# On Pseudo-Superluminal Motion

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**Abstract**: Modern physics confirms the impossibility of Superluminal Motion through the considerations of Special Relativity. In General Relativity we may apply this constraint rigorously only to the Local Inertial Frames where Einstein's Field Equations are linear. This article, incidentally seeks to investigate the possibility of Pseudo-Superluminal motion in the non–local context without violating Special Relativity
**Keywords:** General Relativity, Local Inertial Frames, Manifold, Tangent Plane
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## INTRODUCTION

Finite speed of signal transmission is one of the greatest discoveries that have revolutionized modern physics .Special Relativity[1] through its second postulate claims that the speed of light is independent of its source. Though very much counter-intuitive if viewed through the "classical ideas" it turns out to be an amazing fact. In combination with the first postulate of relativity it leads to the novel aspect of space and time getting mixed up into a composite fabric. One of the fundamental outcomes of all this is the finite speed of signal transmission.

Incidentally all this refers to what we know as Flat Spacetime or Minkowski Space[2]. General Relativity is heavily based on the concept of the Local Inertial frames(LIF) which break up curved space into a set of small inertial territories. Curved Spacetime is governed by Einstein's Field equations which are non-linear in nature. But the Local Inertial frames offer us the advantage of Special Relativity---the Field Equations become linear. Calculations become simpler and comfortable in Flat Spacetime which exists here only in the local context, of course.

## NON-LOCAL CONSIDERATIONS

Now we consider the observation of an event at a point Q from a point P such that they have a finite separation between them, so that both my not be located in the same Inertial local frame. But each point carries its own LIF with it. In our "thought experiment "we have two observers one at P and the other at Q. A light ray flashes across an infinitesimally small, spatial interval at Q. It is observed from both the points P and Q. The spatial interval noted by both is the same. But the time recorded for the passage is different for the observers since their clocks run at different rates, the metric coefficients pertaining to time , generally speaking , are different for the two points.

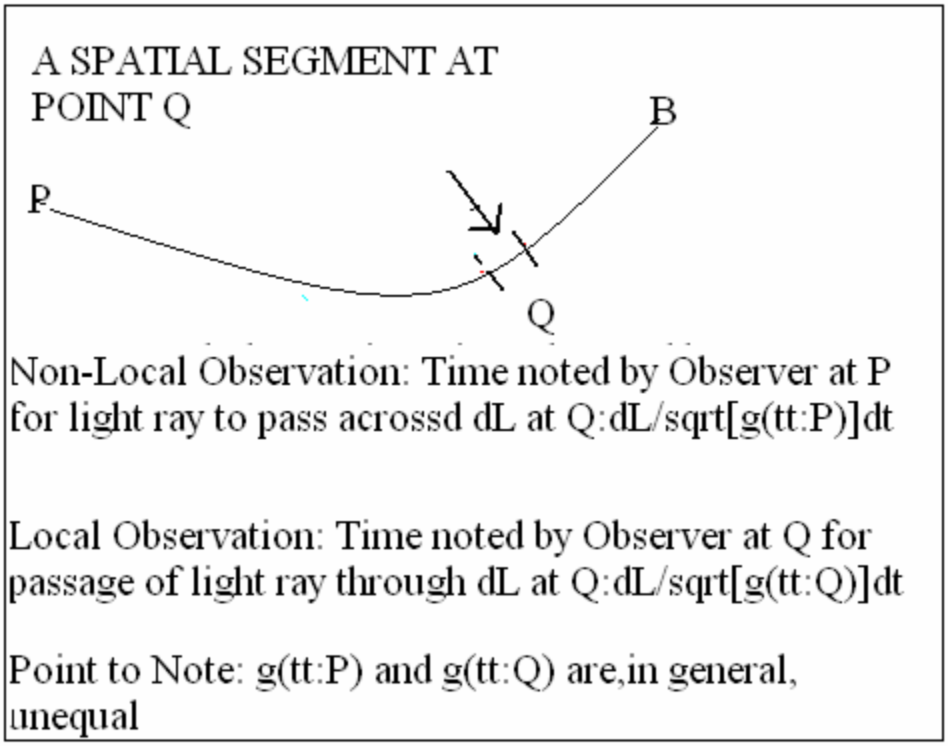


**FIGURE 1**. Local and Non-Local observations

Metric[3]: $ds^2 = g_{tt}dt^2 - g_{zz}dx^2 - g_{yy}dy^2 - g_{zz}dz^2$ (1)

Spatial interval at Q: $dL = \sqrt{g_{xx}(Q)dx^2 + g_{yy}(Q)dy^2 + g_{zz}(Q)dz^2}$

Both the observers record the same value for the above.

Non-Local time interval observed from P: $dT_P = \sqrt{g_{tt}(P)}dt$ (2)

Local time interval observed from Q: $dT_Q = \sqrt{g_{tt}(Q)}dt$ (3)

Non-Local Observation: Speed of light at Q as observed from P: $c_P = \frac{dL}{\sqrt{g_{tt}(P)}dt}$

Local Observation: Speed of light at Q as observed from Q: $c_Q = \frac{dL}{\sqrt{g_{tt}(Q)}dt}$

But the speed of light as observed from Q is the local speed of light that is, $c_Q = c$ ,where c is the standard value for speed of light in vacuum as we know it.

Therefore,

$$\frac{c_P}{c_Q} = \sqrt{\frac{g_{tt}(Q)}{g_{tt}(P)}}$$

Or,

$$c_P = c_Q \times \sqrt{\frac{g_{tt}(Q)}{g_{tt}(P)}}$$

Or,

Non-Local speed of ligh, $c_P$ , is given by:

$$c_P = \sqrt{\frac{g_{tt}(Q)}{g_{tt}(P)}} \times c \qquad (4)$$

[ $c_Q = c$ :Observation being Local]

Therefore the speed of light for non-local observation may be greater than equal to or less than the speed of light “c” as we know it, the standard value, depending on the value of the ratio $g_{tt(Q)} : g_{tt(P)}$

Now let’s consider a *particle* moving across an infinitesimally small spatial interval at Q(instead of a light ray).

Spatial separation: $dL = \sqrt{g_{xx}(Q)dx^2 + g_{yy}(Q)dy^2 + g_{zz}(Q)dz^2}$

Both the observers record the same value for it.

Time interval observed from P: $dT_P = \sqrt{g_{tt}(P)}dt$

Time interval observed from Q: $dT_Q = \sqrt{g_{tt}(Q)}dt$

Non-Local Observation: Speed of particle at Q as observed from P: $v_{(P:particle)} = \frac{dL}{\sqrt{g_{tt}(P)}dt}$

Local Observation: Speed of particle at Q as observed from Q: $v_{(Q:particle)} = \frac{dL}{\sqrt{g_{tt}(Q)}dt}$

Therefore,

$$\frac{v_{(P:particle)}}{v_{(Q:particle)}} = \sqrt{\frac{g_{tt}(Q)}{g_{tt}(P)}}$$

$$v_{(P:particle)} = \sqrt{\frac{g_{tt}(Q)}{g_{tt}(P)}} \times v_{(Q:Particle)} \qquad (5)$$

So the non-local speed of the particle, $v_{(P:particle)}$ , may exceed the standard local value of the speed of light depending on the value of the ratio $g_{tt(Q)} : g_{tt(P)}$

Incidentally the local speed of the particle is always less than the local speed of light, that is,

$$v_{(Q:particle)} < c$$

Therefore from relation (5) we have,

$$v_{(P:particle)} < \sqrt{\frac{g_{tt}(Q)}{g_{tt}(P)}} \times c \tag{6}$$

But the Right-hand-side of relation (6) is the non-local speed of light [see relation (4)] Therefore

$$v_{(P:particle)} < c_P \tag{7}$$

Thus the non-local speed of the particle is less than the non local speed of light, though the non-local speed of the particle can exceed the local standard speed of light in vacuum depending on the value of the ratio: $g_{tt(Q)} : g_{tt(P)}$ .The light ray is *always ahead of the particle* does not matter whether you are concerned with local or non-local observation. *We are not violating relativity in any manner.*

Now the non-local speed of light or some particle is important in deciding the average speed of light coming across a finite interval of space Time of non-local time of travel of a light ray is given by:

$$dT = \frac{dL}{\sqrt{g_{tt}(P) / g_{tt}(A)} \times c}$$

$$Time - Taken:$$

$$T = \int_B^A \frac{dL}{\sqrt{g_{tt}(P) / g_{tt}(A)} \times c} \tag{8}$$

The average speed of light for non-local travel across macroscopic distances:

$$c_{Average} = \frac{\int_B^A dL}{\int_B^A \frac{dL}{\sqrt{g_{tt}(P) / g_{tt}(A)} \times c}}$$

$$= c \times \frac{\int_B^A dL}{\int_B^A \frac{dL}{\sqrt{g_{tt}(P) / g_{tt}(A)}}} \neq c \tag{9}$$

So the average speed of light may be different from the local speed "c"(which corresponds to the known value---the speed of light in vacuum)
When a light ray is coming towards an observer across some interval of space he would be more interested in the average speed of light over the interval than the local speed of light local speed of light for various points traversed by the light ray..

## SYNCHRONIZATION OF CLOCKS:

For the purpose of synchronization[4] of clocks we take the speed of light constant over large macroscopic distances. It it really justified in view of the fact that the speed of light may change in the non-local sense especially when we are considering sensitive experiments like the OPERA[5] or ICARUS[6]?. It would be an interesting reminder for us that the OPERA experiment failed(due to cable fault: loose cable connection) with the condition that the speed of light was taken to be constant with respect to observation stations in disregard of the fact that the light ray traveled over large macroscopic distances in the process of synchronization. The ICARUS experiment succeeded on the basis of the same "aspect" ---the non-local variation of the speed of light was not given a due consideration..

Sample Calculations

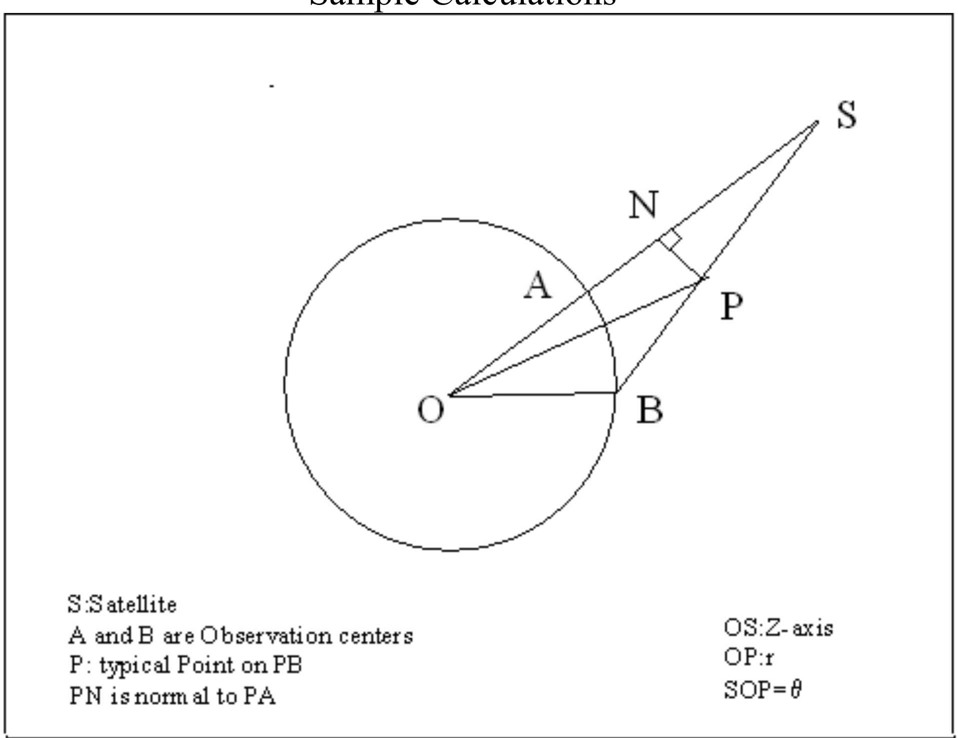


**FIGURE 2**.Transmission of light ray from a satellite to two earth stations at A and B

The above figure shows a "non-rotating " earth-like planet with observation stations at A and B. S is a satellite from where light signals are being sent. These are being received at

the earth stations A and B. O is taken to be the z:axis.OP=r;∠SOP=θ. OS=d,a fixed "coordinate distance".∠OSP=α, a fixed/constant angle. PN is perpendicular to OS.
Now,
ON=rCosθ
SN=OS-ON=d-rCosθ

$$\tan\alpha = \frac{PN}{SN} = \frac{rSin\theta}{d - rCos\theta}$$

$$d\tan\alpha - r\tan\alpha Cos\theta = rSin\theta \quad (10)$$

Taking differentials from (10) we have:

$$dr(Sin\theta + \tan\alpha Cos\theta) = rd\theta(Cos\theta - \tan\alpha Sin\theta) \quad (11)$$

Agein from relation (10) we obtain:

$$r = \frac{d\tan\alpha}{Sin\theta + \tan\alpha Cos\theta} \quad (12)$$

Schwarzschild's Metric:

$$ds^2 = (1 - 2GM/c^2 r)dt^2 - (1 - 2GM/c^2 r)^{-1} dr^2 - r^2(d\theta^2 + Sin^2\theta\, d\varphi^2)$$

The spatial element on the line SB, say at P, is given by:

$$dL = \sqrt{\left(1 - \frac{2GM}{c^2 r}\right)^{-1} \frac{r^2(Cos\theta - \tan\alpha Sin\theta)^2}{(Sin\theta + \tan\alpha Cos\theta)^2} d\theta^2 + r^2 d\theta^2}$$

$$\text{Or, } dL = \sqrt{\left(\left(1 - \frac{2GM}{c^2 r}\right)^{-1} \frac{(Cos\theta - \tan\alpha Sin\theta)^2}{(Sin\theta + \tan\alpha Cos\theta)^2} + 1\right)} \times rd\theta \quad (13)$$

Spatial element on AS is given by:

$$dL = \left(1 - \frac{2GM}{c^2 r}\right)^{-1/2} dr \quad (14)$$

[Since both α and θ are zero on AS]
Time of travel of light ray from B to S:

$$T = \int_{\theta_S}^{\theta_P} \frac{\sqrt{\left(\left(1 - \frac{2GM}{c^2 r}\right)^{-1} \frac{(Cos\theta - \tan\alpha Sin\theta)^2}{(Sin\theta + \tan\alpha Cos\theta)^2} + 1\right)}}{c \times \sqrt{\frac{\left(1 - \frac{2GM}{c^2 r(P)}\right)}{\left(1 - \frac{2GM}{c^2 r(S)}\right)}}} \times rd\theta \quad (15)$$

"r" may be taken from (12)
Time of light ray from S to A:

$$T = \int_{r_S}^{r_P} \frac{\sqrt{\left(1 - \frac{2GM}{c^2 r}\right)^{-1}}}{c \times \sqrt{\frac{\left(1 - \frac{2GM}{c^2 r(P)}\right)}{\left(1 - \frac{2GM}{c^2 r(S)}\right)}}} \times dr \qquad (16)$$

[Incidentally, for this path θ and dθ are both zero. SO we have considered integration wrt dr]

These calculations take care of the "tick rate" at each point on the path of the light ray while in they GPS they consider the tick rates at the point of transmission and reception only.

## NON-LINEARITY OF EINSTEIN'S FIELD EQUATIONS

The fact that Einstein's field equations are non linear is a well known fact in physics. But in the inertial frames of reference the Christoffel symbols[7]evaluate to zero value and the field equations are no more non linear. They become linear. So if you are working in a laboratory you are enjoying the privilege of linearity which is not there outside your laboratory if it(lab) happens to be a local inertial frame. For non local observations the non linearity of the field equations are supposed to play a very big role as in our case of pseudo superluminal motion. One issue becomes important in this respect : To what extent is our lab fixed on the earth's surface is an inertial frame of reference?

### Lab Fixed on the Earth's Surface

You are working in your small laboratory room fixed on the earth so that you may call it a local inertial frame[And you are working for a suitably small interval of time]. Now you may think of a freely falling lift in front of you. That lift is a better approximation of a LIF. Your Lab room does not correspond to the "better approximation". The contrast should would become glaringly conspicuous if you imagine the "gravity" to be a million times stronger---that is if you consider your lab room to be in a region of strong spacetime curvature. The freely falling lift is a LIF while your lab room in this example may be termed as a "*Local Non Inertial Frame*"

The basic advantage provided by the Local Inertial Frames is the Special Relativity context. The point that naturally arises is that to what extent do we expect deviations from SR in the local non-inertial frame?

### The Tangent Plane to the Manifold

Let us consider the tangent plane[8] at the point of contact P with the curved spacetime surface. The tangent surface offers the advantage of the Special Relativity context.*Since time goes on changing in both the tangent plane and the curved surface(though differently)*, our laboratory, its spacetime, location(coordinates) at the most can be at a momentary contact with the point P.Then the space-time point of the laboratory will move along the curved surface unless we make some *technological arrangement* of containing our laboratory on the tangent plane by arranging a freely falling frame.To materialize the local transformation from 4D curved space to Minkowski space we have to arrange a freely falling frame--the falling lift in the simple this case of the earth.

Let the coordinates of the curved 4D surface be $(t,x,y,z)$and the local coordinates on the tangent surface at P: $(\xi_0,\xi_1,\xi_2,\xi_3)$. The first coordinate in parenthesis represents time in each system. If we want to keep the on the tangent surface in order to enjoy the advantage of Special Relativity, the lift should accelerate wrt to the curved surface[generally speaking]

Transformations (Equation Set 17)

$$t=f_1(\xi_0,\xi_1,\xi_2,\xi_3)$$
$$x=f_2(\xi_0,\xi_1,\xi_2,\xi_3)$$
$$y=f_3(\xi_0,\xi_1,\xi_2,\xi_3)$$
$$z=f_4(\xi_0,\xi_1,\xi_2,\xi_3)$$

Our the tangent plane is actually an inertial frame of reference. Consider a world line on it through the point of contact, P a short world line of course . Let us denote the world line by:

$$F(\xi_0,\xi_1,\xi_2,\xi_2)=0 \qquad (18)$$

For the transformed values the quantities , $\frac{d^2x}{dt^2},\frac{d^2y}{dt^2}$ *and* $\frac{d^2z}{dt^2}$, in general will be non zero.

To understand the situation at the point of contact we consider a simpler analogy. We take the parabola given by: $y=x^2$. Gradient: $\frac{dy}{dx}=2a$ The tangent to it at the point M(a,b) is given by:

$$\frac{y-b}{x-a}=2a$$

At the point M(point of contact) the value of $\frac{dy}{dx}$ is identical for both the parabola and the tangent which is a straight line in this case. But what about the second order derivative, $\frac{d^2y}{dx^2}$? For the parabola: $\left[\frac{d^2y}{dx^2}\right]_M=2$. For the straight line: $\left[\frac{d^2y}{dx^2}\right]_M=0$. *The second order derivatives differ even at the point of contact.* You may translate this example to higher dimensions.

Points to Observe:

1. The point of contact P on the manifold and the tangent plane are not identical in so far as the second order derivatives are considered. The first order derivatives on the two planes at the point of contact are identical. But they are not identical(in general) at other neighboring points.
2. To stay on the tangent plane[LIF],even at the point of contact ,P, some acceleration is necessary. We need a *freely falling frame* to stay on the said tangent plane.

**Speed of Light in Local Inertial Frames**

Indeed , we may write the metric:

$$ds^2 = g_{tt}dt^2 - g_{xx}dx^2 \qquad (19)$$

$$ds^2 = dT^2 - dL^2 \qquad (20)$$

In the above metric, that is in (19), the x-axis has been oriented along the infinitesimal path of a light ray. Now $ds^2 = 0$ for the null geodesic. Therefore from relation (20) we have,

$$Mod\left[\frac{dL}{dT}\right] = 1$$

Incidentally c=1 in the natural units and we have the same invariable speed of light in vacuum provided we define physical time interval as: $dT(physical) = \sqrt{g_{tt}}\,dt$

dt is the coordinate time interval.

We are getting the speed of light "c"[standard value] with respect to the tangent plane. Incidentally, equation (20) corresponds to the tangent plane, the LIF.What about equation (19)?It represents curved spacetime. At the point of contact we ,of course, get the same value for the first order derivative for both the surfaces of which the speed of light is an example. But even for short distances this fails—the picture is so tricky even in the contest of the local inertial frames. Pseudo super luminal speed of light in the non-local context is coming into picture! We may try to calculate the acceleration of the light ray at the point of contact of the tangent plane with the manifold(wrt to the manifold).Any deviation from an LIF due to absence of correct amount of acceleration required to stay on the tangent plane will necessitate such an investigation

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