APPLYING SANTILLIAN ISO-MATHEMATICS TO THE TRIPLEX NUMBERS AND THE INOPIN HOLOGRAPHIC RING: PRELIMINARY ASSESSMENT AND NEW HYPOTHESIS

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Abstract

In a preliminary assessment, we begin to apply Santillian iso-numbers to the triplex numbers and the Inopin holographic ring. In doing so, we realize a new hypothesis: the Inopin holographic ring may be one non-linear structure that satisfies the strict iso-curvature and iso-duality constraints of the Santillian iso-number theory, where Inopin's ring delineates Santilli's dual *interior* and *exterior* dynamical systems. Ultimately, in order to prove or disprove this hypothesis, we propose that a thorough and rigorous mathematical investigation should be conducted on these issues.

Keywords: Iso-number; Geno-number; Hyper-number; Iso-dual; Riemann-Santilli iso-space; Triplex number; Holographic ring; Topological deformation.

1 Introduction

A number is a mathematical object that is used to count, label, and measure. Number systems are fundamental to all quantitative sciences because they are used to encode the state space and transition space of experimental and observable features in nature. Such systems are tools that let scientists explore ideas and quantify experimental results. Historically, advances in science, engineering, and technology have pushed the mathematical definition of a number to include additional structures such as 0, negative numbers, integer numbers, rational numbers, irrational numbers, real numbers, complex numbers, and quaternion numbers in order to satisfy the imposed representational demands of these disciplines.

A number is an element, so a set of numbers is a set of elements, where the number set is equipped with addition and multiplication operators to establish a number field that complies with five number field axioms [1]. Let

$$X = \{x_1, x_2, x_3, ...\} \tag{1}$$

be a number field, where operators can be applied to numbers for addition $(+)$ and multiplication $(\times)$ to produce a sum $(x_1 + x_2)$ and product $(x_1 \times x_2)$, respectively, because X satisfies the five number field axioms [1, 2, 3]

1. X permits an element 1, namely the *multiplicative unit*, such that $1 \times x_i = x_i \times 1$, $\forall x_i \in X$;

2. X permits an element 0, namely the *additive unit*, such that $0 + x_i = x_i + 0$, $\forall x_i \in X$;

3. X is *closed* under addition and multiplication, which indicates that the respective sums $(x_i + x_j)$ and products $(x_i \times x_j)$ between elements $x_i, x_j \in X$ produce all possible elements of X;

4. X's addition and multiplication are associative, such that $(x_i + x_j) + x_k = x_i + (x_j + x_k)$ and $(x_i \times x_j) \times x_k = x_i \times (x_j \times x_k)$, respectively; and

5. the combination of X's addition and multiplication is distributive, such that $(x_i + x_j) \times x_k = x_i \times x_k + x_j \times x_k$ and $x_i \times (x_j + x_k) = x_i \times x_j + x_i \times x_k$.

A dominant problem of pure mathematics, in the context of number theory, is to establish a universal number classification, such as the identification of all sets that occur with numeric field axioms. To attack this classification problem, extensive and rigorous studies have been conducted over the course of history [1, 2, 4, 5, 6, 7]. A major result of these studies is the creation of real numbers [8, 9, 10], complex numbers [11, 12], and quaternion numbers [13, 14] with all possible numeric fields [2]. Moreover, it is known that these encoding frameworks bare enormous scientific application to quantifiable and computational implementations of disciplines such as physics, chemistry, biology, and more.

As scientists, it is important to continue to investigate, scrutinize, challenge, and develop new theories and ideas, such as the triplex numbers of [15] and the Santillian iso-numbers of [16, 17, 18, 19]. Thus, in this preliminary paper, we pursue this "universal number classification beast" with an initial application of the Santillian iso-mathematics framework [2, 16, 17, 18, 19] to the triplex numbers [15] and the Inopin holographic ring [20]. In Section 2, we give a brief outline of Santilli's framework [2, 16, 17, 18, 19], the triplex numbers [15], and Inopin's holographic ring [20]. Subsequently, in Section 3, we begin to apply Santilli's framework [2, 16, 17, 18, 19] to the triplex numbers [15] and Inopin's holographic ring [20]. The paper concludes with Sections 4–5, where we briefly recapitulate the results and provide a thankful acknowledgment, respectively.

2 Alignment and background

Here, we prepare by aligning the reader with a background that highlights some aspects of Santilli's framework [2, 16, 17, 18, 19], the triplex numbers [15], and Inopin's holographic ring [20] that pertain to the application of Section 3.

2.1 Santilli's iso-mathematics framework

In [16, 17], Santilli reinspected the historical classification of sets verifying the numeric field axioms [1] and discovered that they equally authorize solutions for an arbitrary unit $\hat{\epsilon}$, generally outside the original fields [1], under the sole condition of being non-singular, and therefore invertible, such that $\hat{\epsilon} = \frac{1}{\hat{t}}$, provided that the conventional associative number multiplication $x_i \times x_j$ is replaced with the associativity-preserving form $x_i \hat{\times} x_j = x_i \times \hat{t} \times x_j$ under which $\hat{\epsilon}$ is indeed the right and left multiplicative unit. In [16, 17], Santilli then classified the new numbers depending on the main topological features of $\hat{\epsilon}$, such as:

1. $\hat{\epsilon}$ is single-valued and Hermitean for the case of *Santillian iso-numbers*;
2. $\hat{\epsilon}$ is single-valued and non-Hermitean for the case of *right Santillian geno-numbers*, characterized by $\hat{\epsilon}$, and *left Santillian geno-numbers* characterized by $\hat{\epsilon}^{\dagger}$; and
3. $\hat{\epsilon}$ is multi-valued and non-Hermitean for the case of *right Santillian hyper-numbers*, characterized by $\hat{\epsilon}$, and *left Santillian hyper-numbers* characterized by $\hat{\epsilon}^{\dagger}$.

Moreover, in [16, 17], Santilli identified the additional number series characterized by the anti-Hermitean image of the preceding generalized numbers via the iso-duality map that is denoted with the upper index $\hat{d} = \hat{\epsilon}^{\dagger}$ called *Santillian iso-dual iso-, geno-, and hyper-numbers.*

In general, Santilli [16, 17] successfully demonstrated that, in addition to conventional numbers, such axioms authorize the existence of four distinct iso-number classes [2]:

1. **Santillian iso-topic numbers ("iso-numbers")**
   - Iso-numbers exist because Santilli [16, 17] showed that the number field axioms [1] *do not* require that the multiplicative unit $\hat{\epsilon}$ is the number 1, so $\hat{\epsilon}$ can be any value provided that [2]:
     (a) the new unit $\hat{\epsilon}$ is positive ($\hat{\epsilon} > 0$) to permit the inverse $\hat{\epsilon} = \frac{1}{\hat{t}} > 0$,

(b) the multiplication $x_i \times x_j$ is changed to the form $x_i \hat{\times} x_j = x_i \times \hat{t} \times x_j = x_i \times \frac{1}{\hat{\epsilon}} \times x_j$, which is always associative, and

(c) the additive unit and its sum are kept unchanged.

- The number elements comprising X in eq. (1) are called *liftings*, such that $x_i = x_i \times \hat{\epsilon}$, which exist for the number field axioms [1] and are therefore numbers that are applicable to quantitative science [2]. The axiom of the multiplicative units is confirmed by the expression

$$1 \times x_i = 1 \times \left(\frac{1}{\hat{\epsilon}}\right) \times x_i = x_i \times \left(\frac{\hat{t}}{1}\right) \times 1 = x_i \times 1 \tag{2}$$

  is valid $\forall x_i \in X$, where X has been "lifted" to become the new iso-topic number set [2].

- The original number field axioms [1] are preserved for X for the new unit $\hat{\epsilon}$, namely the *Santillian iso-unit*, where the new multiplication $x_i \times x_j$ is termed *Santillian iso-multiplication* [2].

- The iso-multiplication led to additional refinement, identification, and usage [16, 17, 18] of the *iso-real numbers*, *iso-complex numbers*, and *iso-quaternion numbers* [2].

- Note the importance that if $x_i = 2$ and $x_j = 3$, then in general the iso-multiplication 2×3 yields a product that is *different* than 6 [2].

2. **Santillian geno-topic numbers ("geno-numbers")**

- Geno-numbers exist because, in addition to iso-numbers, Santilli [16, 17] showed that the number field axioms [1] *do not* require that the iso-multiplication operates on both the right and left directions because the axioms are also tested when all the multiplications (and sums) are restricted to operate right $x_i > x_j$ or to operate left $x_i < x_j$ [2].

- When Santilli [16, 17] restricted all operations to act on the right or left, he was able to construct two different sets $X_>$ and $X_<$ with corresponding units $\hat{\epsilon}_>$ and $\hat{\epsilon}_<$, and multiplications that are compatible with the respective units [2]. The new units $\hat{\epsilon}_>$ and $\hat{\epsilon}_<$ are called *Santillian geno-multiplication* on the right and left, respectively [2].

- The geno-multiplication led to additional refinement, identification, and usage [16, 17, 18] of the *geno-real numbers*, *geno-complex numbers*, and *geno-quaternion numbers* [2].

- Note the importance that if $x_i = 2$ and $x_j = 3$, then in general the geno-multiplications $2 > 3$ and $2 < 3$ yield different products that both differ from 6 because $2 > 3 \neq 2 < 3$ and $\hat{\epsilon} >\neq< \hat{\epsilon}$ [2].

3. **Santillian hyper-topic numbers ("hyper-numbers" or "hyper-Santillian numbers")**

- Hyper-numbers (which are not to be confused with so called hyper-mathematical structures that generally do not have units) exist because, in addition to the geno-numbers, Santilli [16, 17] showed that the multiplicative unit is not limited to a unique value because it can comprise a set of values, such as $\hat{\epsilon}_> = \{1, 1, 2, 3, 5, ...\}$ or $\hat{\epsilon}_> = \{2, \frac{4}{5}, 7, ...\}$, if the set is ordered and defined as being applicable to right or left [2].
- The hyper-multiplication led to additional refinement, identification, and usage [16, 17, 18] of the *hyper-real numbers*, *hyper-complex numbers*, and *hyper-quaternion numbers* [2].
- Note the importance that, in general, the hyper-multiplications $2 > 3$ and $2 < 3$ yield two distinct result sets that both differ from 6 [2].

4. **Santillian iso-dual numbers**

   - *Iso-dual numbers* exist because, in addition to iso-numbers, geno-numbers, and hyper-numbers, Santilli [16, 17] showed that the multiplicative unit $\hat{\epsilon}$ can be any (positive or negative) value except for zero (i.e. $-\hat{\epsilon}$) [2].
   - The "iso-dual" term identifies a duality between positive and negative units in accordance to the original number field axioms [2].
   - The *iso-dual multiplication* led to additional refinement, identification, and usage [16, 17, 18] *iso-dual iso-numbers*, *iso-dual geno-numbers*, and *iso-dual hyper-numbers* [2].

So in total, Santilli's axiomatic iso-mathematics framework [16, 17, 18, 19] reveals *eleven* new data structures [2]:

- iso-numbers,
- geno-numbers (right and left),
- hyper-numbers (right and left),
- conventional iso-dual numbers,
- iso-dual iso-numbers,
- iso-dual geno-numbers (right and left), and
- iso-dual hyper-numbers (right and left);

each data structure is applicable to the real [8, 9, 10], complex [11, 12], and quaternion numbers [13, 14], where each application bares an infinite number of possible units [2].

Subsequently, in a series of physical applications, Santilli [16, 17] then utilized the generalized iso-numbers, geno-numbers, and hyper-numbers to characterize the increasing complexities of matter with regard to non-linearity, non-Hamiltonian features, irreversibility, multi-valuedness, etc., while the iso-dual images were used by Santilli to characterize anti-matter under the corresponding complexity increases. For an in-depth explanation of this framework, we recommend a technical study of the original publications [16, 17, 18, 19].

2.2 Triplex numbers and Inopin's holographic ring

Chronologically, the establishment of the Inopin holographic ring [20] came before the triplex numbers [15]. Inopin's ring was initially introduced in the analytic quark confinement and baryon-antibaryon duality proof of [20]. This ring is a powerful tool because it is topological sphere with an "amplitude-radius" (or "amplitude-modulus") that serves as an *iso-metric*, which may be utilized to attack a wide range of mathematical and physical problems [15, 20].

Initially, in [20], Inopin's *1-sphere holographic ring* of amplitude-radius ϵ, which we term T^1, was used to topologically encode the *Time Zone* (or "non-linear time dimension" and "temporal distance scale"), such that eq. (15) of [20] defines

$$T^1 = \{\vec{x} \in X : |\vec{x}| = \epsilon\} \tag{3}$$

for the *Riemannian dual 3D space-time*, such that T^1 was iso-metrically embedded in a topological complex plane, namely X of eqs. (10–11) of [20], where $T^1 \subset X$. T^1 was equipped with topological deformation order parameter fields of fractional statistics for quasi-particles [20]. The *inside* of T^1 corresponds to an *interior* dynamical system of *superluminal quasi-particle spatial excitations*, namely the *Micro Space Zone* (or "short spatial distance scale") $X_- \subset X$ of eq. (16) in [20], while the *outside* of T^1 corresponds to an *exterior* dynamical system of *luminal or sub-luminal quasi-particle spatial excitations*, namely the *Macro Space Zone* (or "long spatial distance scale") $X_+ \subset X$ of eq. (17) in [20]—T^1 itself is populated with *luminal quasi-particle temporal excitations*, which is simultaneously dual to both X_- and X_+ as in eqs. (20–21) in [20]. Here, $X_-, T^1, X_+ \subset X$ are disjoint and comprise the complete X, such that $X = X_- \cup T^1 \cup X_+$ [20]. Eq. (7) in [15] identifies the complex number form

$$\vec{x} = \vec{x}_{\mathbb{R}} + \vec{x}_{\mathbb{I}} = (\vec{x}) = (|\vec{x}|, \langle\vec{x}\rangle) = (\vec{x}_{\mathbb{R}}, \vec{x}_{\mathbb{I}}, \vec{x}_Z), \tag{4}$$

where $\vec{x}$ is a complex location state in the complex location state space X. Eq. (4) complies with the complex number constraints of eqs. (2–6) in [15]. For this, each complex location $\vec{x} \in X$ was equipped with one or more complex order parameter field states in the generic form $\vec{\psi}(\vec{x})$ within a complex order parameter field state space $\Phi(\vec{x})$ for topological deformations, such that $\vec{\psi}(\vec{x}) \in \Phi(\vec{x})$—see eq. (20) in [15].

For the quark confinement proof of [20], the three distinct quark-antiquark pairs for a baryon-antibaryon pair are confined to T^1 in the upgraded Gribov vacuum on a six-coloring kagome lattice antiferromagnet with correlated order parameters. The transforming wavefunction states of T^1 are directly related to the states of X_- and X_+, which are 2-branes within 3-branes, so their 3D state space is directly inferred from the 2D state space of T^1. But at the time of writing [20], the triplex numbers with triplex multiplication did not yet exist, therefore it was not possible to fully encode the 3D state space of the 3-branes—this genuine need to employ 3D numbers to encode 3D states (with a pertinent 3D number multiplication) fueled the motivation for the triplex implementation of [15], which served as a topological upgrade to [20].

The triplex work of [15] was inspired by the emerging triplex number framework of P. Nylander and D. White [21, 22, 23]. Fortunately, after a thorough and rigorous investigation, the great "White-Nylander mythical beast" of [21, 22, 23] was destroyed through the creation of a well-behaved triplex algebra with triplex multiplication [15]. Thus, the 3D/triplex topological data structure Y was introduced, such that $X \subset Y$ [15]. For this, T^1 was generalized to Inopin's *2-sphere holographic ring*, which is defined in eq. (40) of [15] for the *Riemannian dual 4D space-time* as

$$T^2 = \{\vec{y} \in Y : |\vec{y}| = \epsilon\}, \tag{5}$$

such that T^2 was iso-metrically embedded in Y, where $T^2 \subset Y$ [15]. Hence, given that $X \subset Y$ and $T^1 \subset X \subset Y$, then

$$T^1 = T^2 \cap X, \tag{6}$$

so T^1 is a great circle or Riemannian circle of T^2 [15, 20]. So given $X \to Y$ and $X \subset Y$, T^2 delineates the dual interconnected spatial 3-branes $Y_- \subset Y$ and $Y_+ \subset Y$ in eq. (41) of [15], which supercede the spatial 2-branes $X_- \subset Y_- \subset Y$ and $X_+ \subset Y_+ \subset Y$, where T^2 is simultaneously dual to Y_- and Y_+ in eq. (44) of [15] for $Y = Y_- \cup T^2 \cup Y_+$, recalling that $Y_-, T^2, Y_+ \subset Y$ are disjoint [15]. Thus, eq. (35) in [15] defines a triplex number as

$$\vec{y} = \vec{y}_{\mathbb{R}} + \vec{y}_{\mathbb{I}} + \vec{y}_Z = (\vec{y}) = (|\vec{y}|, \langle\vec{y}\rangle, [\vec{y}]) = (\vec{y}_{\mathbb{R}}, \vec{y}_{\mathbb{I}}, \vec{y}_Z), \tag{7}$$

where $\vec{y}$ is a triplex location state in the triplex location state space Y. Eq. (7) complies with the triplex number constraints of eqs. (29–34) in [15]. In eqs. (70–71) of [15], the triplex multiplication between two distinct triplex numbers $\vec{y}_i, \vec{y}_j \in Y$ in the form of eq. (7) is defined, such that $\vec{y}_i \neq \vec{y}_j$, for the operation $\vec{y}_i \times \vec{y}_j$. Here, each triplex location $\vec{y} \in Y$ was equipped with one or more triplex order parameter field states in the generic form $\vec{\psi}(\vec{y})$ within a triplex order parameter field state space $\Phi(\vec{y})$ for topological deformations, such that $\vec{\psi}(\vec{y}) \in \Phi(\vec{y})$—see eq. (50) in [15]. For multiplying triplex locations and/or order parameter features, eqs. (70–71) in [15] define the requisite triplex multiplication.

Therefore, given that Santilli applied his iso-mathematics [16, 17] to the real [8, 9, 10], complex [11, 12], and quaternion numbers [13, 14], it seems important that Santilli's breakthroughs [16, 17, 18, 19] should also be applied to the triplex numbers [15], where the amplitude radius ϵ of both T^1 and T^2 serves as an iso-metric [15, 20].

3 Application

In Section 3.1, we attack our first objective by presenting an initial application of Santillian iso-numbers [16, 17, 18, 19] to the triplex numbers of [15]. Subsequently, in Section 3.2, we advance to our second objective, which is to provide a preliminary explanation on how Santilli's iso-numbers [16, 17] apply to the Inopin holographic ring of [20]. Consequently, based on these preceding results, we state our new hypothesis in Section 3.3.

3.1 Triplex numbers

Here, in the first phase, we begin to apply Santilli's iso-mathematics framework [16, 17, 18, 19] to the triplex numbers with triplex multiplication in [15]. Here, the initial objective is to demonstrate that Santilli's iso-numbers, geno-numbers, hyper-numbers, and iso-dual numbers apply to $\vec{y}_i, \vec{y}_j \in Y$.

First, for Santilli's iso-topic numbers [16, 17], we select some $\hat{\epsilon} > 0$ with corresponding inverse $\hat{t} = \frac{1}{\hat{\epsilon}}$, such that $\hat{t} > 0$. Next, the triplex multiplication of $\vec{y}_i \times \vec{y}_j$ in eq. (70) of [15] is changed to the Santillian form

$$\vec{y}_i \hat{\times} \vec{y}_j = \vec{y}_i \times \hat{t} \times \vec{y}_j = \vec{y}_i \times \frac{1}{\hat{\epsilon}} \times \vec{y}_j, \tag{8}$$

which is always associative, such that the additive unit and its sum are kept unchanged. At this point, Y's elements become liftings, such that

$$\begin{array}{rcl} \vec{y}_i &=& \vec{y}_i \times \hat{\epsilon} \\ \vec{y}_j &=& \vec{y}_j \times \hat{\epsilon}, \end{array} \tag{9}$$

where the axiom of multiplicative units is confirmed by the expressions

$$\begin{array}{rcccccc} 1 \times \vec{y}_i &=& 1 \times \left(\frac{1}{\hat{\epsilon}}\right) \times \vec{y}_i &=& \vec{y}_i \times \left(\frac{\hat{t}}{1}\right) \times 1 &=& \vec{y}_i \times 1 \\ 1 \times \vec{y}_j &=& 1 \times \left(\frac{1}{\hat{\epsilon}}\right) \times \vec{y}_j &=& \vec{y}_j \times \left(\frac{\hat{t}}{1}\right) \times 1 &=& \vec{y}_j \times 1 \end{array} \tag{10}$$

is valid $\forall \vec{y}_i, \vec{y}_j \in Y$, because Y has been "lifted" to become an iso-topic number set that generalizes eq. (2) to the triplex numbers of [15]. Therefore, the operation $\vec{y}_i \hat{\times} \vec{y}_j$ is known as the triplex version of Santillian iso-multiplication, so we have used the work of [16, 17] to identify *Santillian iso-triplex numbers* in accordance to [2].

Second, for Santilli's geno-topic numbers [16, 17], we establish that the right $\vec{y}_i > \vec{y}_j$ and left $\vec{y}_i < \vec{y}_j$ operations partition Y into the two different sets $Y_>$ and $Y_<$ with corresponding units $\hat{\epsilon}_>$ and $\hat{\epsilon}_<$ for Santillian geno-multiplication to identify *Santillian geno-triplex numbers* in accordance to [2]. Hence, the geno-multiplication applies to the triplex numbers of [15].

Third, for Santilli's hyper-topic numbers [16, 17], we clarify that the multiplicative unit for $\vec{y}_i \hat{\times} \vec{y}_j$ is not limited to a unique value because it can comprise an ordered set of values (i.e. recall $\hat{\epsilon}_> = \{1, 1, 2, 3, 5, ...\}$ or $\hat{\epsilon}_> = \{2, \frac{4}{5}, 7, ...\}$) since the set is applicable to right and left [2]. Thus, it is evident that the Santillian hyper-multiplication applies to the triplex numbers of [15] and identifies the pertinent *Santillian hyper-triplex numbers* in accordance to [2].

And finally, for Santilli's iso-dual numbers [16, 17], it is certainly feasible that the multiplicative unit $\hat{\epsilon}$ can be any value except for zero, so Santilli's method [16, 17] does apply to the triplex numbers of [15] to identify the relevant *Santillian iso-dual triplex numbers* in accordance to [2].

3.2 Inopin's holographic ring

Here, in the second phase, we begin to apply Santilli's iso-mathematics framework [16, 17, 18, 19] to Inopin's holographic ring in [20]. Recall from Section 2.2 that the Inopin holographic ring was first introduced in the quark confinement proof and topology of [20] as T^1 in eq. (3), and subsequently upgraded and refined in the complex and triplex encoding framework of [15] as T^2 in eq. (5).

First, given eq. (5), it is evident that T^2 is the multiplicative group of all non-zero triplex numbers with the positive-definite amplitude-radius $\hat{\epsilon}$, where we recall that T^2 is simultaneously dual to the two 3-branes, namely $Y_- \subset Y$ and $Y_+ \subset Y$ [15]. Immediately, we see the resemblance of Santilli's $Y_<$ and $Y_>$ to the corresponding Y_- and Y_+, with respect to the multiplicative unit $\hat{\epsilon}$ because we already have $\hat{\epsilon}_<$ and $\hat{\epsilon}_>$, respectively.

Hence, Y is the Riemann-Santilli iso-space over the *iso-triplex numbers* of Section 3.1 with a single geodesic for $T^1 \in T^2$, where it becomes evident that T^2 is in fact a *Riemann-Santilli iso-sphere* [16, 17]. How? Because in [15, 20], it was demonstrated that T^1 and T^2 are equipped with topological deformation order parameters to encode the complete elliptical state space projected in Y; this satisfies Santilli's proof that the infinite set of all possible ellipsoids arising from topologically-preserving deformations are mapped to a geodesic of T^1 in the iso-sphere T^2 in the Euclid-Santilli iso-space Y over the iso-triplex numbers (instead of the iso-reals as in [16, 17]). For this, the mechanism consists of embedding all such topological deformations $\hat{t}\delta$, such that $\delta = Diag(1, 1, 1)$ of the Euclidean metric is the iso-unit $\hat{\epsilon} = \frac{1}{\hat{t}}$, where T^2 is the perfect sphere, such that the amplitude-radius $\hat{\epsilon}$ is iso-one. Moreover, if T^2's metric is g, then all infinitely possible Riemannian metrics G can be factorized in the form $G = \hat{t} \times g$, where the reformulation of Y in terms of a Riemann-Santilli iso-space of iso-unit $\hat{\epsilon} = \frac{1}{\hat{t}}$ can be reduced to the study of one circle element, namely T^1, in the 2-sphere element, namely T^2, since all such circle elements are obtained via stereographic projection as in [20].

These above results suggest that all forms of T^1 may be unified into one single iso-linear form on the Riemann-Santilli iso-dual space of Y, where $Y_- \subset Y$ and $Y_+ \subset Y$ are iso-dual. In this context, the existence of the triplex numbers with multiplication in [15] apparently indicates the presence of compact and non-compact *Lie triplex algebras*, where simple Lie triplex algebras of dimension n may be unified into one single *Lie-Santilli triplex isotope* of dimension n.

3.3 Hypothesis

At this point we state our *new hypothesis*: the Inopin holographic ring $T^1 \subset T^2$ of [15, 20] may be one non-linear structure that satisfies the strict iso-curvature and iso-duality constraints of Santilli's iso-mathematics framework [16, 17, 18, 19] for $T^2 \in Y$, where $Y_- \subset Y$ and $Y_+ \subset Y$ may correspond to Santilli's interior and exterior dynamical systems, respectively.

4 Conclusion

In this work, we started by briefly discussing the importance and development of number systems in terms of science. We touched on the five original number field axioms [2, 3] and acknowledged the significance of identifying a universal number classification system. Subsequently, we identified Santilli's four distinct data structure classes, namely the iso-numbers, the geno-numbers, the hyper-numbers, and the iso-dual numbers, [2, 16, 17, 18, 19], which are pertinent to an application assessment of the triplex numbers in [15] and the Inopin holographic ring in [15, 20].

Next, we conducted a preliminary upgrade to the triplex numbers in [15] by engaging Santilli's four iso-number classes [2, 16, 17, 18, 19]. As a result, we found that it is possible to define iso-triplex numbers, geno-triplex numbers, hyper-triplex numbers, and iso-dual triplex numbers in a preparatory context because the triplex numbers [15] do comply with the constraints of Santilli's iso-number classes [2, 16, 17, 18, 19]. Additionally, we provided a preliminary explanation on how Santilli's iso-mathematics [16, 17, 18, 19] apply to the Inopin holographic ring in [15, 20]. These initial results seem to indicate that Santilli's iso-mathematics [2, 16, 17] will enhance the application and usability of Inopin's ring [15, 20] because the ring's amplitude-radius and Santilli's positive-definite multiplicative unit (for the iso-multiplication, iso-curvature, and iso-duality) may in fact be *equivalent* and thus *share identical roles and functional dependencies*. From this observation, we hypothesize that Inopin's ring [15, 20] may be one non-linear structure that satisfies the strict iso-curvature and iso-duality constraints of Santilli's iso-number theory [16, 17], where the Micro Space Zone and Macro Space Zone 3-branes of [15, 20] may correspond to Santilli's interior and exterior dynamical systems, respectively.

Therefore, in order to test the validity our new hypothesis in Section 3.3, we propose that a thorough and rigorous mathematical investigation should be conducted along this research trajectory.

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