POLARISATION OF LIGHT

PACS numbers: 03.50.De; 42.60.Jf; 42.25.Ja
DOI: 10.1070/QE2012v042n12ABEH014850

On the possibility of an experiment on 'nonlocality' of electrodynamics

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Abstract. **It has been known since the 19th century that a circularly polarised electromagnetic wave carries an angular momentum. A simple experiment (Righi, 1882) apparently indicates that the angular momentum is distributed over the entire cross section of the beam. According to some modern ideas, the angular momentum of the beam with the given polarisation is localised near the beam 'surface' and represents a spin of photons, while the energy in the beam is distributed throughout its cross section, which is inconsistent with the principle of locality. For the experimental determination of the localisation of the angular momentum, we propose a new scheme, in which we study the interference pattern of two coherent circularly polarised beams. Each beam first passes through a half-wave plate, one of the plates being divided into two coaxial parts. With (manual) rotation of one parts of the plate we change the frequency of the light passing through it: the plate absorbs the momentum and, therefore, work is done. This change in frequency should cause a movement of the interference fringes and show the distribution of the angular momentum over the beam cross section.**

Keywords: *angular momentum of a light beam, electrodynamic torque, classical spin, interferometer.*

1. Introduction

It is well known that a beam of electromagnetic radiation with circular polarisation [1, 2],

$$\boldsymbol{E} = \omega \exp(\mathrm{i}kz - \mathrm{i}\omega t)[\boldsymbol{x} + \mathrm{i}\boldsymbol{y} + \frac{1}{k}\boldsymbol{z}(\mathrm{i}\partial_x - \partial_y)]u(x,y),$$

$$\boldsymbol{B} = -\mathrm{i}k\boldsymbol{E}/\omega \qquad (1)$$

(the expression is written for the right-hand circular polarisation), carries an angular momentum [1 – 7]. Therefore, the body, which absorbs at least a portion of the beam and/or changes the state of its polarisation, will be subjected to a torque.

The electromagnetic field (1) satisfies the wave equation, which is widely used in the paraxial approximation. This approximation suggests a slow change in the intensity of the beam along its axis ($\partial_z u \ll ku$) and leads to the equation $\partial^2_{xx}u + \partial^2_{yy}u + \mathrm{i}2k\partial_z u = 0$ [2]. In analogy with [1, 5, 6], we consider a wide beam (1) and assume that the amplitude u is constant in the central part of the beam ($u = u_0$) and vanishes in a narrow surface layer at a distance R from its axis (see Figs 1a and 9.3 from [5], and Fig. 1 from [6]).

Beth's experiment [3] and many modern experiments with microparticles [2, 7] confirm the existence of an angular momentum in a circularly polarised beam. Theoretically, this issue was also discussed in papers [8 – 10]. Unfortunately, there are no known experiments in which the distribution of the angular momentum is determined by the beam cross section. However, it is this distribution that is of special interest because of the following.

According to papers [2, 4], the z-component of the angular momentum volume density j_z and the z-component of the angular momentum flux density along the z axis, i.e., the component of the torque density μ_z, are localised near the beam 'surface' and are given by

$$j_z = -\varepsilon_0\omega r\partial_r|u(r)|^2/2, \quad \mu_z = -c\varepsilon_0\omega r\partial_r|u(r)|^2/2 \qquad (2)$$

(by the beam 'surface' is meant a layer in which the radial intensity gradient is very large). These densities are proportional to the radial gradient of the beam intensity, while the energy density w and the Poynting vector $\boldsymbol{S}$ depend on the intensity itself:

$$w = \varepsilon_0\omega^2|u|^2, \quad S = c\varepsilon_0\omega^2|u|^2. \qquad (3)$$

Therefore, the ratio of densities

$$\frac{j_z}{w} = \frac{\mu_z}{S} = -\frac{r\partial_r|u(r)|^2}{2\omega|u(r)|^2} \qquad (4)$$

should vary significantly in the beam cross section.

Allen and his co-authors write: 'Consequently, in a beam that satisfies the paraxial condition, this means that the ratio changes from place to place' [2, p. 300]. 'A different amount of angular momentum might be expected to be transferred at different positions in the wavefront' [11, p. 70]. 'At a particular local point the z-component of angular momentum flux divided by energy flux does not yield a simple value' [7].

Simmonds and Guttman write: 'The skin region of the [beam] is the only place in which the z-component of the angular momentum does not vanish' [5].

Thus, $|\mu_z/S| \gg 1/\omega$ in the surface layer and $\mu_z/S = 0$ at all other points. Hence, it is natural to conclude that a body absorbing the beam under consideration experiences a torque only in places where the surface layer of the beam is absorbed, and most of the inner region of the absorber does not experi-

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Received 18 March 2012; revision received 27 July 2012
Kvantovaya Elektronika **42** (12) 1133 – 1136 (2012)
Translated by I.A. Ulitkin

ence the torque, although according to (3), it absorbs all the power of the beam.

However, Beth [3] explained the emergence of the current torque in his own way: 'The moment of force or torque exerted on a doubly refracting medium by a light wave passing through it arises from the fact that the dielectric constant $\hat{K}$ is a tensor. Consequently the electric intensity $\boldsymbol{E}$ is, in general, not parallel to the electric polarisation $\boldsymbol{P}$ or to the electric displacement $\boldsymbol{D} = \hat{K}\boldsymbol{E} = \boldsymbol{E} + 4\pi\boldsymbol{P}$ in the medium. The torque per unit volume produced by the action of the electric field on the polarisation of the medium is $\boldsymbol{\tau}/V = \boldsymbol{P} \times \boldsymbol{E}$.' According to this reasoning, the torque is distributed evenly over the entire cross section of the beam.

Carrara [12] also wrote: 'If a circularly polarised wave is absorbed by a screen or is transformed into a linearly polarised wave, the angular momentum vanishes. Therefore the screen must be subjected to a torque per unit surface equal to the variation of the angular momentum per unit time. The intensity of the torque is $\pm S/\omega$.'

Loudon [13] is a supporter of the concept described by formulas (2) and (4). Nevertheless, he takes into account the term $\boldsymbol{P} \times \boldsymbol{E}$ in the calculation of the impact of the beam on a dielectric {see equation (7.18) in [13]}.

Feynman et al. [14] used the concept of the spin of photons with circular polarisation of light: '... the resultant electric vector $\boldsymbol{E}$ goes in a circle – as drawn in Fig. 17-5(a). Now suppose that such light shines on a wall which is going to absorb it – or at least some of it – and consider an atom in the wall according to the classical physics... The net result is that the electron moves in a circle, as shown in Fig. 17-5(b). The electron is displaced at some displacement $\boldsymbol{r}$ from its equilibrium position at the origin and goes around with some phase lag with respect to the vector $\boldsymbol{E}$. The relation between $\boldsymbol{E}$ and $\boldsymbol{r}$ might be as shown in Fig. 17-5(b). As time goes on, the electric field rotates and the displacement rotates with the same frequency, so their relative orientation stays the same. Now let's look at the work being done on this electron. The rate that energy is being put into this electron is v, its velocity, times the component of $\boldsymbol{E}$ parallel to the velocity: $\mathrm{d}W/\mathrm{d}t = eE_t v$.

But look, there is angular momentum being poured into this electron, because there is always a torque about the origin. The torque is $\tau = eE_t r$, which must be equal to the rate of change of angular momentum $\mathrm{d}J_z/\mathrm{d}t$:

$$\mathrm{d}J_z/\mathrm{d}t = \tau = eE_t r$$

Remembering that $v = \omega r$, we have that $\mathrm{d}J_z/\mathrm{d}W = 1/\omega$.'

Thus, according to Feynman the density of the torque μ_z refers to the energy flux density on the absorbing surface S in the same way as the net torque refers to the net energy flux and the spin of the photon $\hbar$ refers to the photon energy $\hbar\omega$:

$$|\mu_z/S| = |\mathrm{d}J_z/\mathrm{d}W| = |j_z/w| = 1/\omega. \tag{5}$$

To this end, the density of the torque is constant on the absorbing surface within the illuminated area, and not localised on the boundary of this area, as follows from (2).

In the spring of 1999 the problem of the angular momentum distribution over the cross section of a circularly polarised beam was discussed at the All-Moscow Seminar on Theoretical Physics headed by V.L. Ginzburg and was formulated in terms of a possible experiment [8]. Later, the problem was analysed in detail theoretically in [10].

The analysis consisted in the following. Suppose that the absorber is divided coaxially at a radius $r_1 < R$ on the inner ($r < r_1$) and outer ($r > r_1$) parts so that the surface layer of the light beam is absorbed by the outer part. The question is: Will the inner part experience the action of the torque (and rotate)? This question is crucial.

Indeed, if the inner part does not experience a torque, the spin angular momentum of the photons is absorbed in the periphery of the absorber, while the energy of the photons is absorbed by the inner part. If the inner part of the absorber experiences a torque, it would contradict formulas (2) and (4). In any case, it is interesting to investigate this problem experimentally, because both possible answers suggest a significant 'nonlocality' of electrodynamics. The scheme of the corresponding experiment is proposed and discussed in this paper.

2. The Righi experiment (1882)

Let us consider, as in Beth's experiment [3], instead of an absorbing body, a half-wave plate, which changes the handedness of the circular polarisation into the reversed one, so that the plate experiences the torque density $\mu = 2\mu_z$. In the Righi experiment described in [15], the plate was rotated by hand (in the plane of the plate) with angular velocity Ω. Thus, work was done with the beam, which led to a change in the photon energy. A change in the photon energy means a change in the frequency of light and results in the movement of the interference fringes in the corresponding interference experiment. Interestingly, this effect can be observed in the experiment on a student optical bench with a Fresnel biprism [15].

The change of the Poynting vector $\Delta S = 2\mu_z\Omega$ causes a shift in frequency

$$\Delta\omega = \omega\frac{\Delta S}{S} = 2\Omega\omega\frac{\mu_z}{S}, \tag{6}$$

where ω is the angular frequency of light. The corresponding phase shift for the time t is $\Delta\varphi = \Delta\omega t$; the phase shift per revolution of the plate ($t = 2\pi/\Omega$) has the form

$$\Phi = 4\pi\frac{\mu_z}{S}\omega, \tag{7}$$

and the interference pattern is shifted with the number of fringes

$$N = 2\frac{\mu_z}{S}\omega. \tag{8}$$

According to the concept described by equation (2), the fringes should not shift in the inner part of the illuminated plate, because $\mu_z/S = 0$ in this region, while at the same time an extremely large shift ($N \gg 1$) should be observed in a narrow region of absorption of the surface layer of the beam, because $|\mu_z/S| \gg 1/\omega$ in this region.

3. Modification of the experiment

We hope to answer the question posed in [8], by observing the local shift of the interference fringes (8). To do this, we will use in a two-beam interferometer two half-wave plates, one of

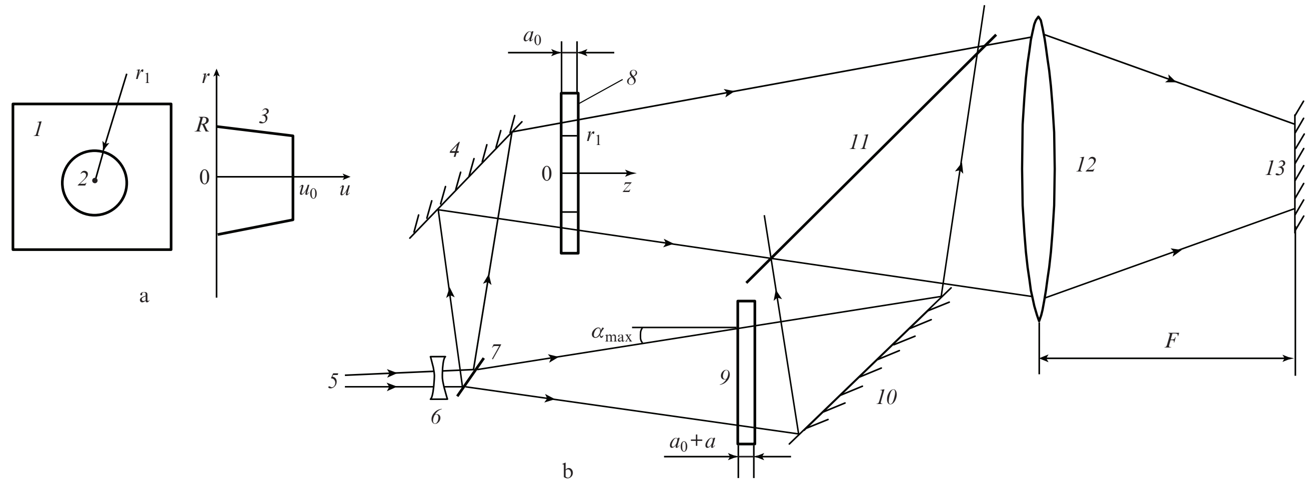

Figure 1. (a) Half-wave plate whose parts can be rotated by hand and (b) scheme of the experimental setup: (*1*) outer part of the plate; (*2*) inner part of the plate; (*3*) beam field profile; (*4*, *10*) mirrors; (*5*) laser beam; (*6*) diffuser; (*7*, *11*) semi-transparent mirrors; (*8*) first half-wave plate whose parts are rotated by hand; (*9*) second half-wave plate; (*12*) collecting lens; (*13*) screen, where the two beams are superimposed.

which is divided into the inner part (in the form of a disk) and the outer annular part (Fig. 1a). For the experiment to be performed, it is necessary to provide independent manual rotation of the two parts of the plate. The half-wave plates are varied in thickness by a small value a. Because of this difference, the interference fringes are observed on the screen where the two beams are superimposed (Fig. 1b).

The calculation of the difference of the optical paths is shown in Fig. 2. If α is the angle of incidence of light, the optical path ABC is equal to $an/\cos\beta + a(\tan\alpha - \tan\beta)\sin\alpha$ (n is the refractive index), and the corresponding path AD through the air is equal to $a/\cos\alpha$. The condition of constructive interference is given by $an/\cos\beta + a(\tan\alpha - \tan\beta)\sin\alpha - a/\cos\alpha = m\lambda$, i.e.

$$n\cos\beta - \cos\alpha = m\lambda/a, \quad m = 0,1,2,\dots \,. \tag{9}$$

If $\sin\alpha \approx \alpha$, and $\cos\alpha \approx 1 - \alpha^2/2$, equation (9) yields

$$n - 1 + \alpha^2(n-1)/(2n) = m\lambda/a. \tag{10}$$

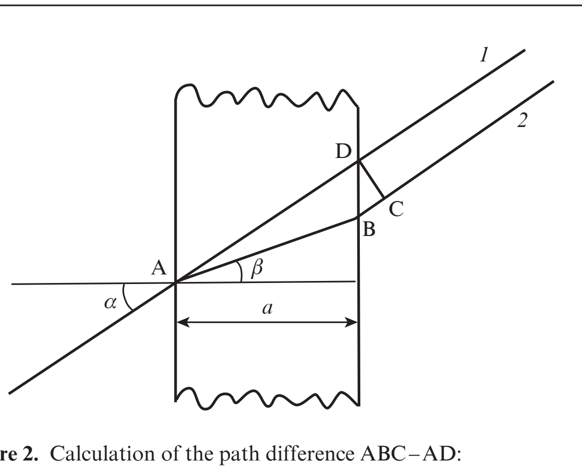

Figure 2. Calculation of the path difference ABC–AD: (*1*) beam passing through the layer of thickness a through air near the first plate, and (*2*) beam passing through the layer of thickness a through the second plate.

Omitting the constant term $n - 1$, we obtain the angular size of the ring with the number m

$$\alpha_m = \sqrt{\frac{2n\lambda m}{(n-1)a}}\,. \tag{11}$$

Let $\lambda = 630$ nm and a quartz half-wave plate be used, i.e., $n = 1.55$, $\Delta n = n_o - n_e = 0.009$. Then, the minimum thickness of the half-wave plate, at which the handedness of the circular polarisation is reversed, is equal to $l_{1/2} = \lambda/(2\Delta n) = 35$ μm. If we put $a = 17\, l_{1/2} = 595$ μm, then $\alpha_m = 0.0772\sqrt{m}$ and $m_{max} \leqslant 167\alpha_{max}^2$. According to Fig. 1b, the angle $\alpha_{max} \approx 10° = 0.175$; therefore, $m_{max} = 5$. These five rings are shown in Fig. 3.

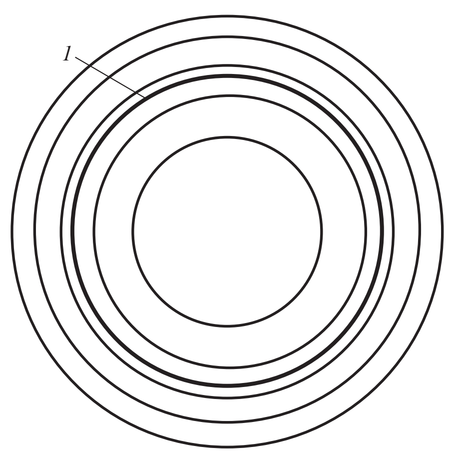

Figure 3. Interference fringes: (*1*) interface between the inner and outer parts of the half-wave plate.

According to (5), we expect the shift of the interference fringes (8) to be equal to 2, when the inner part of the plate makes a complete revolution. According to (4), we expect a large shift of the fringes on the edge of the illuminated area when the outer part of the plate is rotated. As far as we can judge by the report [15], the shift of the interference fringes in the inner illuminated area really was 2 per revolution of the undivided plate. In this case, perhaps, a large shift of the fringes at the boundary of the illuminated region was unnoticed.

Acknowledgements. I am deeply grateful to Prof. Robert H. Romer for valiant publishing of my question [8] (submitted on 7 October 1999) and to Prof. Timo Nieminen who drew my attention to paper [15].

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