AMAZING FACTS IN GENARAL RELATIVITY

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Abstract
This article seeks to investigate certain astonishing issues in relation to General Relativity—(1)The gravitational effects of electric charges(2)Issue of the OPERA neutrinos in a relativity-consistent manner as far as possible.
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INTRODUCTION

The Reissner-Nordstrom Metric involves the effect of charge as well as mass in curving the space around it. To what extent is the electric charge itself responsible for the curvature of space that is, how does it contribute towards “Gravity”? We may easily derive the following result from the RN metric—that the gravitational potential of the charge falls off as the square of the distance of the charge, the constant of proportionality being much smaller than the value of G. The electric charge itself exerts a “Repulsive Gravity” which gets masked by the attractive influence of the mass of the particle.. Regarding the OPERA neutrinos: Are they really super luminal in case the experimental results are correct are accurate enough? There is always a possibility of explaining these results within the confines of relativity if we consider the non-local issue—the speed of a moving particle being observed at a finite distance from the observation point.

SECTION A

[Electric Charges and their Gravitational Effects]

We start of with the Reissner-Nordstrom metric[1]:

$$c^2 d\tau^2 = \left(1 - \frac{r_s}{r} + \frac{{r_Q}^2}{r^2}\right) c^2 dt^2 - \left(1 - \frac{r_s}{r} + \frac{{r_Q}^2}{r^2}\right)^{-1} dr^2 - r^2 (d\theta^2 + Sin^2\theta d\phi^2) \text{ ------------- (A1)}$$

Where,

$${r_Q}^2 = \frac{Q^2 G}{4\pi\varepsilon_0 c^4}$$

$$r_s = \frac{2GM}{c^2}$$

If the mass(M) is allowed to tend to zero the metric reduces to:

$$c^2 d\tau^2 = \left(1 + \frac{{r_Q}^2}{r^2}\right) c^2 dt^2 - \left(1 + \frac{{r_Q}^2}{r^2}\right)^{-1} dr^2 - r^2 (d\theta^2 + Sin^2\theta d\phi^2) \text{ -------------- (A2)}$$

In the above metric, the coefficients are independent of “t” and “φi”
Therefore we have two Killing vectors[2]:

$(t,r,\theta,\varphi)=(1,0,0,0)$
$(t,r,\theta,\varphi)=(0,0,0,1)$
The corresponding conservation laws [3]are:

$$\left(1+\frac{r_Q{}^2}{r^2}\right)c^2\frac{dt}{d\tau}=k \text{ -------------------- (A3)}$$

$$r^2 Sin^2\theta\frac{d\phi}{d\tau}=l \qquad \text{--------------------- (A4)}$$

The first one[eqn (3)] relates to the conservation of energy
The second one[equation (4)] relates to the conservation of angular momentum
From the conservation of angular momentum we may conclude that the motion of the orbiting particle is confined to a plane. We consider the plane of the motion to be the equatorial plane. The axes are oriented so that the plane of the orbiting particle/[moving particle] coincides with the x-y plane. That is, $\theta=\pi/2$ for all points on the equatorial plane.

Therefore $d\theta=0$ for all such points.

And

$Sin\theta=1$

The metric expressed by relation (2) reduces to:

$$c^2d\tau^2=\left(1+\frac{r_Q{}^2}{r^2}\right)c^2dt^2-\left(1+\frac{r_Q{}^2}{r^2}\right)^{-1}dr^2-r^2d\phi^2$$

Dividing both sides of the above relation by $d\tau^2$, we have,

$$c^2=\left(1+\frac{r_Q{}^2}{r^2}\right)c^2\left(\frac{dt}{d\tau}\right)^2-\left(1+\frac{r_Q{}^2}{r^2}\right)^{-1}\left(\frac{dr}{d\tau}\right)^2-r^2\left(\frac{d\phi}{d\tau}\right)^2$$

Using relations(2) and (3) in the above , we obtain:

$$c^2=\left(1+\frac{r_Q{}^2}{r^2}\right)^{-1}\frac{k^2}{c^2}-\left(1+\frac{r_Q{}^2}{r^2}\right)^{-1}\left(\frac{dr}{d\tau}\right)^2-\frac{l^2}{r^2}$$

Or,

$$c^2\left(1+\frac{r_Q{}^2}{r^2}\right)=\frac{k^2}{c^2}-\left(\frac{dr}{d\tau}\right)^2-\left(1+\frac{r_Q{}^2}{r^2}\right)\frac{l^2}{r^2} \text{ ------------------ (A5)}$$

$$\frac{k^2}{c^2}=\left(\frac{dr}{d\tau}\right)^2+\left(1+\frac{r_Q{}^2}{r^2}\right)\left(c^2+\frac{l^2}{r^2}\right)$$

Subtracting c^2 from both sides and multiplying both sides by ½ we have,

$$\frac{1}{2}\left(\frac{k^2}{c^2}-c^2\right)=\frac{1}{2}\left(\frac{dr}{d\tau}\right)^2+\frac{1}{2}\left(1+\frac{r_Q{}^2}{r^2}\right)\left(c^2+\frac{l^2}{r^2}\right)-\frac{1}{2}c^2 \quad \text{------------------- (A6)}$$

On simplification we have:

$$\frac{1}{2}\left(\frac{k^2}{c^2}-c^2\right)=\frac{1}{2}\left(\frac{d\,r}{d\tau}\right)^2+\left(\frac{Q^2G}{4\pi\varepsilon_0c^4r^2}\right)+\frac{1}{2}\left(\frac{Q^2Gl^2}{4\pi\varepsilon_0c^2r^4}\right)+\frac{l^2}{2r^2} \quad \text{-------------------- (A7)}$$

The Left-hand side may be identified a energy per unit rest mass."k" has the unit of speed squared. The first term on the right hand side is the kinetic energy term[per unit rest mass]. The other terms represent what we may interpret as the effective potential[Gravitational] due to electric charge.

The effective potential is given by:

$$V_{eff}=\left(\frac{Q^2G}{4\pi\varepsilon_0c^4r^2}\right)+\frac{1}{2}\left(\frac{Q^2Gl^2}{4\pi\varepsilon_0c^2r^4}\right)+\frac{l^2}{2r^2} \quad \text{-------------------- (A8)}$$

All the terms are positive . Therefore the potential is of repulsive nature.
A similar treatment for the Schwarzschild metric yields:

$$V_{eff}=-\frac{m}{r}-\frac{ml^2}{r^3}+\frac{l^2}{2r^2} \quad \text{---------------------- (A9)}$$

Where, $m->\dfrac{GM}{c^2}$

The first two terms represent attractive potential while the last term represents repulsive potential. The repulsive potential exists only for non-zero angular momentum. We have a repulsive "centrifugal barrier" of finite height in the general relativity [for mass without charge] Incidentally the classical centrifugal barrier is of infinite height[4]. It gets stunted due to GR effects.

But for a charge there is no centripetal admittance as a counterpart to the centrifugal barrier since all the terms in the effective potential expression are positive.

Incidentally we may repeat the previous calculations in consideration of the RN metric in its entirety instead of allowing the mass to tend to zero. Let's go through the main steps.

We start with equation (1). The metric is independent of t and φ. Again we have the following Killing vectors:
(t,r,θ,φ)=(1,0,0,0)
(t,r,θ,φ)=(0,0,0,1)
The corresponding conservation laws are:

$$\left(1-\frac{r_s}{r}+\frac{r_Q{}^2}{r^2}\right)c^2\frac{dt}{d\tau}=k$$

$$r^2 Sin^2\theta\frac{d\phi}{d\tau}=l$$

The above equations are the counterparts of equations(A3) and (4)

$$c^2=\left(1-\frac{r_s}{r}+\frac{r_Q{}^2}{r^2}\right)^{-1}\frac{k^2}{c^2}-\left(1-\frac{r_s}{r}+\frac{r_Q{}^2}{r^2}\right)^{-1}\left(\frac{dr}{d\tau}\right)^2-\frac{l^2}{r^2} \quad \text{----------------------- (A11)}$$

$$\frac{1}{2}\left(\frac{k^2}{c^2}-c^2\right)=\frac{1}{2}\left(\frac{dr}{d\tau}\right)^2+\frac{1}{2}\left(1-\frac{r_s}{r}+\frac{r_Q{}^2}{r^2}\right)\left(c^2+\frac{l^2}{r^2}\right)-\frac{1}{2}c^2 \quad \text{----------------- (A12)}$$

$$\frac{1}{2}\left(\frac{k^2}{c^2}-c^2\right)=\frac{1}{2}\left(\frac{dr}{d\tau}\right)^2+\frac{1}{2}\left(\frac{r_s c^2}{r}-\frac{r_s l^2}{r^3}+\frac{c^2 r_Q{}^2}{r^2}+\frac{r_Q{}^2 l^2}{r^4}+\frac{l^2}{r^2}\right) \quad \text{------------ (A13)}$$

OBSERVATIONS:
In the above the contribution of mass and charge towards gravitation occur independent/separate terms. We may hope to extend the series to incorporate the contribution of other fundamental properties of matter towards gravitation. But there should be background support of the metric which should include properties other than mass and electric charge [ex: strong charge].The aim would be to create a unified metric to explain gravitation as elaborately as possible along with a formula of the type A13 if possible.. One may also try to think in the reverse direction as to how matter itself may have a direct contribution towards electromagnetic phenomena another phenomena. Now a direct look at the metric A1 gives us a clear that for r satisfying notion that for r satisfying: $\frac{r_s}{r}=\frac{r_Q{}^2}{r^2}$ we have flat spacetime . For very small values of r the $1/r^4$ term should dominate and the repulsive gravity of the charge should exceed the attractive gravity of mass. The chargeless neutrino is a suitable candidate[test particle] for such an experience[or investigation] in the vicinity of an electron.

SECTION B.
[The OPERA Neutrinos]

The speed of light in vacuum is a universal constant. Let us review this fact in the light of the following situation. Three different points A,B and C are considered. They are at a finite separation form each other. The observers at A and B see a ray of light flashing across a small spatial interval at C.
The metric[5][6]:

$$ds^2 = g_{tt}dt^2 - g_{11}dx^2 - g_{22}dy^2 - g_{33}dz^2 \text{ ----------------- (B1)}$$

We may write,

$$ds^2 = dT^2 - dL^2$$

Where,

$$dT = \sqrt{g_{tt}}\,dt \text{ and } dL = \sqrt{g_{11}dx^2 - g_{22}dy^2 - g_{33}dz^2}$$

dT is the propertime for an observer fixed at a spatial location.
Importance of the Quantity dT:
$ds^2 = dT^2 - dL^2$
For a null geodesic: $ds^2=0$
Therefore:
$dT^2 = dL^2$
$=> \frac{dL}{dT} = 1$ [c=1 in the natural units]
dT is consistent with the fact that speed of light is locally a constant.

The physical distance dL is the same for all observers in the reference frame. For the situation in consideration, the quantities g(tt),g(11) g(22) and g(33) relate to the point C.
The time interval observed by A:
$dT_A = \sqrt{g_{00}(A)}\,dt$
The time interval observed by B:
$dT_B = \sqrt{g_{00}(B)}\,dt$

The time interval according to C:
$dT_C = \sqrt{g_{00}(C)}\,dt$
The time interval recorded by the three observers are different[in this case for the light ray flashing across C]
Speed of light as observed from A:

$$v_A = \frac{dL}{\sqrt{g_{00}(A)}dt} \quad \text{------------ (B2)}$$

Speed of light as observed from B:

$$v_B = \frac{dL}{\sqrt{g_{00}(B)}dt} \quad \text{------------- (B3)}$$

Local speed of light as observed from C:

$$v_C = \frac{dL}{\sqrt{g_{00}(C)}dt} = c \quad \text{------------ (B4)}$$

We have the following relations:

$$\frac{v_A}{v_C} = \frac{\sqrt{g_{00}(C)}}{\sqrt{g_{00}(A)}}$$

$$v_A = \frac{\sqrt{g_{00}(C)}}{\sqrt{g_{00}(A)}} \times v_C = \frac{\sqrt{g_{00}(C)}}{\sqrt{g_{00}(A)}} \times c \quad \text{----------- (B5)}$$

$$\frac{v_B}{v_C} = \frac{\sqrt{g_{00}(C)}}{\sqrt{g_{00}(B)}}$$

$$v_B = \frac{\sqrt{g_{00}(C)}}{\sqrt{g_{00}(B)}} \times v_C = \frac{\sqrt{g_{00}(C)}}{\sqrt{g_{00}(B)}} \times c \quad \text{----------- (B6)}$$

$$\frac{v_A}{v_B} = \frac{\sqrt{g_{00}(B)}}{\sqrt{g_{00}(A)}} \quad \text{--------------- (B7)}$$

Instead of a light ray we may consider a particle moving across a small spatial interval at C. For such a case we write:

$$v_{CP} = v[Local(P)]$$
$$v_{AP} = v[Non-Local(A,P)]$$
$$v_{BP} = v[Non-Local(B,P)]$$

In the suffixes AP,BP and CP A,B and C denote locations and P stands for particle

$$v_{AP} = \frac{\sqrt{g_{00}(C)}}{\sqrt{g_{00}(A)}} \times v_{CP} = \frac{\sqrt{g_{00}(C)}}{\sqrt{g_{00}(A)}} \times v[Local(C,P)] \quad \text{------------------- (B8)}$$

$$v_{BP}=\frac{\sqrt{g_{00}(C)}}{\sqrt{g_{00}(B)}}\times v_{CP}=\frac{\sqrt{g_{00}(C)}}{\sqrt{g_{00}(B)}}\times v[Local(C,P)] \text{ ------------------ (B9)}$$

The local speed of the particle is less than the local speed of light.

$v[Local(C,P)]<c$

Therefore we have the following:

$$v_{AP}=\frac{\sqrt{g_{00}(C)}}{\sqrt{g_{00}(A)}}\times v_{CP}=\frac{\sqrt{g_{00}(C)}}{\sqrt{g_{00}(A)}}\times v[Local(C,P)]<\frac{\sqrt{g_{00}(C)}}{\sqrt{g_{00}(A)}}\times c \text{ ----- (B10)}$$

But

$$\frac{\sqrt{g_{00}(C)}}{\sqrt{g_{00}(A)}}\times c = non-local-speed-of-light$$

Therefore,

$$v_{AP}<Non-Local-Speed-of-Light \text{ -------------- (B11)}$$

The non-local speed of the particle as observed from A is less than the non local speed of light, the observation point being A. *The particle is never, never getting superluminal*. The light ray is always ahead of the particle. Only the local barrier may be exceeded for non-local observations, the point of observation and the observation-station being separated by a finite distance. The principle of causality is there in tact. The OPERA neutrinos should fall into this category if the experimental results are accurate enough. There is no violation of Special relativity.

I have provided below some sample calculations in relation to a homogeneous non-rotating earth. Special relativity effects in relation to rotation have been considered later.

B1. SAMPLE CALCULATIONS

Approximate length of path traversed by the OPERA[7] neutrinos: 732km=732000m

Latitude at Geneva:46^{o}N 12E

Latitude at San Grosso: 42^0 N 18E

I have taken an average value latitude of 44^{0}N for my Sample Calculations

Theta=90^0-44^0=46^0

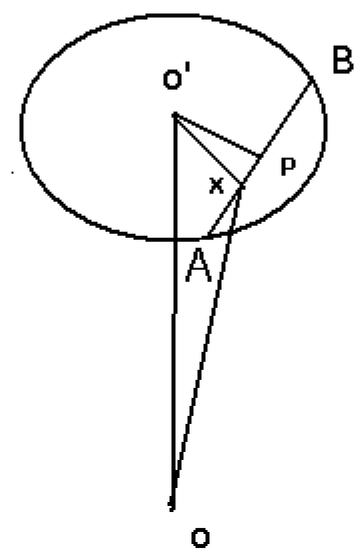
We are writing the Schwarzschild metric in consideration of a non-rotating earth

$$ds^2=(1-\frac{2GM}{c^2r})dt^2-(1-\frac{2GM}{c^2r})^{-1}dr^2-r^2(d\theta^2+Sin^2\theta d\phi^2)$$

Theta changes as the particle moves along AB

The picture is like this:We have two locations [observation stations] at a particular latitude on the earth's surface.. The radius of this small circle=RSinθ , where R is the coordinate radius of the earth. As the neutrino passes from A to B the distance, the distance [coordinate –distance]of some intermediate point on the path from the center of the earth is our participating radius=r’ .The local speed of the neutrino is its speed at some intermediate point(X) wrt to an observer at rest at that point. The non-local speed of the neutrino is its speed as observed from A or B.

The coordinate distance of the center of the path[AB] from the axis[center of the small circle] is denoted by a. In the figure below we have the small circle with its center O’ on the axis:



With reference to the diagram above:

O: Center of the earth.

O’: Center of the Small Circle

O’P=a [It is the radius of the small circle for $\theta=46^0$]

OA=Radius of the earth[the line has not been drawn]

AB is the straight line path from Geneva to Gran Sasso[Italy]. P is the center of the path AB. X is an arbitrary point on the path. A and B are the observation stations.X is the position of the spatially fixed observer at X[an arbitrary point on the path over which the neutrinos pass]. In our calculations we will compare the observations of X with those of A and B

Angle OO'P=θ.

Theta changes as the particle moves along AB. The value of theta at A and B is 46^0

Now

Angle PO'X=φ

O'X=aSec(φ)

PX=a tan(φ)

OO'=RCos46^0

Participating Radius: $r'^2 = OO'^2 + O'X^2$

$$r' = \sqrt{R^2 Cos^2 46^0 + a^2 Sec^2(\varphi)} \quad \text{-----------} \text{ (B12)}$$

Sin(θ)=O'X/XO=

$$Sin(\theta) = \frac{aSec(\varphi)}{\sqrt{R^2 Cos^2 46^0 + a^2 Sec^2(\varphi)}} \quad \text{-------------} \text{ (B13)}$$

$$\tan(\theta) = \frac{aSec(\varphi)}{RCos(46^0)} \quad \text{--------------} \text{ (B14)}$$

$$Sec^2(\theta)d\theta = \frac{aSec(\varphi)\tan(\varphi)}{RCos(46^0)}d\varphi$$

$$d\theta = \frac{aSec(\varphi)\tan(\varphi)}{(1+\tan^2(\theta))RCos(46^0)}d\varphi$$

$$d\theta = \frac{RCos46^0 \times aSec(\varphi)\tan(\varphi)}{[(aSec(\phi))^2 + (RCos(46^0))^2]}d\varphi$$

$$r'^2 d\theta^2 = a^2 RCos46^0[(aSec(\phi))^2 + (RCos(46^0))^2]Sec^2(\varphi)\tan^2(\varphi)d\varphi \text{ ------------ (B15)}$$

$$r'^2 Sin^2(\theta)d\varphi^2 = a^2 Sec^2\varphi d\varphi^2 \text{ ------------------------ (B16)}$$

Local time interval[interval measured by an imaginary observer at X:an arbitrary point on the path AB] for the neutrino as it passes under the ground:

$$dT[Local] = \left(1 - \frac{2GM'}{c^2 r'}\right)^{1/2} dt$$

M' is the Mass is the gravitating mass involved at the particular instant concerned, when the neutrino is moving through the earth and "r' is the participating radius. The extent of this gravitating mass is determined by the participating radius r'.[Gauss law for Gravitation has been assumed in the weak field context]

$$M = \frac{4}{3}\pi R^3 \rho$$

$$M' = \frac{4}{3}\pi r'^3 \rho$$

In the above relations:

R: Coordinate radius of the earth

r': Participating Radius[Coordinate]

ρ: Constant density[=mass per *coordinate* volume]

[A constant Coordinate density has been assumed]

$$\frac{M'}{M} = \frac{R^3}{r'^3}$$

$$M' = \frac{R^3}{r'^3} M$$

[r' is the participating radius at some specific point of the path.]

Local Speed of neutrino:

$$v = \frac{dL}{\left(1 - \frac{GM'}{c^2 r'}\right)^{1/2} dt}$$

Non-Local time interval considered from the observation stations:

$$dT[Non-Local] = \left(1 - \frac{2GM}{c^2 R}\right)^{1/2} dt$$

R: Coordinate Radius of the earth

Non-Local Speed, ie, speed wrt the observation stations.=

$$v[Non-Local] = \frac{dL}{\left(1 - \frac{2GM}{c^2 R}\right)^{1/2} dt}$$

$$v[Non-Local] = \frac{\left(1 - \frac{2GM'}{c^2 r'}\right)^{1/2}}{\left(1 - \frac{2GM}{c^2 R}\right)^{1/2}} \times v[local]$$

The integral is to be computed between the endpoints of the path at Geneva and Gran Sasso
Now,

Observed Time of Travel:

$$T = \int \left(\frac{\left(1 - \frac{2GM}{c^2 R}\right)^{1/2} dx}{\left(1 - \frac{2GM'}{c^2 r'}\right)^{1/2} v[Local]} \right) \text{------------------ (B17)}$$

The limits of integral correspond to the end points of the path.

$$dx^2 = (1 - \frac{2GM'}{c^2 r'})^{-1} dr'^2 + r'^2 Sin^2\theta d\phi^2 + r'^2 d\theta^2$$

The coordinate values are identical with the flat spacetime coordinates and so we may apply Pythagoras theorem with them without any error or approximation.

$$2a^2 Sec^2\phi \tan\phi d\phi = 2r'dr'$$

$$r'^2 dr'^2 = a^4 Sec^4\phi \tan^2\phi d\phi^2$$

$$dr'^2 = \frac{a^4 Sec^4\phi \tan^2\phi d\phi^2}{r'^2}$$

$$dr'^2 = \frac{a^4 Sec^4\phi \tan^2\phi d\phi^2}{[aSec(\phi)]^2 + [RCos(46^0)]^2}$$

$$dx^2 = (1 - \frac{2GM'}{c^2 r'})^{-1} \frac{a^4 Sec^4\phi \tan^2\phi d\phi^2}{r'^2} + r'^2 Sin^2\theta d\phi^2 + r'^2 d\theta^2$$

$$dx = \sqrt{(1 - \frac{2GM'}{c^2 r'})^{-1} \frac{a^4 Sec^4\phi \tan^2\phi d\phi^2}{r'^2} + r'^2 Sin^2\theta d\phi^2 + r'^2 d\theta^2}$$

$$T = \int \left(\frac{\left(1 - \frac{2GM}{c^2 R}\right)^{1/2} \sqrt{(1 - \frac{2GM'}{c^2 r'})^{-1} \frac{a^4 Sec^4\phi \tan^2\phi d\phi^2}{r'^2} + r'^2 Sin^2\theta d\phi^2 + r'^2 d\theta^2}}{\left(1 - \frac{2GM'}{c^2 r'}\right)^{1/2} v[Local]} \right)$$

$$T = \int \left(\frac{\left(1 - \frac{2GM}{c^2 R}\right)^{1/2} \sqrt{\left(1 - \frac{2GM[(aSec(\phi))^2 + (RCos(46^0))^2]}{c^2 R^3}\right)^{-1} \frac{a^4 Sec^4 \phi \tan^2 \phi}{[(aSec(\phi))^2 + (RCos(46^0))^2]} + a^2 RCos 46^0 [(aSec(\phi))^2 + (RCos(46^0))^2] Sec^2(\varphi) \tan^2(\varphi) + a^2 Sec^2 \varphi}}{\left(1 - \frac{2GM[(aSec(\phi))^2 + (RCos(46^0))^2]}{c^2 R^3}\right)^{1/2} v[Local]} \right) d\phi$$

-------------------------------------- (B18)

Limits of Integration:

$$\varphi_1 = \frac{365}{6400} = -0.07928 \text{ radian}$$

$$\varphi_2 = \frac{365}{6400} = +0.07928 \text{ radian}$$

$$\frac{2GM}{c^2} = 8.550 \times 10^{-3}$$

[M=5.972×10^{24}kg;G=6.67×10^{-11}SI Units]

From Schwarzschild's metric we have,

$$R[Physical] = \sqrt{R^2 - 2mR} + 2m \ln[\sqrt{R} + \sqrt{R - 2m}] + C$$

R[Physical]=6400Km

We evaluate the above from limits2m to r[coordinate] and then add 2m to get the physical radius=6400km. Solving :

$$\sqrt{r^2 - 2mr} + 2m \ln[\sqrt{r} + \sqrt{r - 2m}] - 2m \ln \sqrt{2m} + 2m = 6400000$$

$$[\, m = \frac{GM}{c^2} \,]$$

We get r=6399.999km~6400km

a^2=[6400Sin46^0]^2-[730/2]^2

$=2.106\times10^{7}km^{2}$

$=2.106\times10^{13}m^{2}$

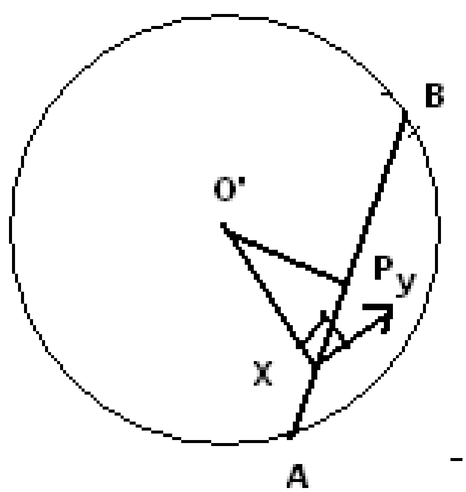
a=4589 km

In the above relation [in the evaluation of a, Pythagoras theorem has been assumed in the weak field limit.

B2. SPECIAL RELATIVITY EFFECTS

[Due to Rotation]

We begin our discussion with reference to the diagram below:



AB is the path from Geneva to San Grosso[730 km]. A constant latitude=44^{0} has been assumed. Corresponding $\theta=46^{0}$.P is the mid point of the path AB and X is any arbitrary point on the path. Angle PO'X=φ. Speed of the earth at X is=ω×O'X along XY [XY is normal to the direction of O'X. Speed of the earth along the path at X= ω×O'X×cos(φ). But O'X×cos(φ)=O'Y=a.
Therefore speed of the earth point along AB=ωa which is a constant along the path.there will be no length contraction along the path wrt to A or B.
The speed of the earth point perpendicular to AB=ω×O'X×sin(φ)=ω×XP=ω×a× tan(φ) which is not constant along the path. This would cause the effect of time dilation.
Lets consider a stationary observer at X. The time dilation wrt to an inertial frame at X is given by:

$$\gamma = \sqrt{1 - \left(\frac{a \times \omega \times \tan(\varphi)}{c}\right)^2}$$

The value of γ at A and B is: $\gamma_0 = \sqrt{1 - \left(\frac{a \times \omega \times 0.7924}{c}\right)^2}$

The integrand gets magnified by a factor: $\frac{\gamma_0}{\gamma}$

The value of $c=3\times10^8$m/s

But the result of integration(numerical)is not changing in an appreciable manner.

With all these considerations the integral B18 works out to 729570/v[local]

That gives a huge value of 1.4 microseconds early arrival[wrt to light]. Small variations in values can convert this to a late arrival. These are sample calculations to highlight that pseudo-superluminal motion, consistent with relativity can explain the OPERA results.

The difference is due to the following reasons:

1. A fixed intermediate latitude has been considered.

2 Gauss Theorem for GR has been considered in view of the weak nature of the field.

3.Approximate use of the Pythagoras theorem in curved spacetime[Schwarzschild]

4.The earth has been considered as an exact sphere. Approximate values of $c=3*10^8$ and R[physical]=6400km have been considered.

5. Gravitational Interaction between the neutrinos and the electronic charge[in close proximity] should be considered statistically. One may get an indication of this from the first section[A]

6. A direct consideration of the Kerr metric can improve the calculations in relation to rotation[frame dragging etc]

B3.NON-LOCAL SPEEDS AND THE WEAK NATURE OF THE EARTH'S FIELD

Non-local speeds in relation to curved space has been demonstrated by the theoretical estimation and subsequent measurement of the speed of the satellites moving round the earth. They are constrained move round the earth by the curvature of spacetime and not by the "force" of gravity producing centripetal acceleration. The weak field nature of the earth should not be under-estimated. If the field[its curvature] was not there the satellites should have moved along a straight line with constant speed.

It's the spacetime Geometry of the earth that causes rivers to flow producing hydroelectricity. The same spacetime property causes landslides to come down. The weak nature of the earth's field has immense capabilities.

B4.OBSERVATIONS:

It is the enormity of the results that have raised eye-brows in regard of the OPERA experiments. Otherwise there would have been no reason for skepticism. Any effort to explain the results in conformity with Special Relativity should improve their credibility in the eyes of physicists ,opening up doors of further research.

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