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On spinors and their transformation

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ABSTRACT

The Dirac spinor and its transformation properties form an important part in the foundation of Relativistic Quantum Mechanics and of course of particle physics. The Dirac spinor is neither a scalar nor a four vector. It has an identity of its own ---- it is a spinor. The article clearly brings out the fact that the spinor components transform individually on the passage from one reference frame to another though the spinor transformation matrix seems to produce the impression that each component in the transformed frame is created by the interaction of the four spinor components in the original frame. Such issues have been handled in this article.

Keywords: Relativistic Quantum Mechanics Dirac Spinor, Klein-Gordon Equation, Dirac equation, Lorentz-Transformation Matrix, Spinor Transformation Matrix

INTRODUCTION

The concept of the Dirac spinor1,2 and its transformation properties form an important entity in the construction of Relativistic Quantum Mechanics and of course of particle physics. The Dirac spinor is neither a scalar nor a four vector. It has its own transformation rules defined by its transformation matrix and as such has an identity of its own ---- it is a spinor. Each component of the transformed Dirac solution is obtained by the interaction of all the components of the original solution and this procedure is quite different from the transformation of scalars. A careful reconsideration reveals that the component solutions of the Dirac equation transform **individually**.

. The matter will be confirmed by proving a new result

$S(\Lambda)\Psi(t, x, E, p) = \Psi[\Lambda(t, x)\ \Lambda(E, p)]$ ---------------- (1)

Where ,

Λ: Lorentz-Transformation Matrix3

S(Λ): Spinor Transformation Matrix 4,5

It is important to note that $\Psi[\Lambda(t, x)\ \Lambda(E, p)]$ is the value of Ψ at (t/, x/, p/, E/)) = [Λ(t, x),Λ(E, p)]

One may also envisage the component solutions of the Dirac equation to be scalars. This is in view of the fact that the individual components of the Dirac solution are also solutions of the Klein-Gordon equation [6,7] and that the Klein –Gordon solution is necessarily a scalar. The matter may be resolved if we assume that the Klein-Gordon Equation has non-scalar solutions apart from the scalar ones.

The Dirac Equation and its Lorentz-Covariance

The Dirac equation [8,9] is Lorentz-covariant. In the unprimed frame it reads:

$(i\gamma^{\mu}\partial_{\mu} - m)\,\Psi(t, x, E, p) = 0$ ----------------------- (2)

In the primed frame it reads:

$(i\gamma^{\mu}\partial'_{\mu} - m)\,\Psi'(t', x', E', p') = 0$ --------------------- (3)

The preserved nature of the equations reveals the fact that they have identical solutions in their respective frames of reference

In particular we may write,

$\Psi(t, x, E, p) = \Psi'(t, x, E, p)$ ----------------------------- (4)

(t ,x,E, p) on the right hand side and the left hand side represents different points in the two reference frames having identical values of the coordinates. By “different points” we mean that they are not the corresponding points of a Lorentz transformation. This result shall be of immense use to us in the future course of developing the article. It is a direct consequence of the principle of relativity.

The transformation matrix re-examined: Let the function Ψ(t, x, E, p) in the unprimed frame transform to $\Psi'(t', x', E', p')$ in the primed frame .Thus we have,

$\Psi'(t', x', E', p') = S(\Lambda)\Psi(t, x, E, p)$ ------------------ (5)

And $(t', x', E', p') = [\Lambda(t, x), \Lambda(E, p)]$ ----- (6)

In the above relations we cannot assume that

$\Psi'(t', x', E', p') = \Psi[\Lambda(t, x),\Lambda(E, p)]$ -------------- (7)

Rather we are defining Ψ' by equation (5) so as to preserve the form of the Dirac equation on transformation.

At this point let us define a new transformation in the following way:

$NT[\Psi(t, x, E, p)] = \Psi[\Lambda(t, x)\ \Lambda(E, p)]$ ------ (8)

In the above relation i.e., (7), NT stands for our new transformation and NT(Ψ) is the value of Ψ at $(t', x', E', p') = [\Lambda(t, x), \Lambda(E, p)]$. We may use the above transformation to move inside the same frame from one point to another inside the same reference frame and determine the psi function at the new location (of the same frame) or we may use it to move from one frame of reference to another. But this is not our spinor transformation. The action of the new transformation has been illustrated in the figure below.

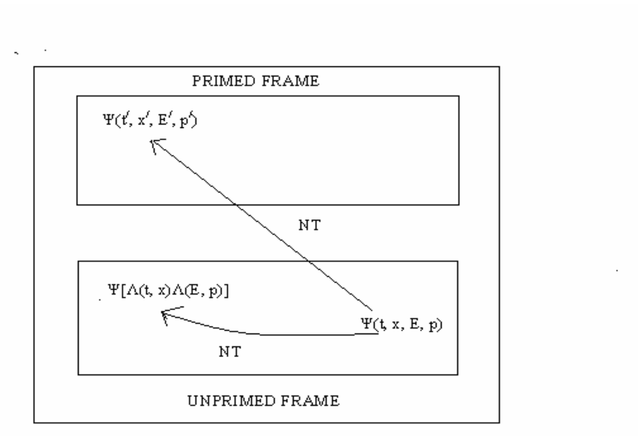

Fig 1. The first interpretation will be applied in the subsequent discussion.

Performing the exercise: Let us perform the following exercise. We apply the new transformation to move from the point (t, x, p) to (t', x', p') inside the unprimed reference frame and calculate the psi function at the new location. At the same time we apply the S matrix on the psi-function at (t, x, p) and pass to the primed reference frame. The action has been portrayed in the diagram below.

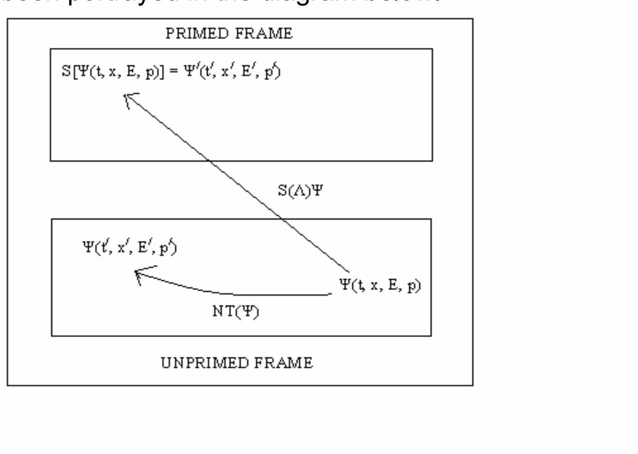

Fig 2.

Now by applying equation (4) we may claim that $\Psi(t', x', p') = \Psi'(t', x', p')$. Indeed by applying the new transformation we have moved within the unprimed frame from (t, x, p) to (t', x', p').According to equation (4) the spinor transformed value at (t', x', p') in the primed frame is equal to the value of the psi function at (t', x', p') in the unprimed frame. $\Psi(t', x', p')$ is the solution of equation (2) at (t', x', p') and $\Psi'(t', x', p')$ is the solution of equation (3) at (t', x', p') and equations (2) and (3) have identical solutions at the "different points" (t', x', p') due to the principle of relativity.

Now,

$\Psi(t', x', E', p') = \Psi[\Lambda(t, x), \Lambda(E, p)]$ -------------------- (9)

$\Psi'(t', x', E', p') = S(\Lambda)\Psi(t, x, E, p)$ ------------------ (10)

Therefore,

$S(\Lambda)\Psi(t, x, E', p) = \Psi[\Lambda(t, x), \Lambda(E, p)]$ ------------ (11)

In fact we have proved (7)

Equation (11) is in perfect conformity with specific examples of the Dirac Spinor. We get the same result by applying either side of equation on the Dirac spinor.

Supporting Calculations : In the above discussion I have proved that

$S(\Lambda)\Psi(t, x, E, p) = \Psi[\Lambda(t, x)\ \Lambda(E, p)]$

Where S(Λ) is the spinor transformation matrix

And Λ is the Lorentz Transformation matrix.

I have claimed that both the left hand side and the right hand side of the formula (11) produce the same results.

Let me illustrate this

For a particle moving along the x-axis (of the unprimed frame) with momentum p_x the Dirac spinnor is given by[10]:

$$\Psi(p_x)=\begin{pmatrix} 1 & 0 & 0 & \frac{p_x}{E+m_0} \\ 0 & 1 & \frac{p_x}{E+m_0} & 0 \\ 0 & \frac{p_x}{E+m_0} & 1 & 0 \\ \frac{p_x}{E+m_0} & 0 & 0 & 1 \end{pmatrix} \times \exp(-ip^{\mu}x_{\mu})$$

Let the primed frame be moving wrt the unprimed frame with speed V along the x-x' direction. Momentum=$p_x' = \Lambda(p_x)$

The Dirac solution is given by:

$\Psi^{/}(p_x^{/})=$

$$\begin{pmatrix} 1 & 0 & 0 & \frac{p^{/}{}_x}{E+m_0} \\ 0 & 1 & \frac{p^{/}{}_x}{E+m_0} & 0 \\ 0 & \frac{p^{/}{}_x}{E+m_0} & 1 & 0 \\ \frac{p^{/}{}_x}{E+m_0} & 0 & 0 & 1 \end{pmatrix} \times \exp(-ip^{/\mu} x^{/}{}_{\mu})$$

Indeed we have S(Λ)Ψ = Ψ[Λ(t, x, E, p)] and the right had side is more convenient to use.

Now this problem takes on a special dimension when we consider particles at rest .The Dirac solution for such particles is given by:

$\Psi^r = \omega^r(0)\exp(-i\varepsilon_r m_0 t)$, r=1,2,3,4

Where, ω^1, ω^2, ω^3 and ω^4 are the usual column matrices containing zeros and ones in proper order.

If the conventional psi function of a moving spinor(momentum=p_x) is known to us the right hand side of formula (1) easily predicts the psi function of the spinor wrt to a frame where it is at rest. But the other way round there is some difficulty if the functional dependence on momentum is not known to us. If the form of dependence of the psi function on momentum is not visible to the formula $\Psi^{/}$= Ψ[Λ(t, x) Λ(E, p)] it cannot perform the conversion work. The left hand side seems to offer us some advantage in this matter. Let us check on this point.

If a component of the rest spinor be zero it could either be a scalar zero or it could be of the form f(p) such f(p)=0 for p=0. A correct knowledge of the situation could be gained by considering the Dirac equation. Let us write the four component equations for motion along the x-axis($p_y = p_z = 0$)

$$-i\frac{\partial\psi_1}{\partial t} - i\frac{\partial\psi_4}{\partial x} + m_0\psi_1 = 0$$
$$-i\frac{\partial\psi_2}{\partial t} - i\frac{\partial\psi_3}{\partial x} + m_0\psi_2 = 0$$
$$i\frac{\partial\psi_3}{\partial t} + i\frac{\partial\psi_2}{\partial x} + m_0\psi_3 = 0$$
$$i\frac{\partial\psi_4}{\partial t} + i\frac{\partial\psi_1}{\partial x} + m_0\psi_4 = 0$$

The above set may be grouped into two identical parts:

$$-i\frac{\partial\psi_1}{\partial t} - i\frac{\partial\psi_4}{\partial x} + m_0\psi_1 = 0$$

------------------------------------ (12)

$$i\frac{\partial\psi_4}{\partial t} + i\frac{\partial\psi_1}{\partial x} + m_0\psi_4 = 0$$

And,

$$-i\frac{\partial\psi_2}{\partial t} - i\frac{\partial\psi_3}{\partial x} + m_0\psi_2 = 0$$

------------------------------------ (13)

$$i\frac{\partial\psi_3}{\partial t} + i\frac{\partial\psi_2}{\partial x} + m_0\psi_3 = 0$$

The second set ie, equations(13) may be given trial solutions of the form

$$\psi_2 = \psi_3 = 0$$

With the first[equations (12)] set we try,

$$\psi_1 = f_1(p_x)\exp(-ip^{\mu}x_{\mu})$$
$$\psi_4 = f_2(p_x)\exp(-ip^{\mu}x_{\mu})$$

The exponential part has been predicted from the rest spinor solution. Substituting these trial solutions into equations (12) we have,

$$f_1(p_x)[E - m_0] = p_x f_2(p_x)$$

$And,$

$$[E + m_0] f_2(p_x) = p_x f_1(p_x)$$

$Or,$

$$f_2(p_x) = f_1(p_x)\frac{[E - m_0]}{p_x}$$

Using the relation,

$$E^2 - m_0{}^2 = p_x{}^2$$

We have,

$$f_2(p_x) = \frac{f_1(p_x)p_x}{[E+m_0]}$$

Writing $f_1(p_x) = [\frac{E+m_0}{2m_0}]^{1/2}$

We have the normal Dirac Solution for a particle moving along the x-direction. The above choice of $f_1(p_x)$ is such that it satisfies the condition that $\psi^\dagger\psi$ is a Lorentz-invariant Our solution is:

$$\psi = [\frac{E+m_0}{2m_0}]^{1/2}(1,0,0,\frac{p_x}{E+m_0})^T \exp(-ip^\mu x_\mu)$$

[T stands for transpose in the above expression]

The solutions for the two-dimensional or three dimensional motion may be calculated from the transformation rules or **directly from the Dirac Equations.**

TWO DIMENSIONAL MOTION

$[p_x \neq 0, p_y \neq 0, p_z = 0]$

The Dirac Equation for two-dimensional motion are:

$$-i\frac{\partial\psi_1}{\partial t} - i\frac{\partial\psi_4}{\partial x} - \frac{\partial\psi_4}{\partial y} + m_0\psi_1 = 0 \text{ ----------- (14)}$$

$$i\frac{\partial\psi_4}{\partial t} + i\frac{\partial\psi_1}{\partial x} - \frac{\partial\psi_1}{\partial y} + m_0\psi_4 = 0$$

$$-i\frac{\partial\psi_2}{\partial t} - i\frac{\partial\psi_3}{\partial x} + \frac{\partial\psi_3}{\partial y} + m_0\psi_2 = 0 \text{ ----------- (15)}$$

$$i\frac{\partial\psi_3}{\partial t} + i\frac{\partial\psi_2}{\partial x} + \frac{\partial\psi_2}{\partial y} + m_0\psi_3 = 0$$

Trial Solution:

$$\psi_2 = \psi_3 = 0, \psi_1 = f_1(E, p_x, p_y)\exp(-ip^\mu x_\mu), \psi_4 = f_2(E, p_x, p_y)\exp(-ip^\mu x_\mu)$$

Substituting the trial solution into equations (14) and using the relation $E^2 - m_0{}^2 = p_x{}^2 + p_y{}^2$ we have,

$$f_2 = f_1\frac{p_x + ip_y}{E+m_0}$$

Solution:

$$\psi = f_1(1,0,0,\frac{p_x+ip_y}{E+m_0})\exp(-ip^\mu x_\mu)$$

If the trial solution is taken as:

$$\psi_1 = \psi_4 = 0, \psi_2 = f_1(E, p_x, p_y)\exp(-ip^\mu x_\mu), \psi_3 = f_2(E, p_x, p_y)\exp(-ip^\mu x_\mu)$$

The Dirac-solution works out to:

$$\psi = f_1(0,1,\frac{p_x-ip_y}{E+m_0},0)^T \exp(-ip^\mu x_\mu)$$

The condition that $\psi^\dagger\psi$ is a Lorentz invariant gives the value of $f_1 = [\frac{E+m_0}{2m_0}]^{1/2}$ in both the above cases.

In the exponential parts we could have used $\exp(ip^\mu x_\mu)$ instead of $\exp(-ip^\mu x_\mu)$ to obtain other types of solution. These results conform to the standard ones. Fo transformation we simply replace $p_{x,} p_y$ by the corresponding Lorentz values of p_x' and p_y'. Our new transformation rule,

$$\psi'(t,x,E,p) = \psi[\Lambda(t,x),\Lambda(E,p)]$$

indeed remains valid. The three dimensional solution may also be evaluated by using suitable trial solutions.

An element of arbitrariness: Now the spinor transformation matrix is defined by

$$S^{-1}\gamma^\nu S = \Lambda^\nu{}_\mu \gamma^\mu \text{ ------------------------------- (16)}$$

Let us look into the proof of the above relation:

We have,

$$(-i\gamma^\mu\partial_\mu + m)\Psi(x) = 0 \text{ ----------------------- (17)}$$

$$\frac{\partial}{\partial x^\mu} = \Lambda^\nu{}_\mu \partial'_\nu \text{ ------------------------- (18)}$$

And

$$S^{-1}\Psi'(x') = \Psi(x) \text{ ------------------------- (19)}$$

Equation (3) may be written as

$(-i\gamma^{\mu}\Lambda^{v}{}_{\mu}\partial'_{v}\Psi'(x') + m) S^{-1}(\Lambda)\Psi'(x') = 0$ ----------- (20)

Left multiplying the above equation by S we have

$-iS\Lambda^{v}{}_{\mu}\gamma^{\mu}S^{-1}\partial'_{v}\Psi'(x') + m\Psi'(x') = 0$

[It is important to note that $\Lambda^{v}{}_{\mu}$ are numbers(matrix elements) while the gammas are matrices themselves.]

In order to preserve the Dirac equation we must have,

$S^{-1}\gamma^{v} S = \Lambda^{v}{}_{\mu}\gamma^{\mu}$ -------------------------- (21)

But it is especially interesting to take note of the matrix operator:

$S'(\Lambda) = S\exp(-i M \omega)$ --------------------------------- (22)

Where ω is the boost angle ($\tanh\omega = v_x/c$) or the rotation angle . Here M is a constant scalar.

[Please take note of the fact that exp (-i M ω) = 1 for ω=0. Also take note of the fact that exp(-I M ω) is a scalar and not a matrix.]

We shall prove that if the transformation matrix S preserves the Dirac equation the matrix $S'(\Lambda)$ also preserves it.

.

Now,

$(S')^{-1} = [S\exp(-iM\omega)]^{-1} = \exp(iM\omega) S^{-1}$ --------(23)

We have, $\Psi(x) = (S')^{-1}\Psi''(x')$

Under the action of $S'(\Lambda)$ equation (3) transforms to

$(-i\gamma^{\mu}\Lambda^{v}{}_{\mu}\partial'_{v}\Psi''(x') + m \exp(iM\omega) S^{-1}(\Lambda)\Psi''(x') = 0$

Then left multiplying by $S' = S\exp(iM\omega)$ we obtain,

$(S')(-i\gamma^{\mu}\Lambda^{v}{}_{\mu}\partial'_{v}\Psi''(x') + m)(S')^{-1}(\Lambda)\Psi''(x') = 0$

Or, $-iS\exp(-iM\omega)\Lambda^{v}{}_{\mu}\gamma^{\mu}\exp(iM\omega)S^{-1}\partial'_{v}\Psi''(x') + m\Psi''(x') = 0$ ----------------- (24)

Or, $-iS\Lambda^{v}{}_{\mu}\gamma^{\mu}S^{-1}\partial'_{v}\Psi''(x') + m\Psi''(x') = 0$

By using (7) that is,

$\Lambda^{v}{}_{\mu}\gamma^{\mu} = S^{-1}\gamma^{v} S$,

We obtain,

$(-i\gamma^{\mu}\partial'_{\mu} + m)\Psi''(x') = 0$ ---------------- (25)

So the Dirac equation is again preserved.

Invariance of norm of the psi function wrt to S implies

$$\Psi^{\dagger}\Psi = \Psi^{\dagger} S^{\dagger} S\Psi$$

That is $S^{\dagger}S = I$

Now,

$\Psi^{\dagger} S'^{\dagger} S'\Psi = \Psi^{\dagger}\exp(iM\omega)S'^{\dagger} S'(-iM\omega)\Psi$

$= \Psi^{\dagger}\exp iM\omega)(-iM\omega)\Psi$

$= \Psi^{\dagger} I\Psi$

$= \Psi^{\dagger}\Psi$

Thus the norm is preserved for S' also.

Indeed S and S' preserve the Dirac equation for the same boost or rotation. In fact the scalar M may contain expressions containing components of the energy-momentum four-vector. Of these several matrices the left hand side of (1) chooses a particular one for transforming the spinor at rest and so the transformation appears to be of a unique nature.

But the transformation is not actually unique if we consider all the matrices given by (8)

Indeed the right hand and the left hand sides of (1) have equal problems in handling spinors at rest if the functional dependence of psi on the momentum components is not known to us.

The Klein-Gordon Connection: It is a well known fact that the individual components of the Dirac solution satisfy the Klein-Gordon equation. Now we know that the solutions of the K-G equation must necessarily be a scalar and therefore the individual components of the Dirac equation must also be scalars[if they transform individually] and hence the four-component Dirac solution should be a scalar.

But this is not true! The Dirac solution Ψ is not a scalar (psi-bar psi is a scalar). Therein lays the contradiction!

Interestingly the Dirac components may be transformed in two ways:

1) By using the rule $\Psi'(t', x', E', p') = \Psi[\Lambda(t, x), \Lambda(E, p)]$.This we have proved in the previous section.

In this case the components transform individually.

2) By using the relation $\Psi'(t', x', E', p') = S(\Lambda)\Psi(t, x, E, p)$. This is the usually followed procedure where all the components are involved. But if the relation $S(\Lambda)[\Psi(t, x,E\ p)]=\Psi[\Lambda(t, x), \Lambda(E, p)]$ is true both the methods should produce the same result. This has been clearly explained in the previous section.

By applying the first method we conclude that the components transform individually. At the same time they should be scalars (each component being a solution of the K-G equation).

Let us first write the Klein Gordon Equation (natural units have been assumed):

$$(\partial^2 + m^2)\psi = 0$$

The Klein Gordon operator itself is a Lorentz-invariant quantity. Therefore to preserve the above relationship (on Lorentz Transformation) we have two options:

1) The function ψ is itself a Lorentz invariant quantity
2) Keeping in mind that the Klein-Gordon operator is a differential operator and that the result of the operation depends not only on the point transformed but also on the neighboring points, we come to an immediate conclusion that other solutions are possible (The Dirac solution is such a solution).In fact the Klein-Gordon equation is a partial differential equation and it may have several solutions depending on the boundary conditions. This strengthens the argument. The Klein Gordon Solution may not be a scalar!

It is important that we investigate the solutions other than the two usual types- the Dirac spinor and the usual zero-spin solution.

CALCULATIONS

The Klein –Gordon equation (Assuming natural units: c=1, $\hbar$ =1):

$(\partial^2 + m^2)\ \psi = 0;$

Or,

$$\frac{\partial^2\psi}{\partial t^2}-\frac{\partial^2\psi}{\partial x^2}-\frac{\partial^2\psi}{\partial y^2}-\frac{\partial^2\psi}{\partial z^2}+m^2=0 \text{ -------------26)}$$

We apply the separation of variables technique.

Let ψ = $\varphi_0(t)\varphi_1(x)\ \varphi_2(y)\ \varphi_3(z)$

Substituting the above trial solution in (1) we obtain,

$$\frac{1}{\varphi_0}\frac{\partial^2\varphi_0}{\partial t^2}-\frac{1}{\varphi_1}\frac{\partial^2\varphi_1}{\partial x^2}-\frac{1}{\varphi_2}\frac{\partial^2\varphi_2}{\partial y^2}-\frac{1}{\varphi_3}\frac{\partial^2\varphi_3}{\partial z^2}+m^2=0$$

The solutions are:

$$\begin{aligned}\varphi_0 &= A_1\exp(ik_0t)+A_2\exp(-ik_0t)\\ \varphi_1 &= B_1\exp(ik_1x)+B_2\exp(-ik_1x)\\ \varphi_2 &= C_1\exp(ik_2y)+C_2\exp(-ik_2y)\\ \varphi_3 &= D_1\exp(ik_3z)+D_2\exp(-ik_3z)\end{aligned} \text{ -----(27)}$$

The separation constants are

$-k_0{}^2, -k_1{}^2, -k_2{}^2 and \quad -k_3{}^2$

General Soln: $\psi = \Sigma const \times \psi_0\psi_1\psi_2\psi_3$ -----(28)

Summation is carried over all the values of the constants

The elementary Klein – Gordon eigen-function may be obtained form (23) by asigning suitable values to the constants. For example putting A_1=0,B_2=0,C_2=0 and D_2= we obtain one of the elementary Klein-Gordon solutions. Also we assume, k_0=E (=ω) .The condition $k_0^2 - k_1^2 - k_2^2 - k_3^2 = m^2$ gets automatically imposed. We may identify k_1,k_2 and k_3 as the momenta p_1, p_2 and p_3(since $\hbar$ =1)

The Dirac Components: First we take note of the fact that the values of energy and individual components of momentum four-vector are constants in the elementary solutions (eigenfunctions) of both the Klein –Gordon and the Dirac equations.(In fact to verify the validity of the solutions wrt the equations we partial differentiate them holding each component of the energy momentum four-vector constant with respect to t, x , y and z separately).Therefore these values may be used as the constants A_1, B_1 etc. This basic information is crucial to what we are to do next.

First let us consider the usual Dirac solution for particles moving along the x axis with momentum p_x and energy E(Displayed previously):

For r=1, the first Dirac component is:

$$\psi(p_x)=[\frac{E+m_0}{2m_0}]^{1/2}\exp(-ip^\mu x_\mu)$$

We may get this result from equation Set (23) by assuming. $A_1=[\frac{E+m_0}{2m_0}]^{1/2}$ and $k_0=m_0$

All other constants are assumed to be zero It may also be noted that E is constant for a particular eigenfunction.

But the above function is not a scalar since E is not an invariant. Moreover one appears in the coefficient while the other in the exponent.

For r=1 the fourth component is:

$$\psi(p_x)=\frac{p_x}{\sqrt{2m_0(E+m_0)}}\exp(-ip^\mu x_\mu)$$

Again we may obtain the above function from Set(23) by assuming $A_1=\frac{p_x}{\sqrt{2m_0(E+m_0)}}$,

$K_0= m_0$ and other constants are zero. It may again be noted that p_x and E being constant fore a particular eigenfunctions may be fitted into the constants. But the above function is not a scalar since p_x ,E and transform without preserving the function.

Similar statements may be made in reference to other Dirac Components. It is difficult to say whether such solutions exist in nature (they are neither zero-spin nor half spin particles!) but the possibility should be explored. We could predict other solutions of the Klein Gordon equation by assigning other suitable types of values to the constants and explore the possibility of existence of such particles!

CONCLUSIONS

We have proved the important relation (1)

$S(\Lambda)\Psi(t, x, E, p) = \Psi[\Lambda(t, x, E, p)]$

Where ,

Λ: Lorentz-Transformation matrix

$S(\Lambda)$: Spinor Transformation matrix

It is important to note that in the above $\Psi[\Lambda(t, x, E, p)]$ is the value of Ψ at $(t^{/}, x^{/}, E^{/}, p^{/}) = \Lambda(t, x)\Lambda(E, p)$

So the Dirac components may transform individually or collectively producing the same result. It has also been shown that the Klein-Gordon solution is not necessarily a scalar

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