On the Divergence of the Negative Energy Density Equation in both Alcubierre and Natario Warp Drive Spacetimes: No Divergence At All

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Abstract

Warp Drives are solutions of the Einstein Field Equations that allows superluminal travel within the framework of General Relativity. There are at the present moment two known solutions: The Alcubierre warp drive discovered in 1994 and the Natario warp drive discovered in 2001. However as stated by both Alcubierre and Natario themselves the warp drive violates all the known energy conditions because the stress energy momentum tensor is negative implying in a negative energy density. While from a classical point of view the negative energy is forbidden the Quantum Field Theory allows the existence of very small amounts of it being the Casimir effect a good example as stated by Alcubierre himself.The major drawback concerning negative energies for the warp drive is the huge amount of negative energy able to sustain the warp bubble.In order to perform interstellar space travel to "nearby" stars at 20 light-years away with potential habitable exo-planets(eg:Gliese 581) at superluminal speeds in a reasonable amount of time a ship must attain a speed of about 200 times faster than light.However the negative energy density at such a speed is directly proportional to the factor 10^{48} which is 1.000.000.000.000.000.000.000.000 times bigger in magnitude than the mass of the planet Earth!!! Some years ago Hiscock published a work in which the composed mixed tensor $\langle T_{\mu}{}^{\nu} \rangle$ obtained from the negative energy density tensor $T_{\mu\nu}$ $\mu = 0, \nu = 0$ of the two-dimensional Alcubierre warp drive metric diverges when the velocity of the ship vs exceeds the speed of light.(see pg 2 in [4]).We demonstrate in this work that in fact this do not happens and the Hiscock result must be re-examined.We introduce here a shape function that defines the Natario warp drive spacetime as an excellent candidate to low the negative energy density requirements from 10^{48} to affordable levels.We also discuss Horizons and Doppler Blueshifts that affects the Alcubierre spacetime but not the Natario counterpart.

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1 Introduction

The Warp Drive as a solution of the Einstein Field Equations of General Relativity that allows superluminal travel appeared first in 1994 due to the work of Alcubierre.([1]) The warp drive as conceived by Alcubierre worked with an expansion of the spacetime behind an object and contraction of the spacetime in front.The departure point is being moved away from the object and the destination point is being moved closer to the object.The object do not moves at all[1].It remains at the rest inside the so called warp bubble but an external observer would see the object passing by him at superluminal speeds(pg 8 in [1])(pg 1 in [2]).

Later on in 2001 another warp drive appeared due to the work of Natario.([2]).This do not expands or contracts spacetime but deals with the spacetime as a "strain" tensor of Fluid Mechanics(pg 5 in [2]). Imagine the object being a fish inside an aquarium and the aquarium is floating in the surface of a river but carried out by the river stream.The warp bubble in this case is the aquarium whose walls do not expand or contract. An observer in the margin of the river would see the aquarium passing by him at a large speed but inside the aquarium the fish is at the rest with respect to his local neighborhoods.

However the major drawback that affects the warp drive is the quest of large negative energy requirements enough to sustain the warp bubble.While from a classical point of view negative energy densities are forbidden the Quantum Field Theory allows the existence of very small quantities of such energies but unfortunately the warp drive requires immense amounts of it.Ford and Pfenning computed the negative energy density needed to maintain a warp bubble and they arrived at the conclusion that in order to sustain a stable configuration able to perform interstellar travel the amount of negative energy density is of about 10 times the mass of the Universe and they concluded that the warp drive is impossible.(see pg 10 in [3] and pg 78 in [5]).

Another drawback that affects the warp drive is the quest of the interstellar navigation:Interstellar space is not empty and from a real point of view a ship at superluminal speeds would impact asteroids,comets,interstellar space dust and photons of Cosmic Background Radiation(COBE).

According to Clark,Hiscock and Larson a single collision between a ship and a COBE photon would release an amount of energy equal to the photosphere of a star like the Sun.(see pg 11 in [9]).And how many photons of COBE we have per cubic centimeter of space??

These highly energetic collisions would pose a very serious threat to the astronauts as pointed out by McMonigal,Lewis and O'Byrne (see pg 10 in [10]).

Another problem:these highly energetic collisions would raise the temperature of the warp bubble reaching the Hawking temperature as pointed out by Barcelo,Finazzi and Liberati.(see pg 6 in in [11]).At pg 9 they postulate that all future spaceships cannot bypass 99 percent of the light speed.

In section 4 we will see that these problems of interstellar navigation affects the Alcubierre warp drive but not the Natario one.

[1] do not violates Relativity

The last drawback raised against the warp drive is the fact that inside the warp bubble an astronaut cannot send signals with the speed of the light to control the front of the bubble because an Horizon(causally disconnected portion of spacetime)is established between the astronaut and the warp bubble.We discuss this in section 4 and in section 5 we discuss a possible way to overcome the Horizon problem using only General Relativity.

Some years ago Hiscock published a work in which the composed mixed tensor $\langle T_{\mu}{}^{\nu}\rangle$ obtained from the negative energy density tensor $T_{\mu\nu}$ $\mu = 0, \nu = 0$ of the two-dimensional Alcubierre warp drive metric diverges when the velocity of the ship vs exceeds the speed of light.(see pg 2 in [4]).We demonstrate in this work that in fact this do not happens and the Hiscock result must be re-examined.

In this work we introduce a shape function that defines the Natario spacetime as an excellent candidate to lower the negative energy density requirements to arbitrary low levels.

We adopted the International System of Units where $G = 6,67 \times 10^{-11}$ $\frac{Newton \times meters^2}{kilograms^2}$ and $c = 3 \times 10^8$ $\frac{meters}{seconds}$ for negative energy density purposes and the Geometrized System of units in which $c = G = 1$ for geometrical purposes.

We consider here a Natario warp drive with a radius $R = 100$ meters a thickness parameter with the value $@ = 50000$ moving with a speed 200 times faster than light implying in a $vs = 2 \times 10^2 \times 3 \times 10^8 = 6 \times 10^{10}$ and a $vs^2 = 3,6 \times 10^{21}$

We also adopt a warp factor as a dimensionless parameter in our Natario shape function with a value $WF = 200$

This work is organized as follows:

- Section 2)-Outlines the problems of the immense magnitude in negative energy density when a ship travels with a speed of 200 times faster than light. The negative energy density for such a speed is directly proportional to the factor 10^{48} which is 1.000.000.000.000.000.000.000.000 times bigger in magnitude than the mass of the planet Earth!!!..
- Section 3)-The most important section in this work.According to Hiscock the composed mixed tensor $\langle T_{\mu}{}^{\nu}\rangle$ obtained from the negative energy density tensor $T_{\mu\nu}$ $\mu = 0, \nu = 0$ of the two-dimensional Alcubierre warp drive metric diverges when the velocity of the ship vs exceeds the speed of light.(see pg 2 in [4]).We demonstrate that in fact this do not happens and the Hiscock result must be re-examined. Also we introduces a shape functions that defines the Natario warp drive spacetime being this function an excellent candidate to lower the energy density requirements in the Natario warp drive to affordable levels completely obliterating the factor 10^{48} which is 1.000.000.000.000.000.000.000.000 times bigger in magnitude than the mass of the planet Earth!!!..
- Section 4)-Outlines the major advantages of the Natario warp drive spacetime when compared to its Alcubierre counterpart.The Natario warp drive can survive to the Horizons and Doppler Blueshift problem.It can also survive against the objections raised by Clark,Hiscock,Larson, McMonigal,Lewis,O'Byrne,Barcelo,Finazzi and Liberati
- Section 5)-Outlines the possibility of how to overcome the Horizon problem from an original point of view of General Relativity.

2 The Problem of the Negative Energy in the Natario Warp Drive Spacetime-The Unphysical Nature of Warp Drive

The negative energy density for the Natario warp drive is given by(see pg 5 in [2])

$$\rho = T_{\mu\nu} u^{\mu} u^{\nu} = -\frac{1}{16\pi} K_{ij} K^{ij} = -\frac{v_s^2}{8\pi} \left[3(n'(rs))^2 \cos^2\theta + \left(n'(rs) + \frac{r}{2} n''(rs) \right)^2 \sin^2\theta \right] \tag{1}$$

Converting from the Geometrized System of Units to the International System we should expect for the following expression:

$$\rho = -\frac{c^2}{G} \frac{vs^2}{8\pi} \left[3(n'(rs))^2 \cos^2\theta + \left(n'(rs) + \frac{rs}{2} n''(rs) \right)^2 \sin^2\theta \right]. \tag{2}$$

Rewriting the Natario negative energy density in cartezian coordinates we should expect for[2]:

$$\rho = T_{\mu\nu} u^{\mu} u^{\nu} = -\frac{c^2}{G} \frac{v_s^2}{8\pi} \left[3(n'(rs))^2 (\frac{x}{rs})^2 + \left(n'(rs) + \frac{r}{2} n''(rs) \right)^2 (\frac{y}{rs})^2 \right] \tag{3}$$

In the equatorial plane:

$$\rho = T_{\mu\nu} u^{\mu} u^{\nu} = -\frac{c^2}{G} \frac{v_s^2}{8\pi} \left[3(n'(rs))^2 \right] \tag{4}$$

Note that in the above expressions the warp drive speed vs appears raised to a power of 2. Considering our Natario warp drive moving with $vs = 200$ which means to say 200 times light speed in order to make a round trip from Earth to a nearby star at 20 light-years away in a reasonable amount of time(in months not in years) we would get in the expression of the negative energy the factor $c^2 = (3 \times 10^8)^2 = 9 \times 10^{16}$ being divided by $6,67 \times 10^{-11}$ giving $1,35 \times 10^{27}$ and this is multiplied by $(6 \times 10^{10})^2 = 36 \times 10^{20}$ coming from the term $vs = 200$ giving $1,35 \times 10^{27} \times 36 \times 10^{20} = 1,35 \times 10^{27} \times 3,6 \times 10^{21} = 4,86 \times 10^{48}$!!!

A number with 48 zeros!!!Our Earth have a mass[3] of about $6 \times 10^{24} kg$

This term is 1.000.000.000.000.000.000.000.000 times bigger in magnitude than the mass of the planet Earth!!!or better:The amount of negative energy density needed to sustain a warp bubble at a speed of 200 times faster than light requires the magnitude of the masses of 1.000.000.000.000.000.000.000.000 planet Earths for both Alcubierre and Natario cases!!!!

And multiplying the mass of Earth by c^2 in order to get the total positive energy "stored" in the Earth according to the Einstein equation $E = mc^2$ we would find the value of $54 \times 10^{40} = 5,4 \times 10^{41} Joules$.

Earth have a positive energy of $10^{41} Joules$ and dividing this by the volume of the Earth(radius $R_{Earth} =$ 6300 km approximately) we would find the positive energy density of the Earth.Taking the cube of the Earth radius $(6300000m = 6,3 \times 10^6)^3 = 2,5 \times 10^{20}$ and dividing $5,4 \times 10^{41}$ by $(4/3)\pi R_{Earth}^3$ we would find the value of $4,77 \times 10^{20} \frac{Joules}{m^3}$. So Earth have a positive energy density of $4,77 \times 10^{20} \frac{Joules}{m^3}$ and we are talking about negative energy densities with a factor of 10^{48} for the warp drive while the quantum theory allows only microscopical amounts of negative energy density.

[2]see Appendix A
[3]see Wikipedia:The free Encyclopedia

So we would need to generate in order to maintain a warp drive with 200 times light speed the negative energy density equivalent to the positive energy density of 10^{28} Earths!!!!

A number with 28 zeros!!!.Unfortunately we must agree with the major part of the scientific community that says:"Warp Drive is impossible and unphysical!!"(eg Ford-Pfenning)(see pg 10 in [3] and pg 78 in [5]).

However looking better to the expression of the negative energy density in the equatorial plane of the Natario warp drive:

$$\rho = T_{\mu\nu} u^{\mu} u^{\nu} = -\frac{c^2}{G} \frac{v_s^2}{8\pi} \left[3(n'(rs))^2 \right] \tag{5}$$

We can see that a very low derivative and hence its square can perhaps obliterate the huge factor of 10^{48} ameliorating the negative energy requirements to sustain the warp drive.

In the next section we will introduce a shape function that defines the Natario warp drive spacetime and this function allows the reduction of the negative energy requirements from 10 times the mass of the Universe that would render the warp drive as impossible and unphysical to arbitrary low values completely obliterating the factor 10^{48} which is 1.000.000.000.000.000.000.000.000 times bigger in magnitude than the mass of the planet Earth!!!...

3 On the Divergence of the Negative Energy Density Equation in both Alcubierre and Natario Warp Drive Spacetimes: No Divergence At All

This is the most important section in this work.According to Hiscock the composed mixed tensor $\langle T_{\mu}{}^{\nu}\rangle$ obtained from the negative energy density tensor $T_{\mu\nu}$ $\mu = 0, \nu = 0$ of the two-dimensional Alcubierre warp drive metric diverges when the velocity of the ship vs exceeds the speed of light.(see pg 2 in [4]).

We will demonstrate that in fact this do not happens and the Hiscock result must be re-examined.

The Alcubierre warp drive equation in four-dimensions using signature $(+, -, -, -)$[4] is given by:(see eq 8 pg 4 in [1],eq 1 pg 3 in [3] and eq 1 pg 3 in [4])

$$ds^2 = dt^2 - [dx - vsf(rs)dt]^2 - dy^2 - dz^2 \tag{6}$$

Hiscock uses the equation of the two-dimensional version of the Alcubierre warp drive metric where $[y^2 + z^2] = 0$ implying in $[dy + dz] = 0$ (see pg 3 and 4 in [4]) given by:

$$ds^2 = dt^2 - [dx - vsf(rs)dt]^2 \tag{7}$$

In the equation above $f(rs)$ is the Alcubierre shape function.Below is presented the equation of the Alcubierre shape function and its derivative square.(see eqs 6 and 7 pg 4 in [1] or top of pg 4 in [2])[5].

$$f(rs) = \frac{1}{2}[1 - tanh[@(rs - R)] \tag{8}$$

$$f'(rs)^2 = \frac{1}{4}[\frac{@^2}{cosh^4[@(rs - R)]}] \tag{9}$$

.

$$rs = \sqrt{(x - xs)^2 + y^2 + z^2} \tag{10}$$

According with Alcubierre any function $f(rs)$ that gives 1 inside the bubble and 0 outside the bubble while being $1 > f(rs) > 0$ in the Alcubierre warped region is a valid shape function for the Alcubierre warp drive.(see eqs 6 and 7 pg 4 in [1] or top of pg 4 in [2]).

In the Alcubierre shape function xs is the center of the warp bubble where the ship resides. R is the radius of the warp bubble and @ is the Alcubierre dimensionless parameter[6] related to the thickness.According to Alcubierre these can have arbitrary values.We outline here the fact that according to pg 4 in [1] the parameter @ can have arbitrary values.

rs is the path of the so-called Eulerian observer that starts at the center of the bubble xs and ends up outside the warp bubble.

[4] Alcubierre,Ford-Pfenning and Hiscock used the signature $(-, +, +, +)$.This comment is meant for introductory readers

[5] $tanh[@(rs + R)] = 1, tanh(@R) = 1$ for the value of the Alcubierre thickness parameter $@ = 50000$

[6] in this work we use numerical plots of a warp bubble with a radius $R = 100$ meters and dimensionless parameter @ with the value $@ = 50000$ and not $@ = 50000$ meters

In our case due to Hiscock we consider the equatorial plane $(x \geq xs, y = 0, z = 0)$ and we have for rs the following expression[7].

$$rs = \sqrt{(x - xs)^2} \tag{11}$$

$$rs = x - xs \tag{12}$$

Now we will derive algebraically the results of Hiscock concerning Horizons(causally disconnected portions of spacetime that appears when the spaceship speed vs exceeds the speed of light).These results algebraic tedious for an experienced reader(but very useful for introductory readers) could be present in an appendix[8] but these results are so important to demonstrate that the the composed mixed tensor $\langle T_\mu{}^\nu \rangle$ do not diverge at superluminal speeds and due to this we decided to include these algebraic derivations in the main text.

Back again to the two-dimensional equation of the Alcubierre warp drive:

$$ds^2 = dt^2 - [dx - vsf(rs)dt]^2 \tag{13}$$

Expanding the squared term we have:

$$[dx - vsf(rs)dt]^2 = dx^2 - 2vsf(rs)dxdt + vs^2 f(rs)^2 dt^2 \tag{14}$$

Inserting these expanded terms in the main equation we get:

$$ds^2 = dt^2 - [dx^2 - 2vsf(rs)dxdt + vs^2 f(rs)^2 dt^2] \tag{15}$$

$$ds^2 = dt^2 - dx^2 + 2vsf(rs)dxdt - vs^2 f(rs)^2 dt^2] \tag{16}$$

Rearranging the common algebraic terms we get the two-dimensional Alcubierre warp drive equation in its expanded form in signature $(+, -, -, -)$ as given below:(see eq 7 pg 4 in [4])

$$ds^2 = [1 - vs^2 f(rs)^2]dt^2 + 2vsf(rs)dxdt - dx^2 \tag{17}$$

The term responsible for the Horizon is the term g_{00} given by:

$$g_{00} = [1 - vs^2 f(rs)^2] \tag{18}$$

An Horizon occurs every time $g_{00} = 0$ when the Alcubierre shape function reaches the values given below:

$$g_{00} = 0 \dashrightarrow [1 - vs^2 f(rs)^2] = 0 \dashrightarrow vs^2 f(rs)^2 = 1 \dashrightarrow vsf(rs) = 1 \tag{19}$$

$$g_{00} = 0 \dashrightarrow f(rs) = \frac{1}{vs} \dashrightarrow 1 > f(rs) > 0 \dashrightarrow vs > 1 \tag{20}$$

[7]see Appendix A for the Natario warp drive

[8]as we did with the Natario warp drive equation in Appendix B

The non-diagonalized spacetime metric components of the Alcubierre metric in two-dimensions are given by:

$$g_{01} = g_{10} = vsf(rs) \tag{21}$$

These terms will be useful later to demonstrate that the the composed mixed tensor $\langle T_\mu{}^\nu \rangle$ do not diverge

Alcubierre wrote the warp drive equations in the so-called "remote frame" which means to say a frame associated to a stationary observer at the rest with respect to the Universe watching the warp bubble passing by him with a speed vs.

Would be nice to write the same warp drive equations from the point of view of the so-called "co-moving coordinates frame" which means to say a frame associated to the astronaut at the rest inside the bubble.This frame of reference is the so-called "ship frame".

The relation between the remote frame dx and the ship frame dx' is given by the following coordinates transformations that allows ourselves to pass equations from one frame to another(and vice-versa):(see pg 4 in [4])[9]

$$dx' = dx - vsdt \tag{22}$$

$$dx = dx' + vsdt \tag{23}$$

Writing the two-dimensional Alcubierre warp drive equation in the ship frame we get:

$$ds^2 = dt^2 - [dx' + vsdt - vsf(rs)dt]^2 \tag{24}$$

$$ds^2 = dt^2 - [dx' + (1 - f(rs))vsdt]^2 \tag{25}$$

Note that $1 - f(rs)$ is the Alcubierre shape function written in ship frame coordinates.While $f(rs) = 1$ inside the bubble and $f(rs) = 0$ outside the bubble in the remote frame coordinates,the shape function in ship frame coordinates gives results exactly opposite:0 inside the bubble and 1 outside the bubble.The relation between the Alcubierre shape function in remote frame coordinates $f(rs)$ and its ship frame counterpart $g(rs)$ is given by the following expressions:

$$g(rs) = 1 - f(rs) \tag{26}$$

$$f(rs) = 1 - g(rs) \tag{27}$$

These expressions will be useful later to pass negative energy density equations from one frame to another(and vice-versa).

[9] dr is the ship frame coordinate equivalent to our dx'.This is meant to introductory readers

Rewriting the two-dimensional Alcubierre warp drive equation in the ship frame coordinates using the ship frame shape function we get:

$$ds^2 = dt^2 - [dx' + vsg(rs)dt]^2 \tag{28}$$

Expanding the squared term we have:

$$[dx' + vsg(rs)dt]^2 = (dx')^2 + 2vsg(rs)dx'dt + vs^2g(rs)^2dt^2 \tag{29}$$

Inserting these expanded terms in the main equation we get:

$$ds^2 = dt^2 - [(dx')^2 + 2vsg(rs)dx'dt + vs^2g(rs)^2dt^2] \tag{30}$$

$$ds^2 = dt^2 - (dx')^2 - 2vsg(rs)dx'dt - vs^2g(rs)^2dt^2 \tag{31}$$

Rearranging the common algebraic terms we get the two-dimensional Alcubierre warp drive equation in its expanded form in signature $(+,-,-,-)$ written in the ship frame coordinates system:

$$ds^2 = [1 - vs^2g(rs)^2]dt^2 - 2vsg(rs)dx'dt - (dx')^2 \tag{32}$$

The term responsible for the Horizon is the term g_{00} given by:

$$g_{00} = [1 - vs^2g(rs)^2] \tag{33}$$

Since $g(rs) = 1 - f(rs)$ the term g_{00} now becomes:(please see eq 10 pg 4 in [4]).

$$g_{00} = [1 - vs^2(1 - f(rs))^2] \tag{34}$$

An Horizon occurs every time $g_{00} = 0$ when the Alcubierre shape function reaches the values given below:

$$g_{00} = 0 \dashrightarrow [1 - vs^2(1 - f(rs))^2] = 0 \dashrightarrow 1 = vs^2(1 - f(rs))^2 \tag{35}$$

$$1 = vs^2(1 - f(rs))^2 \dashrightarrow (1 - f(rs))^2 = \frac{1}{vs^2} \dashrightarrow 1 - f(rs) = \frac{1}{vs} \tag{36}$$

$$1 - f(rs) = \frac{1}{vs} \dashrightarrow f(rs) = 1 - \frac{1}{vs} \tag{37}$$

Please compare the result given above with eq 15 pg 5 in [4]).

The non-diagonalized spacetime metric components of the ship frame Alcubierre metric in two-dimensions are given by:

$$g_{01} = g_{10} = -vsg(rs) \tag{38}$$

Now we are ready to demonstrate that the composed mixed tensor $\langle T_{\mu}{}^{\nu} \rangle$ obtained from the negative energy density tensor $T_{\mu\nu}$ $\mu = 0, \nu = 0$ of the two-dimensional Alcubierre warp drive metric do not diverges when the velocity of the ship vs exceeds the speed of light and the results obtained by Hiscock needs to be re-examined or re.evaluated.(see pg 2 in [4]).

We will use the negative energy density equations.

The expression for the negative energy density in the Alcubierre warp drive spacetime wether in the remote frame or the ship frame are given by:(see eq 8 pg 6 in [3],eq 19 pg 8 and pg 5 in [1],pg 4 eq 11 in [8] and pg 4 in [2])[10][11][12]:

- Negative Energy Density in the Remote Frame)-$f(rs) = 1 - g(rs)$

$$\langle T^{\mu\nu} u_{\mu} u_{\nu} \rangle = \langle T^{00} \rangle = \frac{1}{8\pi} G^{00} = -\frac{1}{8\pi} \frac{v_s^2(t)[y^2 + z^2]}{4r_s^2(t)} \left(\frac{df(r_s)}{dr_s} \right)^2 = -\frac{1}{32\pi} \frac{v_s^2(t)[y^2 + z^2]}{r_s^2(t)} \left(\frac{df(r_s)}{dr_s} \right)^2 \tag{39}$$

- Negative Energy Density in the Ship Frame)-$g(rs) = 1 - f(rs)$

$$\langle T^{\mu\nu} u_{\mu} u_{\nu} \rangle = \langle T^{00} \rangle = \frac{1}{8\pi} G^{00} = -\frac{1}{8\pi} \frac{v_s^2(t)[y^2 + z^2]}{4r_s^2(t)} \left(\frac{dg(r_s)}{dr_s} \right)^2 = -\frac{1}{32\pi} \frac{v_s^2(t)[y^2 + z^2]}{r_s^2(t)} \left(\frac{dg(r_s)}{dr_s} \right)^2 \tag{40}$$

According to eq 12 pg 4 in [8] the contravariant components of the negative energy density tensor are equivalent to its covariant counterparts.Then we have the following result:

$$T^{00} = T^{\mu\nu}\ u_{\mu}\ u_{\nu} = T_{00} = T_{\mu\nu}\ u^{\mu}\ u^{\nu} \dashrightarrow \mu = 0, \nu = 0 \tag{41}$$

So in resume we have:

$$T^{00} = T_{00} \tag{42}$$

[10] $f(rs)$ is the Alcubierre shape function.Equation written in the Geometrized System of Units $c = G = 1$

[11] Alcubierre defined $\rho = \sqrt{y^2 + z^2}$ see pg 5 in [1] while other authors(eg Natario) uses $\rho = T_{\mu\nu} u^{\mu} u^{\nu}$ see pg 3 in [2].When reading works from different authors a conversion of notations must be made.This comment is meant for introductory readers

[12] in pg 4 eq 11 in [8] Lobo and Visser uses the z as the axis of motion while Alcubierre,Natario and Ford-Pfenning uses the x as the axis of motion.When reading Lobo-Visser equations a conversion to the x axis must be made.This comment is also meant for introductory readers

Hiscock worked the Alcubierre warp drive equations in two dimensions with $y^2 + z^2 = 0$ and $dy + dz = 0$.(see pg 3 and 4 in [4]).

But according to the equations of Alcubierre,Natario,Ford-Pfenning,Lobo and Visser a two dimensional Alcubierre warp drive wether in the remote frame or the ship frame have a zero negative energy density given by:(see eq 8 pg 6 in [3],eq 19 pg 8 and pg 5 in [1],pg 4 eq 11 in [8] and pg 4 in [2])[13]

$$T^{00} = T_{00} = -\frac{1}{32\pi}\frac{v_s^2(t)[y^2+z^2]}{r_s^2(t)}\left(\frac{df(r_s)}{dr_s}\right)^2 = 0 \dashrightarrow [y^2+z^2] = 0 \tag{43}$$

$$T^{00} = T_{00} = -\frac{1}{32\pi}\frac{v_s^2(t)[y^2+z^2]}{r_s^2(t)}\left(\frac{dg(r_s)}{dr_s}\right)^2 = 0 \dashrightarrow [y^2+z^2] = 0 \tag{44}$$

So we are left with:

$$T^{00} = T_{00} = 0 \tag{45}$$

Tensors with the same form of the composed mixed tensor $\langle T_\mu{}^\nu \rangle$ obtained from the negative energy density tensor $T_{\mu\nu}$ of the two-dimensional Alcubierre warp drive metric that Hiscock claims to diverge when the velocity of the ship vs exceeds the speed of light(see pg 2 in [4]) are easily obtained applying the basic rules of Tensor Calculus as shown below:(see Carroll eqs 1.61 and 1.62 pg 24 in [17]).

$$\langle T_\mu{}^\nu \rangle = \langle T_{\mu\nu} g^{\nu\nu} \rangle = 0 \dashrightarrow \mu = 0, \nu = 0 \dashrightarrow \langle T_{00} g^{00} \rangle = 0 \dashrightarrow \langle T_{00} \rangle = 0 \tag{46}$$

$$\langle T_\mu{}^\nu \rangle = \langle T_{\mu\mu} g^{\mu\nu} \rangle = 0 \dashrightarrow \mu = 0, \nu = 1 \dashrightarrow \langle T_{00} g^{01} \rangle = 0 \dashrightarrow \langle T_{00} \rangle = 0 \tag{47}$$

$$\langle T^\mu{}_\nu \rangle = \langle T^{\mu\nu} g_{\nu\nu} \rangle = 0 \dashrightarrow \mu = 0, \nu = 0 \dashrightarrow \langle T^{00} g_{00} \rangle = 0 \dashrightarrow \langle T^{00} \rangle = 0 \tag{48}$$

$$\langle T^\mu{}_\nu \rangle = \langle T^{\mu\mu} g_{\mu\nu} \rangle = 0 \dashrightarrow \mu = 0, \nu = 1 \dashrightarrow \langle T^{00} g_{01} \rangle = 0 \dashrightarrow \langle T^{00} \rangle = 0 \tag{49}$$

Independently of the speed of the spaceship vs exceeds the speed of light or not or independently of the Horizon spacetime metric tensor components g_{00} or the non-diagonalized spacetime metric tensor components g_{01} mixed tensors of the forms $\langle T_\mu{}^\nu \rangle$ and $\langle T^\mu{}_\nu \rangle$ are impossible to diverge at least for the Alcubierre warp drive in two dimensions because in this case the negative energy density is simply:zero!!!!

The results obtained by Hiscock for the divergence of the mixed tensor $\langle T_\mu{}^\nu \rangle$ needs to be re-examined or re.evaluated.(see pg 2 in [4]).

[13]in pg 4 eq 11 in [8] Lobo and Visser uses the z as the axis of motion while Alcubierre,Natario and Ford-Pfenning uses the x as the axis of motion.When reading Lobo-Visser equations a conversion to the x axis must be made.This comment is also meant for introductory readers

Now we are ready to compare negative energy density requirements between both Alcubierre and Natario warp drive spacetimes:

The four-dimensional Natario warp drive equation is given by the following expression in ship frame coordinates with signature $(+,-,-,-)$(see Appendix B):

$$ds^2 = [1 - (X^{rs})^2 - (X^\theta)^2]dt^2 + 2[X^{rs}drs + X^\theta rsd\theta]dt - drs^2 - rs^2 d\theta^2 \tag{50}$$

The expressions for X^{rs} and X^θ are given by:

$$X^{rs} = 2v_s n(rs)\cos\theta \tag{51}$$

$$X^\theta = -v_s(2n(rs) + (rs)n'(rs))\sin\theta \tag{52}$$

Like in the case of Alcubierre we are interested in the two-dimensional Natario warp drive given by:

$$ds^2 = [1 - (X^{rs})^2]dt^2 + 2X^{rs}drsdt - drs^2 \tag{53}$$

$$X^{rs} = 2v_s n(rs) \tag{54}$$

According to Natario(pg 5 in [2]) any function that gives 0 inside the bubble and $\frac{1}{2}$ outside the bubble while being $0 < n(rs) < \frac{1}{2}$ in the Natario warped region is a valid shape function for the Natario warp drive.

A Natario warp drive valid shape function can be given by:

$$n(rs) = [\frac{1}{2}][1 - f(rs)^{WF}]^{WF} \tag{55}$$

Its derivative square is :

$$n'(rs)^2 = [\frac{1}{4}]WF^4[1 - f(rs)^{WF}]^{2(WF-1)}[f(rs)^{2(WF-1)}]f'(rs)^2 \tag{56}$$

The shape function above gives the result of $n(rs) = 0$ inside the warp bubble and $n(rs) = \frac{1}{2}$ outside the warp bubble while being $0 < n(rs) < \frac{1}{2}$ in the Natario warped region(see pg 5 in [2]).

Note that the Alcubierre shape function is being used to define its Natario shape function counterpart. The term WF in the Natario shape function is dimensionless too:it is the warp factor that will squeeze the region where the derivatives of the Natario shape function are different than 0.

For the Natario shape function introduced above it is easy to figure out when $f(rs) = 1$(interior of the Alcubierre bubble) then $n(rs) = 0$(interior of the Natario bubble) and when $f(rs) = 0$(exterior of the Alcubierre bubble)then $n(rs) = \frac{1}{2}$(exterior of the Natario bubble).

We consider here a Natario warp drive with a radius $R = 100$ meters a thickness parameter with the value $@ = 50000$ and a warp factor with a value $WF = 200$.

We provide a numerical plot of the values of the derivative squares of both Alcubierre and Natario shape functions for two warp drives in four dimensions.

rs	$f(rs)$	$n(rs)$	$f'(rs)^2$	$n'(rs)^2$
$9,99970000000E+001$	1	0	$2,650396620740E-251$	0
$9,99980000000E+001$	1	0	$1,915169647489E-164$	0
$9,99990000000E+001$	1	0	$1,383896564748E-077$	0
$1,00000000000E+002$	0,5	0,5	$6,250000000000E+008$	$3,872591914849E-103$
$1,00001000000E+002$	0	0,5	$1,383896486082E-077$	0
$1,00002000000E+002$	0	0,5	$1,915169538624E-164$	0
$1,00003000000E+002$	0	0,5	$2,650396470082E-251$	0

Above are the numerical plots for both an Alcubierre and a Natario warp drive spacetimes with a $R=100$ meters bubble radius and a $@=50000$ and $WF=200$ when the Eulerian observer rs initially at the center of the bubble($rs=0$) approaches the end of the bubble ($rs \simeq R$)[14].Inside the bubble($rs<R$) $f(rs)=1$ and $n(rs)=0$ and outside the bubble($rs>R$) $f(rs)=0$ and $n(rs)=0,5$ according to both Alcubierre and Natario shape functions requirements.

Note that when $rs=9,99970000000E+001$ still inside the bubble $f(rs)=1$ but the derivative square is not zero and when $rs=1,00003000000E+002$ already outside the bubble $f(rs)=0$ but the derivative square is also not zero. When $rs<9,99970000000E+001$ or $rs>1,00003000000E+002$ the derivatives of $f(rs)=0$.Since the negative energy density in both Alcubierre and Natario warp drive spacetime is directly proportional to the derivative square of the shape functions we are interested in the region where the derivative squares are not zero.The expressions for the negative energy density in both Alcubierre and Natario warp drive spacetimes are given by:(see eq 8 pg 6 in [3] and pg 5 in [2])[15]:

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{c^2}{G}\frac{1}{8\pi}\frac{v_s^2(t)[y^2+z^2]}{4r_s^2(t)}\left(\frac{df(r_s)}{dr_s}\right)^2 \tag{57}$$

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{c^2}{G}\frac{v_s^2}{8\pi}\left[3(n'(rs))^2(\frac{x}{rs})^2 + \left(n'(rs)+\frac{r}{2}n''(rs)\right)^2(\frac{y}{rs})^2\right] \tag{58}$$

Note that when the derivative square of the Alcubierre shape function is zero(flat spacetime) ($f'(rs)^2=0$) then the derivative square of the Natario shape function is also zero too .

Examining the plot above we can see that both warp drives wether in Alcubierre or Natario case have two warped regions:

- 1)-The warped region where $1>f(rs)>0$ or $0<n(rs)<\frac{1}{2}$ according with both Alcubierre and Natario requirements.-This warped region is known as the Geometrized warped region.
- 2)-The warped region where the derivative squares of both Alcubierre and Natario shape functions are not zero resulting in a non-null negative energy density and in a non-flat spacetime.-This warped region is known as the Energized warped region

[14]the surface of the bubble is delimited by its radius.so the bubble terminates when $rs=R$

[15]Equation written in Cartesian Coordinates.See Appendix A

Note that in the Natario case the Energized warped region is distributed in a very thin layer over the neighborhoods of the bubble radius starting in a region very close to the bubble radius
$(rs <= R, rs \cong R, R - rs \cong 0)$ and terminating also in a region very close to the bubble radius
$(rs >= R, rs \cong R, rs - R \cong 0)$

Calling the point where the Energized warped region begins as point a and the point where the Energized warped region ends as point b then the point b lies almost infinitely closed to the point a resulting in an Energized warped region of almost infinite small thickness.

$(rs <= R, rs \cong R, R - rs \cong 0, R - rs = a, a \cong 0)$
$(rs >= R, rs \cong R, rs - R \cong 0, rs - R = b, b \cong 0)$
$(a \cong 0, b \cong 0, b >= a, b \cong a, b - a \cong 0)$

Why this behavior that contains both the Geometrized and Energized warped regions in the same boundary limits a and b occurs only with the Natario case and not also with its Alcubierre counterpart??

Starting with the square of the derivative of the Natario shape function:

$$n'(rs)^2 = [\frac{1}{4}]WF^4[1 - f(rs)^{WF}]^{2(WF-1)}[f(rs)^{2(WF-1)}]f'(rs)^2 \tag{59}$$

Inside the bubble $f(rs) = 1$ and $[1 - f(rs)^{WF}]^{2(WF-1)} = 0$ resulting in a $n'(rs)^2 = 0$.This is the reason why the Natario shape function dont have derivatives inside the bubble.

Outside the bubble $f(rs) = 0$ and $[f(rs)^{2(WF-1)}] = 0$ resulting also in a $n'(rs)^2 = 0$.This is the reason why the Natario shape function dont have derivatives outside the bubble.

In the Geometrized warped region for the Alcubierre warp drive $1 > f(rs) > 0$.In this region the derivatives of the Natario shape function do not vanish because if $f(rs) < 1$ then $f(rs)^{WF} << 1$ resulting in an $[1 - f(rs)^{WF}]^{2(WF-1)} << 1$ but $[1 - f(rs)^{WF}]^{2(WF-1)} > 0$.Also if $f(rs) < 1$ then $[f(rs)^{2(WF-1)}] << 1$ too but $[f(rs)^{2(WF-1)}] > 0$

Note that if $[1 - f(rs)^{WF}]^{2(WF-1)} << 1$ and $[f(rs)^{2(WF-1)}] << 1$ their product
$[1 - f(rs)^{WF}]^{2(WF-1)}[f(rs)^{2(WF-1)}] <<<< 1$

Note that inside the Alcubierre Geometrized warped region $1 > f(rs) > 0$ when $f(rs)$ approaches 1 $n'(rs)^2$ approaches 0 due to the factor $[1 - f(rs)^{WF}]^{2(WF-1)}$ and when $f(rs)$ approaches 0 $n'(rs)^2$ approaches 0 again due to the factor $[f(rs)^{2(WF-1)}]$.

The maximum value for $n'(rs)^2 \cong 10^{-103}$ occurs in the middle of the Alcubierre Geometrized warped region where $f(rs) = 0,5$.Note that this value for the square of the derivative of the Natario shape function can obliterate the factor 10^{48} resulting in an extremely low level of negative energy density.

Due to the fact that the values of $n'(rs)^2$ grows from 0 to 10^{-103} and decreases to 0 again we choose the Natario warp drive as the best candidate to low the negative energy density requirements completely obliterating the factor 10^{48} which is 1.000.000.000.000.000.000.000.000 times bigger in magnitude than the mass of the planet Earth!!!.

4 Horizon and Infinite Doppler Blueshifts in both Alcubierre and Natario Warp Drive Spacetimes

According to pg 6 in [2] warp drives suffers from the pathology of the Horizons and according to pg 8 in [2] warp drive suffer from the pathology of the infinite Doppler Blueshifts that happens when a photon sent by an Eulerian observer to the front of the warp bubble reaches the Horizon.This would render the warp drive impossible to be physically feasible.

For a complete mathematical demonstration of the Horizon and Doppler Blueshift Problems see pg 20 section 6 in [7](basic) and pg 4 section 2 in [6](advanced).The Horizon occurs in both spacetimes.This means to say that the Eulerian observer cannot signal the front of the warp bubble wether in Alcubierre or Natario warp drive because the photon sent to signal will stop in the Horizon..The solution for the Horizon problem must be postponed until the arrival of a Quantum Gravity theory that encompasses both General Relativity and Non-Local Quantum Entanglements of Quantum Mechanics however in the next section we will present a possible solution for this problem that only encompasses General Relativity.

The infinite Doppler Blueshift happens in the Alcubierre warp drive but not in the Natario one.This means to say that Alcubierre warp drive is physically impossible to be achieved but the Natario warp drive is perfectly physically possible to be achieved.

Consider again the negative energy density distribution in the Alcubierre warp drive spacetime(see eq 8 pg 6 in [3])[16][17]:

$$\langle T^{\mu\nu}u_\mu u_\nu\rangle = \langle T^{00}\rangle = \frac{1}{8\pi}G^{00} = -\frac{1}{8\pi}\frac{v_s^2(t)[y^2+z^2]}{4r_s^2(t)}\left(\frac{df(r_s)}{dr_s}\right)^2, \tag{60}$$

And considering again the negative energy density in the Natario warp drive spacetime(see pg 5 in [2])[18][19]:

$$\rho = T_{\mu\nu}u^\mu u^\nu = -\frac{1}{16\pi}K_{ij}K^{ij} = -\frac{v_s^2}{8\pi}\left[3(n'(rs))^2(\frac{x}{rs})^2 + \left(n'(rs)+\frac{r}{2}n''(rs)\right)^2(\frac{y}{rs})^2\right] \tag{61}$$

In pg 6 in [2] a warp drive with a x-axis only is considered.In this case for the Alcubierre warp drive $[y^2+z^2]=0$

$$\langle T^{\mu\nu}u_\mu u_\nu\rangle = \langle T^{00}\rangle = \frac{1}{8\pi}G^{00} = -\frac{1}{8\pi}\frac{v_s^2(t)[y^2+z^2]}{4r_s^2(t)}\left(\frac{df(r_s)}{dr_s}\right)^2, = 0 \tag{62}$$

And the negative energy density is zero but the Natario energy density is not zero and given by:.

$$\rho = T_{\mu\nu}u^\mu u^\nu = -\frac{1}{16\pi}K_{ij}K^{ij} = -\frac{v_s^2}{8\pi}\left[3(n'(rs))^2(\frac{x}{rs})^2\right] \tag{63}$$

[16] $f(rs)$ is the Alcubierre shape function.Equation written in the Geometrized System of Units $c=G=1$

[17] Equation written in Cartezian Coordinates

[18] $n(rs)$ is the Natario shape function.Equation written the Geometrized System of Units $c=G=1$

[19] Equation written in Cartezian Coordinates.See Appendix A

Note that in front of the ship in the Alcubierre case the spacetime is empty but in the Natario case there exists negative energy density in the front of the ship.

According with Natario in pg 7 before section 5.2 in [14] negative energy density means a negative mass density and a negative mass generates a repulsive gravitational field. This repulsive gravitational field in front of the ship in the Natario warp drive spacetime protects the ship from impacts with the interstellar matter.The objections raised by Clark,Hiscock,Larson,McMonigal,Lewis,O'Byrne,Barcelo,Finazzi and Liberati in the introduction of this work are not valid for the Natario warp drive spacetime.

Since the Alcubierre warp drive dont have negative energy in front of the ship but only empty spacetime it doest not have protection against the interstellar medium making valid the objections raised by Clark,Hiscock,Larson,McMonigal,Lewis,O'Byrne,Barcelo,Finazzi and Liberati in the introduction of this work.

The Alcubierre shape function $f(rs)$ is defined as being 1 inside the warp bubble and 0 outside the warp bubble while being $1 > f(rs) > 0$ in the Alcubierre warped region according to eq 7 pg 4 in [1] or top of pg 4 in [2].

Expanding the quadratic term in eq 8 pg 4 in [1] and solving eq 8 for a null-like interval $ds^2 = 0$ we will have the following equation for the motion of the photon sent to the front(see pg 3 in [12] and pg 22 eqs 146 and 147 in [7])[20]:

$$\frac{dx}{dt} = vsf(rs) - 1 \tag{64}$$

Inside the Alcubierre warp bubble $f(rs) = 1$ and $vsf(rs) = vs$.Outside the warp bubble $f(rs) = 0$ and $vsf(rs) = 0$.

Somewhere inside the Alcubierre warped region when $f(rs)$ starts to decrease from 1 to 0 making the term $vsf(rs)$ decreases from vs to 0 and assuming a continuous behavior then in a given point $vsf(rs) = 1$ and $\frac{dx}{dt} = 0$.The photon stops,A Horizon is established and in the Horizon the Doppler Blueshift occurs rendering the Alcubierre warp drive impossible.This due to the fact that there are no negative energy density in the front of the Alcubierre warp drive in the x-axis to deflect the photon.

Now taking the components of the Natario vector defined in the top of pg 5 in [2] and inserting these components in the first equation of pg 2 in [2] and solving for the same null-like interval $ds^2 = 0$ considering only radial motion we will get the following equation for the motion of the photon sent to the front(see eqs 16 and 17 pg 5 in [6])[21]:

$$\frac{dx}{dt} = 2vsn(rs) - 1 \tag{65}$$

The Natario shape function $n(rs)$ is defined as being 0 inside the warp bubble and $\frac{1}{2}$ outside the warp bubble while being $0 < n(rs) < \frac{1}{2}$ in the Natario warped region according to pg 5 in [2].

[20] The coordinate frame for the Alcubierre warp drive as in [1] is the remote observer outside the ship

[21] The coordinate frame for the Natario warp drive as in [2] is the ship frame observer in the center of the warp bubble $xs = 0$

Inside the Natario warp bubble $n(rs) = 0$ and $2vsn(rs) = 0$.Outside the warp bubble $n(rs) = \frac{1}{2}$ and $2vsn(rs) = vs$. Somewhere inside the Natario warped region $n(rs)$ starts to increase from 0 to $\frac{1}{2}$ making the term $2vsn(rs)$ increase from 0 to vs and assuming a continuous behavior then in a given point we would have a $2vsn(rs) = 1$ and a $\frac{dx}{dt} = 0$ The photon would stops.A Horizon would be established.

However when the photon reaches the beginning of the Natario warped region it suffers a deflection by the negative energy density in front of the Natario warp drive because this negative energy is not null.So in the case of the Natario warp drive the photon never reaches the Horizon and the Natario warp drive never suffer from the pathology of the infinite Doppler Blueshift due to a different distribution of energy density when compared to its Alcubierre counterpart.This negative energy with repulsive gravitational behavior deflects the photon from inside avoiding it to reach the Horizon and protects the Natario warp drive from the dangers of collisions with the interstellar medium at superluminal speeds.

Adapted from the negative energy in Wikipedia:The free Encyclopedia:

"if we have a small object with equal inertial and passive gravitational masses falling in the gravitational field of an object with negative active gravitational mass (a small mass dropped above a negative-mass planet, say), then the acceleration of the small object is proportional to the negative active gravitational mass creating a negative gravitational field and the small object would actually accelerate away from the negative-mass object rather than towards it."

The Natario warp drive as a solution of the Einstein Field Equations of General Relativity that allows faster than light motion is the first valid candidate for interstellar space travel.

5 A Causally Connected Superluminal Natario Warp Drive Spacetime using Micro Warp Bubbles

In 2002 Gauthier,Gravel and Melanson appeared with the idea of the micro warp bubbles.([15],[16])

According to them,microscopical particle-sized warp bubbles may have formed spontaneously immediately after the Big Bang and these warp bubbles could be used to transmit information at superluminal speeds.These micro warp bubbles may exist today.(see abs of [16])

A micro warp bubble with a radius of 10^{-10} meters could be used to transport an elementary particle like the electron whose Compton wavelength is 2.43×10^{-12} meters at a superluminal speed.These micro warp bubbles may have formed when the Universe had an age between the Planck time and the time we assume that Inflation started.(see pg 306 of [15])

Following the ideas of Gauthier,Gravel and Melanson ([15],[16]) a micro warp bubble can send information or particles at superluminal speeds.(abs of [16],pg 306 in [15]).Since the infinite Doppler Blueshift affect the Alcubierre warp drive but not the Natario one and a superluminal micro warp bubble can only exists without Infinite Doppler Blueshifts[22] we consider in this section only the Natario warp drive spacetime.

The idea of Gauthier,Gravel and Melanson ([15],[16]) to send information at superluminal speeds using micro warp bubbles is very interesting and as a matter of fact shows to us how to solve the Horizon problem. Imagine that we are inside a large superluminal warp bubble and we want to send information to the front.Photons sent from inside the bubble to the front would stop in the Horizon but we also know that incoming photons from outside would reach the bubble.[23] The external observer outside the bubble have all the bubble causally connected while the internal observer is causally connected to the point before the Horizon.Then the external observer can create the bubble while the internal observer cannot. This was also outlined by Everett-Roman in pg 3 in [12]. Unless we find a way to overcome the Horizon problem. We inside the large warp bubble could create and send one of these micro warp bubbles to the front of the large warp bubble but with a superluminal speed $vs2$ larger than the large bubble speed $X = 2vsn(rs)$.Then $vs2 >> X$ or $vs2 >> 2vsn(rs)$ and this would allow ourselves to keep all the warp bubble causally connected from inside overcoming the Horizon problem without the need of the "tachyonic" matter.

- 1)- Superluminal micro warp bubble sent towards the front of the large superluminal warp bubble $vs2 = \frac{dx}{dt} > X - 1 > vs - 1 \dashrightarrow X = 2vsn(rs)$

From above it easy to see that a micro warp bubble with a superluminal speed $vs2$ maintains a large superluminal warp bubble with speed vs causally connected from inside if $vs2 > vs$

[22]assuming a continuous growth of the warp bubble speed vs from zero to a superluminal speed at a given time the speed will be equal to c and the Infinite Doppler Blueshift crashes the bubble.The Alcubierre warp drive can only exists for $vs < c$ so it cannot sustain a micro warp bubble able to shelter particles or information at superluminal speeds

[23]true for the Alcubierre warp drive but not for the Natario one because the negative energy density in the front with repulsive gravitational behavior would deflect all the photons sent from inside and outside the bubble effectively shielding the Horizon from the photon avoiding the catastrophical Infinite Doppler Blueshift

From the point of view of the astronaut inside the large warp bubble he is the internal observer with respect to the large warp bubble but he is the external observer from the point of view of the micro warp bubble so he keeps all the light-cone of the micro warp bubble causally connected to him so he can use it to send superluminal signals to the large warp bubble from inside.(Everett-Roman in pg 3 in [12]).

Gauthier,Gravel and Melanson developed the concept of the micro warp bubble but the idea is at least 5 years younger.The first time micro warp bubbles were mentioned appeared in the work of Ford-Pfenning pg 10 and 11 in [3].

According with Natario in pg 7 before section 5.2 in [14] negative energy density means a negative mass density and a negative mass generates a repulsive gravitational field that deflect photons or positive mass-density particles from the interstellar medium or particles sent to the bubble walls by the astronaut inside the bubble.

However while the negative mass deflects the positive mass or photons[24] a negative mass always attracts another negative mass so the astronaut cannot send positive particles or photons to the large warp bubble but by sending micro warp bubbles these also possesses negative masses that will be attracted by the negative mass of the large warp bubble effectively being a useful way to send signals.

[24] by negative gravitational bending of light

6 Conclusion

In this work we demonstrated that the composed mixed tensor $\langle T_\mu{}^\nu \rangle$ obtained from the negative energy density tensor $T_{\mu\nu}$ $\mu = 0, \nu = 0$ of the two-dimensional Alcubierre warp drive metric do not diverges when the velocity of the ship vs exceeds the speed of light and the results obtained by Hiscock needs to be re-examined or re.evaluated.(see pg 2 in [4]).

We also introduced a shape functions for the Natario warp drive spacetime that is very efficient lowering the negative energy densities in the Natario warp drive to affordable levels.

Also we demonstrated that the objections raised by Clark,Hiscock,Larson,McMonigal,Lewis,O'Byrne,Barcelo, Finazzi and Liberati in the introduction of this work and valid for the Alcubierre warp drive making it physically impossible independently from arbitrary lower levels of negative energy do not affect the Natario warp drive which is perfectly possible to be achieved.This was the main reason behind our interest in the derivatives of a particular form of the shape function for the Natario warp drive spacetime.

The Natario warp drive once created can survive against all the obstacles pointed as physical impossibilities that rules out the warp drive as a dynamical spacetime.

Lastly and in order to terminate this work:There exists another problem not covered here:the fact that we still dont know how to generate the negative energy density and negative mass and above everything else we dont know how to generate the shape function that distorts the spacetime geometry creating the warp drive effect.So unfortunately all the discussions about warp drives are still under the domain of the mathematical conjectures.

However we are confident to affirm that the Natario warp drive will survive the passage of the Century XXI and will arrive to the Future.The Natario warp drive as the first Human candidate for faster than light interstellar space travel will arrive to the the Century $XXIV$ on-board the future starships up there in the middle of the stars helping the human race to give his first steps in the exploration of our Galaxy

Live Long And Prosper

7 Appendix A:The Natario Warp Drive Negative Energy Density in Cartezian Coordinates

The negative energy density according to Natario is given by(see pg 5 in [2])[25]:

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{1}{16\pi}K_{ij}K^{ij} = -\frac{v_s^2}{8\pi}\left[3(n'(rs))^2\cos^2\theta + \left(n'(rs) + \frac{r}{2}n''(rs)\right)^2\sin^2\theta\right] \tag{66}$$

In the bottom of pg 4 in [2] Natario defined the x-axis as the polar axis.In the top of page 5 we can see that $x = rs\cos(\theta)$ implying in $\cos(\theta) = \frac{x}{rs}$ and in $\sin(\theta) = \frac{y}{rs}$

Rewriting the Natario negative energy density in cartezian coordinates we should expect for:

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{1}{16\pi}K_{ij}K^{ij} = -\frac{v_s^2}{8\pi}\left[3(n'(rs))^2(\frac{x}{rs})^2 + \left(n'(rs) + \frac{r}{2}n''(rs)\right)^2(\frac{y}{rs})^2\right] \tag{67}$$

Considering motion in the equatorial plane of the Natario warp bubble (x-axis only) then $[y^2 + z^2] = 0$ and $rs^2 = [(x - xs)^2]$ and making $xs = 0$ the center of the bubble as the origin of the coordinate frame for the motion of the Eulerian observer then $rs^2 = x^2$ because in the equatorial plane $y = z = 0$.

Rewriting the Natario negative energy density in cartezian coordinates in the equatorial plane we should expect for:

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{1}{16\pi}K_{ij}K^{ij} = -\frac{v_s^2}{8\pi}\left[3(n'(rs))^2\right] \tag{68}$$

[25] $n(rs)$ is the Natario shape function.Equation written in the Geometrized System of Units $c = G = 1$

8 Appendix B:Mathematical Demonstration of the Natario Warp Drive Equation using the Natario Vector nX for a constant speed vs

The warp drive spacetime according to Natario is defined by the following equation but we changed the metric signature from $(-,+,+,+)$ to $(+,-,-,-)$(pg 2 in [2])

$$ds^2 = dt^2 - \sum_{i=1}^{3}(dx^i - X^i dt)^2 \tag{69}$$

where X^i is the so-called shift vector.This shift vector is the responsible for the warp drive behavior defined as follows(pg 2 in [2]):

$$X^i = X, Y, Z \curvearrowright i = 1, 2, 3 \tag{70}$$

The warp drive spacetime is completely generated by the Natario vector nX(pg 2 in [2])

$$nX = X^i \frac{\partial}{\partial x^i} = X\frac{\partial}{\partial x} + Y\frac{\partial}{\partial y} + Z\frac{\partial}{\partial z}, \tag{71}$$

Defined using the canonical basis of the Hodge Star in spherical coordinates as follows(pg 4 in [2]):

$$e_r \equiv \frac{\partial}{\partial r} \sim dr \sim (rd\theta) \wedge (r\sin\theta d\varphi) \tag{72}$$

$$e_\theta \equiv \frac{1}{r}\frac{\partial}{\partial\theta} \sim rd\theta \sim (r\sin\theta d\varphi) \wedge dr \tag{73}$$

$$e_\varphi \equiv \frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi} \sim r\sin\theta d\varphi \sim dr \wedge (rd\theta) \tag{74}$$

Redefining the Natario vector nX as being the rate-of-strain tensor of fluid mechanics as shown below(pg 5 in [2]):

$$nX = X^r \mathbf{e}_r + X^\theta \mathbf{e}_\theta + X^\varphi \mathbf{e}_\varphi \tag{75}$$

$$nX = X^r dr + X^\theta r d\theta + X^\varphi r\sin\theta d\varphi \tag{76}$$

$$ds^2 = dt^2 - \sum_{i=1}^{3}(dx^i - X^i dt)^2 \tag{77}$$

$$X^i = r, \theta, \varphi \curvearrowright i = 1, 2, 3 \tag{78}$$

We are interested only in the coordinates r and θ according to pg 5 in [2])

$$ds^2 = dt^2 - (dr - X^r dt)^2 - (rd\theta - X^\theta dt)^2 \tag{79}$$

$$(dr - X^r dt)^2 = dr^2 - 2X^r dr dt + (X^r)^2 dt^2 \tag{80}$$

$$(rd\theta - X^{\theta}dt)^2 = r^2d\theta^2 - 2X^{\theta}rd\theta dt + (X^{\theta})^2dt^2 \tag{81}$$

$$ds^2 = dt^2 - (X^r)^2dt^2 - (X^{\theta})^2dt^2 + 2X^r drdt + 2X^{\theta}rd\theta dt - dr^2 - r^2d\theta^2 \tag{82}$$

$$ds^2 = [1 - (X^r)^2 - (X^{\theta})^2]dt^2 + 2[X^r dr + X^{\theta}rd\theta]dt - dr^2 - r^2d\theta^2 \tag{83}$$

making $r = rs$ we have the Natario warp drive equation:

$$ds^2 = [1 - (X^{rs})^2 - (X^{\theta})^2]dt^2 + 2[X^{rs}drs + X^{\theta}rsd\theta]dt - drs^2 - rs^2d\theta^2 \tag{84}$$

According with the Natario definition for the warp drive using the following statement(pg 4 in [2]):any Natario vector nX generates a warp drive spacetime if $nX = 0$ and $X = vs = 0$ for a small value of rs defined by Natario as the interior of the warp bubble and $nX = -vs(t)dx$ or $nX = vs(t)dx$ with $X = vs$ for a large value of rs defined by Natario as the exterior of the warp bubble with $vs(t)$ being the speed of the warp bubble

The expressions for X^{rs} and X^{θ} are given by:(see pg 5 in [2])

$$nX \sim -2v_s n(rs)\cos\theta \mathbf{e}_{rs} + v_s(2n(rs) + (rs)n'(rs))\sin\theta \mathbf{e}_{\theta} \tag{85}$$

$$nX \sim 2v_s n(rs)\cos\theta \mathbf{e}_{rs} - v_s(2n(rs) + (rs)n'(rs))\sin\theta \mathbf{e}_{\theta} \tag{86}$$

$$nX \sim -2v_s n(rs)\cos\theta drs + v_s(2n(rs) + (rs)n'(rs))\sin\theta rsd\theta \tag{87}$$

$$nX \sim 2v_s n(rs)\cos\theta drs - v_s(2n(rs) + (rs)n'(rs))\sin\theta rsd\theta \tag{88}$$

But we already know that the Natario vector nX is defined by(pg 2 in [2]):

$$nX = X^{rs}drs + X^{\theta}rsd\theta \tag{89}$$

Hence we should expect for:

$$X^{rs} = -2v_s n(rs)\cos\theta \tag{90}$$

$$X^{rs} = 2v_s n(rs)\cos\theta \tag{91}$$

$$X^{\theta} = v_s(2n(rs) + (rs)n'(rs))\sin\theta \tag{92}$$

$$X^{\theta} = -v_s(2n(rs) + (rs)n'(rs))\sin\theta \tag{93}$$

9 Epilogue

- "The only way of discovering the limits of the possible is to venture a little way past them into the impossible."-Arthur C.Clarke[26]
- "The supreme task of the physicist is to arrive at those universal elementary laws from which the cosmos can be built up by pure deduction. There is no logical path to these laws; only intuition, resting on sympathetic understanding of experience, can reach them"-Albert Einstein[27][28]

10 Remarks

Reference [15] was online at the time we picked it up for our records.It ceased to be online but we can provide a copy in PDF Acrobat reader of this reference for those interested.

Reference [16] we only have access to the abstract.

We performed all the numerical calculus of our simulations for both Alcubierre and Natario warp drive spacetimes using Microsoft Excel[29].We can provide our Excel files to those interested and although Excel is a licensed program there exists another program that can read Excel files available in the Internet as a freeware for those that perhaps may want to examine our files:the OpenOffice[30] at http://www.openoffice.org

[26]special thanks to Maria Matreno from Residencia de Estudantes Universitas Lisboa Portugal for providing the Second Law Of Arthur C.Clarke

[27]"Ideas And Opinions" Einstein compilation, ISBN $0-517-88440-2$, on page 226."Principles of Research" ([Ideas and Opinions],pp.224-227), described as "Address delivered in celebration of Max Planck's sixtieth birthday (1918) before the Physical Society in Berlin"

[28]appears also in the Eric Baird book Relativity in Curved Spacetime ISBN $978-0-9557068-0-6$

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