that

The above is a right  $\Delta^e$  formed by the lengths '4' and '3'; but we know that,

Area of right  $\Delta^e$  of varied side lengths added to its symmetrical pair forms a rectangle,

Mathematically, expressed as  $ab + ab = 2ab$ ; Put  $a = 4$  and  $b = 3$

We get,  $4 * 3 + 4 * 3 = 2 * 4 * 3 = 24$ ; which is the area of the rectangle

Take  $\sqrt{24} = 4.89 \neq 5$  (Disproved)

However, it holds true for a right triangle of equal side lengths and this is purely a mathematical coincidence and therefore can not be treated as a universal law.

**NOTE 5:** This manuscript clearly proves that the factor "two" needs to be properly accounted in whole of science. I would like to list out few important things here as a result of my new research findings. All Physical Constants in science needs to be doubled, Areas and Volumes of all geometrical figures changes, all the trigonometric ratios and values changes, calculus changes, Pythagoras theorem is proved to be an inequality and the list continues..... Therefore, the necessity to re-write our science.

## 9. CONCLUSION

This paper convincingly proves the invalidity of the " $\pi$ " constant. It also invalidates the current expression for the area of a circle. The percentage unaccounted is by a factor of two. It correctly determines the circle constant which is double of the existing " $\pi$ " constant. It also gives the correct area of a circle which is also the double of the existing value. It discovered the fundamental geometrical ratio. It also discovered a new formula relating all the three basic properties of a circle. At unit radius, the area and circumference equals the fundamental circle constant. And also at present, by underestimating the fundamental circle constant by fifty percent one was over estimating the unit circle by the same amount which in simple mathematical terms means that one was accounting fifty percent more of the dimensions of a unit circle unnecessarily. This means any science law involving " $\pi$ " represented 50% error, therefore the simple inference which one can make is that, any multiples of this constant value is still in error, with the percentage of it being increased. Therefore, the error percentage in Science according to me is  $\{Lower\ Limit = 50\%,\ Upper\ Limit = No\ Idea\}$ . Therefore, it becomes necessary to make the corrections to all laws wherever applicable in the whole of science where circle or the constant " $\pi$ " is involved and otherwise. This paper removes any inequalities (*Importantly Pythagoras theorem, Fermat's Last Theorem, Beal's Conjecture, etc*) & others that may exist in science and it would definitely pave the way for science to be simplified and geometrically consistent. The novel approach adopted in this paper is very simple but at the same time very effective in giving conclusive proofs to any given situation. Therefore, this methodology can be adopted for better mathematical studies and analysis. Current science made a fatal mistake of circumscribing a square where as it actually had to inscribe a circle in a square. This paper certainly has far reaching consequences not only to mathematics but also to all other branches of science. *In the end, " $\pi$ " is put to rest.* And a new beginning should be made with the correct "*fundamental circle constant " $\tau$ "* and correct "*area of the circle*" in our science. *Hence, I conclude my little mathematical philosophy here.*

## ACKNOWLEDGEMENTS

I offer my sincere thanks to all those great mathematicians of our times who have inspired people like me to explore this world. It's their works which has inspired me to do research and as a result, I have comprehended the ultimate. I dedicate this work to all of them. A special note of thanks for Prof. Bob Palais & Michael Hartl for bringing the problem in public domain. And, I also offer my sincere thanks to *Mother Nature*, the true inspiration in my life. If I have succeeded in comprehending anything new, it's because of her help and guidance.

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**ANNEXURE 2**

