PHYSICS OF THE HILBERT BOOK MODEL

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Preface

I started the Hilbert Book Model during my studies in physics in the sixties on the Technical University of Eindhoven (TUE).

In the first two years the lectures concerned only classical physics. In the third year quantum physics was introduced. I had great difficulty in understanding why the methodology of doing physics changed drastically. So I went to the teacher, which was an old nearly retired and very wise professor and asked him:

"Why is quantum mechanics done so differently from classical mechanics?".

His answer was short. He stated":

"The reason is that quantum mechanics is based on the superposition principle".

I quickly realized that this was part of the methodology and could not be the reason of the difference in methodology. So I went back and told him my concern. He told me that he could not give me a better answer and if I wanted a useful answer I should research that myself. So, I first went to the library, but the university was quite new and its library only contained rather old second hand books, which they got as a gift from other institutions. Next I went to the city's book shops. I finally found a booklet from P. Mittelstaedt: (Philosophische Probleme der modernen Physik,

BI Hochschultaschenbücher, Band 50, 1963) that contained a chapter on quantum logic.

It learned me that small particles appear to obey a kind of logic that differs from classical logic. As a result their dynamic behavior differs from the behavior of larger objects, which obey classical logic. I concluded that this produced the answer that I was looking for.

I searched further and encountered papers from Garret Birkhoff and John von Neumann that explained the correspondence between

quantum logic and separable Hilbert spaces. That produced a more conclusive answer to my question.

The lectures also told me that observables were related to eigenvalues of Hermitian operators. These eigenvalues are real numbers. However, it was clearly visible that nature has a 3+1D structure. So I tried to solve that discrepancy as well. After a few days of puzzling I discovered a number system that had this 3+1D structure and I called them compound numbers. I went back to my professor and asked him why such compound numbers were not used in physics. Again he could not give a reasonable answer.

When I asked the same question to a much younger assistant professor he told me that these numbers were discovered more than a century earlier by William Rowan Hamilton when he was walking with his wife over a bridge in Dublin. He was so glad about his discovery that he carved the formula that treats the multiplication of these numbers into the sidewall of the bridge. The inscription has faded away, but it is now molded in bronze and fixed to the same wall by Hamilton's students. The numbers are known as quaternions. So, I went to the library and searched for papers on quaternions.

In those years Constantin Piron wrote his papers on the number systems that can be used by Hilbert spaces. Piron discovered that only members of suitable division rings can be used as coefficients in linear combinations of Hilbert vectors in separable Hilbert spaces. Division rings comprise real numbers, complex numbers and quaternions. That information completed my insight in this subject. I finalized my physics study with an internal paper on quaternionic Hilbert spaces.

The university was specialized in applied physics and not in theoretical physics. This did not stimulate me to proceed with the subject. Next, I went into a career in industry where I used my

knowledge of physics in helping to analyze intensified imaging and in assisting with the design of night vision equipment and X-ray image intensifiers. That put me with my nose on the notion of quanta.

The output window of image intensifiers did not show impinging radiation waves. Instead they showed clouds of impinging quanta. In those times I had not much opportunity to deliberate on that fact. However, after my retirement I started to rethink the matter. That was the instant that the Hilbert Book Model project was started.

In 2009 I started the Hilbert Book Model project. The HBM is a very simple model of physics that is completely deduced and only covers the lowest levels of fundamental physics. For that reason it is strictly based on a solid foundation. For that foundation I choose the lattice structure of traditional quantum logic. The lattice structure of this logic system is isomorphic to the lattice structure of the set of closed subspaces of a separable Hilbert space.

Since neither the logic system nor the Hilbert space can represent dynamics, a full dynamics model is based on an ordered sequence of such static sub-models. This sequence shows great similarity with the set of pages of a book. This has led to the name "Hilbert Book Model"

Thus, in a few words: The Hilbert Book Model tries to explain the existence of quanta. It does that by starting from traditional quantum logic.

You will find the model to be in many aspects controversial and non-conventional. That is why the author took great efforts in order to keep the model self-consistent.

Due to a series of new concepts that are introduced by the HBM and the fact that they lend themselves for a rather pictorial description, will physicists that support conventional physics experience the HBM as a kind of Alice's wonderland. In this manuscript, all of these new concepts will be introduced in a cautious and trustworthy manner. The methodology will directly or indirectly base on the selected foundation.

The main purpose of the Hilbert Book Model is to get insight into the possibilities of the physical toolkit.

Each time that I read this book I encounter small and sometimes big inconsistencies. When I see them I repair them. Due to my sloppy nature there must still be a lot of them left. I apologize to the reader for this inconvenience. I do not consider myself a good and precise mathematician and I consider myself as a horrible physicist. The Great Creator must be a lot better. For a better manuscript you better invite Him. He constructed this structure.

If a mathematical theory is self-consistent, then there is a realistic chance that nature somewhere somehow uses it.

If that theory is compatible with traditional quantum logic, then there is a much larger *chance that nature will use it.*

This drives my intuition.

No model of physics can change physical reality.
Any view on physical reality involves a model.
Drastically different models can still be consistent in themselves.

The Hilbert Book Model is a simple self-consistent model of physics.
This model steps with universe-wide progression steps from one sub-model to the next one. Each of these sub-models represents a static status quo of the universe. The sub-models are strictly based on traditional quantum logic

The HBM is a pure quaternion based model. Conventional physics is spacetime based. When both models are compared, then the progression quantity (which represents the page number in the Hilbert Book model) corresponds to proper time in conventional physics.

In the HBM all proper time clocks are synchronized.
The length of a smallest quaternionic space-progression step in the HBM corresponds to an "infinitesimal" coordinate time step in conventional physics.

HvL

Ir J.A.J. van Leunen

PHYSICS OF THE HILBERT BOOK MODEL

ACKNOWLEDGEMENTS

DETAILS

The Hilbert Book Model is the result of a still ongoing research project.

That project started in 2009.

The continuing status of the project can be followed at
http://www.e-physics.eu

The author's e-print site is:
http://vixra.org/author/j_a_j_van_leunen.
This book is accompanied by a slide show at
http://vixra.org/abs/1302.0125
or
HBM slides
Use a PowerPoint viewer for this .pptx file

The nice thing about laws of physics is that they repeat them-selves. Otherwise they would not be noticed. The task of physicists is to notice the repetition.

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1. Introduction

I present you my personal view on the lower part of the hierarchy of objects that occur in nature. Only fields and elementary particles are treated in some detail. Composite particle objects are treated in a general way. Cosmology is touched.

For the greater part, the model is deduced. For that reason the model is founded on a solid and well accepted foundation. That foundation is traditional quantum logic.

The model does not aim at experimental verification of its results, but it uses experimentally verified results of physics as a guidance. The model uses mathematical tools for extending the foundation. In some cases "new" mathematics is applied.

The paper is founded on three starting points:
- A sub-model in the form of traditional quantum logic that represents a static status quo.
- A correlation vehicle that establishes sufficient cohesion between subsequent members of a sequence of such static sub-models.
- The cosmological principle.

Further it uses a small set of hypotheses. It turns out that the cosmological principle is already a corollary of the first two points.

The correlation vehicle must provide sufficient cohesion between the subsequent members of the sequence. The cohesion must not be too stiff otherwise no dynamics will take place.

The cosmological principle means that at large scales, universe looks the same for whomever and wherever you are. One of the consequences is that at larger scales universe possesses no preferred directions. It is ***quasi-isotropic*** (on average isotropic).

The mathematical concepts are treated in more detail in the second part, which is called Q-formulæ

The HBM refines quantum logic to Hilbert Logic. A Hilbert logic system resembles a separable Hilbert space much closer than quantum logic does. Together with quantum logic this refined logical system represents a new hierarchy that introduces nature's building blocks and their constituents.

The paper explains[1] all features of fundamental physics that are encountered in the discussed hierarchy which ranges from propositions about physical objects until elementary particles and their composites. Amongst them are the cosmological principle, the existence of quantum physics, the existence of a maximum speed of information transfer, the existence of ultra-high frequency carrier waves, the existence of physical fields and how they are generated, the origin of curvature, the origin of inertia, the dynamics of gravity, the existence of elementary particles, the existence of generations of elementary particles, the existence of the Pauli principle and the history of the universe and the consequences of color confinement.

The HBM introduces a new way in which potentials are formed by wave fronts that together form ultra-high frequency carrier

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[1] Or it indicates a possible explanation

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waves[2]. It considers photons and gluons as modulations of these carrier waves. The carrier waves cannot be observed directly.

New mathematics is involved in the dynamic generation of potential functions.

On the other hand the current HBM does not explore further than composites that are directly constructed from elementary particles. It only touches some aspects of cosmology.

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[2] Despite the fact that there are great similarities, these ultra-high frequency waves must be distinguished from the well-known UHF radio waves.

2 The Book Model

The name of the research project that treats this investigation is due to the main starting point on which this study is based.

The Hilbert Book Model (HBM) is based on the assumption[3] that universe steps with universe wide progression steps and that the static status quo of each of these steps can be described by an infinite dimensional separable Hilbert space and its Gelfand triple. With other words a dynamic model will consist of an ordered sequence of these static sub-models.

The sequence of the static models show similarity with the sequence of pages in a book. That is why the name "Hilbert Book Model" is selected for the project.

The progression step size τ_s defines an ultra-high frequency ν_u, which represents the ***basic carrier frequency*** for transport of information.
This document does not produce an estimate of τ_s.

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[3] This assumption is derived from the fact that the foundation on which the HBM is based cannot represent dynamics.

3 General remarks

3.1 Why logic can be used as foundation

In no way a model can give a precise description of physical reality. At the utmost it presents a correct view on physical reality. But, such a view is always an abstraction.

Physical reality is very complicated. It seems to belie Occam's razor. However, views on reality that apply sufficient abstraction can be rather simple and it is astonishing that such simple abstractions exist. Complexity is caused by the number and the diversity of the relations that exist between objects that play a role. A simple model has a small diversity of its relations.

Mathematical structures might fit onto observed physical reality because its relational structure is isomorphic to the relational structure of these observations.

The part of mathematics that treats relational structures is lattice theory[4]. Logic systems are particular versions of lattice theory. Classical logic has a simple relational structure[5]. However since 1936 we know that physical reality cheats classical logic. Since then we think that nature obeys quantum logic, which has a much more complicated relational structure. Mathematics offers structures that are lattice isomorphic to quantum logic. One of them is a separable Hilbert space.

The conclusion of this deliberation is that physical reality is not based on mathematics, but that it happens to feature relational structures that are similar to the relational structure that some mathematical constructs have. That is why mathematics fits so well in the formulation of physical laws. Physical laws formulate repetitive relational structure and behavior of observed aspects of nature.

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[4]The German name for lattice theory is Theorie der Verbände.

[5] It can be represented by Venn diagrams.

3.2   Completely deduced model

The Hilbert Book Model is completely deduced. In general physicists tend not to trust completely or largely deduced models. For that reason the HBM is based on a solid foundation, which is extended by using trustworthy mathematical tools. We want to understand the physics of the developed model. For that reason the first priority of the HBM is to understand how this model works and it is not considered her primary task to verify whether nature behaves that way. This is compensated by pursuing a strong degree of self-consistence of the model. At the same time the knowledge of how nature works is a guide in the development of the model.

For example the HBM uses proper time instead of coordinate time. Proper time is a Lorentz invariant measure of progression. The corresponding clock ticks at the location of the observed item. Our common notion of time is coordinate time. The coordinate time clock ticks at the location of the observer. The HBM adds to this fact that all proper time clocks are synchronized. The HBM does not bother about the fact that in general proper time cannot practicably be measured.

Together with a fixed maximum speed of information transfer this selection of the progression parameter renders most formulas automatically Lorentz invariant.

Further, the model includes lower level objects that cannot be observed as individuals. Only as groups these objects become noticeable behavior.

The result is that the HBM introduces its own methodology that often deviates considerably from the methodology of contemporary physics. The advantage is that this approach enables the researcher to dive deeper into the undercrofts of physics than is possible with conventional methodology.

As a consequence the HBM must be reluctant in comparing these methodologies and in using similar names. Confusions in discussion groups about these items have shown that great care is necessary. Otherwise, the author can easily be accused from stealing ideas from other theories that are not meant to be included in the HBM model.

This again will make it difficult to design measurements. Measuring methods are designed for measuring physical phenomena that are common in contemporary physics. This is best assured when is sought for phenomena that are similar between the model and contemporary physics. This action contradicts the caution not to use similar terms and concepts. This is the main reason why the HBM does not make experimental verification to its first priority.

On the other hand, also contemporary physics contains items that cannot be measured. For example color charge is an item that cannot (yet) be measured. Due to color confinement, quarks have never been detected as separate objects[6].

As indicated above, proper time is a concept that also exists in contemporary physics, but in general it cannot be measured.

Contemporary physics uses the field concept, but except for the cases that the fields are raised by properties of separate particles contemporary physics does not bother what causes the field.

3.3   Generators, spread and descriptors.

The HBM allows very pictorial representations of its fundamental concepts. Let me give you a small preview. Later in this manuscript

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[6] Isolated quarks have a very short live.

these subjects will cautiously be derived from the selected foundation.

In the model, generators produce coherent groups of discrete objects that are spread over a sub-regions of an embedding continuum. The object density distribution and the current density distribution of these coherent groups are continuous functions that describe and categorize these groups.

Depending on a dedicated Green's function, the distributions of discrete objects also correspond to potential functions.

Due to the way in which the gravitation potential is generated, this potential function corresponds to a local curvature of the embedding space.

This can be comprehended when the groups are generated dynamically in a rate of one element per progression step. During its very short existence the element transmits a wave front[7] that slightly folds and thus curves the embedding space. The wave front keeps floating away with light speed from its previous source. It represents a trace of the existence of the element. This trace survives the element when that element is long gone. These traces can be observed without affecting the emitter.

For each coherent group, the elements are generated at a rate of one element per progression step. With other words the wave fronts form ultra-high frequency waves that move with light speed away from their source.

However, each wave front is emitted at a slightly different location. Already at a small distance it appears as if they originate from

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[7] For anisotropic elements the message is transmitted by an anisotropic wave.

the same center location. The transport of the wave fronts is controlled by the Huygens principle.

The coherent group forms a building block. The emitted wave fronts together constitute the potential functions of this building block.

The elements act as step stones and together they form a micro-path for the corresponding group. This micro-movement can be considered as a combination of a quasi-oscillation and a quasi-rotation. The micro-path is completed in a fixed number N_w of progression steps. After the completion a new set is generated that has the same statistical characteristics. It is impossible to know precisely where the next step stone will be located. In a running micro-path it is impossible to detect the start and the end of the path. Only the cycle time $\tau_c = N_w \cdot \tau_s$ is fixed.

Thus, indirectly, the generator influences space curvature. The descriptors only describe the influence of the potentials on the local space curvature.

The ultra-high frequency wave cannot be observed. Only its averaged effect is observable. The resulting potential is an integral and therefore a rather static effect. Modulations of this wave, which are due to oscillations of the emitter, can be observed. These modulation waves possess a much lower frequency than the ultra-high frequency carrier wave has. Photons are examples of modulations of the ultra-high frequency carrier waves. Their emission and absorption take N_w progression steps.

The implementation of the element generator can be described by the convolution of a sharp continuous function and a low scale stochastic spatial spread function that blurs the continuous function.

In this way, the spreading part can be seen as the activator of local space curvature, while the derivative of the sharp part defines a local metric that can be considered as the descriptor of the local curvature.

The two parts must be in concordance. In this way, two kinds of descriptors of local curvature exist. The first is the density distribution that describes the stochastic spatial spread of the discrete objects. It corresponds to a potential function. The second descriptor is the local metric. Since these functions act on different scales, they can usually be treated separately.

The ***origin of the local curvature*** is the dynamic stochastic process that produces the low scale spread of the discrete objects. As described above these objects transmit wave fronts that curve the local space. The HBM suggests the combination of a Poisson process that is coupled to a binomial process, where the attenuation of the binomial process is implemented by a 3D spread function[8]. The stochastic generator process will generate according to a standard plan.

In principle, at each location where it is active the generator produces locally the same kind of patterns. In undisturbed (natal) format, these patterns may only differ in their symmetry properties.

However, these patterns cause space curvature. The local curvature is generated by the considered group and by neighboring groups.

Due to an existing uniform move of the building block and due to the variance in space curvature, the center location of the pattern may

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[8] See: The enumeration process.

become displaced and the pattern becomes distorted. Both effects disturb the natal state of the distributions that are generated by the generating process.

Since the patterns are generated with a single element per progression step, the generation poses a large chance to not generate the target natal shape but instead a distorted shape that in addition is spread over the path that the center location decides to follow.

The produced distribution can still be described by a continuous function, but that function will differ from the continuous function that describes the undisturbed natal state.

So the generation process is characterized by two functions. The first one represents the characteristics of the local generation process. It describes the natal state of the intended distribution. It is more a prospector than a descriptor.

The second one describes the actually produced distribution that is distorted by the local space curvature and spread out by the movement of the center location.

Further the generation of the distribution may not be completely finished, because not enough elements were generated since the generation of the pattern was started.

The generated element only lives during the current progression step. In the next step a newly generated element replaces the previous object.

At any instant the generated distribution consists of only one element. Thus for its most part the distribution can be considered as a set of virtual elements that lived in the past or will live in the future.

The virtual distribution together with its current non-virtual element represents a pattern. The local curvature is partly caused by the pattern itself, but for another part it is caused by neighbor patterns.

The previous description of the natal generation can be imagined visually.

At a rate of one element per progression instant the generator produces step stones that are used by the generated building block.

The step stones are located randomly in a coherent region of 3D space.

The building block walks along these step stones. As a consequence even at rest the building block follows a stochastic micro-path.

Any movement of the building block as a whole, will be superposed on the micro-path.

At every arrival at a step stone, the building block transmits its presence via a wave front that slightly folds and thus curves the embedding continuum.

These wave fronts and the transmitted content constitute the potentials of the building block.

Nobody said that the undercrofts of physics behave in a simple way!

3.3.1   Generation and annihilation cycle

Generation and annihilation of elementary particles and emission and absorption of photons take a fixed number N_w of progression

steps. This number is set by the cycle time of the generation mechanism. This cycle time $\tau_c = N_w \cdot \tau_s$ also determines the duration of other processes, such as the absorption and emission of photons.

This fact also means that all Qpatterns of the same generation contain N_w step stones. It also means that probably any electric charge of a building block is evenly spread over N_w elements[9].

A strong distortion of the planned step stone distribution can start the annihilation cycle of the corresponding particle. This can occur by violent movements and/or by strong space curvature.

Due to conservation laws, with elementary particles annihilation and creation always occurs together for a pair of a particle and an anti-particle.

3.4   Fundamental particles

The HBM takes color confinement as a serious restriction. As a consequence not all elementary particles can be generated as individual particles[10].

Quarks can only appear combined in hadrons.

For that reason the HBM introduces the category of fundamental particles. This category concerns particles that in one coherent cycle are generated by the generation mechanism. (Each such cycle takes a fixed number of progression steps).

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[9] For generations this assumption means that since charges are the same between different generations, N_w must be the same for different generations.

[10] Isolated quarks have a very short live

Inside fundamental particles no interactions take place that are observable from the outside.

3.5 Coupling and events

The HBM introduces the notion of *coupling* of fields. It also means that non-coupled fields exists. Coupling is described by a *coupling equation*, which is a special kind of *differential continuity equation[11]*.

Coupling takes place between *stochastic fields*. Stochastic fields describe *density distributions* and *current density distributions* of *lower order objects*. The distributions are generated by a local generation process that in each progression step produces ONE lower order object per stochastic field.

Coupling is implemented by messages that are transmitted in the embedding continuum by the active elements of the distribution via wave fronts[12] that slightly fold and thus curve this continuum. Together these wave fronts constitute the potentials that are raised by the distribution. It is sensible to presume that the element generator reacts on the potentials that are active in that location.

When the particle is annihilated, the coupling stops. This also means that no further wave fronts are generated that contribute to the potential. However, the existing wave fronts keep flowing away from their original source. They keep extending their reach with light speed. With these wave fronts the potential flees away.

In order to keep the considered group coherent, an inbound or outbound micro-move must on average be followed by a move in a reverse direction. This must hold separately in each spatial dimension.

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[11] See: Coupling

[12] For anisotropic elements the message is transmitted by an anisotropic wave.

Thus in each spatial dimension a kind of quasi oscillation takes place. The synchronization of this quasi oscillation may differ per dimension. In a similar way a quasi-rotation can exist. A certain kind of coupling of fields may be based on induced synchronization of these quasi oscillations and quasi-rotations.

Coupling becomes complicated when it involves coupling dependencies that live in different dimensions. Such cases can no longer be solved by separating the problem per dimension. It also means that the problem is inherently quaternionic and cannot be solved by simple complex number based technology. This occurs in the coupling equation of elementary particles where two quaternionic functions are coupled that belong to different discrete symmetry sets. Dirac has solved this problem by applying spinors and Dirac matrices. The HBM solves this with quaternionic methodology. The HBM applies special indices that identify discrete symmetry flavors.

The wave fronts that constitute the potentials of the building blocks are non-coupled fields. When the source oscillates then these ultra-high frequency carrier waves get modulated. The corresponding modulation frequency is much lower than the carrier frequency[13]. Photons are examples of these modulated waves.

3.6 Wave particle duality

The HBM offers a simple explanation for wave particle duality.

A point-like object can hop along a stochastically distributed set of step stones that together form a micro-path. The step stones form a coherent distribution that can be described by a continuous object density distribution. Via a properly selected Green's function the step stone distribution can also be converted into a potential function.

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[13] Atoms implement these modulations in a peculiar way.

Each suitable Green's function corresponds to a corresponding potential function.

A direct conversion from density distribution to a potential function is also possible and also uses a dedicated Green's function.

These higher level objects are different views of the same thing. Let us call it a building block. However, the potential is not just another view. The underlying mechanism also stands for some extra functionality.

Both the density distribution and the potential function have a Fourier transform and can be considered as a wave package. Problem with this view is the fact that the step stones only are used in a single progression instant. So most of the time the step stones are virtual. This becomes less relevant when the step stone distribution is generated according to a given plan. In that case the plan represents the building block.

The most impressing view is raised by the fact that at every arrival at a step stone the building block emits a wave front that contains information about its presence and about its properties. The flow of these wave fronts are controlled by the Huygens principle. Together the wave fronts constitute an ultra-high frequency carrier wave that cannot be observed directly, but that can show observable lower frequency modulations and that shows its influence in the form of the potentials of the building block. This does not take away that also the distribution of the step stones can behave in a wavy way.

Now we have a higher level object that at the same time is a point-like particle and will act as a wave package and an ultra-high frequency carrier wave

This idea is exploited by the Hilbert Book Model.

3.7 Fuzziness

Much confusion exists about the fundamental fuzziness of nature.

In the HBM the origin of this fuzziness is well defined. The fuzziness is created by the stochastic part of the correlation mechanism, which controls that sufficient, but not too much coherence exists between subsequent members of the sequence of static sub-models.

The fuzziness is implemented in the standard planned form of nature's building blocks. It is described by the natal quantum state function of this standard planned building block.

The standard building block is generated by a stochastic process that consists of the combination of a Poisson process and a binomial process. The binomial process is implemented by a 3D spread function, which has a standard form. This standard form is isotropic and is formed by a fixed number of elements. That number defines the standard deviation of the standard building block.

The quantum state function of the standard building block is a continuous quaternionic function, which has its parameters in configuration space. The spread of the quantum state function in configuration space corresponds to the standard deviation of the building block.

The quantum state function has a Fourier transform. This transformed function has its parameters in momentum space. The spread in the momentum space and the spread in the configuration space are related via Heisenberg's uncertainty relation. This relation is characterized by the standard deviation of the standard building block, which depends on the number of elements in the building block.

In conventional physics this relation is characterized by Planck's constant. Thus, in the HBM Planck's constant relates to the number of elements in the standard building block.

The actual building block is created element by element and is distorted by the local space curvature. Its movement smears the actual building block along its path of movement.

Often the uncertainty principle is indicated as explanation for nature's fuzziness. That is a false interpretation. The uncertainty relation only indicates that location and momentum cannot both be known with arbitrary precision. The actual fuzziness is expressed by the spread of the quantum state function or better said it is characterized by the number of elements of the standard building block.

3.8   Quanta

Quantum physics deserves its name due to the existence of quanta. Quanta are quantized amounts of energy that are carried by photons and gluons or exchanged with these carriers by particles.

The absorption of quanta may cause the release of particles from a bounded situation. The emission of quanta may indicate the binding of free particles into a composite.

These processes can be used to detect quanta.

Thus the observation of photons does not concern the visualization of their wave structure, but instead it concerns the detection event in which the photon is absorbed. On the other hand the spatial configuration of the detection events can reveal wave and interference patterns.

3.9 What image intensifiers reveal

The author spent eighteen years in the development of image intensifier tubes. These devices range from goggles via driver scopes to fourteen inch wide X-ray image intensifiers.

The image intensifiers had one feature in common. They were all capable of turning the impingement of a quantum at their input screen into a visible light spot at their output screen. A hail storm of impinging quanta at the input resulted in a noisy film at the output[14][15][16].

The starlight scopes enable visibility of very low dose scenes under starlight conditions. They turned infrared and visible light quanta into light spots on a luminescent phosphor screen.

The X-ray image intensifiers were designed to deliver a perceptible image of an X-ray shadow picture at the lowest possible X-ray dose for the diagnosed patient[17].

What still astonishes me is that I never saw any indication of a wave entering the input of the image intensifiers. I only saw clouds of quanta. That does not say that these clouds cannot have the shape of waves, but the individually detected quanta did not show that relation.

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[14] See: http://en.wikipedia.org/wiki/File:Moon_in_x-rays.gif . Low dose X-ray image of the moon.

[15] See: http://www.youtube.com/watch?v=U7qZd2dG8uI ; Hail storm. Warning, this is NOT a video of an external object.

[16] Also see: http://en.wikipedia.org/wiki/Shot_noise

[17] A short film of the output of an X-ray image intensifier made at a very low dose.

Provided by Philips Healthcare

4 The logic model

In this chapter the basic phenomena of physics will be deduced from its logical foundation. The HBM choses traditional quantum logic as its most basic foundation. In 1936, this foundation was suggested by Garret Birkhoff and John von Neumann[18].

Founding physics on classical logic seems a suitable solution, however since 1936 the physical community knows that nature cheats classical logic and instead obeys quantum logic. The structure of quantum logic is far more complicated than the structure of classical logic.

4.1 Static status quo

It is astonishing to discover that the foundation of physics that was suggested by Birkhoff and von Neumann cannot implement dynamics. The suggested model can only implement a static status quo of the universe[19].

4.1.1 Quantum logic

The most basic level of objects in nature is formed by the propositions that can be made about the objects that occur in nature. The relations between these propositions appear to be restricted by the

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[18] http://en.wikipedia.org/wiki/John_von_Neumann#Quantum_logics & Stanford Encyclopedia of Philosophy, *Quantum Logic and Probability Theory*, http://plato.stanford.edu/entries/qt-quantlog/

[19] Quantum logic does not offer operators that can install dynamics. That facility is offered by Hilbert logic and by Hilbert spaces. However, the HBM will not exploit the possibility to allow dynamic Hilbert logic operators.

axioms of traditional quantum logic. This set of related propositions can only describe a static status quo. The axioms that specify quantum logic are specified in Q-FORMULÆ 2.

In mathematical terminology the propositions whose relations are described by traditional quantum logic form a lattice. More particular, they form an orthomodular lattice that contains a countable infinite set of atomic (=mutually independent) propositions. Within the same quantum logic system multiple versions of sets of these mutually independent atoms exist. In this phase of the model the content of the propositions is totally unimportant. As a consequence these atoms form principally an unordered set[20]. Only the interrelations between the propositions count.

Traditional quantum logic shows narrow similarity with classical logic, however the modular law, which is one of the about 25 axioms that define the classical logic, is weakened in quantum logic. This is the cause of the fact that the structure of quantum logic is significantly more complicated than the structure of classical logic.

4.1.2   Hilbert logic

The set of propositions of traditional quantum logic is lattice isomorphic with the set of closed subspaces of a separable Hilbert space. However still significant differences exist between this logic system and the Hilbert space. This gap can be closed by a small refinement of the quantum logic system. This refinement leads to a Hilbert logic system.

Step 1: Define *linear propositions* (also called ***Hilbert propositions***) as quantum logical propositions that are characterized by a

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[20] This fact will prove to be the underpinning of the cosmologic principle.

number valued strength or **relevance**. This number is taken from a division ring.

Step 2: Require that linear combinations of Hilbert propositions also belong to the logic system.

Step 3: Introduce the notion of **relational relevance** between two linear propositions. This measure has properties that are similar to the properties of the inner product of Hilbert space vectors. The number value of the relational relevance is again taken from a suitable division ring.

Step 4: Close the subsets of the new logic system with respect to this relational relevance measure.

The resulting logic system will be called **Hilbert logic**.

The Hilbert logic is lattice isomorphic as well as topological isomorphic with the corresponding Hilbert space.

The definition of Hilbert logic is formally specified in Q-FORMULÆ 3.

In this correspondence, Hilbert propositions are the equivalents of Hilbert vectors. General quantum logic propositions are the equivalents of (closed) subspaces of a Hilbert space.

The measure of the relational relevance between two Hilbert propositions is the equivalent of the inner product between two Hilbert vectors.

Due to this similarity the Hilbert logic will also feature operators[21].

In a Hilbert logic, linear operators can be defined that have atomic Hilbert propositions as their eigen-propositions. The eigenspace of these operators is countable.

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[21] In the HBM the Hilbert logic does not feature dynamic operators.

In a Hilbert logic system the ***superposition principle*** holds. A linear combination of Hilbert proposition is again a Hilbert proposition.

5   Dynamic model

A dynamic model can be constructed from an ordered sequence of the above static sub-models[22]. Care must be taken to keep sufficient cohesion between subsequent static models. Otherwise, the model just represents dynamical chaos. However, some deviation must be tolerated, because otherwise, nothing dynamical will happen in this new dynamic model. The cohesion is established by a suitable *correlation vehicle*.

5.1   Correlation vehicle

The correlation vehicle supports and guards the coherence of the dynamics of the model. The correlation vehicle uses a toolkit consisting of an enumerator generator, an embedding continuum and an allocation function that maps the enumerators onto the embedding continuum. The function is a function of both the sequence number of the sub-models and the enumerators that are attached to a member of the selected set of atomic propositions. The enumeration is artificial and is not allowed to structurally add extra characteristics or functionality to the attached proposition. For example, if the enumeration takes the form of a coordinate system, then this coordinate system cannot have a unique origin and it is not allowed to structurally introduce preferred directions. These restrictions lead to an "affine-like space"[23]. The avoidance of preferred directions produces problems in multidimensional coordinate systems. As a consequence, in case of a multidimensional coordinate system the correlation vehicle must use a smooth touch. This means, that at very small scales the coordinate system must get blurred. This means that the guarantee

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[22] Another choice is to apply dynamic operators. That choice is not pursued by the HBM.

[23] We will call a space with these restrictions an affine-like space.

for coherence between subsequent sub-models cannot be made super hard. Instead coherence is reached with an acceptable tolerance. In any case a super hard coherence is unwanted. Thus the blur serves two purposes. At small scales it avoids preferred directions in multi-dimensional enumeration systems and at the same time it introduces sufficient freedom to tolerate the implementation of dynamics.

The correlation vehicle also takes care of the perseverance of the emitted potential. For that reason it uses the Huygens principle. At every progression step the ultra-high frequency waves are re-emitted from locations at the existing wave fronts.

In this way at every progression step the correlation mechanism not only regenerates all building blocks, it also regenerates at every progression step all wave fronts that contribute to potentials. Further on will be shown that these wave fronts together constitute the embedding continuum. Thus the correlation vehicle also regenerates the embedding continuum.

At particular events the regeneration of building blocks may stop or may evolve into another mode. A strong enough modulation of the embedding continuum may start a new generation process.

5.1.1 Smallest rational

In order to create sufficient freedom for the action of the stochastic spatial spread function the HBM introduces a smallest rational number. It means that also the rational complex numbers and the rational quaternions use this lower limit. The result is a mesh that consists of a countable number of knot points.

The smallest rational may vary as a function of progression. This corresponds to an isotropic expansion or compression.

Not all mesh knot points are used and the used points may move along a static mesh. The used mesh knots are imaged by the continuous sharp part of the allocation function. The stochastic spatial spread function will use these mesh knot images as its center point.

The images of the total blurred allocation function present locations of corresponding building blocks.

The derivative of the continuous sharp part of the allocation function defines a local metric. That metric describes the local curvature.

5.2   Dynamic logic

The HBM does not support dynamic logic systems. In the HBM, all dynamic aspects are treated by the correlation vehicle. The correlation vehicle implements a very complicated mechanism. It has little sense to implement that capability into a logic system when it can be done by a dedicated external mechanism.

6 **Isomorphic model**

The natural form of the enumeration system can be derived from the lattice isomorphic companion of the quantum logic sub-model. Or it can be derived via a corresponding Hilbert logic system. Here we follow the historical development that was initialized by Birkhoff and von Neumann.

In the fourth decade of the twentieth century Garret Birkhoff and John von Neumann[24] were able to prove that for the set of propositions in the traditional quantum logic model a mathematical lattice isomorphic model exists in the form of the set of the closed subspaces of an infinite dimensional separable Hilbert space[25]. The Hilbert space is a linear vector space that features an inner vector product. It offers a mathematical environment that is far better suited for the formulation of physical laws than what the purely logic model can provide.

Some decades later Constantin Piron[26] proved that the only number systems that can be used to construct the inner products of the Hilbert vectors must be division rings. Later M.P. Solèr's theorem formulated this discovery more precisely. The only suitable division

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[24] http://en.wikipedia.org/wiki/John_von_Neumann#Quantum_logics & Stanford Encyclopedia of Philosophy, *Quantum Logic and Probability Theory*, http://plato.stanford.edu/entries/qt-quantlog/

[25] In fact the discovery went in the reverse direction. The Hilbert spaces were already in use before quantum logic was formulated.

[26] C. Piron 1964; _Axiomatique quantique_

rings are the real numbers, the complex numbers and the quaternions[27].

Quaternions can be seen as combinations of a real scalar and a 3D (real) vector. The number system of the quaternions represent a 1+3D coordinate system. It can be shown that the eigenvalues of normal operators must also be taken from the same division ring.

Since the set of real numbers is multiple times contained in the set of complex numbers and the set of complex numbers is multiple times contained in the set of quaternions, the most extensive isomorphic model is contained in an infinite dimensional quaternionic separable Hilbert space. For our final model we will choose the quaternionic Hilbert space, but first we study what the real Hilbert space model and the complex Hilbert space model provide. What can be done by using a quaternionic Hilbert space can also be done in a real or complex Hilbert space by adding extra structure[28].

It appears that a cross product of two quaternionic Hilbert spaces no longer equals a quaternionic Hilbert space[29]. The HBM does not use such cross products.

The set of closed subspaces of the Hilbert space represents the set of propositions that forms the static quantum logic system. The quantum logic system can be refined to a corresponding Hilbert logic system. Like the sets of mutually independent atoms in the Hilbert logic system, multiple sets of orthonormal base vectors exist in the Hilbert space.

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[27] Bi-quaternions have complex coordinate values and do not form a division ring.

[28] http://math.ucr.edu/home/baez/rch.pdf

[29] The result is an abstraction to a real Hilbert space.

The base vectors do not form an ordered set. However, a so called normal operator will have a set of eigenvectors that form a complete orthonormal base. The corresponding eigenvalues may provide a means for enumeration and thus for ordering these base vectors. An arbitrary normal operator will in general not fit the purpose of providing an affine-like eigenspace.

Usually the eigenvalues of a normal operator introduce a unique origin and in the case of a multidimensional eigenspace, the eigenspace may structurally contain preferred directions. Still, suitable enumeration operators exist that produce properly ordered enumerations in a subspace[30].

Several things can already be said about the eigenspace of the wanted enumeration operator:
- Its eigenspace is countable.
- It has no unique origin.
- It does not show preferred directions.
- Its eigenvalues can be embedded in an appropriate reference continuum.

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[30] See Cosmology

As part of its corresponding Gelfand triple[31] a selected separable Hilbert space forms a sandwich that features uncountable orthonormal bases and (compact) normal operators with eigenspaces that form a continuum[32].

A reference continuum can be taken as the eigenspace of the corresponding enumeration operator that resides in the Gelfand triple of this reference Hilbert space.

Together with the pure quantum logic model, we now have a dual model that is significantly better suited for use with calculable mathematics. Both models represent a static status quo.

The Hilbert space model suits as part of the toolkit that is used by the correlation vehicle.

As a consequence*, an ordered sequence of infinite dimensional quaternionic separable Hilbert spaces forms the isomorphic model of the dynamic logical model*.

6.1   Hierarchy

The refinement of quantum logic to Hilbert logic also can deliver an enumeration system. However, the fact that the selected separable Hilbert space offers a reference continuum via its Gelfand triple

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[31] See http://vixra.org/abs/1210.0111 for more details on the Hilbert space and the Gelfand triple. See the paragraph on the Gelfand triple.

[32] A similar sandwich can be applied to the Hilbert logic. This possibility is not pursued in the HBM.

make the Hilbert space more suitable for implementing the Hilbert Book Model[33].

The two logic systems feature a hierarchy that is replicated in the Hilbert space. Quantum logic propositions can be represented by closed subspaces of the Hilbert space. Atomic Hilbert propositions can be represented by base vectors of the Hilbert space. The base vectors that span a closed subspace belong to that subspace. This situation becomes interesting when the base vectors are eigenvectors. In that case the corresponding eigenvalues can be used to enumerate the eigenvectors of the Hilbert space operator and the corresponding eigen atoms of the Hilbert logic operator.

A similar hierarchy can be found when a coherent set of lower order objects forms a building block. Here the lower order objects correspond to atomic Hilbert propositions and to corresponding Hilbert base vectors. The building block corresponds to the quantum logical proposition and to the corresponding closed Hilbert subspace.

Isomorphisms			
Logic	Propostion	Hilbert space	Object
Quantum logic	Atomic	Subspace	Building block
Hilbert logic	Atomic	Base vector	Step stone

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[33] The HBM does not pursue the introduction of a sandwich-like construct for the Hilbert logic as the Gelfand triple is for the Hilbert space. In fact it could be done.

6.2   Correspondences

Several correspondences exist between the sub models:

Quantum logic	Hilbert space		Hilbert logic
Propositions: a, b	Subspaces a,b	Vectors: $\lvert a \rangle, \lvert b \rangle$	Hilbert proposi-tions: a, b
atoms c, d		Base vectors: $\lvert c \rangle, \lvert d \rangle$	atoms c, d
Relational complexity: $C_{omplexity}(a \cap b)$	Relational complexity: $C_{omplexity}(a \cap b)$	Inner product: $\langle a \vert b \rangle$	Rela-tional cou-pling meas-ure
Inclusion: $(a \cup b)$	Inclusion: $(a \cup b)$	Linear combination: $\alpha \lvert a \rangle + \beta \lvert b \rangle$	Linear combina-tion: $\alpha a + \beta b$
For atoms c_i: $\bigcup_i c_i$	Subspace $\left\{ \sum_i \alpha_i \lvert c_i \rangle \right\}_{\forall \, \alpha_i}$		Subset $\left\{ \sum_i \alpha_i c_i \right\}$

The distribution

$$a(i) \equiv \{\langle a \vert c_i \rangle\}_{\forall \, i}$$

has no proper definition in quantum logic. It can be interpreted via the Hilbert logic and Hilbert space sub-models.

6.3 Affine-like space

The installation of the correlation vehicle requests the introduction of enumerators. The enumeration may introduce an ordering.

The set of mutually independent atomic Hilbert propositions is represented by an orthonormal set of base vectors in Hilbert space. Both sets span the whole of the corresponding structure. An arbitrary orthonormal base is not an ordered set. It has no start and no end. It can be embedded in an affine-like space. Or otherwise stated: the set can be enumerated by elements of an affine-like space.

The HBM uses a simple definition of an affine-like space. It is a vector space that has no origin and no preferred directions. The space might be curved and it might be divided in compartments. At large and moderate scales these compartments might allow a coordinate system that has a unique origin.

All or a part of the base vectors can be enumerated for example with rational quaternions. Enumeration introduces an artificial origin and may introduce artificially preferred directions. Thus, in order to prevent preferred directions, enumeration will apply only to a part of the affine-like space. As is shown in the last paragraph, this enumeration process defines a corresponding "mostly normal" operator.

Normal operators are defined in Q-FORMULÆ 9. Off course, the indicated "mostly normal" operator is not normal. It has an affine-like eigenspace. In certain subspaces it is similar to a normal operator. There the eigen(sub)space may have a (selected) origin.

If the enumeration introduces an ordering, then the attachment of the numerical values of the enumerators to the Hilbert base vectors

defines a corresponding operator. It must be remembered that the selection of the enumerators and therefore the corresponding ordering is kind of artificial. The eigenspace of the enumeration operator has **no unique origin**[34] and is has **no natural preferred directions**[35]. Thus it has no natural axes. It can only indicate the distance between two or more locations. It will be shown that for multidimensional rational enumerators the distance is not precise. In that case the enumeration can at the utmost represent a blurred coordinate system. Both in the Hilbert space and in its Gelfand triple, the enumeration can be represented by a "mostly normal" enumeration operator.

The HBM suggests a smallest rational number. At very low scales ordered enumeration may be taken over by a stochastic enumeration mechanism. However, that does not occur at all mesh points. Where this stochastic enumeration occurs, the model will show special objects that act as nature's building blocks.

6.4 Continuity
The task of the correlation vehicle is to arrange sufficient cohesion between subsequent members of the sequence. This cohesion translates to a moderate form of continuity.

6.4.1 Arranging dynamics
Embedding the enumerators in a continuum highlights the interspacing between the enumerators. Having a sequence of static submodels is no guarantee that anything happens in the dynamic model. A fixed (everywhere equal) interspacing will effectively lame any

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[34] It might have several origins.

[35] At small scales the enumeration may be installed by a stochastic process, which avoids preferred directions. At larger scales any preferred direction will be disturbed by space curvature.

dynamics. A more effective dynamics can be arranged by playing with the sizes of the interspacing in a stochastic way. This is the task of *a stochastic enumerator generator.*

6.4.2   Establishing coherence

The cohesion between subsequent static models can be established by embedding each of the countable sets in an appropriate continuum and enforcing coherence via a continuous function.

As a first step, the whole Hilbert space can be embedded in its Gelfand triple. The enumerators of the base vectors of the separable Hilbert space or of a subspace can also be embedded in a corresponding continuum.

In the reference Hilbert space that continuum is formed by the values of the enumerators that enumerate a corresponding orthonormal base of the Gelfand triple[36].

For subsequent Hilbert spaces a new appropriate embedding continuum will be used, but that continuum may be curved.

Next a correlation vehicle is established by introducing a continuous allocation function that controls the coherence between subsequent members of the sequence of static models. It does that by creating a moderate relocation in the countable set of the enumerators that act in the separable Hilbert space by mapping them to the embedding continuum. The relocation is controlled by a stochastic process.

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[36] See Gelfand triple

The differential of the allocation function can be used to specify the small scale working space for this stochastic process[37]. The correlation vehicle also takes care of the persistence of the embedding continuum. For this purpose it uses the Huygens principle.

The equivalence of this action for the logic model is that the enumerators of the atomic propositions are embedded in a continuum that is used by an appropriate correlation vehicle. The problem with this view is that currently the Hilbert logic does not possess an equivalent of the Gelfand triple[38].

The allocation function uses a combination of progression and the enumerator id as its parameter value. The value of the progression might be included in the value of the id.

Apart from their relation via the allocation function, the enumerators and the embedding continuum are mutually independent[39]. For the selected correlation vehicle it is useful to use numbers as the value of the enumerators. The type of the numbers will be taken equal to the number type that is used for specifying the inner product of the corresponding Hilbert space and Gelfand triple.

The danger is then that in general a direct relation between the value of the enumerator of the Hilbert base vectors and the embedding continuum is suggested. A positive exception is formed by the selected reference Hilbert space, but this is an exceptional case.

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[37] The differential defines a local metric.

[38] This "logic Gelfand triple" can be constructed, but the HBM will not use it.

[39] This is not the case for the reference Hilbert space in the sequence. There a direct (close) relation exists.

So, for later Hilbert spaces a warning is at its place. Without the allocation function there is no relation between the value of the enumerators and corresponding values in the embedding continuum that is formed by the Gelfand triple.

However, there is a well-defined relation between the images[40] produced by the allocation function and the selected embedding continuum[41].

The relation between the members of a countable set and the members of a continuum raises a serious one-to-many problem. That problem can easily be resolved for real Hilbert spaces and complex Hilbert spaces, but it requires a special solution for quaternionic Hilbert spaces. That solution is treated below when quaternionic Hilbert spaces are discussed.

Together with the selected embedding continuum and the Hilbert base enumeration set, the allocation function defines the evolution of the model.

6.4.3   Structure of the correlation vehicle

At every progression step the correlation vehicle regenerates the eigenspaces of the non-conserved operators[42]. This regeneration runs at an ultra-high frequency. That frequency is set by the progression step size τ_s.

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[40] Later these images will be called Qpatches

[41] Later the nature of this embedding continuum will be revealed. In later Hilbert spaces the embedding continuum is constituted by interfering ultra-high frequency waves.

[42] These operators reside in Hilbert logic, in the corresponding Hilbert space and in the corresponding Gelfand triple.

An important part of the functionality of the correlation vehicle is implemented by the blurred allocation function. This function is the **convolution** of a continuous part and a local blur. The local blur is implemented by the combination of a Poisson process and a binomial process. The binomial process is implemented by a 3D spread function. We will call this combined process a **stochastic spatial spread function**.

The derivative of the continuous part of the allocation function defines a **local metric**.

Another part of the functionality of the correlation vehicle concerns the regeneration of the embedding continuums. This regeneration is governed by **Huygens principle**. This part of the correlation vehicle is implemented by **wave fronts** that flow with the constant maximum speed of information transfer. The wave fronts constitute **ultra-high frequency carrier waves**.

Later we will see that the correlation vehicle is restricted by **color confinement**.

7 Hilbert spaces

Sets of subsets of Hilbert spaces represent quantum logical systems and associated Hilbert logic systems. Closed subspaces of the Hilbert space represent quantum logical propositions and Hilbert space vectors represent Hilbert propositions.

The Hilbert space itself is a static hull. A normal operator with a countable ordered set of rational eigenvalues can be used as a reference operator. This operator will be used for enumeration purposes. These enumerators will be used as parameters for the functions that implement the correlation mechanism. This correlation mechanism controls the evolution of other operators that will carry the progression dependence of the HBM.

Each Hilbert space corresponds to a ***Gelfand triple***. That space features operators which have a continuum as their eigenspace. Also in this space a normal operator with an ordered set of eigenvalues can be used as a reference operator. Continuum eigenspaces will be used to represent the target space of the blurred allocation function. The corresponding operators depend on progression. Partly they reside in the Hilbert space. Other progression dependent operators reside in the Gelfand triple.

The reference operators are static objects[43]. Together with the Hilbert space and the Gelfand triple they form the static hull.

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[43] However, the real part of the eigenvalue of the reference operator is used to store progression.

Several normal operators in the Gelfand triple will be used to deliver target values for functions that implement the correlation mechanism. These operators are dynamic objects. In the HBM they will be re-created at every progression step.

7.1   Real Hilbert space model

When a real separable Hilbert space is used to represent the static quantum logic, then it is sensible to use a countable set of real numbers for the enumeration. A possible selection is formed by the natural numbers. Within the real numbers the natural numbers have a fixed interspacing. Since the rational number system has the same cardinality as the natural number system, the rational numbers can also be used as enumerators. In that case it is sensible to specify *a (fixed) smallest rational number* as the enumeration step size. In this way the notion of interspacing is preserved and can the allocation function do its scaling task[44]. In the realm of the real Hilbert space model, the continuum that embeds the enumerators is formed by the real numbers. The values of the enumerators of the Hilbert base vectors are used as parameters for the allocation function. The value that is produced by the allocation function determines the target location for the corresponding enumerator in the target embedding continuum. The target embedding continuum is taken from an operator that resides in the Gelfand triple. The interspacing freedom is used in order to introduce dynamics in which something happens.

In fact what we do is defining an enumeration operator that has the enumeration numbers as its eigenvalues. The corresponding eigenvectors of this operator are the target of the enumerator.

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[44] Later, in the quaternionic Hilbert space model, this freedom is used to introduce space curvature and it is used for resolving the one to many problem.

With respect to the logic model, what we do is enumerate a previously unordered set of atomic propositions that together span the Hilbert logic system and next we embed the enumerators in an appropriate continuum. The correlation vehicle takes care of the cohesion between subsequent quantum logical systems.

While the progression step is kept fixed, the (otherwise fixed) space step might scale with progression.

Instead of using a fixed smallest rational number as the enumeration step size and a map into a reference continuum we could also have chosen for a model in which the rational numbered step size varies with the index of the enumerator.

7.2   Gelfand triple

The Gelfand triple of a real separable Hilbert space can be understood via the enumeration model of the real separable Hilbert space. This enumeration is obtained by taking the set of eigenvectors of a normal operator that has rational numbers as its eigenvalues.

Let the smallest enumeration value of the rational enumerators approach zero. Even when zero is reached, then still the set of enumerators is countable.

Now add all limits of converging series of rational enumerators to the enumeration set. When appropriate also add a corresponding eigenvector.

After this operation the enumeration set has become a continuum and has the same cardinality as the set of the real numbers. This operation converts the Hilbert space into its Gelfand triple and it converts the normal operator into a new operator that has the real numbers as its eigenspace.

It means that the orthonormal base of the Gelfand triple that is formed by the eigenvectors of the new normal operator has the cardinality of the real numbers.

It also means that linear operators in this Gelfand triple have eigenspaces that are continuums and have the cardinality of the real numbers[45].

The same reasoning holds for complex number based Hilbert spaces and quaternionic Hilbert spaces and their respective Gelfand triples.

7.3   Complex Hilbert space model

When a complex separable Hilbert space is used to represent quantum logic, then it is sensible to use rational complex numbers for the enumeration.

Again a smallest enumeration step size is introduced. However, the imaginary fixed enumeration step size may differ from the real fixed enumeration step size. The otherwise fixed imaginary enumeration step may be scaled as a function of progression.

In the complex Hilbert space model, the continuum that embeds the enumerators of the Hilbert base vectors is formed by the system of the complex numbers. This continuum belongs as eigenspace to the enumerator operator that resides in the Gelfand triple.

It is sensible to let the real part of the Hilbert base enumerators represent progression. The same will happen to the real axis of the embedding continuum. On the real axis of the embedding continuum the interspacing can be kept fixed.

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[45] This story also applies to the complex and the quaternionic Hilbert spaces and their Gelfand triples.

Instead, it is possible to let the allocation function control the interspacing in the imaginary axis of the embedding continuum. The values of the rational complex enumerators are used as parameters for the allocation function.

The complex value of the allocation function determines the target location for the corresponding target value in the continuum.

The allocation function establishes the necessary coherence between the subsequent Hilbert spaces in the sequence.

The difference with the real Hilbert space model is, that now the progression is included into the values of the enumerators. The result of these choices is that the whole model steps with (very small, say practically infinitesimal) fixed progression steps.

In the model that uses complex Hilbert spaces, the enumeration operator has rational complex numbers as its eigenvalues. In the complex Hilbert space model, the fixed enumeration real step size and the fixed enumeration imaginary step size define *a speed of information transfer*.

The fixed imaginary step size may scale as a function of progression. The same will then happen with the speed of information transfer, defined as space step divided by progression step. However, if information steps one step per progression step, then the information transfer speed will be constant.

Progression plays the role of proper time. Now define a new concept that takes the length of the complex path step as the step value. Call this concept the coordinate time step.

Define a new speed as the space step divided by the coordinate time step. This new **maximum speed is a model constant**. Proper time is the time that ticks in the reference frame of the observed item.

Coordinate time is the time that ticks in the reference frame of the observer[46]. Coordinate time is our conventional notion of time.

Again the eigenvectors of the (complex enumeration) operator are the targets of the enumerator whose value corresponds to the complex eigenvalue.

In the complex Hilbert space model the squared modulus of the quantum state function represents the probability of finding the location of the corresponding particle at the position that is defined by the parameter of this function.

If we ignore the case of negative progression, then the complex Hilbert model exist in two forms, one in which the interspacing appears to expand and one in which the interspacing decreases with progression[47].

7.4   Quaternionic Hilbert space model

When a quaternionic separable Hilbert space is used to model the static quantum logic, then it is sensible to use rational quaternions for the enumeration.

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[46] In fact coordinate time is a mixture of progression and space. See paragraph on spacetime metric.

[47] The situation that expands from the point of view of the countable enumeration set, will contract from the point of view of the embedding continuum of enumerators.

Again the fixed enumeration step sizes are applied for the real part of the enumerators and again the real parts of the enumerators represent progression.

The reference continuum that embeds the enumerators is formed by the number system of the quaternions.

The scaling allocation function of the complex Hilbert space translates into an isotropic scaling function in the quaternionic Hilbert space. However, we may instead use a full 3D allocation function that incorporates the isotropic scaling function. This new allocation function may act differently in different spatial dimensions. However, when this happens at very large scales, then it conflicts with the cosmological principle. At those scales the allocation function must be quasi isotropic. The allocation function is not allowed to create preferred directions.

Now the enumeration operator of the Hilbert space has rational quaternions as its eigenvalues. The relation between eigenvalues, eigenvectors and enumerators is the same as in the case of the complex Hilbert space. Again the whole model steps with fixed progression steps.

In the quaternionic Hilbert space model the real part of the quantum state function represents the probability of finding the location of the corresponding particle at the position that is defined by the parameter of this function. It corresponds to a density distribution of the locations where the corresponding building block can/could be found.

7.4.1 Curvature and fundamental fuzziness

The spatially fixed interspacing that is used with complex Hilbert spaces poses problems with quaternionic Hilbert spaces. Any regular

spatial interspacing pattern will introduce preferred directions. Preferred directions are not observed in nature[48] and the model must not create them. A solution is formed by the ***randomization of the interspacing***. Thus instead of a fixed imaginary interspacing we get an average interspacing.

This problem does not play on the real axis. On the real axis we can still use a fixed interspacing.

The result is an ***average speed of information transfer***. This speed is measured as space step per coordinate time step, where the coordinate time step is given by the length of the 1+3D quaternionic path step.

Further, the actual location of the enumerators in the embedding continuum will be determined by the combination of a sharp continuous allocation function (SCAF) $\wp$ and a stochastic spatial spread function (SSSF) $\mathcal{S}$ that specifies the local blur.

The form factor of the blur may differ in each direction and is set by the differential of the sharp allocation function $\wp$.

The total effect is given by the convolution $\mathcal{P} = \wp \circ \mathcal{S}$ of the sharp allocation function $\wp$ and spread function $\mathcal{S}$. The result is a blurred allocation function $\mathcal{P}$.

The result of $\mathcal{S}$ alone is described by a quaternionic probability amplitude distribution (QPAD). This is a descriptor. It describes the ***planned distribution*** of a set of discrete objects that will be generated in a sequence[49].

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[48] Preffered directions are in conflict with the cosmological principle.

[49] Later we will see that this sequence has a fixed length.

The result of $\mathcal{P}$ is the **actual** local QPAD. In the quaternionic Hilbert space model it conforms to the quaternionic quantum state function. It is a close equivalent of the well-known wave function.

> *The requirement that the cosmological principle must be obeyed is one of the causes of a fundamental fuzziness of the quaternionic Hilbert model. Another cause is the requirement that coherence between subsequent progression steps must not be too stiff. These causes are the reason of existence of quantum physics.*

An important observation is that the blur mainly occurs locally. The blur has a very limited extent. On the other hand, due to the emission of potential generating wave fronts, the blur corresponds to a potential function that has an unlimited extent, but its influence decreases with distance.

At medium distances the freedom that is tolerated by the allocation function causes *curvature of observed space*. However, as explained before, at very large scales the allocation function must be quasi isotropic[50]. The local curvature is described by the differential of the sharp part of the allocation function.

The continuous part of the allocation function defines the properties of the current target embedding continuum. In fact it determines the eigenspace of a corresponding operator that resides in the Gelfand triple.

Apart from the exceptional case of the reference Hilbert space, the selection of this operator poses a choice. For the reference Hilbert

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[50] Quasi-isotropic = on average isotropic.

space the eigenspace of the reference operator that resides in the Gelfand triple is selected. For subsequent Hilbert spaces, the HBM selects the superposition of all emitted wave fronts as the proper choice of this embedding continuum.

This picture only tells that space curvature might exist. It does not describe the origin of space curvature. For a more detailed explanation of the origin of space curvature, please see the paragraph on the enumeration process.

7.4.2 Discrete symmetry sets

Quaternionic number systems exist in 16 versions (sign flavors[51]) that differ in their discrete symmetry sets. The same holds for sets of rational quaternionic enumerators and for continuous quaternionic functions. Four members of the set represent isotropic expansion or isotropic contraction of the imaginary interspacing. At large scales two of them are symmetric functions of progression. The other two are at large scales anti-symmetric functions of progression. We will take the symmetrical member that expands with positive progression as the *reference rational quaternionic enumerator set*. Each member of the set corresponds with a quaternionic Hilbert space model. Thus apart from a reference continuum we now have a reference rational quaternionic enumerator set. Both reference sets meet at the reference Hilbert space. Even at the instance of the reference Hilbert space, the allocation function must be a continuous function of progression.

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[51] See paragraph on Qpattern coupling

When the real parts are ignored, then eight sign flavors result. These eight flavors are discerned by their "color" and their handedness. Besides of color, we use special indices in order to mark the sign flavors.

$$\psi^{\textcircled{0}}\, N\, \mathbf{R}$$
$$\psi^{\textcircled{1}}\, R\, \mathbf{L}$$
$$\psi^{\textcircled{2}}\, G\, \mathbf{L}$$
$$\psi^{\textcircled{3}}\, B\, \mathbf{L}$$
$$\psi^{\textcircled{4}}\, \bar{B}\, \mathbf{R}$$
$$\psi^{\textcircled{5}}\, \bar{G}\, \mathbf{R}$$
$$\psi^{\textcircled{6}}\, \bar{R}\, \mathbf{R}$$
$$\psi^{\textcircled{7}}\, \bar{N}\, \mathbf{L}$$

Figure 1: Sign flavors

Eight sign flavors
(discrete symmetries)
Colors N, R, G, B, $\bar{R}$, $\bar{G}$, $\bar{B}$, W
Right or Left handedness **R,L**
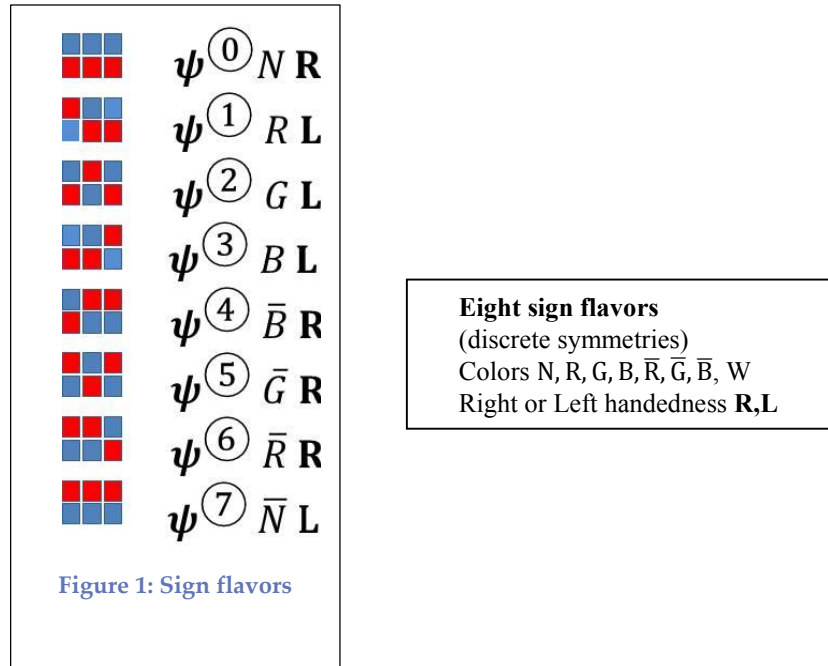
Within a coherent set of enumerators or in the images of such a set that are produced by the allocation function all objects possess the same sign flavor.

A similar split in quaternionic sign flavors as exists with quaternionic number systems occurs with continuous quaternionic functions. In the picture they are listed as $\psi^{\circledgot}...\psi^{\circledgot}$. Apart from their discrete symmetry set, these functions are equal.

In the picture the color N and the continuous function version $\psi^{\circledgot}$ represent the reference sign flavor.

For each discrete symmetry set of their parameter space, the function values of the continuous quaternionic distribution exist in 16 versions that differ in their discrete symmetry set. Within the target domain of the continuous quaternionic distribution the symmetry set will stay constant.

7.4.3 Generations and Qpatterns

Depending on its characteristics, the local generator of enumerators can generate a certain distribution of randomized enumerators. A Poisson generator combined by a binomial process that is implemented by a suitable 3D isotropic spread function can implement a suitable distribution. The planned distribution is described by a local QPAD. The local QPAD corresponds to the characteristics of the generator, but depending on its starting condition the stochastic generator can generate different distributions. Thus, different distributions may correspond to a single QPAD. The QPAD is a continuous quaternionic function that describes in its real part the density of the elements of the described distribution. In its imaginary part the QPAD describes the associated current density distribution.

If generators with different characteristics exist, then several generations[52] of local QPAD's exist.

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[52] See the later paragraph on generations

Since different generations of the same type have the same electric charge and if charge is evenly distributed over all N_w elements, it is quite probable that that for all generations N_w is the same.

HYPOTHESIS 1: *For a selected generation the following holds: Apart from the discrete symmetry set of the QPAD, the natal QPAD's are everywhere in the model the same.*

Therefore we will call the distribution of objects that is described by this basic form of the selected QPAD generation a ***natal Qpattern***. For each generation, *QPAD's exist in 16 versions that differ in their discrete symmetry set*. Each Qpattern has a weighted center location, which is called ***Qpatch***.

At each progression step, all generators produce only a single element of the distribution. ***This means that each Hilbert space contains only one element of the Qpattern.*** That element is called ***Qtarget***.

Due to the influence of local curvature and due to movement of the Qpatch the Qtargets form an ***actual Qpattern*** that differs from the natal Qpattern.

7.4.4 Microstate

A Qpattern corresponds with the statistic mechanical notion of a microstate. A microstate of a gas is defined as a set of numbers which specify in which cell each atom is located, that is, a number labeling the atom, an index for the cell in which atom s is located and a label for the microstate[53].

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[53]   http://www.intechopen.com/books/theoretical-concepts-of-quantum-mechanics/quantum-mechanical-ensembles-and-the-h-theorem

7.5 Optimal ordering

In the Hilbert space it is possible to select a base that has optimal ordering for the eigenvalues of a normal operator. Optimally ordered means that these sections are uniformly distributed and that stochastic properties of these sections are the same. In the Hilbert logic system a similar selection is possible for the set of mutually independent atomic propositions. There the atomic propositions are enumerated by the same set of rational quaternionic values.

For the Hilbert spaces it means that in the Gelfand triple a corresponding operator exist whose eigen space maps onto the well-ordered eigenspace of the operator that resides in the Hilbert space.

We will call these operators "reference operators".

7.6 The reference Hilbert space

The reference Hilbert space is taken as the member of the sequence of Hilbert spaces at the progression instance where the allocation function is a symmetric function of progression that expands in directions that depart from the progression value of the reference Hilbert space.

At large and medium scales the reference member of the sequence of quaternionic Hilbert spaces is supposed to have a quasi-uniform[54] distribution of the enumerators in the embedding continuum. This is realized by requiring that the eigenspace of the enumeration operator that acts in the Gelfand triple of the zero progression value Hilbert space represents the reference embedding continuum.

At this instance of progression, the target embedding continuum is *flat*. For the reference Hilbert space the isotropic scaling function is symmetric at zero progression value. Thus for the reference Hilbert

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[54] quasi-uniform = on average uniform.

space at the reference progression instance the distribution of the enumerators will realize a ***densest packaging[55]of the target images***.

For all subsequent Hilbert spaces the embedding continuum will be taken from the superposition of wave fronts that are initiated in earlier Hilbert spaces.

The (reference) Hilbert space together with its Gelfand triple and the reference operators in both structures form a static block that reappears in all later members of the sequence.

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[55] The densest packaging will also be realized locally when the geometry generates black regions.

7.7   The embedding continuum

For the reference Hilbert space the embedding continuum is taken from a flat normal location operator that resides in its Gelfand triple. That continuum is the virginal reference continuum.

Later we will see that it is possible to construct an embedding continuum from fields that are emitted from other compartments of the universe that have their own evolution cycle.

For subsequent Hilbert spaces the embedding continuum for fermions is formed by the superposition of all wave fronts that are generated by objects that lived in previous Hilbert spaces. The result is a curved equivalent embedding continuum that replaces the virginal reference continuum. The curvature is caused by the mechanism that emits the ultra-high frequency waves that constitute the gravitation potentials. These waves also constitute other potentials.

Bosons use an embedding continuum that is formed by the wave fronts that are emitted locally in previous Hilbert spaces.

The correlation vehicle takes care of the persistence of the UHF waves.

7.8   The cosmological principle revisited

The enumeration process attaches an artificial content to each of the members in the unordered set of atomic propositions. The unrestricted enumeration with rational quaternions generates an artificial origin and it generates artificial preferred directions that are not present in the original set of atomic propositions. The correlation vehicle is not allowed to attach this extra functionality to the original propositions. However, the vehicle must still perform its task to establish cohesion between subsequent sub-models. One measure is to turn the

enumeration space into an affine-like space or to restrict the enumeration to a closed subset of a larger affine-like space where the subset has a unique origin. An affine-like space has no unique origin. The next measure is to randomize the enumeration process sufficiently such that an acceptable degree of cohesion is reached and at the same time a quasi-isotropy of this affine-like space is established. This measure requires the freedom of some interspacing, which is obtained by assigning a lowest rational number. In principle, a lowest rational number can be chosen for the real part and a different smallest base number can be chosen for the imaginary part. This choice defines a basic notion of speed. The resulting (imaginary) space is on average isotropic. The randomization results in a local blur of the continuous function that regulates the enumeration process.

The result of these measures is that roughly the cosmologic principle is installed. Thus, in fact *the cosmological principle is a corollary of the other two starting points*.

However, according to this model, apart from the low scale randomization, at the start the universe would be quite well ordered. After a myriad of progression steps this medium to large scale ordering is significantly disturbed.

Looking away[56] from any point in universe is in fact looking back in proper time. Looking as far as is physically possible will open the view at a reference member of the Hilbert Book Model. This reference member represents a densest and well-ordered packaging. This will result in a uniform background at the horizon of the universe.

The well-known microwave background radiation is not fully uniform and is expelled by members that are close to the densest packaged member.

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[56] Looking away = receiving messages from other objects.

7.9   Fourier transform

The Fourier transform switches between two views of the Hilbert space. It converts a set of base vectors into a new set of base vectors such that none of the old base vectors is retained and all new base vectors can be written as linear combinations of the old base vectors in which the coefficients all have the same norm.

The Fourier transform retains the values of inner products between any two vectors. Thus, it is a unitary transform.

8 **The HBM picture**

In the early days of quantum physics two views on quantum physics existed, the Schrödinger picture and the Heisenberg picture. The HBM adds two extra pictures.

8.1   The Schrödinger picture

The Schrödinger picture describes a dynamic implementation in Hilbert space in which the quantum states carry the time dependence. The operators are static[57].

8.2   The Heisenberg picture

The Heisenberg picture describes a dynamic implementation in Hilbert space in which the operators (represented by matrices) carry the time dependence. The quantum states are static[58].

8.3   The Hilbert Book Model picture

In the HBM picture an ordered sequence of Hilbert spaces and their corresponding Gelfand triples are used. Each of these spaces represent a static status quo.

In the HBM the whole Hilbert space carries the *** proper time dependence***. Both the enumeration operator and the patterns that represent the quantum state functions depend on the progression parameter. Other operators describe the target images of these enumerators. These target images form the Qtargets. For each Qpattern the Hilbert space contains only the actual element, the current Qtarget. Thus if only a single Hilbert space is considered, then the Qpatterns cannot be recognized. The virtual elements are not actually present in any

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[57] http://en.wikipedia.org/wiki/Schr%C3%B6dinger_picture
[58] http://en.wikipedia.org/wiki/Heisenberg_picture

member of the sequence of Hilbert spaces. The virtual elements can only exist as place holders.

The potentials of Qpatterns act as traces of the existing and passed Qpatterns and the corresponding wave fronts form traces of the Qtargets. They affect the embedding continuum that is formed by wave fronts that are emitted by particles that existed in the past. This embedding continuum is reflected in eigenspaces of operators that reside in the Gelfand triple.

The correlation vehicle ensures the cohesion between subsequent Hilbert spaces and takes care of the persistence of the emitted wave fronts. In order to achieve this, at each progression step the correlation vehicle uses the Huygens principle[59].

The wave fronts survive the extinction of the sources that created them. Their amplitude diminishes with distance and the wave fronts interfere, but they exist forever.

Emitted wave fronts do not compensate each other. This fact renders the HBM into a never ending story.

8.4   The operational picture

In the operational picture only a single Hilbert space and its Gelfand triple are used.

An operator that resides in the Hilbert space acts as the reference operator. It has an equivalent in the Gelfand triple and the eigenspaces of these operators map onto each other in an orderly fashion.

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[59] If the potentials are emitted in two dimensions, then the situation is more complicated.

Together with the Hilbert space and Gelfand triple these reference operators represent the static part of the model[60].

The eigenvalues of the reference operators represent the progression value in their real part.

In the Hilbert space and in its Gelfand triple the correlation vehicle supports the existence of progression dependent operators. This concerns a stochastically operating operator in the Hilbert space and for each potential type a compact normal operator that installs the temporal behavior of these potentials.

The correlation vehicle uses the eigenspaces of the reference operators as its parameter spaces. It uses eigenspaces of other operators as its target space. As a consequence these target operators depend on progression.

This picture comes close to the Heisenberg picture, but it does not keep states static.

8.5   Discussion

Obviously the HBM selects the Hilbert Book Model picture. According to the feel of the author this picture offers the cleanest view. In this picture the difference between virtual and actual elements of a building block can be clearly explained.

The Hilbert space and Gelfand triple hulls together with the reference operators form the static part of both the HBM picture and the operational picture. In the HBM picture this static part is represented

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[60] An exception holds for the real parts of the eigenvalues. They represent progression.

by the reference Hilbert space, its Gelfand triple and the reference operators. There is one small exception to this static behavior: the eigenvalues of the reference operators represent the progression value in their real parts.

Not all of the eigenvectors of the Hilbert space reference operator are constantly in use. Annihilation and (re)creation events regulate this usage. Virtual elements of building blocks are not used. Only the Qtarget is used, which is an actual element.

For a local view the models only use a huge subspace of the Hilbert space(s). For a cosmological view the full Hilbert space is used.

Enumeration is considered to be an artificial action and the enumerators must be seen as to be embedded in an affine-like space.

The correlation vehicle controls all aspects of dynamics. It does that both in Hilbert space and in the Gelfand triple. Since the Hilbert space and the Gelfand triple are static hulls, the correlation vehicle controls a selected set of operators that reside in these spaces. The tools of the correlation vehicle are the allocation function (in the Hilbert space) and the Huygens principle (in the Gelfand triple). Its actions are coordinated.

8.6   Quantum state function

In contemporary physics the "quantum state function" is used in its complex format. There it is a complex probability amplitude distribution (CPAD). It is also called "wave function". The squared modulus of the quantum state function is interpreted as the probability to be able to detect the corresponding building block at the loca-

tion that is specified by the parameter of the wave function. The complex phase of the wave function can be freely selected. This freedom is used in gauge transforms.

In quaternionic quantum physics the quaternionic quantum state function is defined as a continuous quaternionic function. Its real part equals the squared modulus of the complex quantum state function and has the same interpretation. This part has no complex phase. In fact the real part can be interpreted as an object density distribution, where the objects are the locations where the corresponding building block can be detected. The imaginary part of the quaternionic quantum state function can be interpreted as the associated current density distribution. In fact it registers the displacement of the described building block since its last location. At every progression instant the building block gets a new location.

The displacement is the sum of the displacement that is due to the movement of the building block as a whole and the displacement that is caused by the stochastic spatial spread. This last category of displacements cause the walk of the building block along a random micro-path.

In quaternionic quantum physics the gauge transformation reduces to a mathematical trick.

The characterization of the quaternionic quantum state function as a quaternionic probability amplitude distribution (QPAD) is used in analogy to the characterization of the complex quantum state function as a complex probability amplitude distribution (CPAD), but care must be taken because the interpretations of the CPAD and the QPAD slightly differ.

9 **The enumeration process**

It is not yet made clear how Qpatterns will be shaped. This information can be derived from the requirements that are set for the correlation vehicle. We will start with a suggestion for the enumeration process that for this vehicle will lead to the wanted functionality.

HYPOTHESIS 2: *At small scales the enumeration process is governed by a Poisson process. The lateral spread that goes together with the low scale randomization of the interspacing plays the role of a binomial process. The combination of a Poisson process and a binomial process is again a Poisson process, but locally it has a lower efficiency than the original Poisson process. The binomial attenuation is implemented by a continuous 3D spread function.*

As an ***example***, we consider the special situation that this combination produces a 3D normal distribution. For a large number of enumerator generations the resulting spatial Poisson distribution resembles a Gaussian distribution[61]. If the generated enumerators are considered as charge carriers, then the corresponding potential has the shape of an Error function divided by r. Already at a short distance from its center location the potential function starts decreasing with distance r as a $1/r$ function[62].

9.1   **New mathematics**
No mathematical solution is known for the conversion of an ultra-high frequency train of wave fronts to a rather static potential function. Normally the relation between a set of charges and a potential

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[61]   http://en.wikipedia.org/wiki/Poisson's_equation#Potential_of_a_Gaussian_charge_density

[62] http://farside.ph.utexas.edu/teaching/em/lectures/node28.html

function is regulated by a dedicated Green's function. We can try a similar solution by letting the wave front play the role of the a charge. We can also use the fact that a building block contains a fixed number of step stones. Thus, instead of an integral a sum over N_w step stones can be used.

9.2 Gravity and electrostatics

Potentials depend on the Green's function that is used to convert the corresponding density distribution into a potential function. Apart from their Green's function, gravity and electrostatics can be treated by similar equations. We use the fact that charge Q is spread over N_w step stones that have charge $q = Q/N_w$.

Description	Gravity	Electrostatics
Field	$\boldsymbol{g} = -\boldsymbol{\nabla}\varphi$	$\boldsymbol{E} = -\boldsymbol{\nabla}\varphi$
Force	$\boldsymbol{F} = m\boldsymbol{g}$	$\boldsymbol{F} = Q\boldsymbol{E}$
Gauss law	$\langle \boldsymbol{\nabla}, \mathrm{g}\rangle = -4\pi G\rho$	$\langle \boldsymbol{\nabla}, \mathrm{E}\rangle = \dfrac{Q}{\varepsilon}$
Poisson law $\Delta\varphi = \langle \boldsymbol{\nabla}, \boldsymbol{\nabla}\varphi\rangle$	$\Delta\varphi = 4\pi G\rho$	$\Delta\varphi = -\dfrac{Q}{\varepsilon}$
Greens function	$\dfrac{-\rho(\boldsymbol{r}')}{|\boldsymbol{r} - \boldsymbol{r}'|}$	$\dfrac{q}{|\boldsymbol{r} - \boldsymbol{r}'|}$
Single charge potential	$\varphi = -\dfrac{4\pi Gm}{|\boldsymbol{r}|}$	$\varphi = \dfrac{Q}{4\pi\varepsilon|\boldsymbol{r}|}$
Single charge field	$g = -\dfrac{4\pi Gm}{|\boldsymbol{r}|^2}\boldsymbol{r}$	$E = \dfrac{Q}{4\pi\varepsilon|\boldsymbol{r}|^2}\boldsymbol{r}$
Two charge force	$\boldsymbol{F} = -\dfrac{4\pi Gm_1 m_2}{|\boldsymbol{r}|^3}\boldsymbol{r}$	$\boldsymbol{F} = \dfrac{Q_1 Q_2}{4\pi\varepsilon|\boldsymbol{r}|^3}\boldsymbol{r}$
Mode	**attracting**	**repelling**

The table shows that the Greens functions of both fields differ in sign. For the gravitation potential the Green's function is charged with the local "charge" density $\rho(\boldsymbol{r'})$. For the electrostatic potential the Green's function is charged with a (constant) electric charge Q.

The Yukawa potential[63] uses a short range Green's function:

$$\frac{-\rho(\boldsymbol{r'})}{|\boldsymbol{r} - \boldsymbol{r'}|} \exp(-\mu|\boldsymbol{r} - \boldsymbol{r'}|) \tag{1}$$

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[63] http://en.wikipedia.org/wiki/Yukawa_potential

In this example we use the gravitational Green's function.

Since the items are carriers with charge ρ_i, the density distribution $\rho_f(\mathbf{r})$ correspond to a potential $\varphi(\mathbf{r})$.

Every item contributes a term $\varphi_i(\mathbf{r} - \mathbf{r_i}) = \frac{-\rho_i}{|r-r_i|}$

$$\varphi(\mathbf{r}) = \sum_i \varphi_i(\mathbf{r} - \mathbf{r_i}) = \sum_i \frac{-\rho_i}{|\mathbf{r} - \mathbf{r_i}|}$$

Example: If there is a static spherically symmetric Gaussian charge density

$$\rho_g(r) = \frac{\rho_c}{\sigma^3\sqrt{2\pi}^3} \exp(\frac{-r^2}{2\sigma^2})$$

where ρ_c is the total charge, then the solution $\varphi(r)$ of Poisson's equation,

$$\nabla^2\varphi = \rho_g$$

is given by

$$\varphi(r) = \frac{\rho_c}{4\pi\varepsilon r}\operatorname{erf}\left(\frac{r}{\sqrt{2\sigma}}\right) = \frac{-1}{4\pi\varepsilon}\int\frac{\rho_g(\mathbf{r'})}{|\mathbf{r} - \mathbf{r'}|}d^3\mathbf{r'}$$

where $erf(x)$ is the error function.

Note that, for r much greater than σ, the erf function approaches unity and the potential $\varphi(r)$ approaches the point charge potential

$$\varphi(r) \approx \frac{-\rho_c}{4\pi\varepsilon r}$$

as one would expect. Furthermore the erf function approaches 1 extremely quickly as its argument increases; in practice for $r > 3\sigma$ the relative error is smaller than one part in a thousand.

9.2.1   Bertrand's theorem

Now we remember Bertrand's theorem.[64] :

Bertrand's theorem states that only two types of central force potentials produce stable, closed orbits:

(1) an inverse-square central force such as the gravitational or electrostatic potential

$$V(r) = \frac{-k}{r}$$ (1)

and

(2) the radial harmonic oscillator potential

$$V(r) = \tfrac{1}{2}\, k\, r^2$$ (2)

According to this investigation it becomes acceptable to assume that the undisturbed shape of the Qpatterns can be characterized by something that comes close to a 3D Gaussian distributions. Since such a distribution produces the correct shape of the gravitation potential*, the underlying mechanism would explain the origin of curvature*.

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[64] http://en.wikipedia.org/wiki/Bertrand's_theorem.

9.3   The internal dynamics of Qpatterns

A Qpattern is generated in a rate of one element per progression step. A corresponding allocation operator that resides in the Hilbert space will reflect these Qtargets in its eigenspace.

During each progression step for each potential type an increment is added to the relatively static potential function. This is performed by transmitting a message to the environment of the Qtarget. The Qtarget is the element, which is currently active.

Depending on the discrete symmetric difference with the embedding continuum to which the building block couples, the wave front is either spherical or anisotropic. In the latter case, it is isotropic in less than three dimensions.

For full 3D isotropic coupling Qtargets[65] the message is sent in the form of a 3D tsunami-like spherical wave front . The wave front folds the embedding continuum. This is the mechanism, which is used in order to transport the message.

By repeating that message for every new Qtarget a constant stream of messages is produced that together form a wave pattern that oscillates with ultra-high frequency[66].

If the Qpattern does not move, then at some distance the situation looks as if an oscillating spherical wave is transmitted from a single source. If the Qpattern oscillates as a whole, then that ultra-high temporal frequency wave gets a lower temporal frequency amplitude and phase modulation.

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[65] See Discrete symmetry sets.

[66] That frequency is determined by the progression step size τ_s.

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The geometry of the emitted wave fronts may depend on the symmetry properties of the emitting Qtarget.
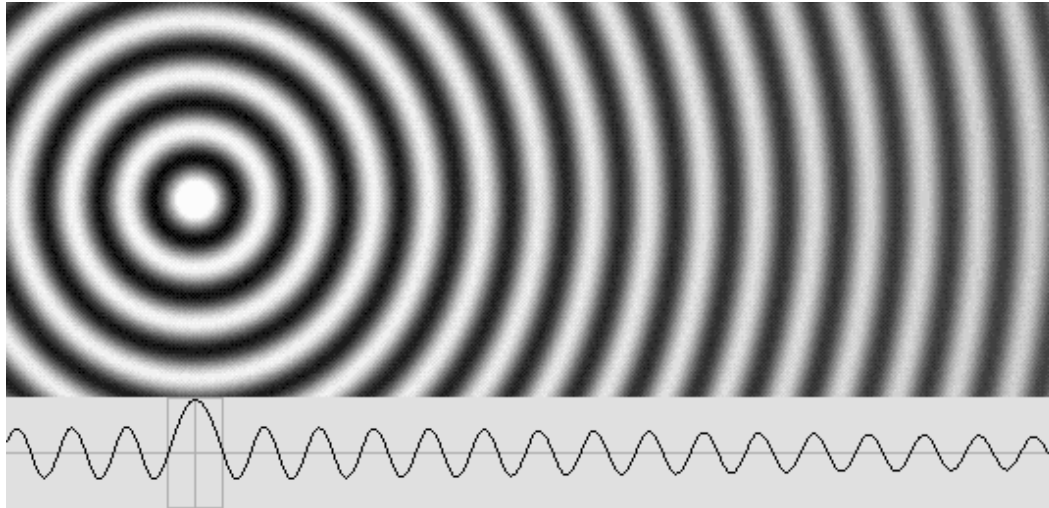
The wave fronts curve the embedding continuum. The effect on local curvature diminishes with distance from the Qtargets. This can be comprehended by accepting that the transport of the wave fronts is controlled by the Huygens principle. The resulting effect is described by the corresponding potential function[67].

The sharp continuous part of the allocation function registers the effect on the embedding continuum and stores this data for the creation of the next version of the embedding continuum.

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[67] See: Waves that spread information.

The correlation vehicle applies the Huygens principle for recreating the embedding continuum at every progression step.

A corresponding operator that resides in the Gelfand triple will reflect the resulting embedding continuum in its eigenspace.

9.4 Qpatterns

9.4.1 Natal and actual Qpatterns

The Qpattern is a dynamic building block. Qpatterns extend over many progression steps[68]. A Qtarget lasts only during a single progression step.

A Qpattern is a coherent collection of objects that are distributed in space by a stochastic process. It means that each Qpattern is created differently.

This coherent distribution can be described by two density distributions. The first one is a scalar function that describes the distribution of the density of the spatial locations.

The second one describes the corresponding current density distribution. It administers the displacement since the last element generation.

The two descriptions combine in a single Quaternionic Probability Amplitude Distribution (QPAD).

The QPAD is a continuous quaternionic function. According to the hypothesis, Qpatterns of a given generation have a QPAD with a fixed natal shape.

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[68] In fact, depending on their generation, they extend over a fixed number of progression steps. See Atoms and their electrons.

The distribution of discrete objects corresponds to several potential functions. For each suitable Green's function a corresponding potential function exists.

In this way the scalar density distribution corresponds to a set of scalar potential functions and the current density distribution corresponds to a set of 3D vector potential functions.

A direct conversion from density distribution to a potential function uses a dedicated Green's function. Each suitable Green's function gives a corresponding potential function. The reverse conversion is only possible when the design plan of the Qpattern is known.

Each natal Qpattern corresponds to a plan. Not all enumerations that are required for generating the planned Qpattern must be used during the life of the actual Qpattern.

Per progression step the generator creates only a single member of the Qpattern and that member is replaced in the next step by another member.

At every instant of progression, Qpatterns contain one actual member and for the rest it consists of virtual members.

The actual member is a location where an event can happen. This actual element is called Qtarget.

That event may be the annihilation of the Qpattern. After that the generation of new elements stops. Or it can be a sudden change of the energy of the Qpattern. In that case a photon is emitted or absorbed.

In any case at every progression instant at the location of the Qtarget, small contributions to the potentials of the Qpattern are generated by the current Qtarget.

Each realization of a Qpattern corresponds to a micro-path that runs along step stones. The Qpatch may move and/or oscillate. The actual distribution of Qtargets spreads along the *actual path* of the building block. In general, this actual path differs from the planned micro-path.

The contributions to the potentials are transmitted by Qtargets at the halts along the actual path.

The natal Qpattern can be described by a function of progression that produces a stochastic spatial location at every subsequent progression step.

Since the collection is generated in a rate of one element per progression step, the contributions to the potential functions are also generated in that rate and at the locations of the Qtargets, which form the current actual element.

It is shown above that the potential functions are generated with the help of wave fronts[69] that with light speed move away from the locations of the elements that generated them.

These wave fronts are emitted with a fixed ultra-high frequency. In the HBM no higher frequency exists.

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[69] The isotropy of the wave front depends on the isotropy of the emitting Qtarget.

Only if the Qpattern stays fixed at a single location in an non-curved part of the embedding continuum, then that location will see the generation of a virtual Qpattern that takes a shape that approaches the planned target distribution. It will take a huge number of progression steps to reach that condition. It is the number of steps that it takes to walk the micro-path.

A moving Qpattern will be spread along the path of the corresponding building block.

A move of the building block may affect the life of the realizable part of the Qpattern[70].

9.4.2　Micro-paths

Qpatterns are representatives of nature's building blocks. They are coherent collections of lower order objects that each can be considered as a location where the building block can be. These objects are generated in a rate of one element per progression step.

The situation can be interpreted as if the building block hops from step stone to step stone. These micro-movements form a micro-path in the form of a random string.

At each arrival at a step stone the building block emits a message. That emission contributes to the potentials of the building block.
The emission does not affect the Qpattern.

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[70] http://en.wikipedia.org/wiki/Particle_decay

In order to stay at the same position, a step in a given direction will on average be followed by a step in the reverse direction. Otherwise the average location will move away or the pattern will implode or explode.

This means that the particle moves along a micro-path and this path is characterized by quasi-oscillations. Similarly the micro-path may show quasi-rotations.

The same micro-path can exist in two directions. This is probably connected with the direction of spin.

9.4.3   Characteristics of the micro-path

The micro-path is a stochastic object and has corresponding characteristics.

A building block type has a fixed number (N_w) of step stones.

The sum of steps results in a building block step (S_b).

This defines the building block speed (S_b/N_w).

The step between subsequent step stones has an average length (l_s) and a step length variance (v_s).[71]

9.4.4   Advantages of QPAD's

The QPAD is a quaternionic function that for all planned step stones of a natal Qpattern describes a static situation, where that description also includes the planned (last) displacement of the step stone. The same holds after the completion of the micro-path for the QPAD of an actual Qpattern.

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[71] The computation of the step length variance has much in common with the computation of Feynman's path integral.

9.4.5   Isotropic space coverage

During generation the natal Qpattern must obtain an isotropic shape. This can be obtained by combining the Poisson process with two uniform random angular rotations. The rotations are mutual perpendicular and have a range of π radians. The Poisson process produces a one dimensional distribution that approaches a normal distribution. It determines the off center distance. The first rotation replaces the point along a circle with central axes A_1, which is perpendicular to the axis A_0 on which the normal distribution spreads. The second rotation uses axis A_2, which is perpendicular to both axis A_0 and A_1. All axes cross at the center location.

Axis A_0 acts as the spin axis of the Qpattern.

Together these actions produce a 3D normal distribution. The choice of the axes offers a sign freedom. Also the direction of rotation is not specified. These sign selections correspond to the discrete symmetry sets of quaternionic numbers.

The direction in which the micro-path is travelled may be connected with the direction of spin. The half and full integer spin value may be related to whether the creation of the distribution covers π or 2π radians.

9.4.6   Qpattern history

A Qpattern can be created and it can be annihilated. If a Qpattern is annihilated, then the generator stops producing new elements. Thus, also the generation of new potential wave fronts will stop. However, existing potential waves will keep proceeding.

A fixed number N_w of wave fronts will signal the annihilation in the form of a modulation of the ultra-high carrier wave. That modulation corresponds to the emission of a photon.

The last generated wave front closes a train of previous wave fronts. This edge moves away with light speed. A previously rather "static" potential will be replaced by a dynamic phenomenon. During a fixed number N_w of progression steps the emitted wave fronts will be modulated. The modulation represents the emission of a photon.

The annihilation frees the identifier of the Qpattern and makes it available for reuse. In this way the identifiers of the Qpatterns refer to their virgin equivalents that were born in the reference Hilbert space.

A sudden change in the energy of the Qpattern will result in the emission or absorption of a photon.

A slow oscillation of the whole Qpattern results in a corresponding oscillation of its potentials. These oscillations become visual as radio waves. The criterion for emitting radio waves is set by the fact whether the micro-path can cover the full oscillation path.

If the micro-path stretches along the path of the oscillation, then the potentials are not affected. It means that the particle emits its usual potentials. This occurs with electrons that take part in the spherical harmonic oscillations inside atoms. In atoms photons are emitted or absorbed when the mode of the spherical harmonic oscillation changes.

9.4.6.1   Looking away

We will define "looking away" as receiving messages from distant objects. Looking away is looking back in proper time.

Looking back as far as is possible is looking back at the virginal state of the historic Qpattern. Looking as far away as is possible is looking at the virginal state. In this way a Qpattern can be coupled both to its past and to its distant background.

On the other side this means that the wave fronts that are transmitted from this virgin state reach the current local Qpattern.

The superposition of all transmitted wave fronts that were emitted in the past and that contribute via superposition to the local potential results in huge background field that acts as a (curved) embedding continuum (for fermions). This effect installs inertia[72].

9.4.7   Qpattern cycle

Despite of the fact that the regeneration process is cyclic, due to the fact that the generator is stochastic, the next cycle need not generate the same natal Qpattern. Only the statistical parameters of this new natal Qpattern will be the same. Depending on its generation, it will correspond to a fixed natal QPAD. For each generation the cycle time τ_c is fixed.

The duration of the cycle is only interesting at the occasions when photons are emitted or absorbed. Emission and absorption of photons takes a full regeneration cycle.

Generation and annihilation can start at any progression instant. The duration of photon emission and photon absorption is fixed. It equals a full Qpattern re-generation cycle.

During the undisturbed life of a Qpattern it will be impossible to determine where the regeneration cycle starts.
Further, the micro-path may exist in two directions.

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[72] See inertia

9.4.8　Fourier transform

A QPAD that has the form of a QPAD of a Gaussian distribution has a Fourier transform that also has the form of a QPAD of a Gaussian distribution. However, the characteristics of the distributions will differ.

The QPAD of a coupled Qpattern is compact in configuration space and wide spread in canonical conjugated space.

The Fourier transform of the QPAD of a Qpattern is its characteristic function[73]. It is a quaternionic function.

9.5　Qtargets

In fact the actual elements, called Qtargets, are represented by *three* different rational quaternions. These rational quaternions define locations or displacements relative to an embedding continuum. That continuum might be curved.

1. For each Qtarget, the first quaternion plays the role of the corresponding parameter. This also holds at zero progression value.
   The real part of this quaternion represents progression. Its imaginary part acts as the identifier of the element. The Qtargets walk through a path as a function of progression.

2. The imaginary part of the *second quaternion* defines the location of the Qtarget in its current embedding continuum. Its real part specifies the local density. It also acts as the relevance factor of the corresponding Hilbert proposition.

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[73] http://en.wikipedia.org/wiki/Characteristic_function_(probability_theory)

3. The imaginary part of the *third quaternion* defines the displacement with respect to the previous Qtarget. The discrete symmetry set of this quaternion determines the "charge" of the Qtarget.
The effective charge is set by the difference between the discrete symmetry set of the Qtarget and the discrete symmetry set of the embedding continuum.
Apart from the discrete symmetry set this third quaternion contains no new information.

The planned and the actual distribution of Qpattern elements can be described by a charged carrier density distribution and a corresponding current density distribution. Via appropriate Green's functions these density distributions correspond to a scalar potential and a corresponding vector potential. The potentials reflect the transmittance of the existence and the discrete properties of the Qtarget via ultra-high frequency information carrier waves.

Since Qtargets are the actual elements of Qpatterns and their identifier is also Qtarget of a Qpattern that existed at zero progression value, the two patterns are connected as well.

9.6   New mathematics

The idea that wave fronts[74] implement the contribution that Green's functions add to the potential functions, represents new mathematics. This is quite clear for the gravitational potential. The emitted wave folds and thus curves the embedding continuum. In this way curvature can be explained.

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[74] For anisotropic Qpatterns the message is transmitted by an anisotropic wave.

It is less clear for other potentials. Especially the encoding of electric charge information in the emitted information is not yet properly established. This encoding uses the difference in discrete symmetry between the Qtarget and the embedding continuum[75].

9.6.1   Waves that spread information

A Qtarget exists during a single progression step. Even when they belong to the same Qpattern will subsequent Qtargets be generated at different locations. If the Qtarget is generated, then in the embedding continuum the Qtarget corresponds to a tsunami-like wave front that has its source at the location of the Qtarget. After the disappearance of the Qtarget the wave front keeps spreading out. The wave fronts that belong to preceding Qtargets and the wave fronts that belong to other Qpatterns will interfere with that wave front. If the Qpatch is stationary, then at sufficient distance it will look as if the waves are generated by a single source. The train of emitted wave fronts will resemble an ultra-high frequency oscillating wave. The amplitude of this oscillating wave decreases with distance from the source. For isotropic spherical waves, this is the reason of the contribution of the term $\frac{Q_i}{|\boldsymbol{r}-\boldsymbol{r}_i|}$ to the static potential integral.

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[75] See elementary particle properties

Example: Generation process with one element per progression instant. Here we use the electrostatic Green's function.

- Poisson process coupled to a binomial process

  - Binomial process implemented by a 3D spread function

  - Produces a 3D distribution

- Which approaches a 3D Gaussian distribution

  - $\rho_f(r) = \frac{Q}{\sigma^3\sqrt{2\pi}^3} \exp\left(\frac{-r^2}{2\sigma^2}\right)$

- This corresponds to a *scalar potential* of the form

$$\varphi(r) = \frac{Q}{4\pi\varepsilon r} \operatorname{erf}\left(\frac{r}{\sqrt{2}\sigma}\right) = \frac{1}{4\pi\varepsilon} \int \frac{\rho_f(\boldsymbol{r'})}{|\boldsymbol{r}-\boldsymbol{r'}|} d^3\boldsymbol{r'} \approx \frac{Q}{4\pi\varepsilon r}\ (r \gg \sigma)$$

- And a *vector potential* of the form

$$\frac{\boldsymbol{Q}}{4\pi\varepsilon r}\ (r \gg \sigma)$$

  - Charge **Q** represents the discrete symmetry set difference between the carrier and the embedding continuum.

If an annihilation event occurs, then the generator stops generating Qtargets for this Qpattern. However the wave fronts that have been started will proceed spreading over the embedding continuum.

When the local generator stops generating then no new wave fronts will be formed. The last wave front and foregoing wave fronts proceed spreading with light speed. The last trail of wave fronts is modulated and carries a photon.

The fact that the wave fronts keep spreading is a consequence of the characteristics of the correlation vehicle, which is implemented by the enumerator generating mechanism. That mechanism also re-generates the embedding continuum for use in the next progression step. The mechanism uses the Huygens principle in order to establish persistence of the floating wave fronts.

The scalar potential functions and vector potential functions that correspond to the charge and current density distributions reflect the transmission of the information that is transmitted by the Qtargets.

The potential functions reveal the existence and the properties of the Qpattern. The potentials can be observed without affecting the Qpattern.

9.6.2   Waves that shrink space

The tsunami-like wave fronts appear to shrink space. The local shrinkage diminishes when the distance from the source increases. As a consequence, for 3D spherical information carrier waves, the influence diminishes as 1/r. Also this fact is a consequence of the actions of the correlation vehicle, which uses the Huygens principle in order to control the flow of the wave fronts.

All quaternionic quantum state functions are fields (they are qua-ternionic probability amplitude distributions) that extend over a lim-ited region of the embedding space.

The corresponding potentials extend over a part of universe that falls within the information horizon of the corresponding particles.

The potential functions act as traces of Qpatterns. When a particle annihilates, then the information about its existence keeps spreading. However, no new information is generated.

The tsunami-like wave front that spreads this information appears to shrink the space where it passes. However, its influence diminishes with distance. For spherical waves the influence diminishes with distance r as 1/r.

As long as a particle lives, it keeps sending these tsunami-like wave fronts. This might be the way that gravitation/space curvature is implemented.

9.6.3   Information carrier waves

Information carrying wave fronts are emitted by Qtargets. The corresponding building block emits these wave fronts at an ultra-high frequency that is set by the progression step size τ_s. The wave fronts move with "light speed". This speed is the highest possible speed that can be achieved for information transmission. Even when the Qtargets belong to the same Qpattern will subsequent Qtargets emit their information carrier wave fronts from different locations.

The spread of information carrier waves is governed by the Huygens principle. The correlation vehicle uses this principle in order to retransmit the waves at every progression step. This holds for transmission in odd numbers of dimensions. For transmission in two dimensions the situation is more complicated.

Information carrier waves pass unblocked through the embedding continuum. These waves are only influenced via interference by other information carrier waves. In that case, the information that they carry combines into a new information set.

If the emitting building block moves, then the new sources of the emitted wave fronts move as well.

If the emitting building block oscillates, then the information carrier wave gets an amplitude and/or phase modulation. The frequency of that modulation will be much lower than the ultra-high frequency of the carrier.

A train of emitted carrier wave fronts constitute a potential field. The interrelation is set by an appropriate Green's function.

9.6.4   Spreading electric charge information

The Qtarget also contains information about the electric charge of the corresponding particle. The process of spreading that information corresponds to the way that gravitational information is transmitted. In this case not the existence and local density, but the charge is transmitted. The charge is determined by the discrete symmetry of the Qtarget in comparison to the discrete symmetry of the embedding continuum. Only the symmetries of the imaginary parts that encode displacement are relevant.

9.6.5   Huygens principle

The correlation vehicle applies the Huygens principle. It means that in every progression step, every location on a wave front can be

seen as a source of a new wave. The Huygens principle acts differently for waves that operate in different numbers of dimensions[76]. The Green's function differs accordingly. For odd dimensions the mechanism works in the commonly understood way.

The Huygens principle acts on ultra-high frequency waves that transmit the information that is contained in potential fields. The corresponding wave fronts proceed with light speed.

9.7   Quasi oscillations and quasi rotations

In order to keep the distribution on average coherent in each dimension, any step in positive direction must be followed by a step in negative direction. With other words a kind of quasi oscillation takes place. This oscillation can be synchronous to a reference or it can be asynchronous. This (a)synchrony may differ per dimension. In a similar way a quasi-rotation can exist.

A special kind of coupling/interaction between fields can be the result of these induced quasi oscillations and or quasi rotations, where distant sources of oscillating potentials induce this coupling with local oscillations.

9.8   Distant Qtargets

The Qtargets of distant Qpatterns also send messages that encode their presence in tsunami-like wave fronts. These waves contribute to a huge local background potential. This effect represents the origin of inertia[77]. Together the potentials of **all** Qpatterns constitute a local potential that can act as an embedding continuum.

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[76] An interesting discussion is given at: http://www.math-pages.com/home/kmath242/kmath242.htm

[77] See Inertia

It is a bit strange that electrostatic potential plays no role in this effect.

In this respect the link http://en.wikipedia.org/wiki/Common_integrals_in_quantum_field_theory may show interesting.

9.9   Spurious elements

Qtargets need not be generated in coherent distributions as is the case with Qpatterns. Coherent distributions correspond to potential functions that are constructed dynamically in a large series of steps.

In extreme cases the distribution consists of a single element that pops up and disappears in a single progression step. During its existence the element still produces a tsunami-like signal in the form of a wave front[78] that travels in the embedding continuum. Again this wave front causes a local curvature.

In large numbers these spurious elements may cause a noticeable effect.

9.10   The tasks of the correlation vehicle

The primary task of the element generator is the generation of Qtargets that are part of Qpatterns.

After the generation and vanishing of the Qtarget the correlation vehicle takes care of the transmission of the information about the generation incident over the embedding continuum in which the Qtarget was produced. This is done in the form of the described tsunami-like wave fronts. This is the second task of the correlation vehicle.

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[78] For anisotropic Qtargets the message is transmitted by an anisotropic wave.

Some tasks are completed before the correlation vehicle stops with the current job. For example a micro-walk is completed before the generator stops generating new step stones. Some related jobs behave similarly. For example the emission or absorption of photons also finish their task after completion of the job.

When the generator stops generating Qtargets for the current Qpattern, then it does not transmit new information but the correlation mechanism keeps supporting the existing flow of information. This means that a third task of the correlation mechanism is the care for the survival of the embedding continuum when the Qtargets vanish.

The transmission of incident information causes space curvature. The sharp part of the allocation function *describes* the strength of the local space curvature. It does this via its differential which specifies a local metric.

Apart from describing the curvature, the correlation mechanism also recreates at every progression step the corresponding embedding continuum. For that purpose it uses the Huygens principle.

Geometric model

10 **Geometrics**

Geometrics enters the model as soon as numerical enumerators are applied. These enumerators are taken from the eigenspaces of operators. This can happen in Hilbert logic and in the Hilbert space and its Gelfand triple.

The geometric model applies the quaternionic Hilbert space model. From now on the complex Hilbert space model and the real Hilbert space model are considered to be abstractions of the quaternionic model. It means that the special features of the quaternionic model bubble down to the complex and real models. For example both lower dimensional enumeration spaces will show blur at small enumeration scales. Further, both models will show a simulation of the discrete symmetry sets that quaternionic systems and functions possess. This can be achieved with spinors and Dirac matrices or with the combination of Clifford algebras, Grassmann algebras and Jordan algebras[79].

The real and complex models suit in situations where multidimensional phenomena can be decoupled from the dimensions in which they appear.

At large scales the model can properly be described by the complex Hilbert space model. After a sufficient number of progression steps, at very large scales the quaternionic model is quasi isotropic.

We will place the reference Hilbert space at zero progression value. This reference Hilbert space can be a subspace of a much larger Hilbert space. However, in the reference Hilbert subspace a state of well-ordered densest packaging must reside.

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[79] See: http://math.ucr.edu/home/baez/rch.pdf

Quaternionic numbers exist in 16 discrete symmetry sets. When used as enumerators, half of this set corresponds with ***negative progression and will not be used in this geometric model.***

As a consequence we will call the Hilbert space at zero progression value the ***start*** of the model.

This model does not start with a Big Bang. Instead it starts in a state that is characterized by densest packaging of the Qpatches. This reference sub-model is well-ordered.

11 Distributions of quaternions

11.1 Continuous quaternionic distributions

Quaternionic distributions consist of a real scalar distribution and an imaginary 3D vector distribution.

It is the sum of a symmetric distribution and an asymmetric distribution.

The complex Fourier transform of a symmetric (complex) function is a cosine transform. It is a real function.

The complex Fourier transform of an anti-symmetric (complex) function is a sine transform. It is an imaginary function.

This cannot directly be translated to quaternionic functions. The simplest solution is to consider the symmetric parts and asymmetric parts separately.

An asymmetric quaternionic function is always anisotropic. A symmetric function can be isotropic.

As shown before the continuous quaternionic distributions can be interpreted as descriptors of the density distribution of a coherent distribution of discrete objects. However the potential functions that can be derived from coherent distributions of discrete objects are also quaternionic functions.

In the HBM these associated potentials can be considered to be generated dynamically.

11.2 RQE's

In principle the base vectors of the Hilbert space can be enumerated by members of a countable affine-like space. Here we concentrate on a **_huge subsp_**ace in which the base vectors are enumerated by rational quaternions.

The huge subspace is covered by a large number of *small dedicated subspaces* that all are identified by a ***Qpatch region***. The (closed) subspaces represent Qpatterns.

The ordering and the corresponding origin of space become relevant when an observer object considers one or more observed objects.

The real parts of the enumerators define progression. In conventional physics progression conforms to proper time. In the HBM all proper time clocks are synchronized. As a consequence according to our model, the equivalent of proper time steps with a fixed step.

RQE stands for ***Rational Quaternionic Enumerator***. This lowest geometrical level is formed by the enumerators of a selected base of a selected member of the sequence of Hilbert spaces. The selected base vectors represent atoms of the Hilbert logic system.

In this level, the embedding continuum plays a secondary role.

The sequence number corresponds with the progression value in the real part of the value of the RQE. In principle the enumerators enumerate a previously unordered set.

The dedicated subspaces are spanned by eigenvectors whose eigenvalues form the elements of ***Qpatterns***. Qpatterns are identified by a ***Qpatch***, which is the weighted center and by a ***Qtarget***, which is the currently actual element. All other elements of the Qpattern and all other vectors of the dedicated subspace are virtual. Virtual means: "reserved, but currently not in use".

Two types of RQE's exist.

- The first type of RQE plays the role the parameter that via the continuous part of the allocation function determine the "origins" of planned Qpatterns. We will call these RQE's *"parameter RQE"*.
- The second type of RQE is defined relative to these origin RQE's. We will call these RQE's *"relative RQE"*

The relative RQE's are targets of the stochastic function that defines the relative locations of the elements of the natal Qpattern. The relative RQE's can be considered to be the target values of the separate stochastic part of the allocation function. They define a natal Qpattern.

The actual Qtarget is the image produced by the total allocation function of the parameter RQE. The total allocation function is the ***convolution*** $\mathcal{P} = \wp \circ \mathcal{S}$ of the continuous part $\wp$ of the allocation function and the stochastic part $\mathcal{S}$ of the allocation function. It maps a parameter RQE onto a selected embedding continuum. For the reference Hilbert space its Gelfand triple delivers the reference continuum as embedding continuum. For later Hilbert spaces the role of the reference embedding continuum is taken over by the superposition of wave fronts.

11.2.1 Reference Hilbert space

A zero value of the real part of an RQE indicates its role in the reference Hilbert space. In the reference Hilbert space the parameter RQE's are well ordered and embedded in a reference continuum that

is taken from the eigenspace of a reference operator that resides in the Gelfand triple of that reference Hilbert space.

The considered huge subspace of the selected reference member of the sequence of Hilbert spaces represents a state of densest packaging of the parameter RQE's. This means that in this subspace of the selected Hilbert space a normal *allocation operator* exists whose discrete and countable eigenspace has eigenvalues that are parameter RQE's, while in the Gelfand triple of this Hilbert space an allocation operator exists whose continuous eigenspace embeds the values of these parameter RQE's in a well-ordered and relative dense way. The relative density is limited by a lowest size of rational quaternions.

Due to this restriction the parameter RQE-space is not afflicted with splits and ramifications[80].

Thus, both the parameter RQE's and the reference continuum are taken from the eigenspace of a corresponding normal allocation operator. These operators will be called reference operators.

In the reference Hilbert space the continuous part of the allocation function is a unity map. The Qpatches in the reference Hilbert space are linear combinations of a coherent set of relative RQE's that together with the parameter RQE of that set correspond to eigenvectors, which together span the dedicated subspace. This dedicated subspace corresponds to a building block.

In the reference Hilbert space the notion of an absolute RQE makes sense. It is the sum of a parameter RQE and a relative RQE. The Qpatch is the average value of all absolute RQE's that belong to the building block.

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[80]   http://en.wikipedia.org/wiki/Quaternion_algebra#Quaternion_algebras_over_the_rational_numbers

11.2.2 Later Hilbert spaces

In each Hilbert space each planned Qpattern has its own local origin. In later Hilbert spaces the embedding continuum is no longer flat as it is in the reference Hilbert space. Also the parameter RQE may have another location (has another imaginary value) than it had in the reference Hilbert space. With other words the parameter RQE's may move.

Still, the actual Qpatch is the average value of all target RQE's that belong to the corresponding building block. The continuous part of the allocation function images the current parameter RQE on a temporary target. This temporary target is taken as the parameter of the stochastic part of the allocation function. This second part produces the Qtarget as a location in the selected embedding continuum. The actual Qpatch can be recalculated after a full generation cycle.

The selected embedding continuum is formed by superposed potentials and is represented by the eigenspace of a dedicated operator that resides in the Gelfand triple. The corresponding potential is a special type. It is the gravitation potential.

Relative RQE's act as planned target vales for elements of actual Qpatterns. They are planned target values for a corresponding parameter RQE of the complete allocation function. The actual target values are the Qtargets.

The Qpatch of the actual building block will become the expectation value of the Qtargets. Thus, at higher progression values, it no longer corresponds to the average value of the undistorted absolute RQE's that characterize the natal Qpattern.

In general, Qtargets are locations in a curved space. Only in the reference Hilbert space, that space is flat.

HYPOTHESIS 3: *At the start of the life of the considered huge subspace the HBM used only one discrete symmetry set for its lowest level of geometrical objects. This discrete symmetry set is the same set that characterizes the reference continuum. This situation stays throughout the history of the model. This set corresponds with the set of eigenvalues of an RQE allocation operator that resides in the reference quaternionic Hilbert space model.*

For each building block, in the reference Hilbert space one of the relative RQE's becomes after adaptation the actual element and will be called Qtarget. In each subsequent Hilbert space another relative RQE will be selected whose image becomes the Qtarget. The selection of the relative RQE occurs via a random process.

In subsequent Hilbert spaces a new eigenvalue of the reference allocation operator becomes the parameter RQE of the new Qtarget of the building block. This goes together with the selection of a new relative RQE. The relative RQE will differ in a random way from the original relative RQE. Thus Qtargets are for a part a continuous function ($\wp$) of the corresponding parameter RQE's and for another part the function result is blurred by a random generator function ($\mathcal{S}$). The convolution ($\mathcal{P}$) of the continuous function and the random generator function ($\mathcal{S}$) determines the location of the current Qtarget.

$$\mathcal{P} = \wp \circ \mathcal{S} \tag{1}$$

($\mathcal{S}$) stands for stochastic spatial spread function. The assignment of the target value of the random function ($\mathcal{S}$) occurs according to a given ***plan***. The natal (undisturbed) result of ($\mathcal{S}$) is a natal Qpattern that is described by a fixed natal quaternionic probability amplitude

distribution (natal QPAD) ψ. A significant difference may exist between the planned building block and the actually realized building block.

11.3   Potentials

Relative RQE's are the (relative) identifiers of the elements of a Qpattern.

Parameter RQE's are parameters of Qtargets. Qpatterns exist during a series of subsequent Hilbert spaces. They represent nature's building blocks.

The absolute RQE's reside in the reference Hilbert space, which occurred in the past.

The real part of the RQE's reflect the current progression value. The parameter RQE's reside in each of the subsequent Hilbert spaces.

Qpatches are linear combinations of the values of elements of a Qpattern. They represent the expectation values of the Qtargets.

The elements of the Qpatterns correspond to base vectors of dedicated Hilbert subspaces.

The Qtargets emit contributions to the potentials of the Qpatterns.

Potentials depend on their Green's function. Apart from that, two kinds of potentials exist: scalar potentials and vector potentials. Potentials of the same type superpose.

The potentials that possess sufficient reach may together add up to huge local potentials[81]. Locally the superposition of wave fronts that form the potentials constitute a curved continuum that can be used to embed localizable objects. This continuum installs inertia for the embedded Qpatterns.

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[81] See Inertia

For all continuous quaternionic functions and for each discrete symmetry set of its parameter space, the function exists in 16 different discrete symmetry sets for its function values.

In the HBM the discrete symmetry set of the parameter RQE's is fixed.

The quaternionic potentials are continuous functions. Their superpositions constitute embedding continuums. This means that for vector potentials also 16 different embedding continuums exist.

Also the allocation function exists in 16 different discrete symmetry sets for its function values. The sharp continuous part of the allocation function describes an embedding continuum. The allocation function keeps its discrete symmetry set throughout its life.

Discrete symmetry sets do not influence the scalar potentials that are connected to object density distributions. The superposition of wave fronts that form these scalar potentials constitutes a special embedding continuum. This continuum characterizes the **Palestra**. It is described by the gravitation potential field. This does not say that in the realm of the Palestra no other potentials play their role.

11.3.1   Diluted potential identity

In the HBM the embedded continuum is thought to be composed by the superposition of emitted wave fronts of ALL elementary particles. In this superposition the potentials lose their identity. The only thing that stays important is that these potentials are formed by superposed wave fronts. During their propagation through universe their amplitude diminishes with travelled distance. However, the

number of sources that contribute to the local superposed value increases with distance. The result is a huge background potential that has a diluted identity. It is no longer a gravitation potential and it is also no electrostatic potential. It is a diluted potential. It has many aspects of the gravitation potential, but it forms the medium in which the new wave fronts are formed, while modulations of these wave fronts represent photons. In contemporary physics photons are considered to be EM waves.

The gravitation potential and the electrostatic potential have different Green's functions. This means that in the diluted potential the Green's function has lost its distinguishing effect.

11.4 Palestra

The second geometric level is a curved space, called Palestra. As ingredients, it consists of an embedding continuum, the embedded Qtarget set and a sharp continuous quaternionic allocation function.

The local curvature is defined via the differential of the continuous (sharp) quaternionic allocation function. The parameter space of the allocation function embeds the parameter RQE-set. Thus since the parameter RQE-set is countable, the Palestra contains a countable set of images of the sharp allocation function.

We have called these images "local origins" of Qpatterns.

The Qpatches represent the expectation values of the corresponding Qtarget values. The allocation function exists in 16 versions. The version determines the discrete symmetry set of the Qpattern and of the corresponding Qtargets.

The allocation function may include an isotropic scaling function.

The differential of the allocation function defines an infinitesimal quaternionic step. In physical terms the length of this step is the infinitesimal coordinate time interval. The differential is a linear combination of sixteen partial derivatives. It defines a quaternionic metric[82].

The enumeration process adds a coordinate system. The selection of the coordinate system is arbitrary. The origin and the axes of this coordinate system only become relevant when the distance between locations must be handled. The origin is taken at the location of the current observer.

The underlying space is an affine-like space. It does not have a unique origin. We only consider an enumerated compartment of the affine-like space.

11.5   Qpatch regions

The third level of geometrical objects consists of a countable set of space patches that occupy the Palestra. We already called them Qpatch regions.

Qpatches are expectation values of the Qtarget images of the parameter RQE's that house in the first geometric object level. The set of parameter RQE's is used for the part of the allocation function that produces the local Qpattern origins.

Apart from the rational quaternionic value of the corresponding local origin, the discrete symmetry set of that origin will be shared by all elements of the corresponding Qpattern.

The curvature of the second level space relates to the density distribution of the local origins of the Qpatterns and to the total energy of the corresponding Qpatterns.

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[82] See the paragraph on the spacetime metric.

The Qpatches represent the weighted centers of the **regions**[83] where next level objects can be detected.

The name Qpatch stands for space patches with a quaternionic value. The charge of the Qpatches can be named Qsymm, Qsymm stands for discrete symmetry set of a quaternion. However, we already established that the value of the enumerator is also contained in the property set that forms the Qsymm charge.

The enumeration problems that come with the quaternionic Hilbert space model indicate that the Qpatches are in fact centers of a fuzzy environment that houses the potential locations where the actual parameter RQE images (the Qtargets) can be found. The subsequent Qtargets form a micro-path.

11.6 QPAD's and Qtargets

The fuzziness in the sampling of the enumerators and their images in the embedding continuum is described by a **quaternionic probability amplitude distribution** (QPAD).

The squared modulus of the **complex probability amplitude distribution** (CPAD) represents the probability that an image of a parameter RQE will be detected on the exact location that is specified by the value of the target of the blurred allocation function.

In the QPAD this location probability is represented by the real part of the QPAD. The imaginary part describes a corresponding displacement probability. The real part is an object density distribution and the imaginary part is the associated current density distribution. The real part is a scalar function and the imaginary part is a 3D vector function.

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[83] Not the exact locations.

Both a CPAD and a QPAD can describe a Qpattern. A QPAD gives a more complete description.

A natal Qpattern is generated in a rate of one element per progressions step.

A natal Qpattern is generated via a fixed statistical plan and is not disturbed by space curvature or a moving local origin. Since a Qpattern is generated by a stochastic process, the same natal QPAD can correspond to different natal Qpatterns.

The ***QPAD's that describe natal Qpatterns have a flat target space in the form of a quaternionic continuum.***

This natal QPAD describes the planned blur (ψ) to the image of the sharp allocation function ($\wp$). The blurred allocation function ($\mathcal{P}$) is formed by the ***convolution*** of the sharp allocation function ($\wp$) with stochastic generator function ($\mathcal{S}$). The results of this generator function are described by the natal QPAD (ψ) that on its turn describes the natal Qpattern.

The parameter space of the blurred allocation function ($\mathcal{P}$) is a flat quaternionic continuum. The parameter RQE's form points in that continuum.

The generator function ($\mathcal{S}$) is a stochastic function of progression. Its anchor point is the image by the continuous part ($\wp$) of the allocation function ($\mathcal{P}$) of the selected parameter RQE. Its target domain is an embedding continuum. The natal Qtarget is one of the function values of ($\mathcal{S}$). Usually, the actual Qtarget is displaced with respect to the natal Qtarget.

In this way the local form of the actually realized QPAD describes a deformed Qpattern. The adaptation concerns the form factor and the gradual displacement of the deformed QPAD. The form factor may differ in each direction. It is determined by the local differential ($d\wp$) of the sharp allocation function ($\wp$).

The image of a parameter RQE that is produced by the *blurred allocation function* ($\mathcal{P}$) is a ***Qtarget***. ***Qtargets only live during a single progression step***. Qtargets mark the location where (higher level) objects may be detected.

In this way QPAD's exist in two types. The natal QPAD type describes the undisturbed natal Qpattern. It describes a fixed plan. The second QPAD type describes the collection of potential Qtargets that at a rate of one element per progression step are or will be[84] ***locally*** generated by the blurred allocation function. That is why this second QPAD type is also called an actual local QPAD.

The natal Qpattern can also be described by a function ($\mathcal{S}$) that produces a stochastic spatial location at every subsequent progression interval. That natal Qpattern describes a natal micro-path.

The fact that Qtargets only exist during a single progression step means that on the instant of an event the generation of the Qpattern might stop or might proceed in a different mode. Only if the Qpattern stays untouched, a rather complete Qpattern will be generated at that location.

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[84] Adding to the QPAD Qtargets that still have to be generated can be considered as an odd decision.

When the Qpatch moves, then the corresponding actual Qpattern smears out. With other words the natal QPAD is a plan rather than reality.

An event means that a Qpattern stops being generated or is generated in a different mode. Being generated means that it is coupled to an embedding continuum. The generator will create a relatively small pattern in that continuum. Coupling means that the generated Qpattern is coupled via its Qpatch to a mirror Qpattern that houses in the embedding continuum. This is reflected in the coupling equation[85].

Local QPAD's are quaternionic distributions that contain a scalar density distribution in their real part that describes a density distribution of *potential* Qtargets. Further they contain a 3D vector function in their imaginary part that describes the associated current density distribution of these potential Qtargets.

Continuous quaternionic distributions exist in sixteen different discrete spatial symmetry sets. However, the QPAD's inherit the discrete symmetry of their connected sharp allocation function.

The Qpatterns may mingle and then the QPAD's will superpose. However the spatial extent of Qpatterns is quite moderate. In contrast, the potentials of their Qtargets reach very far. Quite probably these potentials will superpose. Together the potentials of distant building blocks form a background potential.

Depending on the Green's functions, the local QPAD's correspond to several types of quaternionic potential functions. These

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[85] See coupling equation.

quaternionic potential functions combine a scalar potential and a vector potential.

Qpatterns form coherent countable discrete sets.
A natal Qpattern conforms to a plan.
A Qpattern is a mostly virtual object.
QPAD's are continuous functions.
A QPAD may describe a Qpattern.
A natal QPAD describes a plan.
A natal QPAD describes a natal Qpattern.
A natal QPAD describes a mostly virtual object.
A Qtarget is an actually existing object.
A Qtarget is an element of a Qpattern that is described by an actual QPAD.

11.6.1 Inner products of QPAD's

Each Qpattern is a representative of a Hilbert subspace and indirectly the Qpattern represents a quantum logic proposition. The corresponding Qpatch is represented by a linear combination of Hilbert base vectors and is represented by a Hilbert proposition. These base vectors are eigenvectors of the location operator. The coefficients are determined by the values of the real part of the QPAD. The Qpatch vector represents some aspects of the QPAD.

Two QPAD's a and b have an inner product defined by

$$\langle a|b \rangle = \int_V a \, b \, dV \tag{1}$$

Since the Fourier transform $\mathcal{F}$ preserves inner products, the Parseval equation holds for the inner product:

$$\langle a|b \rangle = \langle \mathcal{F}a|\mathcal{F}b \rangle = \langle \tilde{a}|\tilde{b} \rangle = \int_{\tilde{V}} \tilde{a} \, \tilde{b} \, d\tilde{V} \tag{2}$$

QPAD's have a norm

$$|a| = \sqrt{\langle a|a \rangle} \tag{3}$$

11.7 Blurred allocation functions

The blurred allocation function $\mathcal{P}$ has a flat parameter space that is formed by rational quaternions. It is the convolution of the sharp allocation function $\wp$ with a stochastic spatial spread function $\mathcal{S}$ that generates a blur that is represented by a planned natal Qpattern and is described by planned natal QPAD ψ. $\wp$ has a flat parameter space that is formed by real quaternions.

Natal QPAD ψ has rational quaternionic parameters and corresponds via the sharp allocation function $\wp$ to an actual QPAD ϕ.

$$\mathcal{P} = \wp \circ \mathcal{S} \tag{1}$$

$\wp$ describes the long range variation and ψ describes the short range variation.

Due to the fact that Qpatterns are mostly virtual, the relation between ψ and ϕ is not easily described. However, since $\wp$ has mainly long range effects and $\mathcal{S}$ has mainly short range effects, it is possible to describe the effect of $\mathcal{P}$ on the actual local QPAD ϕ as a deformed and displaced natal QPAD ψ, where the form factor is controlled by the differential $d\wp$ of the sharp allocation function.

The sharp part of the allocation function specifies the current embedding continuum. In fact this function defines the eigenspace of a corresponding operator that resides in the Gelfand triple of the current Hilbert space.

The planned Qpattern is the result of a Poisson process that is coupled to a binomial process, while the binomial process is implemented by a 3D spread function. This second part $\mathcal{S}$ of the allocation function $\mathcal{P}$ influences the local curvature. The differential $d\wp$ of the first part $\wp$ defines a **quaternionic metric** that describes the **local spatial curvature**. This means that the two parts must be in concordance with each other.

Fourier transforms cannot be defined properly for functions with a curved parameter space, however, the blurred allocation function $\mathcal{P}$ has a well-defined Fourier transform $\tilde{\mathcal{P}}$, which is the product of the Fourier transform $\widetilde{\wp}$ of the sharp allocation function and the Fourier transform $\tilde{\mathcal{S}}$ of the stochastic spatial spread function $\mathcal{S}$.

$$\tilde{\mathcal{P}} = \widetilde{\wp} \times \tilde{\mathcal{S}} \tag{2}$$

$\tilde{\mathcal{S}}$ corresponds to a Fourier transform $\tilde{\psi}$ of the planned natal QPAD ψ.

The Fourier transform pairs and the corresponding canonical conjugated parameter spaces form a double-hierarchy model.

16 blurred allocation functions exist that together cover all Qpatches. One of the 16 blurred allocation functions acts as reference. The corresponding sharp allocation function and thus the corresponding actual QPAD ϕ have the same discrete symmetry set as the lowest level space.

The fact that the blur ψ mainly has a local effect makes it possible to treat $\wp$ and ψ separately[86].

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[86] ψ concerns quantum physics. $\wp$ concerns general relativaty.

11.8 Local QPAD's and their superpositions

The model uses Qpatterns in order to implement the fuzziness of the local interspacing. After adaptation of the form factor to the differential of the sharp allocation function a local QPAD is generated. The non-deformed natal QPAD describes a natal Qpattern.

Each Qpattern possess a private inertial reference frame[87].

Local QPAD's may superpose.

Each of the natal Qpatterns extends over a restricted part of the embedding continuum. The probability amplitude of the elements of these Qpatterns quickly diminishes with the distance from their center point[88].

Natal Qpatterns do not feature potentials, but actual Qpatterns and their local QPAD's will feature potentials.

The gravitation potential of an actual Qpattern extends over the whole embedding continuum. As a consequence superpositions of such potentials may cover the whole embedding continuum. In fact they may constitute the embedding continuum.

11.9 Generations

Photons and gluons correspond to a special kind of fields. They differ in temporal frequency from the ultra-high frequency carrier waves that constitute the potentials of particles.

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[87] See the paragraph on inertial reference frames.
[88] See the paragraph on the enumeration process.

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Photons and gluons can be interpreted as amplitude modulations of the potential generating waves. Two photon types and six[89] gluon types exist[90].

For fermions, three[91] generations of Qpatterns exist that have non-zero extension and that differ in their basic form factor. This paper does not in detail explain the existence of these generations.

The generator of enumerators is for a part a random number generator. That part is implemented by a Poisson process and a subsequent binomial process. Generations correspond to different characteristics of the enumerator generator.

All generated Qpatterns may differ in their quasi-oscillations and quasi-rotations.

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[89] In the Standard Model gluons appear as eight superpositions of the six base gluons.

[90] Bertrand's theorem indicates that under some conditions, photons and gluons might be described as radial harmonic oscillators.

[91] At least three generations are known.

12 **Coupling**

According to the coupling equation, coupling may occur because the two QPAD's that constitute the coupling take the same location. Several reasons can be given for this coupling. The strongest reason is that the Qpattern generator produces two patterns that subsequently are coupled.

Other reasons are:
1. Coupling between Qpatterns can be achieved by coupling to each other's potential functions.
    a) Coupling may occur between the local Qpattern and the potentials of very distant Qpatterns. This kind of coupling causes inertia. The coupling products appear to be fermions.
    b) Coupling may occur between the local Qpattern and the potentials of locally situated Qpatterns. These coupling products appear to be bosons.

The fermion coupling uses the gravitation potential, which is a scalar potential. On itself this does not enforce a discrete symmetry. (Suggestion: That symmetry can be enforced/induced by involving the discrete symmetry of the parameter space and/or the discrete symmetry of the virgin Qpattern).

2. Coupling can also occur via induced quasi oscillations and or induced quasi rotations. These quasi-oscillations and quasi-rotations occur in the micro-paths of the Qpatterns. Because they differ in their discrete symmetry they may take part in a local oscillation

where an outbound move is followed by an inbound move and vice versa[92].

3. For fermions coupling also occurs with the parameter RQE and with the historic Qpattern that belongs to this RQE.

12.1 Background potential

We use the ideas of Denis Sciama[93][94][95].

The superposition of all real parts of potentials[96] of distant Qpatterns that emit potential contributions in the form of spherical waves produces a uniform background potential. At a somewhat larger distance r these individual scalar potentials diminish in their amplitude as $1/r$. However, the number of involved Qpatterns increases with the covered volume. Further, on average the distribution of the Qpatterns is isotropic and uniform. The result is a huge (real) local potential Φ

$$\Phi = -\int_V \frac{\bar{\rho}_0}{r} dV = -\bar{\rho}_0 \int_V \frac{dV}{r} = 2\pi\, R^2 \bar{\rho}_0 \tag{1}$$

$$\bar{\rho} = \bar{\rho}_0; \ \overline{\boldsymbol{\rho}} = \mathbf{0} \tag{2}$$

Apart from its dependence on the average value of $\bar{\rho}_0$, Φ is a huge constant. Sciama relates Φ to the gravitational constant G.

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[92] See: Coupling Qpatterns.

[93] http://arxiv.org/abs/physics/0609026v4.pdf

[94] http://www.adsabs.harvard.edu/abs/1953MNRAS.113...34S

[95] http://rmp.aps.org/abstract/RMP/v36/i1/p463_1

[96] In fact it is the superposition of all wave fronts that does this.

$$G = (-c^2) / \Phi \qquad (3)$$

If a local Qpattern moves in this background potential relative to the universe with a uniform speed $\boldsymbol{v}$, then a vector potential $\boldsymbol{A}$ is generated.

$$\boldsymbol{A} \ = \ -\int_V \frac{\boldsymbol{v} \, \bar{\rho}_0}{c \, r} \, dV \qquad (4)$$

Both $\bar{\rho}_0$ and $\mathbf{v}$ are independent of r. The product $\boldsymbol{v} \, \bar{\rho}_0$ represents a current. Together with the constant c they can be taken out of the integral. Thus

$$\boldsymbol{A} \ = \ \Phi \frac{\boldsymbol{v}}{c} \qquad (5)$$

Field theory learns[97]:

$$\mathfrak{E} = \ -\boldsymbol{\nabla}\Phi - \frac{1}{c} \cdot \dot{\boldsymbol{A}} \qquad (6)$$

If we exclude the first term because it is negligible small, we get:

$$\mathfrak{E} = \ -\frac{\Phi}{c^2} \, \dot{\boldsymbol{v}} = G \, \dot{\boldsymbol{v}} \qquad (7)$$

The fields Φ and $\boldsymbol{A}$ together form a quaternionic potential. However, this time the fields Φ and $\boldsymbol{A}$ do not represent the potential of a Qpattern.

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[97] Q-FORMULÆ 13

12.2 Interpretation

As soon as an acceleration of a local Qpattern occurs, an extra component $\dot{\boldsymbol{A}}$ of field $\mathfrak{E}$ appears that corresponds to acceleration $\dot{\boldsymbol{v}}$.[98]

In our setting the component $\boldsymbol{\nabla}\Phi$ of the field $\mathfrak{E}$ is negligible. With respect to this component the items compensate each other's influence. This means that if the influenced subject moves with uniform speed $\boldsymbol{v}$, then $\mathfrak{E} \approx 0$. However, a vector potential $\boldsymbol{A}$ is present due to the movement of the considered local Qpattern. Any acceleration of the considered local item goes together with an extra non-zero $\mathfrak{E}$ field. In this way the universe of particles causes inertia in the form of a force that acts upon the scalar potential of the accelerating item.

The amplitude of Φ says something about the number of coupled Qpatterns of the selected generation that exist in universe. If it is constant and the average interspacing grows with progression, then the universe dilutes with increasing progression. Also the volume of the reference continuum over which the integration must be done will increase with progression. The total energy of these coupled Qpatterns that is contained in universe equals:

$$E_{total} = \sqrt{\int_V \left|\frac{\bar{\rho}_0}{r}\right|^2 dV}$$

(1)

The background potential Φ is the superposition of the contributions of waves that are emitted by distant particles. The emission occurred with ultra-high frequency. This is the highest frequency that exists in the HBM. The background potential constitutes an embedding continuum.

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[98] See: Inertia from the coupling equation.

Waves that oscillate with a lower frequency, such as photons and radio waves, can be considered as amplitude modulations of the ultra-high frequency (potential) field.

The enumerator generator uses the background potential as the embedding continuum for its embedded products.

The allocation function describes this embedding continuum and takes care of its permanence.

12.3   Isotropic vector potential

The scalar background potential may be accompanied by a similar background vector potential that is caused by the fact that the considered volume that was investigated in order to calculate the scalar background potential is enveloped by a surface that delivers a non-zero surface integral. The isotropic background vector potential corresponds to an isotropic scaling factor. This factor was already introduced in the first phases of the model.

12.4   Quantum fluid dynamics

12.4.1   Quaternionic nabla

The quaternionic nabla stands for

$$\nabla \overset{\text{def}}{=} \left\{ \frac{\partial}{\partial \tau}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\} \tag{1}$$

$$\psi \overset{\text{def}}{=} \psi_0 + \boldsymbol{\psi} \tag{2}$$

Here τ stands for the progression parameter.

$$\phi = \nabla \psi \tag{3a}$$

$$\phi_0 = \nabla_0 \psi_0 - \langle \boldsymbol{\nabla}, \boldsymbol{\psi} \rangle \tag{3b}$$

$$\boldsymbol{\phi} = \nabla_0 \boldsymbol{\psi} + \boldsymbol{\nabla}\psi_0 + \boldsymbol{\nabla} \times \boldsymbol{\psi} \tag{3c}$$

(3a) is the differential equation for continuous quaternionic distributions. Rearranging shows:

$$\nabla\psi = \phi \tag{4}$$

This is the differential continuity equation. It holds for QPAD's

12.4.2 The differential and integral continuity equations

Let us approach the balance equation from the integral variety of the balance equation. Balance equation is another name for continuity equation.

We replace ψ by ρ, ψ_0 by ρ_0 and $\boldsymbol{\psi}$ by $\boldsymbol{\rho} = \rho_0 \boldsymbol{v}/c$.

$$\rho \overset{\text{def}}{=} \rho_0 + \boldsymbol{\rho} \tag{1}$$

When ρ_0 is interpreted as a charge density distribution, then the conservation of the corresponding charge[99] is given by the continuity equation:

Total change within V = flow into V + production inside V (2)

In formula this means:

$$\frac{d}{d\tau}\int_V \rho_0 \, dV = \oint_S \boldsymbol{\hat{n}}\rho_0 \frac{\boldsymbol{v}}{c} \, dS + \int_V s_0 \, dV \tag{3}$$

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[99] Also see Noether's laws: http://en.wikipedia.org/wiki/Noether%27s_theorem

$$\int\limits_V \nabla_0 \rho_0 \, dV = \int\limits_V \langle \mathbf{\nabla}, \boldsymbol{\rho} \rangle \, dV + \int\limits_V s_0 \, dV \tag{4}$$

The conversion from formula (2) to formula (3) uses the Gauss theorem[100].

Here $\hat{\boldsymbol{n}}$ is the normal vector pointing outward the surrounding surface S,

$\boldsymbol{v}(\tau, \boldsymbol{q})$ is the velocity at which the charge density $\rho_0(\tau, \boldsymbol{q})$ enters volume V and s_0 is the source density inside V.

In the above formula $\boldsymbol{\rho}$ stands for

$$\boldsymbol{\rho} = \rho_0 \boldsymbol{v}/c \tag{5}$$

It is the flux (flow per unit area and unit time) of ρ_0 .

The combination of $\rho_0(q)$ and $\boldsymbol{\rho}(q)$ is a quaternionic skew field $\rho(q)$ and can be seen as a probability amplitude distribution (QPAD).

ρ is a function of q.

$$q \stackrel{\text{def}}{=} q_0 + \boldsymbol{q}; \; q_0 = \tau \tag{6}$$

$\rho(q)\rho^*(q)$ can be seen as an overall probability density distribution of the presence of the carrier of the charge.

$\rho_0(q)$ is a charge density distribution.

$\boldsymbol{\rho}(q)$ is the current density distribution.

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[100] http://en.wikipedia.org/wiki/Divergence_theorem

This results in the law of charge conservation:

$$s_0(q) = \nabla_0 \rho_0(q) \mp \langle \nabla, (\rho_0(q)v(q) + \nabla \times a(q)) \rangle \tag{7}$$

$$= \nabla_0 \rho_0(q) \mp \langle \nabla, \boldsymbol{\rho}(q) + \boldsymbol{A}(q) \rangle$$

$$= \nabla_0 \rho_0(q) \mp \langle v(q), \nabla \rho_0(q) \rangle \mp \langle \nabla, v(q) \rangle \rho_0(q)$$

$$\mp \langle \nabla, \boldsymbol{A}(q) \rangle$$

The blue colored $\pm$ indicates quaternionic sign selection through conjugation of the field $\rho(q)$.

The field $a(q)$ is an arbitrary differentiable vector function.

$$\langle \nabla, \nabla \times a(q) \rangle = 0 \tag{8}$$

$A(q) \stackrel{\text{def}}{=} \nabla \times a(q)$ is always divergence free. In the following we will neglect $A(q)$.

Equation (6) represents a balance equation for charge density. What this charge actually is, will be left in the middle. It can be one of the properties of the carrier or it can represent the full ensemble of the properties of the carrier.

Up to this point the investigation only treats the real part of the full equation. The full continuity equation runs:

$$s(q) = \nabla \rho(q) = s_0(q) + s(q) \tag{9}$$

$$= \nabla_0 \rho_0(q) \mp \langle \nabla, \boldsymbol{\rho}(q) \rangle \pm \nabla_0 \boldsymbol{\rho}(\tau, \boldsymbol{q})$$
$$+ \nabla \rho_0(\tau, \boldsymbol{q}) \pm (\pm \nabla \times \boldsymbol{\rho}(\tau, \boldsymbol{q}))$$

$$= \nabla_0 \rho_0(\tau, \boldsymbol{q}) \mp \langle \boldsymbol{v}(q), \nabla \rho_0(q) \rangle \mp \langle \boldsymbol{\nabla}, \boldsymbol{vq} \rangle \rho_0(q)$$

$$\pm \nabla_0 \boldsymbol{v}(q) + \nabla_0 \rho_0(q) + \boldsymbol{\nabla} \rho_0(q)$$

$$\pm \left(\pm \left(\rho_0(q) \, \boldsymbol{\nabla} \times \boldsymbol{v}(q) - \boldsymbol{v}(q) \times \boldsymbol{\nabla} \rho_0(q) \right) \right)$$

After splitting into real and imaginary equations, this leads to:

$$s_0(q) = 2\nabla_0 \rho_0(q) \mp \langle \boldsymbol{v}(q), \nabla \rho_0(q) \rangle \mp \langle \boldsymbol{\nabla}, \boldsymbol{v}(q) \rangle \rho_0(q) \tag{10}$$

$$\boldsymbol{s}(q) = \pm \nabla_0 \boldsymbol{v}(q) \pm \boldsymbol{\nabla} \rho_0(q) \tag{11}$$
$$\pm \left(\pm \left(\rho_0(q) \, \boldsymbol{\nabla} \times \boldsymbol{v}(q) - \boldsymbol{v}(q) \times \boldsymbol{\nabla} \rho_0(q) \right) \right)$$

The red sign selection indicates a change of handedness by changing the sign of one of the imaginary base vectors. Conjugation also causes a switch of handedness. It changes the sign of all three imaginary base vectors.

In its simplest form the full continuity equation runs:

$$s(q) = \nabla \rho(q) \tag{12}$$

Thus the full continuity equation specifies a quaternionic distribution s as a flat differential $\nabla \rho$.

When we go back to the integral balance equation, then holds for the imaginary parts:

$$\frac{d}{d\tau} \int_V \boldsymbol{\rho} \, dV = -\oint_S \hat{\boldsymbol{n}} \rho_0 \, dS - \oint_S \hat{\boldsymbol{n}} \times \boldsymbol{\rho} \, dS + \int_V \boldsymbol{s} \, dV \tag{13}$$

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$$\int_V \nabla_0 \, \boldsymbol{\rho} \, dV = -\int_V \nabla \rho_0 \, dV - \int_V \nabla \times \boldsymbol{\rho} \, dV + \int_V \boldsymbol{s} \, dV \tag{14}$$

For the full integral equation holds:

$$\frac{d}{d\tau}\int_V \rho \, dV + \oint_S \hat{\boldsymbol{n}}\rho \, dS = \int_V s \, dV \tag{15}$$

$$\int_V \nabla \rho \, dV = \int_V s \, dV \tag{16}$$

Here $\hat{\boldsymbol{n}}$ is the normal vector pointing outward the surrounding sur-face S, $\boldsymbol{v}(q)$ is the velocity at which the charge density $\rho_0(q)$ enters volume V and s_0 is the source density inside V. In the above formula ρ stands for

$$\rho = \rho_0 + \boldsymbol{\rho} = \rho_0 + \frac{\rho_0 \boldsymbol{v}}{c} \tag{17}$$

It is the flux (flow per unit of area and per unit of progression) of ρ_0 .

τ stands for progression (not coordinate time).

12.5   The coupling equation

The coupling equation is a special form of the continuity equation. ψ is a normalized quaternionic distribution.

$$\langle\psi|\psi\rangle = \int_V |\psi|^2 \, dV = 1 \tag{1}$$

$$\nabla\psi = \phi \tag{2}$$

We also normalize a replacement φ for ϕ by dividing a by a real factor m

$$\phi = m \, \varphi \tag{3}$$

$$\langle\varphi|\varphi\rangle = \int_V |\varphi|^2 \, dV = 1 \tag{4}$$

This results in the coupling equation (7), which holds for coupled field pairs $\{\psi, \varphi\}$

$$\langle\phi|\phi\rangle = \int_V |\phi|^2 \, dV = m^2 \tag{5}$$

$$\langle\nabla\psi|\nabla\psi\rangle = \int_V |\nabla\psi|^2 \, dV = m^2 \tag{6}$$

This equation (6) does not directly depend on φ, thus it also holds for composites. Finally, the coupling equation reads:

$$\nabla\psi = m \, \varphi \tag{7}$$

The quaternionic format of the Dirac equation for the electron is a special form of the coupling equation.

$$\nabla \psi = m \, \psi^*$$ (8)

The coupling equation appears to hold for elementary particles and simple composite particles. For anti-particles hold.

$$(\nabla \psi)^* = m \, \varphi^*$$ (9)

Due to the fact that the parameter space is not conjugated, equation (9) differs from equation (7).

The quaternionic format of the Dirac equation for the positron is a special form of the coupling equation for anti-particles.

$$(\nabla \psi)^* = m \, \psi$$ (10)

12.6 Path integral

The coupling factor m is related to the standard deviation of the step length $\sigma_s = \sqrt{v_s}$.[101]

A large collection of micro-paths can correspond to the same quaternionic density distribution. A smaller set will show a direct relation of the standard deviation σ_s of the step length with the coupling factor m. The formulation and the success of Feynman's path integral indicates that the micro-path with minimal σ_s will be used by nature.

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[101] The computation of the step length variance has much in common with the computation of Feynman's path integral.

12.7 How to apply the coupling equation

The coupling equation can be applied to quaternionic functions that have a flat parameter space and that are both differentiable via the nabla operation and can be normalized, while also the result of the nabla operation can be normalized.

Both the QPAD that describes a planned Qpattern and the QPAD that describes an actual Qpattern fulfill these requirements. However, the QPAD of the actual Qpattern has a curved parameter space that must be converted to a flat parameter space by the sharp part of the allocation function.

It can also be applied to QPAD's that represent simple composites.

12.8 Energy

The above deliberation makes $|\phi|$ to the distribution of the ***local energy*** and m to the ***total energy*** of the quantum state function. The coupling equation can be split in a real equation and an imaginary equation.

$$\nabla_0 \psi_0 - \langle \boldsymbol{\nabla}, \boldsymbol{\psi} \rangle = m \, \varphi_0 \tag{1}$$

$$\nabla_0 \boldsymbol{\psi} + \boldsymbol{\nabla} \psi_0 + \boldsymbol{\nabla} \times \boldsymbol{\psi} = m \, \boldsymbol{\varphi} \tag{2}$$

Bold characters indicate imaginary quaternionic distributions and operators. Zero subscripts indicate real distributions and operators.

The quantum state function of a particle moving with uniform speed $\boldsymbol{v}$ is given by

$$\psi = \chi + \chi_0 \, \boldsymbol{v} \tag{3}$$

$$\chi_0 = \psi_0 \tag{4}$$

Here χ stands for quantum state function of the particle at rest.

We introduce new symbols. In order to indicate the difference with Maxwell's equations we use Gothic capitals:

$$\mathfrak{E} = \nabla_0\boldsymbol{\psi} + \nabla\psi_0 \tag{5}$$

$$\mathfrak{B} = \nabla \times \boldsymbol{\psi} \tag{6}$$

The local field energy E is given by:

$$E = |\phi| = \sqrt{\phi_0\phi_0 + \langle\boldsymbol{\phi},\boldsymbol{\phi}\rangle} \tag{7}$$

$$= \sqrt{\phi_0\phi_0 + \langle\mathfrak{E},\mathfrak{E}\rangle + \langle\mathfrak{B},\mathfrak{B}\rangle + 2\langle\mathfrak{E},\mathfrak{B}\rangle}$$

The total energy is given by the volume integral

$$E_{total} = \sqrt{\int_V |\phi|^2 \, dV} \tag{8}$$

In a static situation the local energy E reduces to

$$E_{static} = \sqrt{\langle\nabla,\boldsymbol{\psi}\rangle^2 + \langle\mathfrak{E},\mathfrak{E}\rangle + \langle\mathfrak{B},\mathfrak{B}\rangle} \tag{9}$$

12.8.1 Fourier transform

In a region of little or no space curvature the Fourier transform of the local QPAD can be taken.

$$\nabla\psi = \phi = m\,\varphi \tag{1}$$

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$$\mathcal{M}\tilde{\psi} = \tilde{\phi} = m\,\tilde{\varphi} \tag{2}$$

$$\langle\tilde{\psi}|\mathcal{M}\tilde{\psi}\rangle = m\,\langle\tilde{\psi}|\tilde{\varphi}\rangle \tag{3}$$

$$\mathcal{M} = \mathcal{M}_0 + \boldsymbol{M} \tag{4}$$

$$\mathcal{M}_0\tilde{\psi}_0 - \langle\boldsymbol{M},\widetilde{\boldsymbol{\psi}}\rangle = m\,\tilde{\varphi}_0 \tag{5}$$

$$\mathcal{M}_0\boldsymbol{\psi} + \boldsymbol{M}\tilde{\psi}_0 + \boldsymbol{M}\times\widetilde{\boldsymbol{\psi}} = m\,\widetilde{\boldsymbol{\varphi}} \tag{6}$$

$$\int_{\tilde{V}}\tilde{\phi}^2\,d\tilde{V} = \int_{\tilde{V}}\left(\widetilde{\mathcal{M}\psi}\right)^2 d\tilde{V} = m^2 \tag{7}$$

In general $|\tilde{\psi}\rangle$ is not an eigenfunction of operator $\mathcal{M}$. That is only true when $|\tilde{\psi}\rangle$ and $|\tilde{\varphi}\rangle$ are equal. For elementary particles they are equal apart from their difference in discrete symmetry.

13 Elementary particles

The vision of the HBM on elementary particles is derived from the quaternionic format of Dirac's equation for the electron, which is a special form of the coupling equation.

$$\nabla\psi = m\,\psi^* \tag{1}$$

This equation is extended to a more general equation that holds for all elementary particles.

The coupling uses pairs $\{\psi^x, \psi^y\}$ of two sign flavors of the same basic Qpattern and its corresponding QPAD, which is indicated by $\psi^{\circledcirc}$. The special coupling equation runs:

$$\nabla\psi^x = m\,\psi^y \tag{2}$$

Corresponding anti-particles obey

$$(\nabla\psi^x)^* = m\,(\psi^y)^* \tag{3}$$

Elementary particles are constituted by the coupling of two QPAD's that belong to the same generation. One of the QPAD's is the quantum state function of the particle. The other QPAD can be interpreted to implement the effect of inertia.

Apart from their sign flavors these constituting QPAD's form the same quaternionic distribution. However, the sign flavor may differ and their progression must have the same direction. It means that the object density distribution is the same, but the signs of the flows of the concerned objects differ between the two distributions.

The second QPAD only simulates a Qpattern. It represents the coupling of the quantum state function to the embedding continuum, which is used in constructing the potentials of the particle. Coupling of elementary particles is governed by the special coupling equation.

The quantum state function is a description of a mostly virtual distribution of discrete objects. Only one element is actual. The second QPAD describes a completely virtual distribution of discrete objects.

As claimed earlier, coupling (also) occurs by embedding the message waves in the potential(s) of other particles.

In this specification, the form of the quaternionic Dirac equations play a significant, but at the same time a very peculiar role. The fact that ψ^x and ψ^y must be equal apart from a discrete symmetry difference is very strange and it is highly improbable that this strong relation is constituted by accident.

On the other hand it is known that the step stones couple to the embedding continuum. Two different types of this embedding continuum exists.

The first embedding continuum is formed by the superposition of the potentials of distant particles. This type of binding produces fermions.

The second embedding continuum is formed by the superposition of the potentials of local particles. This type of binding produces bosons.

It appears as if the correlation mechanism creates two rather than one distribution of step stones in which the descriptor of the first one plays the role of the quantum state function, while the descriptor of

the second one plays the role of a mirror that has the sign flavor of the embedding continuum.

If the first Qpattern (quasi) oscillates, then the second Qpattern oscillates asynchronous or partly in synchrony. This situation may differ per dimension. This results in 64 elementary particle types and 64 anti-particle types. Besides of that exist 8 potential types.

The coupling has a small set of observable properties:
- coupling strength,
- electric charge,
- color charge and
- spin.

Due to the fact that the enumerator creation occurs in configuration space, the coupling affects the local curvature of the involved Palestras.

Qpattern QPAD's that belong to the same generation have the same shape. This is explained in the paragraph on the enumeration process. The difference between the coupling partners resides in the discrete symmetry sets.

Thus, the properties of the coupled pair are completely determined by the sign flavors of the partners.

HYPOTHESIS 4: *If the quaternionic quantum state function of an elementary particle couples to an embedding continuum that is formed by the potentials of distant particles, then the particle is a* **fermion**, *otherwise it is a* **boson**. *The quantum state functions of anti-*

particles are coupled to canonical conjugates of the corresponding embedding continuums.

The fact that for fermions both the reference continuum and the reference enumerator set play a crucial role may indicate **that the Pauli principle is based on this fact.**

This paper does not give an explanation for the influence on the spin by the fact that the quantum state function is connected to an isotropic or an anisotropic Qpattern.

Photons and gluons are not coupled. They modulate the ultra-high frequency waves that constitute particle potentials.

In the standard model the eight gluons are constructed from superpositions of the six base gluons.

13.1   Reference frames

Each Qpattern possesses a reference frame that represents its current location, its orientation and its discrete symmetry. The reference frame corresponds with a Cartesian coordinate system that has a well-defined origin.

Reference frames of different Qpatterns have a relative position.

A Qpattern does not move with respect to its own reference frame. However, reference frames of different Qpatterns may move relative to each other.

The reference frames reside in an affine-like space. Interaction can take place between reference frames that reside in different HBM pages and that are within the range of the interaction speed.

Within the same HBM page no interaction is possible. Interaction runs from a reference frame to a frame that lays in the future of the sender.

Coupling into elementary particles puts the origins of the reference frames of the coupled Qpatterns at the same location. At the same location reference frames are parallel. That does not mean that the axes have the same sign.

13.2   Coupling Qpatterns

This section uses the fact that coupling is caused by interfering with the embedding continuum.

Fermions couple to the embedding continuum that is formed by the superposition of the potentials of distant particles.

Bosons couple to the embedding continuum that is formed by the superposition of the potentials of local particles.

The coupling is represented by pairs $\{\psi^x, \psi^y\}$ of two sign flavors of the same basic QPAD $\psi^{\circledcirc}$. Thus the corresponding coupling equation runs:

$$\nabla \psi^x = \psi^y \tag{1}$$

The corresponding anti-particles obey

$$(\nabla \psi^x)^* = m \, (\psi^y)^* \tag{2}$$

The partial anti-phase couplings must use different sign flavors.

The coupling and its effect on local curvature is treated in the section on the enumeration process.

In the figure below $\psi^{\textcircled{0}}$ and color N act as the reference sign flavor.

■■ ■	$\psi^{\textcircled{0}}$ N **R**
■■ ■	$\psi^{\textcircled{1}}$ R **L**
■■ ■	$\psi^{\textcircled{2}}$ G **L**
■■ ■	$\psi^{\textcircled{3}}$ B **L**
■■ ■	$\psi^{\textcircled{4}}$ $\bar{B}$ **R**
■■ ■	$\psi^{\textcircled{5}}$ $\bar{G}$ **R**
■■ ■	$\psi^{\textcircled{6}}$ $\bar{R}$ **R**
■■ ■	$\psi^{\textcircled{7}}$ $\bar{N}$ **L**

Figure 2: Sign flavors
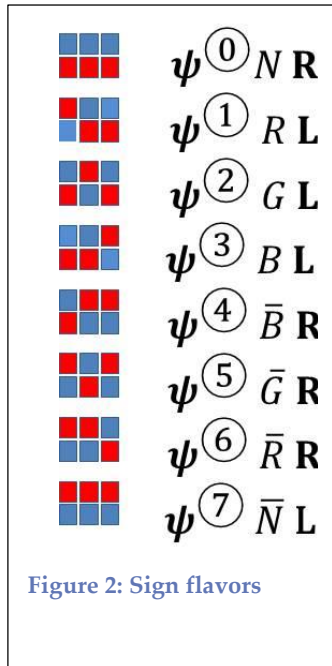
Eight sign flavors
(discrete symmetries)
Colors N, R, G, B, $\bar{R}$, $\bar{G}$, $\bar{B}$, W
Right or Left handedness **R,L**

13.3   Elementary particle properties

Elementary particles retain their discrete properties when they are contained in composite particles.

13.3.1 Spin

HYPOTHESIS 5: *The size of the spin relates to the fact whether the coupled Qpattern is the reference Qpattern. The reference Qpattern QPAD has the reference sign flavor $\psi^{\textcircled{0}}$.*

Each generation has its own reference Qpattern.

Fermions couple to the reference Qpattern. Fermions have half integer spin.

Bosons have integer spin.

The spin of a composite equals the sum of the spins of its components.

13.3.2 Electric charge

HYPOTHESIS 6: *Electric charge depends on the difference and direction of the imaginary base vectors for the Qpattern pair. Each sign difference stands for one third of a full electric charge. Further it depends on the fact whether the handedness differs. If the handedness differs then the sign of the count is changed as well.*

The electric charge of a composite is the sum of the electric charge of its components.

Any electric charge of a building block is evenly distributed over the N_w step stones.

13.3.3 Color charge

HYPOTHESIS 7: *Color charge is related to the direction of the anisotropy of the considered Qpattern with respect to the reference Qpattern. The anisotropy lays in the discrete symmetry of the imaginary part. The color charge of the reference Qpattern is white. The corresponding anti-color is black. The color charge of the coupled pair is determined by the colors of its members.*

All composite particles are black or white. The neutral colors black and white correspond to Qpatterns that are isotropic with respect to the reference sign flavor.

Currently, color charge cannot be measured. In the Standard Model the existence of color charge is derived via the Pauli principle.

The ***color confinement rule***[102] forbids the generation of individual particles that have non-neutral color charge.

13.3.4 Mass

Mass is related to the internal energy of the Qpattern. More precisely stated, mass is related to the square root of the volume integral of the square of the local field energy $E^2 = |\nabla\psi|^2$. Any internal kinetic energy is included in E.

$$m^2 = \langle\nabla\psi|\nabla\psi\rangle = \int_V |\nabla\psi|^2 \, dV \tag{1}$$

The same mass rule holds for composite particles. The fields of the composite particles are dynamic superpositions of the fields of their components.

13.4 Elementary object samples

With these ingredients we can look for agreements with the standard model. It appears that the coverage is (over)complete. The larger diversity of this HBM table appears to be not (yet) measurable. Also the color confinement restriction can reduce observation of elementary particles.

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[102] See Color confinement

For the same generation, the real parts of the natal QPAD's (that contain the scalar density distribution) are all born the same way! In this way the Qpatterns become micro states.

Elementary particles are represented by couplings of two QPAD's that may differ in their discrete symmetries. The differences between the discrete symmetries determine the discrete properties of the particle.

13.4.1 Photons and gluons

Photons and gluons are not particles. They are mentioned here because they are often categorized as elementary particles. The reason for that is that they represent energy quanta.

Photons and gluons modulate the ultra-high frequency fields that constitute particle potentials. Once emitted, the underlying carrier wave fronts flow freely.

Photons are emitted or absorbed when an elementary particle suddenly changes its energy.

If a potential emitting particle oscillates, then radio waves modulate the carrier waves that leave the particle.

Photons are quantized. Radio waves are not quantized.
Photons are emitted and absorbed in a fixed number N_w of progression steps. Radio waves have no restriction on their emission or absorption time.

When the potential emitting particles annihilate, then the potentials keep spreading and flee from their original source. In that way special kinds of photons or gluons are created.

In the standard model the eight gluons are constructed from superpositions of the six HBM base gluons.

type	s-type	e-charge	c-charge	Hand-edness	SM Name
$\{\psi^{⑦}\}$	boson	0	N	**R**	photon
$\{\psi^{⓪}\}$	boson	0	W	**L**	photon
$\{\psi^{⑥}\}$	boson	0	$\overline{R}$	**R**	gluon
$\{\psi^{①}\}$	boson	0	R	**L**	gluon
$\{\psi^{⑤}\}$	boson	0	$\overline{G}$	**R**	gluon
$\{\psi^{②}\}$	boson	0	G	**L**	gluon
$\{\psi^{④}\}$	boson	0	$\overline{B}$	**R**	gluon
$\{\psi^{③}\}$	boson	0	B	**L**	gluon

Only at the instant of their generation or annihilation photons and gluons couple to the emitter or absorber.

Two types of photons exist. One fades away from its point of generation. The other concentrates until it reaches the absorber.

13.4.2 Leptons and quarks

According to the Standard Model both leptons and quarks comprise three generations. They form 22 particles. Neutrinos will be treated separately.

13.4.2.1 Electrons and positrons

Pair	s-type	e-charge	c-charge	Hand-edness	SM Name
$\{\psi^{⑦},\psi^{⓪}\}$	fermion	-1	N	**LR**	electron

$\{\psi^{⓪}, \psi^{⑦}\}$	Anti-fermion	+1	W	**RL**	posi-tron

The generations contain the muon and tau generations of the electrons. The coupled lepton Qpatterns quasi-oscillate asynchronously in three dimensions.

13.4.2.2 Neutrinos

Neutrinos are fermions and have zero electric charge. They are leptons, but they seem to belong to a separate low-weight family of (three) generations. Their quantum state function couples to a QPAD that has the same sign-flavor. The lowest generation has a very small rest mass.

type	s-type	e-charge	c-charge	Hand-edness	SM Name
$\{\psi^{⑦}, \psi^{⑦}\}$	fermion	0	NN	**RR**	neu-trino
$\{\psi^{⓪}, \psi^{⓪}\}$	anti-fer-mion	0	WW	**LL**	neu-trino

13.4.2.3 Boso-neutrinos

type	s-type	e-charge	c-charge	Handedness	Name
$\{\psi^{⑥}, \psi^{⑥}\}$	boson	0	$\overline{\text{R}}\overline{\text{R}}$	**RR**	bneu-trino
$\{\psi^{①}, \psi^{①}\}$	anti-boson	0	RR	**LL**	bneu-trino
$\{\psi^{⑤}, \psi^{⑤}\}$	boson	0	$\overline{\text{G}}\overline{\text{G}}$	**RR**	bneu-trino
$\{\psi^{②}, \psi^{②}\}$	anti-boson?	0	GG	**LL**	bneu-trino
$\{\psi^{④}, \psi^{④}\}$	boson?	0	$\overline{\text{B}}\overline{\text{B}}$	**RR**	bneu-trino
$\{\psi^{③}, \psi^{③}\}$	anti-boson?	0	BB	**LL**	bneu-trino

13.4.3 Quarks

13.4.3.1 Quarks

Pair	s-type	e-charge	c-charge	Handedness	SM Name
$\{\psi^{①},\psi^{⓪}\}$	fermion	-1/3	R	**LR**	down-quark
$\{\psi^{⑥},\psi^{⑦}\}$	Anti-fermion	+1/3	$\overline{R}$	**RL**	Anti-down-quark
$\{\psi^{②},\psi^{⓪}\}$	fermion	-1/3	G	**LR**	down-quark
$\{\psi^{⑤},\psi^{⑦}\}$	anti-fermion	+1/3	$\overline{G}$	**RL**	Anti-down-quark
$\{\psi^{③},\psi^{⓪}\}$	fermion	-1/3	B	**LR**	down-quark
$\{\psi^{④},\psi^{⑦}\}$	anti-fermion	+1/3	$\overline{B}$	**RL**	Anti-down-quark
$\{\psi^{④},\psi^{⓪}\}$	fermion	+2/3	$\overline{B}$	**RR**	up-quark
$\{\psi^{③},\psi^{⑦}\}$	anti-fermion	-2/3	B	**LL**	Anti-up-quark
$\{\psi^{⑤},\psi^{⓪}\}$	fermion	+2/3	$\overline{G}$	**RR**	up-quark
$\{\psi^{②},\psi^{⑦}\}$	Anti-fermion	-2/3	G	**LL**	Anti-up-quark
$\{\psi^{⑥},\psi^{⓪}\}$	fermion	+2/3	$\overline{R}$	**RR**	up-quark

$\{\psi^{①}, \psi^{⑦}\}$	anti-fermion	-2/3	R	**LL**	Anti-up-quark

The generations contain the charm and top versions of the up-quark and the strange and bottom versions of the down-quark. The Qpatterns quasi-oscillate asynchronous in one or two dimensions.

13.4.3.2   Reverse quarks

Pair	s-type	e-charge	c-charge	Handedness	SM Name
$\{\psi^{⓪}, \psi^{①}\}$	boson	+1/3	R	**RL**	down-rquark
$\{\psi^{⑦}, \psi^{⑥}\}$	anti-boson	-1/3	$\overline{R}$	**LR**	anti-down-rquark
$\{\psi^{⓪}, \psi^{②}\}$	boson	+1/3	G	**RL**	down-rquark
$\{\psi^{⑦}, \psi^{⑤}\}$	anti-boson	-1/3	$\overline{G}$	**LR**	anti-down-rquark
$\{\psi^{⓪}, \psi^{③}\}$	boson	+1/3	B	**RL**	down-rquark
$\{\psi^{⑦}, \psi^{④}\}$	anti-boson	-1/3	$\overline{B}$	**LR**	anti-down-rquark
$\{\psi^{⓪}, \psi^{④}\}$	boson	-2/3	$\overline{B}$	**RR**	up-rquark
$\{\psi^{⑦}, \psi^{③}\}$	anti-boson	+2/3	B	**LL**	anti-up-rquark
$\{\psi^{⓪}, \psi^{⑤}\}$	boson	-2/3	$\overline{G}$	**RR**	up-rquark
$\{\psi^{⑦}, \psi^{②}\}$	anti-boson	+2/3	G	**LL**	anti-up-rquark
$\{\psi^{⓪}, \psi^{⑥}\}$	boson	-2/3	$\overline{R}$	**RR**	up-rquark
$\{\psi^{⑦}, \psi^{①}\}$	anti-boson	+2/3	R	**LL**	anti-up-rquark

13.4.4 W-particles

The 18 W-particles have indiscernible color mix. W_+ and W_- are each other's anti-particle.

Pair	s-type	e-charge	c-charge	Hand-edness	SM Name
$\{\psi^{\textcircled{6}}, \psi^{\textcircled{1}}\}$	boson	-1	$\overline{R}R$	**RL**	W_-
$\{\psi^{\textcircled{1}}, \psi^{\textcircled{6}}\}$	anti-boson	+1R	$R\overline{R}$	**LR**	W_+
$\{\psi^{\textcircled{6}}, \psi^{\textcircled{2}}\}$	boson	-1	$\overline{R}G$	**RL**	W_-
$\{\psi^{\textcircled{2}}, \psi^{\textcircled{6}}\}$	anti-boson	+1	$G\overline{R}$	**LR**	W_+
$\{\psi^{\textcircled{6}}, \psi^{\textcircled{3}}\}$	boson	-1	$\overline{R}B$	**RL**	W_-
$\{\psi^{\textcircled{3}}, \psi^{\textcircled{6}}\}$	anti-boson	+1	$B\overline{R}$	**LR**	W_+
$\{\psi^{\textcircled{5}}, \psi^{\textcircled{1}}\}$	boson	-1	$\overline{G}G$	**RL**	W_-
$\{\psi^{\textcircled{1}}, \psi^{\textcircled{5}}\}$	anti-boson	+1	$G\overline{G}$	**LR**	W_+
$\{\psi^{\textcircled{5}}, \psi^{\textcircled{2}}\}$	boson	-1	$\overline{G}G$	**RL**	W_-
$\{\psi^{\textcircled{2}}, \psi^{\textcircled{5}}\}$	anti-boson	+1	$G\overline{G}$	**LR**	W_+
$\{\psi^{\textcircled{5}}, \psi^{\textcircled{3}}\}$	boson	-1	$\overline{G}B$	**RL**	W_-
$\{\psi^{\textcircled{3}}, \psi^{\textcircled{5}}\}$	anti-boson	+1	$B\overline{G}$	**LR**	W_+
$\{\psi^{\textcircled{4}}, \psi^{\textcircled{1}}\}$	boson	-1	$\overline{B}R$	**RL**	W_-
$\{\psi^{\textcircled{1}}, \psi^{\textcircled{4}}\}$	anti-boson	+1	$R\overline{B}$	**LR**	W_+
$\{\psi^{\textcircled{4}}, \psi^{\textcircled{2}}\}$	boson	-1	$\overline{B}G$	**RL**	W_-
$\{\psi^{\textcircled{2}}, \psi^{\textcircled{4}}\}$	anti-boson	+1	$G\overline{B}$	**LR**	W_+

$\{\psi^{④}, \psi^{③}\}$	boson	-1	$\overline{B}B$	**RL**	W_-
$\{\psi^{③}, \psi^{④}\}$	anti-boson	+1	$B\overline{B}$	**LR**	W_+

The Qpatterns oscillate differently in multiple dimensions.

13.4.5 Z-candidates

The 12 Z-particles have indiscernible color mix.

Pair		s-type	e-charge	c-charge	Handedness	SM Name
$\{\psi^{②},\psi^{①}\}$		boson	0	GR	**LL**	Z
$\{\psi^{⑤},\psi^{⑥}\}$		anti-boson	0	$\overline{G}\overline{R}$	**RR**	Z
$\{\psi^{③},\psi^{①}\}$		boson	0	BR	**LL**	Z
$\{\psi^{④},\psi^{⑥}\}$		anti-boson	0	$\overline{R}\overline{B}$	**RR**	Z
$\{\psi^{③},\psi^{②}\}$		boson	0	BR	**LL**	Z
$\{\psi^{④},\psi^{⑤}\}$		anti-boson	0	$\overline{R}\overline{B}$	**RR**	Z
$\{\psi^{①},\psi^{②}\}$		boson	0	RG	**LL**	Z
$\{\psi^{⑥},\psi^{⑤}\}$		anti-boson	0	$\overline{R}\overline{G}$	**RR**	Z
$\{\psi^{①},\psi^{③}\}$		boson	0	RB	**LL**	Z
$\{\psi^{⑥},\psi^{④}\}$		anti-boson	0	$\overline{R}\overline{B}$	**RR**	Z
$\{\psi^{②},\psi^{③}\}$		boson	0	RB	**LL**	Z
$\{\psi^{⑤},\psi^{④}\}$		anti-boson	0	$\overline{R}\overline{B}$	**RR**	Z

The Qpatterns oscillate differently in multiple dimensions.

14 Fields

Field theory exists independent of what it describes. It describes fields varying from fluid dynamics, via electromagnetism to gravitation. You can describe scalar fields and vector fields separately or combined in a quaternionic field. Apart from that, tensor fields exist.

The most basic types of fields can be seen as representing the distribution of the density of discrete objects and the corresponding current densities. These density distributions are the primary field types. They can be combined in quaternionic functions.

Fields can also represent the potentials of the coherent distributions of discrete objects. Examples of this last category are gravitation fields and electrostatic fields.

The type of the potential is set by its Green's function. All these fields have many similarities and some differences.

Object density distributions correspond to scalar potentials.

Current density distributions correspond to vector potentials.

Again these potentials can be combined in quaternionic functions.

Only in case of quaternionic density distributions and corresponding potentials the fields describe the same objects, which form the coherent distribution of discrete objects that underlies these fields.

The elements of this distribution are treated as anonymous objects.

However, it is also possible to enumerate them and allow each individual object to possess a series of properties. For example its location, its displacement and its discrete symmetry. The elements can also share properties. These shared properties will characterize the distribution and the corresponding fields. Typical shared properties are the discrete symmetries.

Some types of fields, such as photons and gluons, can be seen as variations (modulations) of an embedding continuum. Here the embedding continuum is formed by superposed potentials that on their turn are constituted by ultra-high frequency carrier waves.

The corresponding wave fronts are emitted by objects that are elements of the discussed coherent distributions. These wave fronts combine in ultra-high frequency carrier waves. Photons and gluons modulate the carrier waves that constitute the potentials.

14.1 Physical fields

Elementary particles conserve their properties in higher level bindings. These properties are sources to fields that are exposed as dedicated potentials. Examples are the gravitational potential field and the electrostatic potential field. As stated above these potentials are constituted from ultra-high frequency waves and these are constituted by emitted wave fronts.

As soon as they leave the particle, the corresponding wave fronts start their own life and keep flowing away with light speed from their original source. These wave fronts form waves that feature a fixed ultra-high frequency.

If the particle oscillates or annihilates or suddenly changes its energy, then their amplitude of these carrier waves can be modulated. We know these amplitude modulations as photons, gluons and radio waves.

If the source stays at rest, then the ultra-high frequency waves superpose as a static potential. If the source oscillates, then the emitted stream oscillates as well. The corresponding amplitude modulation has a lower frequency than the frequency of the carrier waves. A

special kind of quantized modulation occurs when particles annihilate or when it suddenly changes its energy. The completion of the corresponding action takes a fixed number N_w of progression steps.

14.1.1 Secondary fields

If in a certain region a coherent distribution of property carriers (charged particles) exist, then that distribution can again be described by a density distribution. These fields are secondary fields. When the coherent distribution covers large numbers of particles it may be described by a quaternionic distribution that contains a scalar potential and a vector potential like the QPAD's that describe elementary particles.

In elementary particles the elements are step stones that are used one by one and disappear immediately. In the secondary fields the elements are existing and (relative) persistent charged particles.

Besides the photons, the gluons and the radio waves these secondary fields are the dynamic physical fields that we know in contemporary physics.

14.2 Gravitation field

One of the physical fields, the gravitation field, describes the local curvature of the reference Palestra. It equals the scalar potential field that corresponds to the real part of the quantum state function.

Now let ϕ represent the quaternionic potential of a set of massive particles. It is a superposition of single charge potentials.

$$\phi = \phi_0 + \boldsymbol{\phi} = \sum_i \phi_i = \sum_i m_i \, \varphi_i \tag{1}$$

Potential ϕ represents a secondary field. The particles may represent composites. In that case the mass m_i includes the internal kinetic energy of the corresponding particle. All massive particles attract each other. In superpositions, gravitational fields tend to enforce each other.

14.3 Electromagnetic fields

The electric charge e_i is represented similarly as m_i, but where m_i is always positive, the electric charge e_i can be either positive or negative. Equal signs repel, opposite signs attract each other. Superposition of the fields must include the sign. In superpositions, arbitrary electromagnetic fields tend to neutralize each other. Moving electric charges correspond to a vector potential and the curl of this vector potential corresponds to a magnetic field.

$$\phi = \phi_0 + \boldsymbol{\phi} = \sum_i e_i \, \varphi_i \tag{1}$$

Here ϕ is the quaternionic electro potential. It is a superposition of single charge potentials ϕ_i. ϕ_0 is the scalar potential. $\boldsymbol{\phi}$ is the vector potential. Potential ϕ represents a secondary field. The values of the electric charge sources e_i are included in ϕ.

$$\boldsymbol{E} = \nabla_0 \boldsymbol{\phi} \; + \; \boldsymbol{\nabla}\phi_0 \tag{2}$$

$$\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{\phi} \tag{3}$$

14.4 Photons and gluons

Photons and gluons can be described by quaternionic functions. Their energy is quantized.

In configuration space they obey

$$\nabla^2 \psi = 0 \tag{1}$$

Ensembles of photons and/or gluons are better considered as QPAD's in the canonical conjugated space of the configuration space.

14.5 Radio waves

Radio waves discern from photons and gluons in the fact that their energy is not quantized. Their generation and absorption is not restricted by a fixed number of progression steps.

14.6 Isotropic and anisotropic potentials

The propagation of waves is governed by the Huygens principle. The correlation vehicle uses this mechanism in order to regenerate all wave fronts at every progression step.

14.6.1 Huygens principle for odd and even number of spatial dimension

The following is taken from
http://www.mathpages.com/home/kmath242/kmath242.htm

The spherically symmetrical wave equation in n spatial dimensions can be written as

$$\frac{\partial^2 \psi}{\partial r^2} + \frac{n-1}{r} \frac{\partial \psi}{\partial r} = \frac{\partial^2 \psi}{\partial t^2} \tag{1}$$

Now suppose we define a new scalar field ϕ by the relation

$$\phi(r,t) = r^{(n-1)/2} \psi(r,t) \tag{2}$$

This leads to

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{(n-1)(n-3)}{4r^2}\phi = \frac{\partial^2 \phi}{\partial t^2} \tag{3}$$

If n equals 1, meaning that we have just a single space dimension, then the second term on the left hand side vanishes, leaving us with a one-dimensional wave equation, with has the well-known general solution

$$\psi(r,t) = f(r-t) + g(r+t) \tag{4}$$

for arbitrary functions f and g.

if n equals 3, i.e., in the case of three spatial dimensions, the spherically symmetrical wave equation reduces again to a one-parametric wave equation, in the modified wave function $\phi = r\psi$. Hence the general solution in three space dimensions is

$$\psi(r,t) = \frac{f(r-t)}{r} + \frac{g(r+t)}{r} \tag{5}$$

The fact that this solution is divided by r signifies that the magnitude of the wave tends to drop as r increases (unlike the one-dimensional case, in which a wave would theoretical propagate forever with non-diminished strength). Focusing on just the "retarded" component of the wave, $f(r-t)/r$, the fact that the time parameter t appears only in the difference $r-t$ implies that the (attenuated) wave propagates in time with a phase velocity of precisely 1, because for any fixed phase β we have $r-t = \beta$ and so dr/dt for this phase

point is 1. Consequently if f is a single pulse, it will propagate outward in a spherical shell at precisely the speed 1, i.e., on the light cone. Conversely, it can be shown that the wave function at any point in space and time is fully determined by the values and derivatives of that function on the past light cone of the point.

Any wave equation for which this is true (i.e., for which disturbances propagate at a single precise speed) is said to satisfy Huygens' Principle. The connection with Huygens' original statement about secondary wavelets is that each wavelet - with the same speed as the original wave - represents a tiny light cone at that point, and Huygens' principle asserts that light is confined to those light cones.

For n equals 2 the extra term in equation (3) does not vanish. We can still solve the wave equation, but the solution is not just a simple spherical wave propagating with unit velocity. Instead, we find that there are effectively infinitely many velocities, in the sense that a single pulse disturbance at the origin will propagate outward on infinitely many "light cones" (and sub-cones) with speeds ranging from the maximum down to zero. Hence if we lived in a universe with two spatial dimensions (instead of three), an observer at a fixed location from the origin of a single pulse would "see" an initial flash but then the disturbance "afterglow" would persist, becoming less and less intense, but continuing forever, as slower and slower subsidiary branches arrive.

14.6.2 The case of even spatial dimensions

Now again start from equation (1) and try a solution in the form:

$$\psi(r,t) = f(r)g(t) \tag{1}$$

Inserting this into the wave equation and expanding the derivatives by the product rule gives

$$g\frac{\partial^2 f}{\partial r^2} + \frac{n-1}{r}g\frac{\partial f}{\partial r} = f\frac{\partial^2 g}{\partial t^2} \tag{2}$$

Dividing through by fg gives

$$\frac{1}{f}\frac{\partial^2 f}{\partial r^2} + \frac{n-1}{fr}\frac{\partial f}{\partial r} = \frac{1}{g}\frac{\partial^2 g}{\partial t^2} \tag{3}$$

This decouples into two equations

$$\frac{\partial^2 f}{\partial r^2} + \frac{n-1}{r}\frac{\partial f}{\partial r} = k\,f \tag{3}$$

And

$$\frac{\partial^2 g}{\partial t^2} = k\,g \tag{4}$$

If k is positive or zero the right hand equation gives "run-away" solutions for $g(t)$, whereas if k is negative we can choose scaling so that $k = -1$ and then $g(t)$ satisfies the simple harmonic equation, whose solutions include functions of the form $sin(ct)$ and $cos(ct)$. In that case equation (9) can be re-written in the form

$$r\frac{\partial^2 f}{\partial r^2} + (n-1)\frac{\partial f}{\partial r} + r\,f = 0 \tag{5}$$

This is the form of a Bessel's equation. In fact for n=2 the solution is the zero order Bessel function $J_0(r)$.

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$$J_0(r) = \frac{2}{\pi} \int_0^\infty \sin(\cosh(\theta) \, r) \, d\theta \qquad (6)$$

A plot of $J_0(r)$ is shown below.
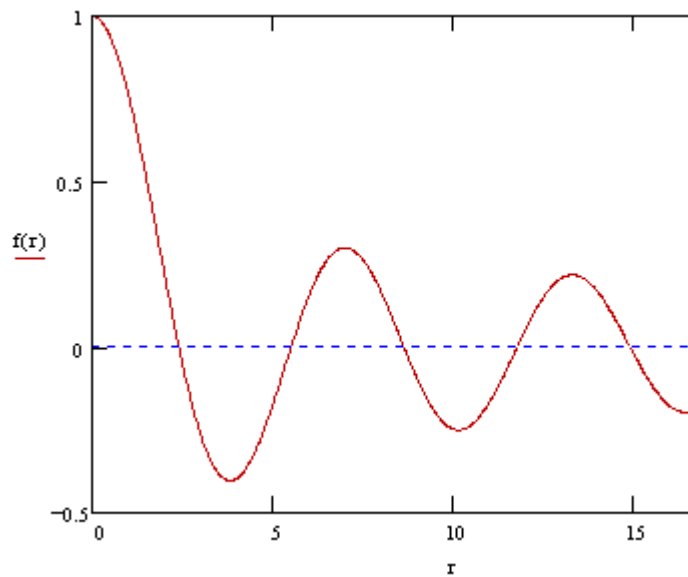
Inserting $g(t) = \sin(ct)$ gives

$$\psi(r,t) = \frac{1}{\pi} \int_0^\infty [\cos(\cosh(\theta) \, r - ct) \\ - \cos(\cosh(\theta) \, r + ct)] d\theta \qquad (7)$$

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Hence, instead of the solution being purely a function of $r \pm ct$ as in the case of odd dimensions, we find that it is an integral of functions of $cosh(\theta)r \pm ct$. Each value of θ corresponds to a propagation speed of $c/cosh(\theta)$, so the speeds vary from c down to zero. This signifies that the wave function at any event is correlated not just with the wave function on its "light cone", but with the wave function at every event inside its light cone.

In two dimensions the Huygens principle corresponds to a centripetal force[103] with potential

$$V = -\frac{\hbar}{8Mr^2}.$$ (8)

14.6.3 Huygens principle applied

HYPOTHESIS 8: *Particles transmit waves in dimensions where the discrete symmetry of the quantum state function differs from the discrete symmetry of the embedding background.*

The correlation mechanism uses the Huygens principle in order to restore the potentials at each progression step. The Huygens principle works differently depending on the number of dimensions in which the waves are transmitted.

The characteristics of the potentials that are emitted or absorbed by elementary particles are determined by the differences between the discrete symmetry set of the quantum state function of the particle and the symmetry set of the coupled QPAD that represents the

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[103] http://cds.cern.ch/record/514621/files/0108083.pdf

embedding continuum. This difference determines whether the potentials act in 1, 2 or 3 dimensions. In odd dimensions the persistence of the potentials can be explained by the common interpretation of the Huygens principle. This common interpretation is that at every point of each wave front new waves are generated. This does not work for particles that send their waves in two dimensions. This includes quarks, W-particles and Z-particles. The corresponding messengers are gluons. For these objects the potentials also act in two dimensions. In even dimensions the Huygens principle does not act in its normal way.

The same conditions that determine whether waves are emitted in 1, 2, or 3 dimensions also determine whether the particle has 1/3, 2/3 or 3/3 integer electric charge.

The re-emitted waves consist out of a retarded component and an advanced component. These components correspond to outbound interactions and inbound interactions.

14.7 Discussion

The particular behavior of the Huygens principle for potential contributions that cover even dimensions might explain the exceptional strength of the corresponding strong force mechanism.

It appears that fermions with electric charges of $\pm n/3$ e produce n dimensional waves that contribute to their electrostatic potential.

For n=3 the Green's function is of form 1/r.

For n=2 the Green's function is a zero order Bessel function.

For n=1 the Green's function is a constant.

On the other hand the color confinement principle[104] prevents that the even dimensional actions of the Huygens principle will ever become observable.

The gravitation potential is not influenced by the discrete symmetries. The corresponding potential contributions are always transmitted isotropic in three dimensions.

The electrostatic potential is controlled by the discrete symmetry sets. Depending on the resulting electric charge of the particle the electric potential contributions are transmitted in 1, 2 or 3 dimensions.

The correlation mechanism applies the Huygens principle for the recreation in each progression step of the corresponding potentials.

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[104] See color confinement.

15 Inertia

We use the ideas on inertia of Denis Sciama[105][106][107].

15.1 Inertia from coupling equation

In order to discuss inertia we must reformulate the coupling equation.

$$\nabla \psi = m\, \varphi \tag{1}$$

$$\nabla_0 \psi_0 - \langle \nabla, \psi \rangle = m\, \varphi_0 \tag{2}$$

$$\nabla_0 \psi + \nabla \psi_0 + \nabla \times \psi = \mathfrak{E} + \mathfrak{B} = m\, \varphi \tag{3}$$

We will write ψ as a superposition

$$\psi = \chi + \chi_0\, \boldsymbol{v} \tag{4}$$

$$\psi_0 = \chi_0 \tag{5}$$

$$\boldsymbol{\psi} = \boldsymbol{\chi} + \chi_0\, \boldsymbol{v}$$

χ represents the rest state of the object. With respect to progression, it is a constant.

$$\nabla_0 \chi = 0 \tag{6}$$

For the elementary particles the coupled distributions $\{\,\psi\,,\,\varphi\,\}$ have the same real part.

[105] http://arxiv.org/abs/physics/0609026v4.pdf

[106] http://www.adsabs.harvard.edu/abs/1953MNRAS.113...34S

[107] http://rmp.aps.org/abstract/RMP/v36/i1/p463_1

$$\psi_0 = \varphi_0 \tag{7}$$

$$\nabla_0 \boldsymbol{\psi} = \chi_0 \, \dot{\boldsymbol{v}} \tag{8}$$

Remember

$$\mathfrak{E} = \nabla_0 \boldsymbol{\psi} + \boldsymbol{\nabla}\psi_0 \tag{9}$$
$$\tag{10}$$
$$\chi_0 \, \dot{\boldsymbol{v}} = \mathfrak{E} - \boldsymbol{\nabla}\psi_0$$

In static conditions $\boldsymbol{v}$ represents a uniform speed of linear movement. However, if the uniform speed turns into acceleration $\dot{\boldsymbol{v}} \neq \boldsymbol{0}$, then an extra field of size $\chi_0\dot{\boldsymbol{v}}$ is generated that counteracts the acceleration. The Qpattern does not change, thus $\boldsymbol{\nabla}\psi_0$ does not change. Also $\mathfrak{B}$ does not change. This means that the acceleration of the particle corresponds to an extra $\mathfrak{E}$ field that counteracts the acceleration. On its turn it corresponds with a change of the coupling partner φ. That change involves the coupling strength m. The counteraction is felt as inertia.

15.2 Information horizon

The terms in the integral continuity equation

$$\Phi = \int_V \nabla\psi \, dV = \int_V \phi \, dV \tag{1}$$

can be interpreted as representing the influence of a local object onto the rest of the universe or as the influence of the rest of the universe onto a local object. In the second case the influence diminishes with distance and the number of influencers increases such that the most distant contributors together poses the largest influence.

These influencers sit at the information horizon. In the history of the model they are part of the birth state of the current episode of the universe. This was a state of densest packaging.

The local Qpattern that is described by ψ couples to the historic Qpattern φ for which the RQE acts as a Qpatch and as a Qtarget. This historic Qpattern resided in the reference page of the HBM.

16 **Lorentz transformation**

Differences between positions in subsequent members of the sequence of HBM pages can be interpreted as displacements. The displacement is a coordinate transformation. For the properties of this transformation it does not matter where the displacement starts or in which direction it is taken.

The same holds for displacements that concern sequence members that are located further apart. The corresponding displacements form a group. The displacement is a function of both the position and the sequence number. The displacement $z, \tau \rightarrow z', \tau'$ can be interpreted as a coordinate transformation and can be described by a matrix. Here τ is progression.

$$\begin{bmatrix} \tau' \\ z' \end{bmatrix} = \begin{bmatrix} \gamma & \delta \\ \beta & \alpha \end{bmatrix} \begin{bmatrix} \tau \\ z \end{bmatrix} \tag{1}$$

The matrix elements are interrelated. When the displacement concerns a uniform movement, the interrelations of the matrix elements become a function of the speed v. Here v is the speed measured as displacement per progression interval. The group properties together with the isomorphism of space fix the interrelations.

$$\begin{bmatrix} \tau' \\ z' \end{bmatrix} = 1/\sqrt{1 + kv^2} \begin{bmatrix} 1 & kv \\ -v & 1 \end{bmatrix} \begin{bmatrix} \tau \\ z \end{bmatrix} \tag{2}$$

If k is positive, then there may be transformations with $kv^2 \gg 1$ which transform time into a spatial coordinate and vice versa. This is considered to be unphysical. The Hilbert book model also supports that vision.

The condition k = 0 corresponds to a Galilean transformation

$$\begin{bmatrix} \tau' \\ z' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -v & 1 \end{bmatrix} \begin{bmatrix} \tau \\ z \end{bmatrix} \tag{3}$$

The condition $k < 0$ corresponds to a Lorentz transformation. We can set $kc^2 = -1$, where c is an invariant speed that corresponds to the maximum of v.

$$\begin{bmatrix} \tau' \\ z' \end{bmatrix} = 1/\sqrt{1 - v^2/c^2} \begin{bmatrix} 1 & -v/c^2 \\ -v & 1 \end{bmatrix} \begin{bmatrix} \tau \\ z \end{bmatrix} \tag{4}$$

The Lorentz transformation corresponds with the situation in which a maximum speed occurs.

Since in each progression step photons step with a non-zero space step and both step sizes are fixed, the speed of the photon at microscopic scale is fixed. No other particle goes faster, so in the model a maximum speed occurs. With other words when sequence members at different sequence number are compared, then the corresponding displacements can be described by Lorentz transformations.

Lorentz transformations introduce the phenomena that go together with relativity, such as length contraction, time dilatation and relativity of simultaneity that occur when two inertial reference frames are considered.

$$\Delta\tau_c = \left(\Delta\tau_p - \Delta z_p \, v/c^2\right)/\sqrt{1 - v^2/c^2} \tag{5}$$

$$(\Delta\tau_c)^2 (1 - v^2/c^2) = \left(\Delta\tau_p - \Delta z_p \, v/c^2\right)^2$$

The term $\Delta z_p \, v/c^2$ introduces time dilatation. If $\Delta \tau_p = 0$ then depending on v and Δz_p the time difference $\Delta \tau_c$ is non-zero.

These phenomena occur in the Hilbert Book Model when different members of the sequence of Hilbert spaces are compared. Usually the inertial frames are spread over a range of Hilbert book pages.

Since the members of the sequence represent static status quos, the relativity of simultaneity restricts the selection of the inertial frames. Only one of the inertial frames can be situated completely in a single member of the sequence. In that case the other must be taken from a range of sequence elements.

17 Gravitation as a descriptor

The gravitation field describes the local curvature. The sharp allocation function can act as the base of a quaternionic gravitation theory. The sharp allocation function has sixteen partial derivatives that combine in a differential.

17.1 Palestra

All quantum state functions share their parameter space as affine-like spaces. Due to the fact that the coupling of Qpatterns affects this parameter space, the Palestra is curved. The curvature is not static. With other words the Qpatches in the parameter space move and densities in the distribution of these patches change. For potential observers, the Palestra is the place where everything classically happens. The Palestra comprises the whole universe.

17.1.1 Spacetime metric

The Palestra is defined with respect to a flat parameter space, which is spanned by the ***rational*** quaternions[108]. We already introduced the existence of a smallest rational number, which is used to arrange interspace freedom. The specification of the set of Qpatches is performed by a continuous quaternionic distribution $\wp(x)$ that acts as a (partial) allocation function. This allocation function defines a quaternionic infinitesimal interval ds. On its turn this definition defines a metric[109].

$$ds(x) = ds^{\nu}(x)e_{\nu} = d\wp = \sum_{\mu=0\ldots3} \frac{\partial \wp}{\partial x_{\mu}} dx_{\mu} = q^{\mu}(x)dx_{\mu} \tag{1}$$

$$= \sum_{\mu=0\ldots3} \sum_{\nu=0,\ldots3} e_{\nu} \frac{\partial \wp_{\nu}}{\partial x_{\mu}} dx_{\mu}$$

$$= \sum_{\mu=0\ldots3} \sum_{\nu=0,\ldots3} e_{\nu} q_{\nu}^{\mu} dx_{\mu}$$

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[108]   http://en.wikipedia.org/wiki/Quaternion_algebra#Quaternion_algebras_over_the_rational_numbers

[109] The intervals that are constituted by the smallest rational numbers represent the infinitesimal steps. Probably the hair of mathematicians are raised when we treat the interspacing as an infinitesimal steps. I apologize for that.

The base e_ν and the coordinates x_μ are taken from the **flat** parameter space of $\wp(x)$. That parameter space is spanned by the quaternions. The definition of the quaternionic metric uses a full derivative $d\wp$ of the sharp allocation function $\wp(x)$. This full derivative differs from the quaternionic nabla ∇, which ignores the curvature of the parameter space. On its turn $d\wp$ ignores the blur of $\mathcal{P}$.

The allocation function $\wp(x)$ may include an isotropic scaling function $a(\tau)$ that only depends on progression τ. It defines the expansion/compression of the Palestra.

ds is the infinitesimal quaternionic step that results from the combined real valued infinitesimal dx_μ steps that are taken along the e_μ base axes in the (flat) parameter space of $\wp(x)$.

$dx_0 = c\, d\tau$ plays the role of the infinitesimal space time interval ds_{st}[110]. In conventional physics it is considered to be a physical invariant. $d\tau$ plays the role of the infinitesimal proper time interval and in the HBM it equals the infinitesimal progression interval. The progression step is an HBM invariant. Without curvature, dt in

$$\|ds\| = c\, dt$$

plays the role of the infinitesimal coordinate time interval.

$$c^2\, dt^2 = ds\, ds^* = dx_0^2 + dx_1^2 + dx_2^2 + dx_3^2 \tag{2}$$

$$dx_0^2 = ds_{st}{}^2 = c^2\, dt^2 - dx_1^2 - dx_2^2 - dx_3^2 \tag{3}$$

In conventional physics, dx_0^2 is used to define the local spacetime metric tensor. With that metric the Palestra is a pseudo-Riemannian manifold that has a Minkowski signature. When the metric is based

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[110] Notice the difference between the quaternionic interval ds and the spacetime interval ds_{st}

on ds^2, then the Palestra is a Riemannian manifold with a Euclidean signature. The Palestra comprises the whole universe. It is the arena where everything happens.

For the sharp allocation function holds

$$\frac{\partial^2 \wp}{\partial x_\mu \partial x_\nu} = \frac{\partial^2 \wp}{\partial x_\nu \partial x_\mu} \tag{4}$$

For higher-order derivatives hold similar equations.

Due to the spatial continuity of the sharp allocation function $\wp(x)$, the quaternionic metric as it is defined above is far more restrictive than the metric tensor that that is used in General Relativity:

$$ds^2 = g_{ik} \, dx^i \, dx^k \tag{5}$$

Still

$$g_{ik} = g_{ki} \tag{6}$$

17.1.2 The Palestra step

When nature steps with universe (Palestra) wide steps during a progression step Δx_0, then in the Palestra a quaternionic step $\Delta s_\wp$ will be taken that differs from the corresponding flat step Δs_f

$$\Delta s_f = \Delta x_0 + \boldsymbol{i} \, \Delta x_1 + \boldsymbol{j} \, \Delta x_2 + \boldsymbol{k} \, \Delta x_3 \tag{1}$$

$$\Delta s_\wp = q^0 \Delta x_0 + q^1 \, \Delta x_1 + q^2 \, \Delta x_2 + q^3 \, \Delta x_3 \tag{2}$$

The coefficients q^μ are quaternions. The Δx_μ are steps taken in the (flat) parameter space of the sharp allocation function $\wp(x)$.

17.1.3 Pacific space and black regions.

If we treat the Palestra as a continuum, then the parameter space of the allocation function is a flat space that it is spanned by the number system of the quaternions. This parameter space gets the name "*Pacific space*". This is the space where the parameter RQE's live. If in a certain region of the Palestra no matter is present, then in that region the Palestra is hardly curved. It means that in this region the Palestra is nearly equal to the parameter space of the allocation function.

The Pacific space has the advantage that if distributions are converted to this parameter space, then the Fourier transform of the converted distributions is not affected by curvature.

In a region where the curvature is high, the Palestra step comes close to zero. At the end where the Palestra step reaches the smallest rational value, an information horizon is established.

For a distant observer, nothing can pass that horizon. The information horizon encloses a *black region*. Inside that region the quantum state functions are so densely packed that they lose their identity.

However, they do not lose their sign flavor. The result is the formation of a single quantum state function that consists of the superposition of all contributing quantum state functions.

The resulting black body has mass, electric charge and angular momentum.

The quantum state function of a black region is quantized.

Due to the fact that no information can escape through the information horizon, the inside of the horizon is obscure. No experiment can reveal its content. It does not contain a singularity at its center. All characteristics of the black region are contained in its quantum state function[111].

The sharp allocation function $\wp(x)$ is a continuous quaternionic distribution. Like all continuous quaternionic distributions it contains two fields. It is NOT a QPAD. It does not contain density distributions.

17.1.4 Start of the universe.

At the start of the universe the package density was so high that also in that condition only one mixed QPAD can exist. That QPAD was a superposition of QPAD's that have different sign flavors. Only when the universe expands enough, multiple individual Qpatterns may have been generated. In the beginning, these QPAD's where uncoupled.

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[111] See Cosmological history

19 **Modularization**

A very powerful influencer is modularization. Together with the corresponding *encapsulation* it has a very healthy influence on the *relational complexity* of the ensemble of objects on which modularization works.

The encapsulation takes care of the fact that most relations are kept internal to the module.

When relations between modules are reduced to a few types, then the module becomes *reusable*.

The most influential kind of modularization is achieved when modules can be *configured from lower order modules*.

Elementary particles can be considered as the lowest level of modules. All composites are higher level modules.

When *sufficient resources* in the form of reusable modules are present, then modularization can reach enormous heights.
On earth it was capable to generate *intelligent species*.

19.1 Complexity

Potential complexity of a set of objects is a measure that is defined by the number of potential relations that exist between the members of that set.

If there are n elements in the set, then there exist n*(n-1) potential relations.

Actual complexity of a set of objects is a measure that is defined by the number of relevant relations that exist between the members of the set.

In human affairs and with intelligent design it takes time and other resources to determine whether a relation is relevant or not. Only an expert has the knowledge that a given relation is relevant.
Thus it is advantageous to have as little irrelevant potential relations as is possible, such that mainly relevant and preferably usable relations result.

Physics is based on relations. Quantum logic is a set of axioms that restrict the relations that exist between quantum logical propositions.

Via its isomorphism with Hilbert spaces quantum logic forms a fundament for quantum physics.

Classical logic is a similar set of restrictions that define how we can communicate logically. Like classical logic, quantum logic only describes static relations.

Traditional quantum logic does not treat physical fields and it does not touch dynamics. However, the model that is based on traditional quantum logic can be extended such that physical fields are included as well and by assuming that dynamics is the travel along subsequent versions of extended quantum logics, also dynamics will be treated.

The set of propositions of traditional quantum logic is isomorphic with the set of closed subspaces of a Hilbert space. The Hilbert space is a mathematical construct in which quantum physicists do their investigations and calculations.

In this way fundamental physics can be constructed. Here holds very strongly that only relevant relations have significance.

19.2   Relational complexity

We define ***relational complexity*** as the ratio of the number of actual relations divided by the number of potential relations.

19.3   Interfaces

Modules connect via interfaces.
Interfaces are used by interactions.
Interactions run via (relevant) relations.

Relations that act within modules are lost to the outside world of the module.

Thus interfaces are collections of relations that are used by interactions.
Inbound interactions come from the past.
Outbound interactions go to the future.
Two-sided interactions are cyclic. They are either oscillations or rotations of the inter-actor.

In physics interactions are implemented by potentials. The solutions in the Huygens principle cover both outgoing as well as incoming waves.
The outbound waves implement ***outbound interfaces*** of elementary particles.
The inbound waves implement ***inbound interfaces*** of elementary particles.

19.4 Interface types

Apart from the fact that they are inbound, outbound or cyclic the interfaces can be categorized with respect to the type of relations that they represent.

Each category corresponds to an ***interface type***.

An interface that possesses a type and that installs the possibility to couple the corresponding module to other modules is called a ***standard interface***.

19.5 Modular subsystems

Modular subsystems consist of connected modules. They need not be modules. They become modules when they are encapsulated and offer standard interfaces that makes the encapsulated system a reusable object.

The cyclic interactions bind the corresponding modules together.

Like the coupling factor of elementary particles characterizes the binding of the pair of Qpatterns will a similar characteristic characterize the binding of modules.

This binding characteristic directly relates to the total energy of the constituted sub-system.

Let ψ represent the renormalized superposition of the involved distributions.

$$\nabla\psi = \phi = m\,\varphi \tag{1}$$

$$\int_V |\psi|^2\, dV = \int_V |\varphi|^2\, dV = 1 \tag{2}$$

$$\int_V |\phi|^2 \, dV = m^2 \tag{3}$$

Here again m represents total energy.

The binding factor is the total energy of the sub-system minus the sum of the total energies of the separate constituents.

19.6 Relational complexity indicators

The inner product of two Hilbert vectors is a measure of the relational complexity of the combination.

A Hilbert vector represents a linear combination of atomic Hilbert propositions.

When all coefficients are equal, then the vector represents an assembly of atoms.

When the coefficients are not equal, then the vector represents a **weighted assembly** of atoms.

For two normalized vectors $|a\rangle$ and $|b\rangle$:

$$\langle a|a\rangle = 1 \tag{1}$$

$$\langle b|b\rangle = 1 \tag{2}$$

$\langle a|b\rangle = 0$ means $|a\rangle$ *and* $|b\rangle$ are not related. (3)

$\langle a|b\rangle \neq 0$ means $|a\rangle$ *and* $|b\rangle$ are related. (4)

$|\langle a|b\rangle| = 1$ means $|a\rangle$ *and* $|b\rangle$ are optimally related. (5)

19.7   Modular actions

Subsystems that have the ability to choose their activity can choose to organize their actions in a modular way.

As with static relational modularization the modular actions reduce complexity and for the decision maker it eases control.

19.8   Random design versus intelligent design

At lower levels of modularization nature designs modular structures in a stochastic way. This renders the modularization process rather slow. This way of modularization is called random design.

It takes a huge amount of progression steps in order to achieve a relatively complicated structure.

Still the complexity of that structure can be orders of magnitude less than the complexity of an equivalent monolith.

As soon as more intelligent subsystems arrive, then these systems can design and construct modular systems in a more intelligent way. They use resources efficiently.

This speeds the modularization process in an enormous way.

20 Principles

20.1 Huygens principle

The Huygens principle is applied by the correlation vehicle in order to establish the persistence of the emitted wave fronts.

20.2 Pauli principle

The Pauli principle is applied by the correlation vehicle to sets of particles and composites that the vehicle considers as coherent sets.

Within these sets the identical fermions cannot occupy the same location.

20.3 Color confinement

No free particles have been detected that do not show neutral colors charges. This is the consequence of color confinement.

It appears that the correlation vehicle is restricted in its construction of elementary particles and composites and can only generate and support particles that feature neutral colors.

This has direct consequences for the particles that can exist.

It means that a large part of the particles of the HBM table do not exist as individuals. At the utmost they can appear in color neutral composites.

Thus quarks exist only[112] in mesons and baryons that have neutral color charge. These composites can reconfigure, but cannot disintegrate into separate quarks.

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[112] Isolated quarks have a very short live

Contemporary physics has translated this habit of the correlation vehicle into the existence of a strong force.

This view implies that the correlation vehicle can and will create hadrons directly.

Where elementary particles are created with one single element per progression step, the elements of hadrons may be created in pairs or triples.

The correlation mechanism uses the Huygens principle in order to restore the potentials at each progression step. The Huygens principle works differently depending on the number of dimensions in which the waves are transmitted.

The HBM suggests that quarks emit their wave fronts in less than three dimensions.

If color confinement restricts particle generation, then emission in less than three dimensions might be prohibited or it is confined within the realm of the composite.

This means that the wave fronts that are emitted by quarks are quickly combined into 3D spherical wave fronts. During this process the wave fronts strongly fold and thus curve the embedding continuum and thus forces the particles to stay together. After combination this curvature is relaxed and the wave front behaves as a normal 3D spherical wave front.

21 Fundamental particles

Fundamental particles obey the rules of color confinement.

Fundamental particles are particles that are generated in one integrated action by the correlation vehicle.

Quarks are not fundamental particles. Electrons, positrons, neutrinos, W-particles and Z-particles are fundamental.

The full generation of these fundamental particles takes the same number of progression steps as is taken by the full generation of an elementary particle.

The set of fundamental particles comprises all elementary particles that have neutral color charge and all hadrons.

At every progression step all actual step stones of a fundamental particles are generated. Mesons contain two Qtargets and baryons contain three Qtargets.

Since these multi-Qtarget generations are completely governed by the correlation vehicle, it has little sense to consider the interactions between the elementary particles that populate mesons or baryons.

Of the strong force interactions only the interactions between hadrons result.

22 Events

22.1 Generations and annihilations

An event indicates that the enumeration generator changes its generation mode and will locally generate another set of Qpatterns or no Qpattern at all.

The generation and annihilation process are both restricted by color confinement.

For example it means that an electron-positron pair that get intermixed will be replaced by a muon-anti-muon pair or by a series of mesons. But it is also possible that no new Qpattern is created. Instead a photon is produced that is carried by the waves that constitute the fleeing potential.

Since the Qpattern is generated with a Qtarget at each progression step the event has immediate consequences.

Conservation laws govern the annihilation and creation processes.

22.2 Absorption versus emission

The Huygens principle allows absorption as well as emission of wave fronts.

Emission of wave fronts takes place at the arrival of a building block at the Qtarget.

It is imaginable that this wave front is borrowed from the wave fronts that constitute the embedding continuum.

This can be accomplished by making use of the fact that the embedding continuum is the superposition of the potentials of distant building blocks.

The amplitude of the new wave front must be initialized at a normalized level. However, events may cause a modulation of this level. That modulation occurs during a fixed number N_w of progression steps. The modulation appears as a photon or as a gluon.

A similar mechanism may also describe the absorption of photons or gluons, but here it affects the subsequent Qtargets. This influences the movement of the building block as a whole. For example it can influence the harmonic oscillation of the building block[113].

If the emitting building block oscillates then the emitted carrier waves will be modulated. This feature describes radio waves

22.3   Oscillating interactions

Oscillating interactions are implemented by cyclic interfaces. They consist of a sequence of annihilations and generations, where the locations alternate.

22.4   Movements

The fact that a particle moves, and the fact that a Qpattern is generated with only one Qtarget per progression step means that during a movement the Qpattern is spread along the path of movement.

It means that the micro-path is stretched along the path of the Qpatch.

23 Atoms and their electrons

Atoms are composites that are (re)generated under a coherent action of the correlation vehicle. The electrons in the atom follow one of a collection of planned paths that can be described as spherical

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[113] See Atoms and their electrons.

harmonics. Their natal micro-path is stretched along that selected path. As long as the electron keeps the selected path, then they behave like free electrons that follow a selected geodesic path. Its total energy is affected by this extra movement. This total energy is reflected in its mass and thus in its gravitation potential.

If for some reason the electron changes the selection of the spherical harmonic path into a lower energy version , then a cycle of wave fronts that are emitted starting from that instant get an impulse and becomes modulated. As a consequence, a corresponding photon is emitted.

If the local embedding continuum gets modulated, then the corresponding energy can be used to change the selected spherical harmonic path into a higher energy version. A corresponding photon is absorbed. Again this action takes a full generation cycle. In fact it is sufficient when a sufficiently long cycle of wave fronts in the embedding continuum are modulated.

According to this picture the spherical harmonics path is not traveled in a continuous way, but instead in a stochastic up and down way, similarly as the micro-path is walked when the electron is at rest. In this way it becomes comprehensible that during its walk along the spherical harmonics path no extra EM signal is transmitted.

23.1 Photon emission and absorption.
The energy of the emitted or absorbed photon corresponds with the energy level step. This involves a single energy quant. According to observations in conventional physics the involved energy is

$$E = \hbar\nu \tag{1}$$

This suggests that the natal Qpattern is generated in N_w steps.
N_w might vary per generation. Thus this cycle concerns N_w step
stones. In this cycle the emitting or absorbing particle walks along
its micro-path.

In that case the emission of each photon involves a fixed number
of progression steps and thus a fixed number N_w of wave fronts that
carry the photon which modulates the corresponding ultra-high fre-
quency carrier wave.

The above formula also indicates that Planck's constant is di-
rectly related to the number N_w of progression steps in a full cycle.

A higher energy photon is produced in a higher number of mod-
ulation periods. Since this occurs in the same Qpattern cycle. The
frequency grows with energy, as formula (1) shows.

Heisenberg's uncertainty relation guides the dimensional rela-
tion between Planck's constant $\hbar$, proper cycle time $\Delta\tau$ and energy
quant ΔE.

$$\Delta\tau \cdot \Delta E \propto \hbar$$
$$\Delta\tau = \tau_c = N_w \cdot \tau_s$$

(2)

23.2 Photon propagation and interference

The emitted photon rides somewhere on the ultra-high fre-
quency carrier wave. Its presence is described by an object density
distribution that describes the probability for the photon of being
at that location. Not the photons, but instead these object density
distributions control the interference of multiple photons.

The photon keeps its energy. At large distance the probability of detecting the particle diminishes but not its capability to trigger a suitable detector. This explains the difference between photons and gravitation. A distant supernova can be seen, but cannot be felt as a gravitation ripple. Still both messages must arrive at the same location at the same instant.

23.3 Chunks of energy

The fact that photons are energy quanta and encode their energy in their frequency leads to the suggestion that the energy quantum is divided in a discrete set of chunks. These chunks have a fixed size and are evenly spread over the step stones that configure the micro-path. It means that in the simplest model in each micro-walk a participating step stone at the utmost can change its energy by a single energy chunk.

This also means that a lowest and a largest photon energy exist. Their ratio is given by the number N_w of step stones that belong to a building block.

23.4 Radio waves

Radio waves have much in common with photons. On the other hand its generation process differs. Oscillating electrons produces radio waves via their potentials. Here the generation cycle is unlimited.

23.5 Creation and annihilation

Creation and annihilation of Qpatterns also take N_w progression steps. It corresponds to the (proper) time that it takes to generate or annihilate a photon.

On the other hand, contrary to the proper time clocks, the creation and annihilation cycles appear in general not to be synchronized. The

corresponding cycle can start at any progression step. Only its duration is fixed.

Qpatterns exist in three[114] generations and the cycle period (defined by N_w) might differ between these generations.

23.6 Basic frequencies

Besides the existing ultra-high frequency of the carrier wave, which is set by the size of the progression step, for each generation a second basic frequency of the HBM is set by the cycle period of the generation of a building block.

24 Entanglement

In the Hilbert Book Model, entanglement is due to the fact that nature's building blocks have a set of discrete properties that can be observed via indirect means that does not touch their state, while the building block may extend over rather large distances.

So measuring the same property at nearly the same instant at quite different locations will give the same result.

If shortly before these measurements were performed the property is changed, then it might give the impression that an "instant action at a distance" occurred, because neither light nor the wave fronts that constitute the potentials could bridge these locations in the period between the two measurements.

The explanation is that the building block at each progression instant moves to a different step stone and that these step stones may lay far apart.

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[114] For fermions at least three generations are known.

Apart from the property measurements, in this process no information transfer needs to take place.

At least the first measurement must be done without affecting the state of building block. At each arrival at a step stone the building block transmits contributions to its potentials. If the measurement uses these potentials, then the building block is not affected.

According to this explanation, at least one progression step must separate the two measurements.

25 Cosmology

25.1 Cosmological view

Even when space was fully densely packed with matter (or another substance) then nothing dynamic would happen. Only when sufficient interspacing comes available dynamics becomes possible.

The Hilbert Book Model exploits this possibility. It sees black regions as local returns to the original condition.

The HBM considers fundamental physics as a kind of fluid dynamics.

For the description of quantum physical features it uses the differential equations of fluid dynamics.

For the description of cosmologic features it uses the integral equations of fluid dynamics.

25.2 The cosmological equations

The integral equations that describe cosmology are:

$$\frac{d}{d\tau}\int_V \rho\, dV + \oint_S \widehat{\boldsymbol{n}}\rho\, dS = \int_V s\, dV \qquad (1)$$

$$\int_V \nabla\rho\, dV = \int_V s\, dV \qquad (2)$$

Here $\widehat{\boldsymbol{n}}$ is the normal vector pointing outward the surrounding surface S, $\boldsymbol{v}(\tau, \boldsymbol{q})$ is the velocity at which the charge density $\rho_0(\tau, \boldsymbol{q})$ enters volume V and s_0 is the source density inside V. In the above formula ρ stands for

$$\rho = \rho_0 + \boldsymbol{\rho} = \rho_0 + \frac{\rho_0\boldsymbol{v}}{c} \qquad (3)$$

It is the flux (flow per unit of area and per unit of progression) of ρ_0 . t stands for progression (not coordinate time).

25.3 Inversion surfaces

An inversion surface S is characterized by:

$$\oint_S \widehat{\boldsymbol{n}}\rho\, dS = 0 \qquad (1)$$

Potentials and their constituting wave fronts can still pass this inversion surface.

25.4 Entropy

As a whole, universe expands.

Locally regions exist where contraction overwhelms the global expansion.

These regions are separated by inversion surfaces. These regions are characterized by their inversion surface.

Within these regions the holographic principle resides.

The fact that the universe as a whole expands means that the average size of the encapsulated regions increases.

The *holographic principle* says that the total entropy of the region equals the entropy of a black region that would contain all matter in the region.

Black regions represent regions where entropy is optimally packed.

Thus entropy is directly related to the interspacing between enumerators. With other words, local entropy is related to local curvature.

25.5   Cosmological history

The inversion surfaces divide universe into compartments. Think that these universe pockets contain matter that is on its way back to its natal state.

If there is enough matter in the pocket this state forms a black region. The rest of the pocket is cleared from its mass content.

Still the size of the pocket may increase. This corresponds to the expansion of the universe.

Inside the pocket the holographic principle governs. The black region represents the densest packaging mode of entropy.

The pockets may merge. Thus finally a very large part of the universe may return to its birth state, which is a state of densest packaging of entropy.

Then the resulting mass which is positioned at a huge distance will enforce a uniform attraction. This uniform attraction will install an isotropic extension of the central package.

This will disturb the densest packaging quality of that package.

The motor behind this is formed by the combination of the attraction through distant massive particles, which installs an isotropic expansion and the influence of the small scale random localization which is present even in the state of densest packaging.

This describes an eternal process that takes place in and between the pockets of an affine-like space.

25.6 Recapitulation

The model starts by taking quantum logic as its foundation. Next quantum logic is refined to Hilbert logic.

It could as well have started by taking an infinite dimensional separable Hilbert space as its foundation. However, in that case the special role of base vectors would not so easily have been brought to the front.

It appears that the atomic propositions of the logic system and the base vectors of the Hilbert space play a very crucial role in the model. They represent the lowest level of objects in nature that play the theater of our observation.

The atoms are only principally unordered at very small "distances".
They have content.

The Hilbert space offers built-in enumerator machinery that defines the distances and that specifies the content of the represented atoms.
The same can be achieved in a refined version of quantum logic that we call Hilbert logic.

In fact we focus on a compartment of universe, while the whole universe is considered to be an affine-like space.

The isotropic scaling factor that was assumed in the early phases of the model appears to relate to mass carrying particles that exist in other compartments at huge distances.

In the considered compartment an enumeration process establishes a kind of coordinate system.

The master of the enumeration process is the blurred allocation function $\mathcal{P}$. This function has a flat parameter space.

$$\mathcal{P} = \wp \circ \mathcal{S} \tag{1}$$

At small scales this function becomes a stochastic spatial spread function $\mathcal{S}$ that governs the quantum physics of the model.

The whole function $\mathcal{P}$ is a convolution of a sharp part $\wp$ and the stochastic spatial spread function $\mathcal{S}$.

The differential of $\wp$ delivers a local metric.

The spread function appears to be generated by a Poisson generator which produces Qpatterns.

After a myriad of progression steps the original ordering of the natal state of the model is disturbed so much that the natal large and medium scale ordering is largely lost.

However, this natal ordering is returning in the black regions that constitute pockets that surround them in universe.

When the pockets merge into a huge black region, the history might restart enforced by the still existing low scale randomization and by the isotropic expansion factor, which is the consequence of the existence of massive particles at huge distances in the affine-like space, which emit gravitation potentials.

The model uses a first part where elementary particles are formed by the representatives of the atomic propositions of the logic systems.

In a second part the formation of composites is described by a process called modularization. In that stage, in places where sufficient resources are present, the modularization process is capable of forming intelligent species.

This is the start of a new phase of evolution in which the intelligent species get involved in the modularization process and shift the mode from random design to intelligent design.

Intelligent design runs much faster and may use its resources in a more efficient and conscientious way.

Other subjects

26 Functions that are invariant under Fourier transformation.

A subset of the (quaternionic) distributions have the same shape in configuration space and in the linear canonical conjugated space.

We call them dual space distributions. It are functions that are invariant under Fourier transformation[115]. These functions are not eigenfunctions of the Fourier operator.

The Qpatterns and the harmonic and spherical oscillations belong to this class.

Fourier-invariant functions show iso-resolution, that is, $\Delta_p = \Delta_q$ in the Heisenberg's uncertainty relation.

26.1 Natures preference

Nature seems to have a preference for quaternionic distributions that are invariant under Fourier transformation.

A possible explanation is the two-step generation process, where the first step is realized in configuration space and the second step is realized in canonical conjugated space. The whole pattern is generated two-step by two-step.

The only way to keep coherence between a distribution and its Fourier transform that are both generated step by step is to generate them in pairs.

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[115] Q-Formulæ contains a section about functions that are invariant under Fourier transformation.

27 Conclusion

With respect to conventional physics, this simple model contains extra layers of individual objects.

The most interesting addition is formed by the RQE's, the Qpatches, the Qtargets and the Qpatterns. They represent the atoms of the quantum logic sub-model.

Another addition is formed by the wave fronts that constitute the potentials and form ultra-high frequency carrier waves that are modulated by EM waves.

The model gives an acceptable explanation for the existence of an (average) maximum velocity of information transfer. The two prepositions:

- Atomic quantum logic fundament
- Correlation vehicle

lead to the existence of fuzzy interspacing of enumerators of the Hilbert space base vectors and to dynamically varying space curvature when compared to a flat reference continuum.

Without the freedom that is introduced by the interspacing fuzziness and which is used by the dynamic curvature, no dynamic behavior would be observable in the Palestra.

In the (re)generation of the model the enumeration process plays a crucial role, but we must keep in mind that the choice of the enumerators and therefore the choice of the type of correlation vehicle is to a large degree arbitrary.

It means that the Palestra has no natural origin. It is an affine-like space.

The choice for quaternions as enumerators seems to be justified by the fact that the sign flavors of the quaternions explain the diversity of elementary particles.

The generation process is restricted by color confinement. This reduces the number of elementary particles and composites that can be observed.

Physicist that base their model of physics on an equivalent of the Gelfand triple which lacks a mechanism that creates the freedom that flexible interspaces provide, are using a model in which no natural curvature and fuzziness can occur. Such a model cannot feature dynamics.

Attaching a progression parameter to that model can only create the illusion of dynamics. However, that model cannot give a proper explanation of the existence of space curvature, space expansion, quantum physics or even the existence of a maximum speed of information transfer.

Contemporary physics made its greatest misstep after the nineteen thirties when it turned away from the fundamental work of Garret Birkhoff and John von Neumann.
This deviation did not prohibit pragmatic use of the new methodology. However, it did prevent deep understanding of that technology because the methodology is ill founded.

Doing quantum physics in continuous function spaces is possible, but it makes it impossible to find the origins of dynamics, curvature and inertia. Most importantly it makes it impossible to find the reason of existence of quantum physics.

Only the acceptance of the fact that ***physics is fundamentally countable*** can solve this dilemma.

Q-FORMULÆ

1 Introduction

This compilation starts with sections on quantum logic and Hilbert Logic.

Next lists of formulas for quaternionic algebra and quaternionic differentials will be given. These formulae are for a significant part derived from Bo Thidé's book "Electromagnetic Field Theory"; http://www.plasma.uu.se/CED/Book. I have merely converted the vector formula into quaternionic format.

Two types of quaternionic differentiation exist.

- Flat differentiation uses the quaternionic nabla and ignores the curvature of the parameter space.
- Full differentiation uses the allocation function $\wp(x)$ that defines the curvature of the parameter space.

The text focuses at applications in quantum mechanics, in electrodynamics and in fluid dynamics.

2 Quantum logic

Elementary particles behave non-classical. They can present themselves either as a particle or as a wave. A measurement of the particle properties of the object destroys the information that was obtained from an earlier measurement of the wave properties of that object.

With elementary particles it becomes clear that that nature obeys a different logic than our old trusted classical logic. The difference resides in the modularity axiom. That axiom is weakened. The classical logic is congruent to an orthocomplemented modular lattice. The quantum logic is congruent to an orthocomplemented weakly modular lattice. Another name for that lattice is orthomodular lattice.

2.1   Lattices

A subset of the axioms of the logic characterizes it as a half ordered set. A larger subset defines it as a lattice.

A lattice is a set of elements $a, b, c, ...$ that is closed for the connections $\cap$ and $\cup$. These connections obey:

- The set is partially ordered. With each pair of elements a, b belongs an element c, such that $a \subset c$ and $b \subset c$.
- The set is a $\cap$ half lattice if with each pair of elements a, b an element c exists, such that $c = a \cap b$.
- The set is a $\cup$ half lattice if with each pair of elements a, b an element c exists, such that $c = a \cup b$.
- The set is a lattice if it is both a $\cap$ half lattice and a $\cup$ half lattice.

The following relations hold in a lattice:

$$a \cap b = b \cap a \tag{1}$$

$$(a \cap b) \cap c = a \cap (b \cap c) \tag{2}$$

$$a \cap (a \cup b) = a \tag{3}$$

$$a \cup b = b \cup a \tag{4}$$

$$(a \cup b) \cup c = a \cup (b \cup c) \tag{5}$$

$$a \cup (a \cap b) = a \tag{6}$$

The lattice has a partial order inclusion $\subset$:

$$a \subset b \Leftrightarrow a \subset b = a \tag{7}$$

A complementary lattice contains two elements n and e with each element a an complementary element a' such that:

$$a \cap a' = n \tag{8}$$

$$a \cap n = n \tag{9}$$

$$a \cap e = a \tag{10}$$

$$a \cup a' = e \tag{11}$$

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$$a \cup e = e \tag{12}$$

$$a \cup n = a \tag{13}$$

An orthocomplemented lattice contains two elements n and e and with each element a an element $a"$ such that:

$$a \cup a" = e \tag{14}$$

$$a \cap a" = n \tag{15}$$

$$(a")" = a \tag{16}$$

$$a \subset b \Leftrightarrow b" \subset a" \tag{17}$$

e is the unity element; n is the null element of the lattice

A distributive lattice supports the distributive laws:

$$a \cap (b \cup c) = (a \cap b) \cup (a \cap c) \tag{18}$$

$$a \cup (b \cap c) = (a \cup b) \cap (a \cup c) \tag{19}$$

A modular lattice supports:

$$(a \cap b) \cup (a \cap c) = a \cap (b \cup (a \cap c)) \tag{20}$$

A weak modular lattice supports instead:

There exists an element d such that

$$a \subset c \Leftrightarrow (a \cup b) \cap c \\ = a \cup (b \cap c) \cup (d \cap c) \tag{21}$$

where d obeys:

$$(a \cup b) \cap d = d \tag{22}$$

$$a \cap d = n \tag{23}$$

$$b \cap d = n \tag{24}$$

$$[(a \subset g) \, and \, (b \subset g) \Leftrightarrow d \subset g \tag{25}$$

In an atomic lattice holds

$$\exists_{p \, \epsilon \, L} \, \forall_{x \, \epsilon \, L} \, \{x \subset p \Rightarrow x = n\} \tag{26}$$

$$\forall_{a \, \epsilon \, L} \, \forall_{x \, \epsilon \, L} \, \{(a < x < a \cap p) \tag{27}$$

$$\Rightarrow (x = a \, or \, x = a \cap p)\}$$

p is an atom

Both the set of propositions of quantum logic and the set of subspaces of a separable Hilbert space **Ң** have the structure of an orthomodular lattice. In this respect these sets are congruent.

In Hilbert space, an atom is a pure state (a ray spanned by a single vector).

Classical logic has the structure of an orthocomplemented distributive modular and atomic lattice.

Quantum logic has the structure of an orthomodular lattice. That is an orthocomplented weakly modular and atomic lattice. The set of closed subspaces of a Hilbert space also has that structure.

2.2   Proposition

In Aristotelian logic a proposition is a particular kind of sentence, one which affirms or denies a predicate of a subject. Propositions have binary values. They are either true or they are false.

Propositions take forms like "*This is a particle or a wave*". In quantum logic "*This is a particle.*" is not a proposition.

In mathematical logic, propositions, also called "propositional formulas" or "statement forms", are statements that do not contain quantifiers. They are composed of well-formed formulas consisting entirely of atomic formulas, the five logical connectives[116], and symbols of grouping (parentheses etc.).

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[116] http://en.wikipedia.org/wiki/Logical_connective

Propositional logic is one of the few areas of mathematics that is totally solved, in the sense that it has been proven internally consistent, every theorem is true, and every true statement can be proved. Predicate logic is an extension of propositional logic, which adds variables and quantifiers.

In Hilbert space a vector is either inside or not inside a closed subspace. A proper quantum logical proposition is *"Vector |f> is inside state s"*.

In Hilbert space, an atomic predicate corresponds with a subspace that is spanned be a single vector.

Predicates may accept attributes and quantifiers. The predicate logic is also called first order logic. A dynamic logic can handle the fact that predicates may influence each other when atomic predicates are exchanged.

2.3   Observation

In physics, particularly in quantum physics, a system **observable** is a property of the system state that can be determined by some sequence of physical operations. An observable can exist without being observed. This paper distinguishes between measurement data and observables.

- The state is considered as a linear combination of eigenvectors of an observable. The value of an observable returns the statistical expectation value of the eigenvalue of the observable.
- A measurement transforms the observed state to one of the eigenvectors of the observable. What happens depends on the characteristics of the measuring equipment. The measurement results in one or more measurement data.

A particle can reveal its existence in the form of potentials. Measuring a potential does not affect the state of the particle. In general, measuring an eigenvalue will alter the state of the particle. This can go as far as the annihilation of the particle.

3 Hilbert logic

The set of propositions of traditional quantum logic is lattice iso-morphic   with the set of closed subspaces of a separable Hilbert space. However there exist still significant differences between this logic system and the Hilbert space. This gap can be closed by a small expansion of the quantum logic system.

Step 1: Require that linear combinations of atomic propositions also belong to the logic system. Call such propositions *linear prop-ositions*.

Step 2: introduce the notion of *relational coupling* between two linear propositions. This measure has properties that are similar to the properties of the inner product of Hilbert space vectors.

Step 3: Close the subsets of the new logic system with respect to this relational coupling measure.

The relational coupling measure can have values that are taken from a suitable division ring. The resulting logic system will be called Hilbert logic.

The Hilbert logic is lattice isomorphic as well topological isomor-phic with the corresponding Hilbert space.

Due to this similarity the Hilbert logic will also feature opera-tors[117].

In a Hilbert logic linear operators can be defined that have linear atoms as their eigen-propositions. The eigenspace of these operators is countable.

Linear propositions are the equivalents of Hilbert vectors. General quantum logic propositions are the equivalents of (closed) subspaces of a Hilbert space.

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[117] The Hilbert logic does not feature dynamic operators.

The measure of the relational coupling between two linear propositions is the equivalent of the inner product between two Hilbert vectors.

In a Hilbert logic system the ***superposition principle*** holds. A linear combination of linear proposition is again a linear proposition.

4 Hilbert space isomorphism

The set of propositions in a quantum logic system is lattice isomorphic with the set of closed subspaces of an infinite dimensional separable Hilbert space.

The set of Hilbert propositions in a Hilbert logic system is isomorphic with the set of Hilbert vectors of an infinite dimensional separable Hilbert space.

The set of eigenvectors of a normal operator in Hilbert space is isomorphic to the set of eigen-atoms of a corresponding operator in the Hilbert logic system.

A coherent distribution of objects that is represented by a QPAD corresponds to a set of Hilbert logic propositions that are eigen-atoms of a normal operator that resides in the Hilbert logic. In this way it also corresponds to a set of Hilbert space base vectors that are eigenvectors of a normal operator that resides in the Hilbert space. The coherent distribution corresponds to a closed subspace of the Hilbert space. It also corresponds to a quantum logic proposition. That quantum logic proposition concerns a building block.

The atomic Hilbert propositions that span this quantum logic proposition form the constituents of the building block. In this way it also corresponds to a set of Hilbert space base vectors that are eigenvectors of a normal operator that resides in the Hilbert space. The coherent distribution corresponds to a closed subspace of the Hilbert space. It also corresponds to a quantum logic proposition. That quantum logic proposition concerns a building block.

5 About quaternions

5.1 Notation

Let **x** be the position vector (radius vector, coordinate vector) from the origin of the Euclidean space $\mathbb{R}^3$ coordinate system to the coordinate point $(x_1; x_2; x_3)$ in the same system and let $|x|$ denote the magnitude ('length') of x. Let further $\alpha(x), \beta(x), \gamma(x), ...,$ be arbitrary scalar fields, $a(x), b(x), c(x), ...,$ arbitrary vector fields, and $A(x), B(x), C(x), ...,$ arbitrary rank two tensor fields in this space.

Let q be the position relative to the origin of the space $\mathbb{H}$ that is spanned by the quaternions and that is given by the coordinate point $(q_0; q_1; q_2; q_3))$ and let $|q|$ denote the norm of q.

Let * denote complex or quaternionic conjugate and † denote Hermitian conjugate (transposition and, where applicable, complex or quaternionic conjugation).

5.2 Cayley-Dickson construction

The Cayley-Dickson construction formula enable the generation of a quaternion from two complex numbers:

$$p = a_0 + a_1\mathbf{k} + \mathbf{i}(b_0 + b_1\mathbf{k}) \tag{1}$$

$$q = c_0 + c_1\mathbf{k} + \mathbf{i}(d_0 + d_1\mathbf{k}) \tag{2}$$

$$(a, b)\,(c, d) = (ac - db^*;\ a^*d + cb) \tag{3}$$

$$r = pq \tag{4}$$

$$r_0 = a_0c_0 - a_1c_1 - b_0d_0 - b_1d_1 \tag{5}$$

$$r_\mathbf{k} = a_0c_1 - a_1c_0 - b_0d_1 + b_1d_0 \tag{6}$$

$$r_\mathbf{i} = a_0d_0 + a_1d_1 + b_0c_0 - b_1c_1 \tag{7}$$

$$r_\mathbf{j} = -a_1d_0 + a_0d_1 + b_0c_1 + b_1c_0 \tag{8}$$

5.3 Warren Smith's numbers

All hyper-complex numbers are based on real numbers. Two main construction formulas for hyper-complex numbers exist. The Cayley-Dickson construction is the most widely known. The Warren-Smith construction gives best algorithmic properties at higher dimensions. Until the octonions both construction formulas deliver the same results.

The quaternions are the highest dimensional hyper-complex numbers that deliver a division ring.

5.3.1 2^n-on construction

The 2^n-ons use the following doubling formula

$$(a, b)(c, d) = (a\,c - (b\,d^*)^*, (b^*c^*)^* + (b^*(a^*((b^{-1})^*d^*)^*)^*)^*) \tag{1}$$

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Up until the 16-ons the formula can be simplified to

$$(a, b)(c, d) \ = \ (a\,c - b\,d^*, c\,b \ + \ (a^*\,b^{-1})\,(b\,d))$$ (2)

Up to the octonions the Cayley Dickson construction delivers the same as the 2^n-on construction. From $n > 3$ the 2^n-ons are 'nicer'.

5.3.1.1 2^n-ons

Table of properties of the 2^n-ons.
See http://www.scorevoting.net/WarrenSmithPages/homepage/nce2.pdf.

Type	*name*	**Lose**
1-ons	*Reals*.	
2-ons	*Complex numbers*	$z^* = z$ (the * denotes conjugating); the ordering properties that both $\{z > 0, -z > 0,$ or $z = 0\}$ and $\{w > 0, z > 0$ implies $w + z > 0$, $wz > 0\}$.
4-ons	*Quaternions*	commutativity $ab = ba$; the algebraic closedness property that every univariate polynomial equation has a root.
8-ons	*Octonions*	associativity $ab \cdot c = a \cdot bc$.
16-ons	(*not Sedenions!*)	right-alternativity $x \cdot yy = xy \cdot y$; right-cancellation $x = xy \cdot y^{-1}$; flexibility $x \cdot yx = xy \cdot x$; left-linearity $(b + c)a = ba + ca$; anti-automorphism $ab = ba$, $(ab)^{-1} = b^{-1}\,a^{-1}$; left-linearity $(b + c)a = ba + ca$; continuity of the map $x \rightarrow xy$; Moufang and Bol identities; diassociativity

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32-ons		generalized-smoothness of the map $x \to xy$;
		right-division properties that $xa = b$ has (generically) a solution x, and the uniqueness of such an x;
		the "fundamental theorem of algebra" that every polynomial having a unique "asymptotically dominant monomial" must have a root; Trotter's formula: $$\lim_{n\to\infty}\left[e^{x/n}e^{y/n}\right]^n = \lim_{n\to\infty}\left(1 + \frac{x+y}{n}\right)^n = e^{x+y}$$

Type	Retain
2^n-ons	Unique 2-sided multiplicative & additive identity elements 1 & 0;
	Norm-multiplicativity $|xy|^2 = |x|^2 \cdot |y|^2$;
	Norm-subadditivity $|a + b| \le |a| + |b|$;
	2-sided inverse $a^{-1} = a^*/|a|^2$ ($a \# 0$);
	$a^{**} = a$;
	$(x \pm y)^* = x^* \pm y^*$;
	$(a^{-1})^{-1} = a$;
	$(a^*)^{-1} = (a^{-1})^*$;
	$|a|^2 = |a|^2 = a^*a$;
	Left-alternativity $yy \cdot x = y \cdot yx$;
	Left-cancellation $x = y^{-1} \cdot yx$;
	Right-linearity $a(b + c) = ab + ac$;
	r^{th} power-associativity $a^n a^m = a^{n+m}$;
	Scaling $s \cdot ab = sa \cdot b = as \cdot b = a \cdot sb = a \cdot bs = ab \cdot s$ (s real); Power-distributivity $(ra^n + sa^m)b = ra^n b + sa^m b$ (r, s real);
	Vector product properties of the imaginary part: ab - re(ab) of the product for pure-imaginary 2^n-ons a,b regarded as (2^n - 1)-vectors;

	$\langle xa,b \rangle = \langle a, x^*b \rangle$, $\langle xa,xb \rangle = |x|^2 \cdot \langle a,b \rangle$ and $\langle x,y \rangle = \langle x^*,y^* \rangle$ Numerous weakened associativity, commutativity, distributivity, antiautomorphism, and Moufang and Bol properties including 9-coordinate ``niner'' versions of most of those properties; contains 2^{n-1}-ons as subalgebra.

5.3.1.1.1 The most important properties of 2^n-ons

If a,b,x,y are 2^n-ons, $n \geq 0$, and s and t are scalars (i.e. all coordinates are 0 except the real coordinate) then

unit: A unique 2^n-on 1 exists, with $1 \cdot x = x \cdot 1 = x$.

zero: A unique 2^n-on 0 exists, with $0 + x = x + 0 = x$ and $0 \cdot x = x \cdot 0 = 0$.

additive properties: $x+y = y+x$, $(x+y)+z = x+(y+z)$; $-x$ exists with $x + (-x) = x - x = 0$.

norm: $|x|^2 = xx^* = x^*x$.

norm-multiplicativity: $|x|^2 \cdot |y|^2 = |x \cdot y|^2$.

scaling: $s \cdot x \cdot y = s \cdot x \cdot y = x \cdot s \cdot y = x \cdot s \cdot y = x \cdot y \cdot s$.

weak-linearity: $(x + s) \cdot y = x \cdot y + s \cdot y$ and $x \cdot (y + s) = x \cdot y + x \cdot s$.

right-linearity: $x \cdot (y + z) = x \cdot y + x \cdot z$.

inversion: If $x \neq 0$ then a unique x^{-1} exists, obeying $x^{-1} \cdot x = x \cdot x^{-1} = 1$. It is $x^{-1} = x \cdot |x|^{-2}$.

left-alternativity: $x \cdot xy = x^2 \cdot y$.

left-cancellation: $x \cdot x^{-1} \cdot y = y$.

effect on inner products: $\langle x \cdot a, b \rangle = \langle a, x^* \cdot b \rangle$, $\langle x,y \rangle = \langle x^*, y^* \rangle$, $\langle x^* \cdot a, x^{-1} \cdot b \rangle = \langle a,b \rangle$, and $\langle x \cdot a, x \cdot b \rangle = |x|^2 \cdot \langle a,b \rangle$.

Conjugate of inverse: $(x^{-1})^* = (x^*)^{-1}$.

Near-anticommutativity of unequal basis elements: $e_k^2 = -1$ and $e_k \cdot e_l^* = -e_l \cdot e_k^*$ if $k \neq l$.

(Note: the case $k; l > 0$ shows that unequal pure-imaginary basis elements anticommute.)

Alternative basis elements: $e_k \cdot e_l \cdot e_k = e_k \cdot e_l \cdot e_k$, $e_l \cdot e_k \cdot e_k = e_l \cdot e_k \cdot e_k$, and $e_k \cdot e_k \cdot e_l = e_k \cdot e_k \cdot e_l$. (However, when $n \geq 4$ the 2^n-ons are not flexible i.e. it is not generally true that $x \cdot y \cdot x = x \cdot y \cdot x$ if x and y are 16-ons that are not basis elements. They also are not right-alternative.)

Quadratic identity: If x is a 2^n-on (over any field F with charF $\neq$ 2), then $x^2 + |x|^2 = 2 \cdot x$ re x

Squares of imaginaries: If x is a 2^n-on with re $x = 0$ ("pure imaginary") then $x^2 = -|x|^2$ is nonpositive pure-real.

Powering preserves im$x$ direction

5.3.1.1.2 Niners

Niners are 2n-ons whose coordinates with index > 8 are zero. The index starts with 0.

9-**flexibility** $xp \cdot x = x \cdot px$, $px \cdot p = p \cdot xp$.

9-**similitude unambiguity** $xp \cdot x^{-1} = x \cdot px^{-1}$, $px \cdot p^{-1} = p \cdot xp^{-1}$.

9-**right-alternativity** $xp \cdot p = x \cdot p^2$, $px \cdot x = p \cdot x^2$.

9-**right-cancellation** $xp^{-1} \cdot p = x$, $px^{-1} \cdot x = p$.

9-**effect on inner products** $\langle x, yp \rangle = \langle xp, y \rangle$, $\langle xp, yp \rangle = |p|^2 \langle x, y \rangle$.

9-**left-linearity** $(x + y)p = xp + yp$, $(p + q)x = px + qx$.

9-**Jordan-identity** $xp \cdot xx = x(p \cdot xx)$, $py \cdot pp = p(y \cdot pp)$.

9-**coordinate-distributivity** $([x + y]z)_{0;\ldots;8} = (xz + yz)_{0;\ldots;8}$.

9-**coordinate-Jordan-identity** $[xy \cdot xx]_{0;\ldots;8} = [x(y \cdot xx)]_{0;\ldots;8}$.

9-**anticommutativity for orthogonal imaginary 2^n-ons**

If $\langle p, x \rangle = $ re $p = $ re $x = 0$ then $px = -xp$.

9-**reflection** If $|a| = 1$ and the geometric reflection operator is defined below then $-($refl$[a](y))_{0;\ldots;8} = (a \cdot y^*a)_{0;\ldots;8}$, and $-\{$refl$[a](y)\}^*_{0;\ldots;8} = (a^*y \cdot a^*)_{0;\ldots;8}$, and

if either a or y is a niner then $-\text{refl}[a](y) = a \cdot y^* a$ and $-\text{refl}[a](y)$
$= a^* y \cdot a^*$.

$$\text{refl}[\vec{x}](\vec{t}) \overset{\text{def}}{=} \vec{t} - \frac{2\langle \vec{x}, \vec{t} \rangle}{|\vec{x}|^2} \vec{x} \tag{1}$$

What holds for the niners, also holds for the octonions.

5.4   Waltz details

The 16-ons lose the continuity of the map $x \Rightarrow xy$. Also, in general holds $(x\,y)x \neq x\,(y\,x)$ for 16-ons. However, for all 2^n-ons the base numbers fulfill $(e_i\,e_j)\,e_i = e_i\,(e_j\,e_i)$. All 2^n-ons feature a conjugate and an inverse. The inverse only exists for non-zero numbers. The 2^n-ons support the **number waltz**

$$c = a\,b/a. \tag{1}$$

Often the number waltz appears as a unitary number waltz

$$c = u^* b\,u \tag{2}$$

where u is a unit size number and u^* is its conjugate $u\,u^* = 1$.

In quaternion space the **quaternion waltz** $a\,b/a$ can be written as

$$a\,b\,/\,a = exp(2\,\pi\,\tilde{\imath}\,\varphi)\,b\,exp(-2\,\pi\,\tilde{\imath}\,\varphi) \tag{3}$$

$$= b - \boldsymbol{b}_\perp + exp(2\,\pi\,\tilde{\imath}\,\varphi)\,\boldsymbol{b}_\perp\,exp(-2\,\pi\,\tilde{\imath}\,\varphi)$$

$$= b - \boldsymbol{b}_\perp + exp(4\,\pi\,\tilde{\imath}\,\varphi)\boldsymbol{b}_\perp$$

$$\Delta b = (exp(4\,\pi\,\tilde{\imath}\,\varphi) - 1)\boldsymbol{b}_\perp \tag{4}$$

$$= (cos(4 \pi \varphi) + \tilde{\imath} \, sin(4 \pi \varphi) - 1) \, \boldsymbol{b}_\perp$$

$$= exp(2 \pi \tilde{\imath} \varphi) \, 2 \, \tilde{\imath} \, sin \, (2 \pi \varphi) \, \boldsymbol{b}_\perp$$

$$\|\Delta b\| = \|2 \, sin(2 \pi \varphi) \, \boldsymbol{b}_\perp\| \tag{5}$$
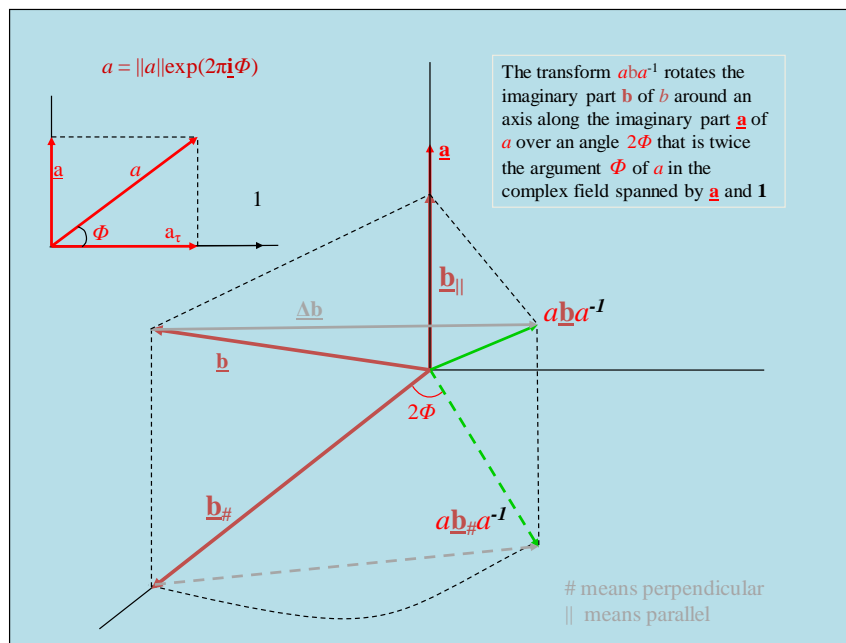
Figure 1. The rotation of a quaternion by a second quaternion.

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Another way of specifying the difference is:

$$\Delta b \ = \ (a \cdot b - b \cdot a)/a \ = \ 2 \cdot (\boldsymbol{a} \times \boldsymbol{b})/a \tag{6}$$

$$\|\Delta b\| \ = 2 \, \|\boldsymbol{a} \times \boldsymbol{b}\| / \, \|a\| \tag{7}$$
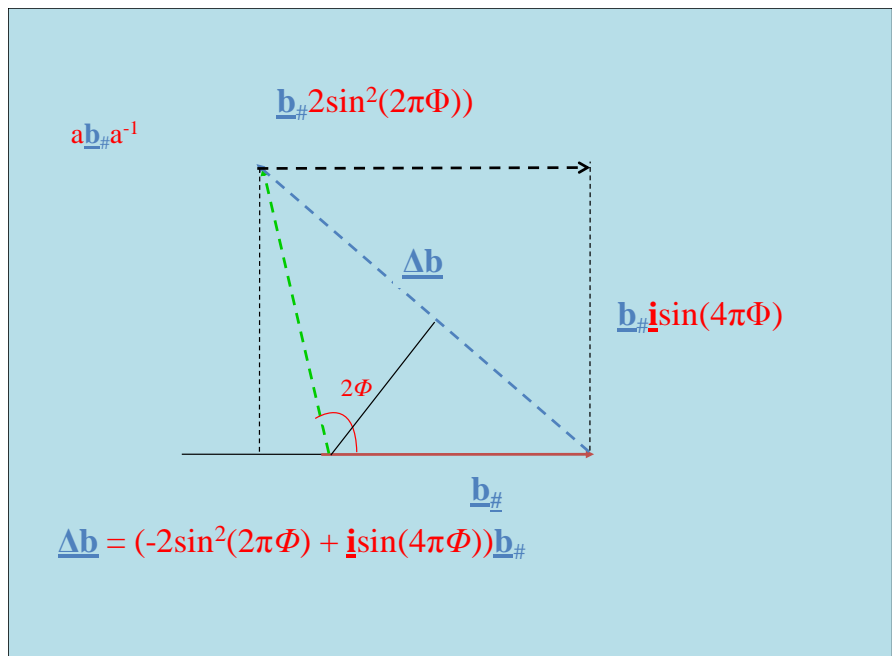
Figure 2: The difference after rotation

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5.5   Spinors and matrices

In contemporary physics complex probability amplitude distributions (CPAD's) are used rather than quaternionic probability amplitude distributions (QPAD's). Spinors and matrices are used to simulate QPAD behavior for CPAD's.

5.5.1 Symmetries

The quaternionic number system exists in sixteen discrete symmetry sets (sign flavors). When the real part is ignored, then eight different symmetry sets result. The values of a continuous function all belong to the same symmetry set. The parameter space of the function may belong to a different symmetry set.

Figure 3: Sign flavors

Eight sign flavors
(discrete symmetries)
Colors $N, R, G, B, \bar{R}, \bar{G}, \bar{B}, W$
Right or Left handedness **R,L**
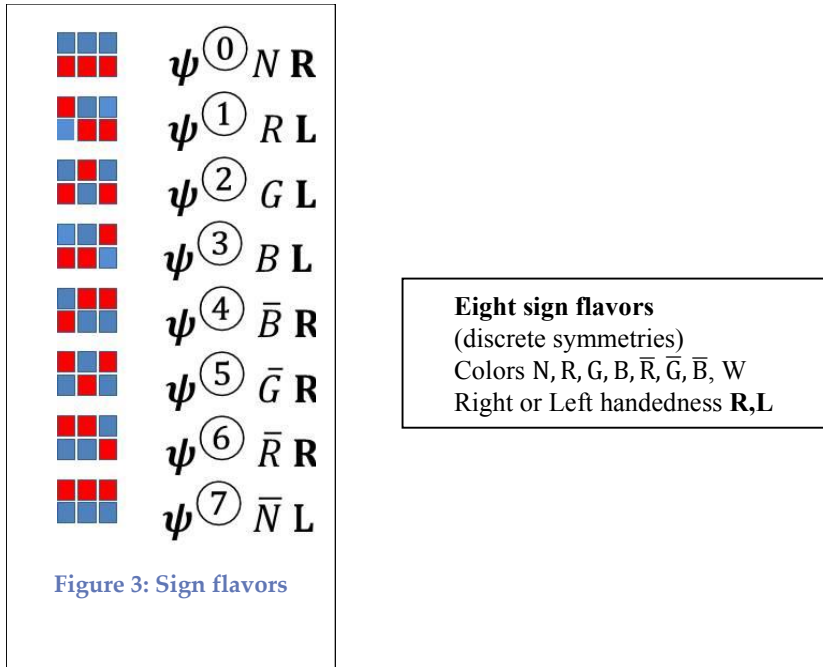
The red blocks indicates sign up or down with respect to the base sign flavor. For quaternionic distributions the (quaternionic) parameter space acts as base sign flavor.

Quaternionic functions can be interpreted as the combination of a scalar function and a 3D vector function. The scalar part can be interpreted as the representation of an object density distribution. In that case the vector function can be thought to correspond to an associated current density distribution. The discrete symmetry values control the direction of the currents. This must be determined relative to a reference.

If we ignore the real part, then only eight discrete symmetries result. The next table lists these symmetries in text format;

```
    ||ddd||n          u=up;d=down;
||RH||                n=neutral;r=red;g=green;b=blue;
||udd||r||L           B=anti.blue;G=anti.green;R=anti.r
H||                 ed;N=anti-neutral
||dud||g||L           RH=right handed; LH= left handed.
H||
||ddu||b||L
H||
||duu||B||R
H||
||udu||G||R
H||
||uud||R||R
H||
||uuu||N||L
H||
```

---

[118] This picture has been changed!

The 3D Kronecker delta tensor

$$\delta_{ij} = \begin{cases} 1 \text{ if } i = j \\ 0 \text{ if } i \neq j \end{cases} \tag{1}$$

The fully antisymmetric Levi-Civita tensor

$$\epsilon_{ijk} = \begin{cases} 1 \text{ if } i, j, k \text{ is an even permutation of 1,2,3} \\ 0 \text{ if at least two of } i, j, k \text{ are equal} \\ -1 \text{ if } i, j, k \text{ is an odd permutation of 1,2,3} \end{cases} \tag{2}$$

5.5.2 Spinor

We use square brackets for indicating spinors. Spinors use real component functions ψ_i. . Complex component functions ψ_i would result in spinor representations of bi-quaternions. Bi-quaternions do not form a division ring[119].

A 2×2 spinor is defined by the row:

$$[\psi] \equiv \big[[\psi_0][\boldsymbol{\Psi}]\big] \tag{1}$$

$$[\psi]^{\ddagger} \equiv \big[[\boldsymbol{\Psi}][\psi_0]\big] \tag{2}$$

Where

$$[\psi_0] \equiv \begin{bmatrix} \Psi_0 & 0 \\ 0 & \Psi_0 \end{bmatrix} \tag{3}$$

$$[\boldsymbol{\Psi}] \equiv \begin{bmatrix} \Psi_3 & \Psi_1 - i\Psi_2 \\ \Psi_1 + i\Psi_2 & -\Psi_3 \end{bmatrix} \tag{4}$$

---

[119] The author uses its own notation for spinors and sign flavors

Spinors obey[120]

$$[\boldsymbol{\Psi}] + [\boldsymbol{\phi}] = 2[\langle \boldsymbol{\Psi}, \boldsymbol{\phi} \rangle] \tag{5}$$

$$[\boldsymbol{\Psi}] - [\boldsymbol{\phi}] = 2\,i[\boldsymbol{\Psi} \times \boldsymbol{\phi}] \tag{6}$$

5.5.2.1 Sign flavors

The relation with the sign flavors is

$$[\boldsymbol{\Psi}] = [\boldsymbol{\Psi}]^{\textcircled{0}} = \left[\boldsymbol{\psi}^{\textcircled{0}}\right] \tag{1}$$
$$[\boldsymbol{\Psi}]^{\textcircled{1}} = \left[\boldsymbol{\psi}^{\textcircled{1}}\right] \tag{2}$$
$$[\boldsymbol{\Psi}]^{\textcircled{2}} = \left[\boldsymbol{\psi}^{\textcircled{2}}\right] \tag{3}$$
$$[\boldsymbol{\Psi}]^{\textcircled{3}} = \left[\boldsymbol{\psi}^{\textcircled{3}}\right] \tag{4}$$
$$[\boldsymbol{\Psi}^*] = [\boldsymbol{\Psi}^*]^{\textcircled{0}} = \left[\boldsymbol{\psi}^{\textcircled{7}}\right] \tag{5}$$
$$[\boldsymbol{\Psi}^*]^{\textcircled{1}} = \left[\boldsymbol{\psi}^{\textcircled{6}}\right] \tag{6}$$
$$[\boldsymbol{\Psi}^*]^{\textcircled{2}} = \left[\boldsymbol{\psi}^{\textcircled{5}}\right] \tag{7}$$
$$[\boldsymbol{\Psi}^*]^{\textcircled{3}} = \left[\boldsymbol{\psi}^{\textcircled{4}}\right] \tag{8}$$

5.5.3 Dirac spinors

The 4×4 spinors target the application in the Dirac equation.
A general 4×4 spinor is defined by the column:

$$\begin{bmatrix} [\psi] \\ [\phi^*]^{\ddagger} \end{bmatrix} \equiv \begin{bmatrix} [\Psi_0] & [\boldsymbol{\Psi}] \\ [-\boldsymbol{\phi}] & [\phi_0] \end{bmatrix} \tag{1}$$

A compacted spinor $]\Psi[$ is a 1×4 matrix consisting of real functions that represent all sixteen sign flavors of a QPAD.

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[120] http://en.wikipedia.org/wiki/Spinors_in_three_dimensions

$$]\Psi[\equiv \begin{bmatrix} [\psi] \\ [\psi^*]^\ddagger \end{bmatrix} = \begin{bmatrix} [\Psi_0] & [\boldsymbol{\Psi}] \\ [-\boldsymbol{\Psi}] & [\Psi_0] \end{bmatrix} \tag{2}$$

$$= \begin{bmatrix} \Psi_0 & 0 & \Psi_3 & \Psi_1 - i\Psi_2 \\ 0 & \Psi_0 & \Psi_1 + i\Psi_2 & -\Psi_3 \\ -\Psi_3 & -\Psi_1 + i\Psi_2 & \Psi_0 & 0 \\ -\Psi_1 - i\Psi_2 & +\Psi_3 & 0 & \Psi_0 \end{bmatrix}$$

5.5.4 Spinor base

The $\boldsymbol{\alpha}$ and β matrices form the base of spinor $]\Psi[$ and its elements

$$\alpha_1 \equiv \begin{bmatrix} 0 & \boldsymbol{i} \\ -\boldsymbol{i} & 0 \end{bmatrix} \tag{1}$$

$$\alpha_2 \equiv \begin{bmatrix} 0 & \boldsymbol{j} \\ -\boldsymbol{j} & 0 \end{bmatrix} \tag{2}$$

$$\alpha_3 \equiv \begin{bmatrix} 0 & \boldsymbol{k} \\ -\boldsymbol{k} & 0 \end{bmatrix} \tag{3}$$

$$\beta \equiv \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \tag{4}$$

$\boldsymbol{i}, \boldsymbol{j}$ and $\boldsymbol{k}$ represent imaginary base vectors of the simulated quaternion. β represents the conjugation action for the spinor.

A relation exist between $\alpha_1, \alpha_2, \alpha_3$ and the [Pauli][121] matrices $\sigma_1, \sigma_2, \sigma_3$:

$$\sigma_1 \equiv \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \qquad \sigma_2 \equiv \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \qquad \sigma_3 \equiv \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \tag{5}$$

---

[121] http://en.wikipedia.org/wiki/Pauli_matrices

$$1 \mapsto I, \qquad \boldsymbol{i} \mapsto \sigma_1, \qquad \boldsymbol{j} \mapsto \sigma_2, \qquad \boldsymbol{k} \mapsto \sigma_3 \tag{6}$$

5.5.5 Gamma matrices

This combination is usually represented in the form of gamma matrices.

In Dirac representation, the four contravariant gamma matrices are

$$\gamma^0 \equiv \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}, \qquad \gamma^1 \equiv \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}, \tag{1}$$

$$\gamma^2 \equiv \begin{bmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{bmatrix}, \qquad \gamma^3 \equiv \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

It is useful to define the product of the four gamma matrices as follows:

$$\gamma^5 \equiv i\,\gamma^0\,\gamma^1\,\gamma^2\,\gamma^3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \tag{2}$$

The gamma matrices as specified here are appropriate for acting on Dirac spinors written in the Dirac basis; in fact, the Dirac basis is defined by these matrices. In the Dirac basis[122]:

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[122] http://en.wikipedia.org/wiki/Gamma_matrices#Dirac_basis

$$\gamma^0 \equiv \begin{bmatrix} I & 0 \\ 0 & -I \end{bmatrix}, \qquad \gamma^k = \begin{bmatrix} 0 & \sigma^k \\ -\sigma^k & 0 \end{bmatrix}, \tag{3}$$

$$\gamma^5 = \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix}$$

This corresponds with $\alpha_k = \gamma^k, \beta = \gamma^5$.
Apart from the Dirac basis, a Weyl basis exists

$$\gamma^0 = \gamma^\beta = \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix}, \qquad \gamma^k = \begin{bmatrix} 0 & \sigma^k \\ -\sigma^k & 0 \end{bmatrix}, \tag{4}$$

$$\gamma^5 = \begin{bmatrix} -I & 0 \\ 0 & I \end{bmatrix}$$

The Weyl basis has the advantage that its [chiral projections][123] take a simple form:

$$\psi_L = \tfrac{1}{2}(1 - \gamma^5)[\psi] = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}[\psi] \tag{5}$$

$$\psi_R = \tfrac{1}{2}(1 + \gamma^5)[\psi] = \begin{bmatrix} 0 & 0 \\ 0 & I \end{bmatrix}[\psi] \tag{6}$$

$$[\psi^*] = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}[\psi] \tag{7}$$

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[123] http://en.wikipedia.org/wiki/Chirality_(physics)

6 Quaternionic differentiation

6.1 Differentiation in flat space

We treat quaternionic distributions as if they possess a continuous parameter space. The differential vector operator ∇ is in Cartesian coordinates given by

$$\nabla = \sum_{i=1}^{3} e_i \frac{\partial}{\partial x_i} \tag{1}$$

The flat quaternionic differential operator ∇ is in Cartesian coordinates given by

$$\nabla = \sum_{i=0}^{3} e_i \nabla_i = \sum_{i=0}^{3} e_i \frac{\partial}{\partial x_i}; \quad e = (1, \boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}) \tag{2}$$

$$\nabla f = \sum_{i=0}^{3} \sum_{j=0}^{3} e_i e_j \frac{\partial f_j}{\partial x_i} \tag{3}$$

6.2   Differentiation in curved space

The allocation function $\wp(x)$ has a flat parameter space that is spanned by the rational or the real quaternions[124]. However, in this section we treat the E-type $\wp(x)$ as if it has a continuous parameter space[125]. That makes it possible to use regular differential calculus. The full quaternionic difference operator $d\wp$ is given by

$$d\wp = \sum_{\mu=0}^{3} q^{\mu}\, dx_{\mu} = \sum_{\mu=0}^{3} \frac{\partial \wp}{\partial x_{\mu}}\, dx_{\mu}$$

$$= \sum_{\nu=0}^{3} e_{\mu} \sum_{\mu=0}^{3} \frac{\partial \wp_{\nu}}{\partial x_{\mu}}\, dx_{\mu}$$

(1)

Here the coefficients q^{μ} are quaternionic coefficients, which are determined by the quaternionic allocation function $\wp(x)$.

$\wp(x)$ defines a curved target space. This curved space can act as parameter space to other quaternionic distributions.

(2)

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[124]   http://en.wikipedia.org/wiki/Quaternion_algebra#Quaternion_algebras_over_the_rational_numbers

[125] See section on quaternionic distributions.

$$q^\mu = \frac{\partial \wp}{\partial x_\mu}; \quad \wp = \sum_{\nu=0}^{3} e_\nu \wp_\nu$$

The allocation function $\wp(x)$ may include an isotropic scaling function $a(\tau)$ that only depends on progression τ. It defines the expansion/compression of the curved space.

The quaternionic infinitesimal interval $d\wp$ defines the quaternionic metric of the curved space that is defined by $\wp(x)$.

7 Coordinate systems

7.1 Cylindrical circular coordinates

7.1.1 Base vectors

7.1.2 Cartesian to cylindrical circular

$$\rho = x_1 \, cos(\,\theta\,) + x_2 \, sin(\,\theta\,) \tag{1}$$

$$\varphi = -x_1 \, sin(\,\theta\,) + x_2 \, cos(\,\theta\,) \tag{2}$$

$$z = x_3 \tag{3}$$

7.1.3 Cylindrical circular to Cartesian

$$x_1 = \boldsymbol{\rho} \, cos(\,\theta\,) - \boldsymbol{\varphi} \, sin(\,\theta\,) \tag{1}$$

$$x_2 = \rho \, sin(\,\theta\,) + \boldsymbol{\varphi} \, cos(\,\theta\,) \tag{2}$$

$$x_3 = z \tag{3}$$

7.1.4 Directed line element

$$dl \; = \; dx \, \frac{\boldsymbol{x}}{|\boldsymbol{x}|} \; = \; \boldsymbol{e}_\rho d\rho \; + \; \boldsymbol{e}_\varphi \rho d\varphi \; + \; \boldsymbol{e}_z dz \tag{1}$$

7.1.5 Solid angle element

$$d\Omega = \sin(\theta) \, d\theta \, d\varphi \tag{1}$$

7.1.6   Directed area element

$$dS = e_r\, r^2\, d\Omega + e_\theta\, r\, sin(\theta)\, dr\, d\varphi + e_\varphi\, r\, dr\, d\theta \tag{1}$$

7.1.7   Volume element

$$dV = dx^3 = dr\, r^2\, d\Omega \tag{1}$$

7.1.8   Spatial differential operators

$$\alpha \;=\; \alpha(r, \theta, \varphi) \tag{1}$$
$$a = a(r, \theta, \varphi) \tag{2}$$

Gradient

$$\nabla \alpha = e_r\, \frac{\partial \alpha}{\partial r} + e_\theta\, \frac{1}{r}\frac{\partial \alpha}{\partial \theta} + \mathbf{e}_\varphi\, \frac{1}{r\, sin(\theta)}\frac{\partial \alpha}{\partial \varphi} \tag{3}$$

Divergence

$$\langle \nabla, a \rangle = \frac{1}{r^2}\frac{\partial (r^2 \alpha_r)}{\partial r} + \frac{1}{r\, sin(\theta)}\frac{\partial (a_\theta\, sin(\theta))}{\partial \theta} + \frac{1}{r\, sin(\theta)}\frac{\partial a_\varphi}{\partial \varphi} \tag{4}$$

Curl

$$\nabla \times a = e_r\, \frac{1}{r\, sin(\theta)}\left(\frac{\partial (a_\varphi\, sin(\theta))}{\partial \theta} - \frac{\partial a_\varphi}{\partial \varphi} \right) \tag{5}$$

$$+ \; e_\theta\, \frac{1}{r}\left(\frac{1}{sin(\theta)}\frac{\partial \alpha_r}{\partial \varphi} - \frac{\partial a_\varphi}{\partial r} \right)$$

$$+ e_\varphi\, \frac{1}{r}\left(\frac{\partial r\, a_\varphi}{\partial r} - \frac{\partial a_r}{\partial \theta} \right)$$

The Laplacian

$$\tag{6}$$

$$\nabla^2 \alpha = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial \alpha}{\partial r} \right) + \frac{1}{r^2 \sin(\theta)} \frac{\partial}{\partial \theta} \left(\sin(\theta) \frac{\partial \alpha}{\partial \theta} \right)$$
$$+ \frac{1}{r^2 \sin^2(\theta)} \frac{\partial^2 \alpha}{\partial \varphi^2}$$

7.2 Polar coordinates

The equivalent to rectangular coordinates in quaternion space is (a_τ, a_x, a_y, a_z)

$$a = a_\tau + i\, a_x + j\, a_y \pm i\, j\, a_z \qquad (1)$$

The equivalent to polar coordinates in quaternion space is

$$a_\tau = \|a\| \cos(\psi) \qquad (2)$$

$$a_x = \|a\| \sin(\psi) \sin(\theta) \cos(\varphi) \qquad (3)$$

$$a_y = \|a\| \sin(\psi) \sin(\theta) \sin(\varphi) \qquad (4)$$

$$a_z = \|a\| \sin(\psi) \cos(\theta) \qquad (5)$$

$sin(\psi)$, where $\psi = (0, \pi)$, is known as the (imaginary) amplitude of the quaternion.

Angle $\theta = (0, \pi)$ is the (co-)latitude and angle $\varphi = (0, 2\pi)$ is the longitude.

For any fixed value of ψ, θ and φ parameterize a 2-sphere of radius $sin(\psi)$, except for the degenerate cases, when ψ equals 0 or π, in which case they describe a point.

This suggests the following structure of the argument $\Lambda = \tilde{\imath} \cdot \psi$

$$a = \|a\| \, exp(\tilde{\imath} \cdot \psi) \tag{6}$$

$$= \|a\| \, (cos(\psi) + \tilde{\imath} \, sin(\psi)) \tag{7}$$

$$= a_\tau + \|a\| \, \tilde{\imath} \, sin(\psi) = a_\tau + \boldsymbol{a} \tag{8}$$

The imaginary number $\tilde{\imath}$ may take any direction. This shows that for quaternions exponential functions only work for (local) abstractions to complex number sub-systems. It also means that the notions of Lie groups works in complex number systems, but not in general in quaternionic number systems.

7.3 3 sphere

A 3-sphere is a compact, connected, 3-dimensional manifold without boundary. It is also simply-connected. What this means, loosely speaking, is that any loop, or circular path, on the 3-sphere can be continuously shrunk to a point without leaving the 3-sphere. The Poincaré conjecture[126] proposes that the 3-sphere is the only three dimensional manifold with these properties (up to homeomorphism)[127].

The round metric on the 3-sphere in these coordinates is given by

$$ds^2 = d\psi^2 + sin^2(\psi) \, (d\theta^2 + sin^2(\theta)d\varphi^2) \tag{1}$$

The volume form is given by

$$dV = sin^2(\psi) \, sin(\theta) \, d\psi \wedge d\theta \wedge d\varphi \tag{2}$$

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[126] http://en.wikipedia.org/wiki/Poincar%C3%A9_conjecture
[127] http://en.wikipedia.org/wiki/3-sphere

The 3-dimensional volume (or **hyperarea**) of a 3-sphere of radius r is

$$2\,\pi^2\,r^3 \tag{3}$$

The 4-dimensional **hypervolume** (the volume of the 4-dimensional region bounded by the 3-sphere) is

$$\tfrac{1}{2}\,\pi^2\,r^4 \tag{4}$$

The 3-sphere has constant positive sectional curvature equal to $1/r^2$.

The 3-sphere has a natural Lie group structure SU(2) given by quaternion multiplication.

The 3-sphere admits non-vanishing vector fields (sections of its tangent bundle). One can even find three linearly-independent and non-vanishing vector fields. These may be taken to be any left-invariant vector fields forming a basis for the Lie algebra of the 3-sphere. This implies that the 3-sphere is parallelizable. It follows that the tangent bundle of the 3-sphere is trivial.

There is an interesting action of the circle group $\mathbb{T}$ on $\mathbb{S}^3$ giving the 3-sphere the structure of a principal circle bundle known as the Hopf bundle. If one thinks of $\mathbb{S}^3$ as a subset of $\boldsymbol{C}^2$, the action is given by

$$(z_1, z_2)\,\lambda \;=\; (z_1\,\lambda, z_2\,\lambda)\;\forall_{\lambda\,\in\,\mathbb{T}}. \tag{5}$$

The orbit space of this action is homeomorphic to the two-sphere $\mathbb{S}^2$. Since $\mathbb{S}^3$ is not homeomorphic to $\mathbb{S}^2 \times \mathbb{S}^1$, the Hopf bundle is nontrivial.

7.4 Hopf coordinates

Another choice of hyperspherical coordinates, (η, ξ_1, ξ_2), makes use of the embedding of $\mathbb{S}^3$ in $\boldsymbol{C}^2$. In complex coordinates $(z_1, z_2) \in \boldsymbol{C}^2$ we write

$$z_1 = exp(\text{ĩ} \, \xi_1) \, sin(\eta) \tag{1}$$

$$z_2 = exp(\text{ĩ} \, \xi_2) \, cos(\eta) \tag{2}$$

Here η runs over the range 0 to $\pi/2$, and ξ_1 and ξ_2 can take any values between 0 and 2π. These coordinates are useful in the description of the 3-sphere as the Hopf bundle

$$\mathbb{S}^1 \to \mathbb{S}^3 \to \mathbb{S}^2 \tag{3}$$

For any fixed value of η between 0 and $\pi/2$, the coordinates (ξ_1, ξ_2) parameterize a 2-dimensional torus. In the degenerate cases, when η equals 0 or $\pi/2$, these coordinates describe a circle.

The round metric on the 3-sphere in these coordinates is given by

$$ds^2 = d\eta^2 + sin^2(\eta) \, (d\zeta_1^2 + cos^2(\eta) \, d \, \zeta_2^2) \tag{4}$$

and the volume form by

$$dV = sin(\eta) \, cos(\eta) \, d\eta \wedge d\zeta_1 \wedge d\zeta_2 \tag{5}$$

7.5 Group structure

Because the set of unit quaternions is closed under multiplication, $\mathbb{S}^3$ takes on the structure of a group. Moreover, since quaternionic multiplication is smooth, $\mathbb{S}^3$ can be regarded as a real Lie group. It is a non-abelian, compact Lie group of dimension 3. When thought of as a Lie group $\mathbb{S}^3$ is often denoted $Sp(1)$ or $U(1, \mathbb{H})$.

It turns out that the only spheres which admit a Lie group structure are $\mathbb{S}^1$, thought of as the set of unit complex numbers, and $\mathbb{S}^3$, the set of unit quaternions. One might think that $\mathbb{S}^7$, the set of unit octonions, would form a Lie group, but this fails since octonion multiplication is non-associative. The octonionic structure does give $\mathbb{S}^7$ one important property: *parallelizability*[128]. It turns out that the only spheres which are parallelizable are $\mathbb{S}^1$, $\mathbb{S}^3$, and $\mathbb{S}^7$.

By using a matrix representation of the quaternions, $\mathbb{H}$, one obtains a matrix representation of $\mathbb{S}^3$. One convenient choice is given by the Pauli matrices:

$$
\begin{aligned}
\left(a_\tau + a_x \cdot \mathbf{i} + a_y \cdot \mathbf{j} + a_z \cdot \mathbf{k} \right) & \\
= \begin{bmatrix} a_\tau + \tilde{\imath} \cdot a_x & a_y + \tilde{\imath} \cdot a_z \\ -a_y + \tilde{\imath} \cdot a_z & a_\tau - \tilde{\imath} \cdot a_x \end{bmatrix} &
\end{aligned}
\tag{1}
$$

This map gives an injective algebra homomorphism from **H** to the set of 2×2 complex matrices. It has the property that the absolute value of a quaternion q is equal to the square root of the determinant of the matrix image of q.

The set of unit quaternions is then given by matrices of the above form with unit determinant. This matrix subgroup is precisely the special unitary group SU(2). Thus, $\mathbb{S}^3$ as a Lie group is isomorphic to SU(2).

Using our hyperspherical coordinates (η, ξ_1, ξ_2) we can then write any element of SU(2) in the form

$$
\begin{bmatrix} \exp(\tilde{\imath} \cdot \xi_1) \cdot \sin(\eta) & \exp(\tilde{\imath} \cdot \xi_2) \cdot \cos(\eta) \\ -\exp(\tilde{\imath} \cdot \xi_2) \cdot \cos(\eta) & \exp(-\tilde{\imath} \cdot \xi_1) \cdot \sin(\eta) \end{bmatrix}
\tag{2}
$$

---

[128] http://en.wikipedia.org/wiki/Parallelizability

Another way to state this result is if we express the matrix representation of an element of SU(2) as a linear combination of the Pauli matrices. It is seen that an arbitrary element U ∈ SU(2) can be written as

$$U = \alpha_\tau \cdot 1 + \sum_{n=x,y,z} \alpha_n \, \boldsymbol{I_n} \tag{3}$$

The condition that the determinant of U is +1 implies that the coefficients α_n are constrained to lie on a 3-sphere.

7.6   Versor

Any **unit quaternion** u can be written as a **versor**:

$$u = exp(\tilde{\imath}\ \psi) = cos(\psi) + \tilde{\imath}\ sin(\psi) \tag{1}$$

This is the quaternionic analogue of Euler's formula. Now the unit imaginary quaternions all lie on the unit 2-sphere in Im ℍ so any such $\tilde{\imath}$ can be written:

$$\tilde{\imath} = \boldsymbol{i}\ cos(\varphi)\ sin(\theta) + \boldsymbol{j}\ sin(\varphi)\ sin(\theta) + \boldsymbol{k}\ cos(\theta) \tag{2}$$

7.7   Symplectic decomposition

Quaternions can be written as the combination of two complex numbers and an imaginary number k with unit length.

$$q = a + b\boldsymbol{j}; \text{ where } a = w + x\boldsymbol{i}; \text{ and } b = y + z\boldsymbol{i}$$

$$q = w + x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$$

7.8 Quaternionic algebra

$$a = (a_0, a_1, a_2, a_3) = \sum_{\mu=0}^{3} e_\mu \, a_\mu \tag{1}$$

$$= a_0 + \boldsymbol{i}\, a_1 + \boldsymbol{j}\, a_2 + \boldsymbol{k}\, a_3 = a_0 + \boldsymbol{a}$$

$$a^* = a_0 - \boldsymbol{a} \tag{2}$$

$$a^* a = a\, a^* = |a|^2 \tag{3}$$

$$\langle \boldsymbol{a}, \boldsymbol{b} \rangle = \sum_{\mu=1}^{3} a_\mu \, b_\mu = \delta_{\mu\nu} a_\mu b_\nu = |\boldsymbol{a}||\boldsymbol{b}| \cos(\theta) \tag{4}$$

$$\boldsymbol{a} \times \boldsymbol{b} = -\boldsymbol{b} \times \boldsymbol{a} = \pm\left(\epsilon_{ijk} \, \boldsymbol{e}_i a_j b_k\right) \tag{5}$$

$$a\, b = a_0 \boldsymbol{b} + b_0 \, \boldsymbol{a} - \langle \boldsymbol{a}, \boldsymbol{b} \rangle \pm \boldsymbol{a} \times \boldsymbol{b} \tag{6}$$

The colored $\pm$ indicates the handedness of the vector cross product.

$$\boldsymbol{a}\, \boldsymbol{b} = -\langle \boldsymbol{a}, \boldsymbol{b} \rangle \pm \boldsymbol{a} \times \boldsymbol{b} \tag{7}$$

$$a\,(b + c) = a\, b + a\, c \tag{8}$$

$$(a + b)\, c = a\, c + b\, c \tag{9}$$

$$(a\, b)c = a(b\, c) \tag{10}$$

$$\langle \boldsymbol{a}, \boldsymbol{b} \times \boldsymbol{c} \rangle = \langle \boldsymbol{a} \times \boldsymbol{b}, \boldsymbol{c} \rangle \tag{11}$$

$$\boldsymbol{a} \times (\boldsymbol{b} \times \boldsymbol{c}) = \boldsymbol{b}\langle \boldsymbol{a}, \boldsymbol{c} \rangle - \boldsymbol{c}\langle \boldsymbol{a}, \boldsymbol{b} \rangle \tag{12}$$

$$(\boldsymbol{a} \times \boldsymbol{b}) \times \boldsymbol{c} = \boldsymbol{b}\langle \boldsymbol{a}, \boldsymbol{c} \rangle - \boldsymbol{a}\langle \boldsymbol{b}, \boldsymbol{c} \rangle \tag{13}$$

$$\boldsymbol{a} \times (\boldsymbol{b} \times \boldsymbol{c}) + \boldsymbol{b} \times (\boldsymbol{c} \times \boldsymbol{a}) + \boldsymbol{c} \times (\boldsymbol{a} \times \boldsymbol{b}) = 0 \tag{14}$$

$$\langle \boldsymbol{a} \times \boldsymbol{b}, \boldsymbol{c} \times \boldsymbol{d} \rangle = \langle \boldsymbol{a}, \boldsymbol{b} \times (\boldsymbol{c} \times \boldsymbol{d}) \rangle \tag{15}$$

$$= \langle \boldsymbol{a}, \boldsymbol{c} \rangle \langle \boldsymbol{b}.\boldsymbol{d} \rangle - \langle \boldsymbol{a}, \boldsymbol{d} \rangle \langle \boldsymbol{b}.\boldsymbol{c} \rangle$$

$$(\boldsymbol{a} \times \boldsymbol{b}) \times (\boldsymbol{c} \times \boldsymbol{d}) = \langle \boldsymbol{a} \times \boldsymbol{b}, \boldsymbol{d} \rangle \boldsymbol{c} - \langle \boldsymbol{a} \times \boldsymbol{b}, \boldsymbol{c} \rangle \boldsymbol{d} \tag{16}$$

8 Quaternionic distributions

We consider four kinds of quaternionic distributions
- A. Distributions of rational quaternions with a discrete parameter space. That parameter space must be flat and it is spanned by the rational quaternions[129]. The A-type quaternionic distribution has a countable set of values.
- B. Distributions of rational quaternions with a continuous parameter space. That parameter space may be curved. The curvature is defined by a continuous quaternionic function. The B-type quaternionic distribution has a countable set of values. It inherits the sign flavor of the quaternionic function that defines the curvature of its parameter space.
- C. Continuous quaternionic distributions with a continuous parameter space. That parameter space may be curved. The curvature is defined by a continuous quaternionic function. The C-type quaternionic distributions inherit the sign flavor of the quaternionic distribution that defines the curvature of their parameter space. The C-type quaternionic function can be split in a real scalar function and a real 3D vector

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[129] http://en.wikipedia.org/wiki/Quaternion_algebra#Quaternion_algebras_over_the_rational_numbers

function. The real scalar function can be interpreted as an object density distribution and the 3D vector function can be interpreted as the associated current density distribution.

D. A convolution of a continuous quaternionic function with a discrete flat parameter space consisting of rational quaternions and a 3D stochastic generator of rational quaternionic target values. The D-type quaternionic distribution has a countable set of values. It corresponds to a collection of coherent B-type distributions, where the continuous function generates the curvature of the parameter space for the B-type distributions, which are generated by the stochastic process.

8.1   Basic properties of continuous quaternionic distributions

For simplicity we confine to quaternionic distributions with flat parameter space. A continuous quaternionic distribution contains a scalar field in its real part and a vector field in its imaginary part.

$$f(x) = f_0(x) + \boldsymbol{f}(x) \tag{1}$$

$$a\, f(x) = a_0 \boldsymbol{f}(x) + f_0(x)\, \boldsymbol{a} - \langle \boldsymbol{a}, \boldsymbol{f}(x) \rangle \pm \boldsymbol{a} \times \boldsymbol{f}(x) \tag{2}$$

$$f(x)\, b = f_0(x)\boldsymbol{b} + b_0\, \boldsymbol{f}(x) - \langle \boldsymbol{f}(x), \boldsymbol{b} \rangle \pm \boldsymbol{f}(x) \times \boldsymbol{b} \tag{3}$$

The distributions follow the rules for the quaternion algebra.

$$a\, (f(x) + g(x)) = a\, f(x) + a\, g(x) \tag{4}$$

$$(a\, + b)f(x) = a\, f(x) + b\, f(x) \tag{5}$$

$$\tag{6}$$

$$f(x)\,g(x) = f_0(x)\boldsymbol{g}(x) + g_0(x)\boldsymbol{f}(x) - \langle \boldsymbol{f}(x), \boldsymbol{g}(x)\rangle$$
$$\pm \boldsymbol{f}(x) \times \boldsymbol{g}(x)$$
$$\big(f(x)g(x)\big)h(x) = f(x)\big(g(x)\,h(x)\big) \tag{7}$$

8.1.1 Symmetries

Continuous quaternionic distributions keep the same discrete symmetries (sign flavor) throughout their domain. The sign flavor of the parameter space acts as reference sign flavor.

8.1.2 Differentials

The quaternionic nabla acts similarly as a normal quaternion
$$\nabla\,(f(x) + g(x)) = \nabla\,f(x) + \nabla\,g(x) \tag{1}$$
$$\nabla\,f(x) = \nabla_0\boldsymbol{f}(x) + \nabla f_0(x) - \langle \nabla, \boldsymbol{f}(x)\rangle \pm \nabla \times \boldsymbol{f}(x) \tag{2}$$
However
$$\nabla(b\,c) \neq (\nabla\,b)c \tag{3}$$
and
$$\nabla(b\,c) \neq (\nabla\,b)c + b\,\nabla\,c \tag{4}$$
Further
$$\langle \nabla, \nabla\rangle \alpha \equiv \nabla^2 \alpha \tag{5}$$
$$\langle \nabla \times \nabla, \mathbf{a}\rangle = 0 \tag{6}$$
$$\langle \nabla, \nabla \times \boldsymbol{a}\rangle = 0 \tag{7}$$
$$\nabla \times \nabla \alpha = \mathbf{0} \tag{8}$$
$$\nabla\,b = -\langle \nabla, \boldsymbol{b}\rangle \pm \nabla \times \boldsymbol{b} \tag{9}$$
$$\nabla\,(\alpha\,\beta) = \alpha\nabla\,\beta + \beta\nabla\,\alpha \tag{10}$$
$$\nabla\,(\alpha\,\boldsymbol{a}) = \alpha\nabla \times \boldsymbol{a} - \alpha\langle \nabla, \boldsymbol{a}\rangle + (\nabla\,\alpha)\boldsymbol{a} \tag{11}$$
$$\langle \nabla, \alpha\,\boldsymbol{a}\rangle = \boldsymbol{a}\nabla\alpha + \alpha\langle \nabla, \boldsymbol{a}\rangle \tag{12}$$
$$\langle \nabla, \boldsymbol{a} \times \boldsymbol{b}\rangle = \langle \boldsymbol{b}, \nabla \times \boldsymbol{a}\rangle - \langle \boldsymbol{a}, \nabla \times \boldsymbol{b}\rangle \tag{13}$$
$$\langle \nabla\,\alpha, \nabla\,\beta\rangle = \langle \nabla, \alpha\nabla\,\beta\rangle - \alpha\nabla^2\beta \tag{14}$$
$$\langle \nabla\,\alpha, \nabla \times \boldsymbol{a}\rangle = -\nabla, \boldsymbol{a} \times \nabla\alpha \tag{15}$$
$$\langle \nabla \times \boldsymbol{a}, \nabla \times \boldsymbol{b}\rangle = \langle \boldsymbol{b}, \nabla \times (\nabla \times \boldsymbol{a})\rangle - \langle \boldsymbol{a}, \nabla \times (\nabla \times \boldsymbol{b})\rangle \tag{16}$$
$$\nabla \times (\alpha\mathbf{a}) = \alpha\nabla \times \boldsymbol{a} - \boldsymbol{a} \times \nabla\alpha \tag{17}$$

$$\boldsymbol{\nabla} \times (\alpha\boldsymbol{\nabla}\beta) = (\boldsymbol{\nabla}\alpha) \times \nabla\beta \tag{18}$$

9 The separable Hilbert space Ħ

We will specify the characteristics of a generalized quaternionic infinite dimensional separable Hilbert space. The adjective "quaternionic" indicates that the inner products of vectors and the eigenvalues of operators are taken from the number system of the quaternions. Separable Hilbert spaces can be using real numbers, complex numbers or quaternions. These three number systems are division rings.

9.1 Notations and naming conventions

$\{f_x\}_x$ means ordered set of f_x . It is a way to define functions.

The use of bras and kets differs slightly from the way Dirac uses them.

|f> is a ket vector, f> is the same ket
<f| is a bra vector, <f is the same bra

A is an operator.
|A is the same operator
A† is the adjoint operator of operator A.
A| is the same operator as A†
| on its own, is a nil operator
|A| is a self-adjoint (Hermitian) operator

We will use capitals for operators and lower case for quaternions, eigenvalues, ket vectors, bra vectors and eigenvectors. Quaternions and eigenvalues will be indicated with *italic* characters. Imaginary and anti-Hermitian objects are often underlined and/or indicated in **bold** text.

$\sum_k$ means sum over all items with index k.

$\int_x$ means: integral over all items with parameter x.

9.2 Quaternionic Hilbert space

The Hilbert space is a **linear space**. That means for the elements $|f>$, $|g>$ and $|h>$ and numbers a and b:

9.2.1 Ket vectors

For **ket** vectors hold

$$|f> + |g> = |g> + |f> = |g + f> \tag{1}$$

$$(|f> + |g>) + |h> = |f> + (|g> + |h>) \tag{2}$$

$$|(a + b)\, f> = |f> \cdot a + |f> \cdot b \tag{3}$$

$$(|f> + |g>) \cdot a = |f> \cdot a + |g> \cdot a \tag{4}$$

$$|f> \cdot 0 = |0> \tag{5}$$

$$|f> \cdot 1 = |f> \tag{6}$$

Depending on the number field that the Hilbert space supports, a and b can be real numbers, complex numbers or (real) quaternions.

9.2.2 Bra vectors

The **bra** vectors form the dual Hilbert space $\mathbf{H}^\dagger$ of $\mathbf{H}$.

$$<f| + <g| = <g| + <f| = |g + f> \tag{1}$$

$$(<f| + <g|) + <h| = <f| + (<g| + <h|) \tag{2}$$

$$<f\,(a + b)> = <f| \cdot a + <f| \cdot b = a^* \cdot <f| + b^* \cdot <f|$$

$$(<f| + <g|)\cdot a = <f|\cdot a + <g|\cdot a = a^*\cdot<f| + a^*\cdot<g| \tag{3}$$
$$\tag{4}$$

$$0\cdot<f| = <0| \tag{5}$$

$$1\cdot<f| = <f| \tag{6}$$

9.2.3 Scalar product

The Hilbert space contains a **scalar product,** also called **inner product,** $<f|g>$ that combines $\mathbf{H}$ and $\mathbf{H}^\dagger$ in a direct product that we also indicate with $\mathbf{H}$.

For Hilbert spaces the values of inner products are restricted to elements of a division ring.

The scalar product $<f|g>$ satisfies:

$$<f|g + h> = <f|g> + <f|h> \tag{1}$$

$$<f|\{|g>\cdot a\}_g = \{<f|g>\}_g\cdot a \tag{2}$$

With each ket vector $|g>$ in $\mathbf{H}$ belongs a bra vector $<g|$ in $\mathbf{H}^\dagger$ such that for all bra vectors $<f|$ in $\mathbf{H}^\dagger$

$$<f|g> = <g|f>^* \tag{3}$$

$$<f|f> = 0 \text{ when } |f> = |0> \tag{4}$$

$$<f|a\ g> = <f|g>\cdot a = <g|f>^*\cdot a = <g\ a|f>^* = (a^*\cdot<g|f>)^* = \tag{5}$$
$$<f|g>\cdot a$$

In general is $\langle f|a\ g\rangle \neq \langle f\ a|g\rangle$. However for real numbers r holds $\langle f|r\ g\rangle = \langle f\ r|g\rangle$

Remember that when the number field consists of quaternions, then also $\langle f|g\rangle$ is a quaternion and a quaternion q and $\langle f|g\rangle$ do in general not commute.

The scalar product defines a **norm**:

$$\|f\| = \sqrt{(\langle f|f\rangle)} \tag{6}$$

And a **distance**:

$$D(f,g) = \|f - g\| \tag{7}$$

The Hilbert space **H** is closed under its norm. Each converging row of elements of converges to an element of this space.

9.2.4   Separable

In mathematics a topological space is called separable if it contains a countable dense subset; that is, there exists a sequence $\{x_n\}_{n=1}^{\infty}$ of elements of the space such that every nonempty open subset of the space contains at least one element of the sequence.

Every continuous function on the separable space **H** is determined by its values on this countable dense subset.

9.2.5   *Base vectors*

The Hilbert space **H** is **separable**. That means that a countable row of elements $\{f_n\rangle\}$ exists that **spans** the whole space.

If $\langle f_n|f_m\rangle = \delta(m,n) = [1$ when $n = m$; 0 otherwise] then $\{|f_n\rangle\}$ forms an **orthonormal base** of the Hilbert space.

A ket base $\{|k\rangle\}$ of $\mathcal{H}$ is a minimal set of ket vectors $|k\rangle$ that together span the Hilbert space $\mathcal{H}$.

Any ket vector $|f\rangle$ in $\mathcal{H}$ can be written as a linear combination of elements of $\{|k\rangle\}$.

$$|f\rangle = \sum_k (|k\rangle \cdot \langle k|f\rangle) \tag{1}$$

A bra base $\{\langle b|\}$ of $\mathcal{H}^\dagger$ is a minimal set of bra vectors $\langle b|$ that together span the Hilbert space $\mathcal{H}^\dagger$.

Any bra vector $\langle f|$ in $\mathcal{H}^\dagger$ can be written as a linear combination of elements of $\{\langle b|\}$.

$$\langle f| = \sum_b (\langle f|b\rangle \cdot \langle b|) \tag{2}$$

Usually base vectors are taken such that their norm equals 1. Such a base is called an othonormal base.

9.2.6   *Operators*

Operators act on a subset of the elements of the Hilbert space.

9.2.6.1   Linear operators

An operator Q is linear when for all vectors $|f\rangle$ and $|g\rangle$ for which Q is defined and for all quaternionic numbers a and b:

$$|Q \cdot a\, f\rangle + |Q \cdot b\, g\rangle = |a \cdot Q\, f\rangle + |b \cdot Q\, g\rangle = |Q\, f\rangle \cdot a + |Q\, g\rangle \cdot b \tag{1}$$

$=$

$$Q (|f>\cdot a + |g>\cdot b) = Q (|a\ f> + |b\ g>) \tag{2}$$

B is **colinear** when for all vectors $|f>$ for which B is defined and for all quaternionic numbers a there exists a quaternionic number c such that:

$$|B\cdot a\ f> = |a\cdot B\ f> = |B\ f> c\cdot a\cdot c^{-1} \tag{3}$$

If $|f>$ is an eigenvector of operator A with quaternionic eigenvalue a, then is $|b\ f>$ an eigenvector of A with quaternionic eigenvalue $b\cdot a\cdot b^{-1}$.

$A| = A^\dagger$ is the **adjoint** of the **normal** operator A. $|A$ is the same as A.

$$<f\ A|\ g> = <fA^\dagger|g>^* \tag{4}$$

$$A^{\dagger\dagger} = A \tag{5}$$

$$(A\cdot B)^\dagger = B^\dagger\cdot A^\dagger \tag{6}$$

$|B|$ is a **self adjoint** operator.
$|$ is a nil operator.

The construct $|f><g|$ acts as a linear operator. $|g><f|$ is its adjoint operator.

$$\sum_n \{|f_n>\cdot a_n\cdot<f_n|\}, \tag{7}$$

where a n is real and acts as a density function.

The set of eigenvectors of a normal operator form an orthonormal base of the Hilbert space.

A self adjoint operator has real numbers as eigenvalues.

$\{<q|f>\}_q$ is a function f(q) of parameter q.
$\{<g|q>\}_q$ is a function g(q) of parameter q.

When possible, we use the same letter for identifying eigenvalues, eigenvalues and the corresponding operator.

So, usually |q> is an eigenvector of a normal operator Q with eigenvalues q.

$\{q\}$ is the set of eigenvalues of Q.
$\{q\}_q$ is the ordered field of eigenvalues of q.
$\{|q>\}_q$ is the ordered set of eigenvectors of Q.
$\{<q|f>\}_q$ is the **Q view** of |f>.

9.2.6.2   Normal operators

The most common definition of continuous operators is:

A **continuous** operator is an operator that creates images such that the inverse images of open sets are open.

Similarly, a **continuous** operator creates images such that the inverse images of closed sets are closed.

If |a> is an eigenvector of normal operator A with eigenvalue a then

$$< a|A|a > \; = \; < a|a|a > \; = \; < a|a > \, a$$

indicates that the eigenvalues are taken from the same number system as the inner products.

A normal operator is a continuous linear operator.

A normal operator in **Ħ** creates an image of **Ħ** onto **Ħ.** It transfers closed subspaces of **Ħ** into closed subspaces of **Ħ**.

Normal operators represent continuous quantum logical observables.

The normal operators N have the following property.

$$N: \mathbf{Ħ} \Rightarrow \mathbf{Ħ} \tag{1}$$

N commutes with its **(Hermitian) adjoint** $N^\dagger$

$$N \cdot N^\dagger = N^\dagger \cdot N \tag{2}$$

Normal operators are important because the spectral theorem holds for them.

Examples of normal operators are

- **unitary** operators: $U^\dagger = U^{-1}$, unitary operators are bounded;
- **Hermitian** operators (i.e., self-adjoint operators): $N^\dagger = N$;
- **Anti-Hermitian** or anti-self-adjoint operators: $N^\dagger = -N$;
- **Anti-unitary** operators: $I^\dagger = -I = I^{-1}$, anti-unitary operators are bounded;
- **positive operators**: $N = MM^\dagger$
- **orthogonal projection** operators: $N = N^\dagger = N^2$

9.2.6.3 Spectral theorem

For every compact self-adjoint operator *T* on a real, complex or quaternionic Hilbert space **Ħ**, there exists an orthonormal basis of **Ħ** consisting of eigenvectors of T. More specifically, the orthogonal

complement of the kernel (null space) of T admits, either a finite orthonormal basis of eigenvectors of T, or a countable infinite orthonormal basis {en} of eigenvectors of T, with corresponding eigenvalues $\{\lambda_n\} \subset R$, such that $\lambda_n \rightarrow 0$. Due to the fact that **Ħ** is separable the set of eigenvectors of T can be extended with a base of the kernel in order to form a complete orthonormal base of **Ħ**.

If T is compact on an infinite dimensional Hilbert space **Ħ**, then T is not invertible, hence $\sigma(T)$, the spectrum of T, always contains 0. The spectral theorem shows that $\sigma(T)$ consists of the eigenvalues $\{\lambda_n\}$ of T, and of 0 (if 0 is not already an eigenvalue). The set $\sigma(T)$ is a compact subset of the real line, and the eigenvalues are dense in $\sigma(T)$.

A normal operator has a set of eigenvectors that spans the whole Hilbert space **Ħ**.

In quaternionic Hilbert space a normal operator has quaternions as eigenvalues.

The set of eigenvalues of a normal operator is NOT compact. This is due to the fact that **Ħ** is separable. Therefore the set of eigenvectors is countable. As a consequence the set of eigenvalues is countable. Further, in general the eigenspace of normal operators has no finite diameter.

A continuous bounded linear operator on **Ħ** has a compact eigenspace. The set of eigenvalues has a closure and it has a finite diameter.

9.2.6.4   Eigenspace

The set of eigenvalues $\{q\}$ of the operator Q form the eigenspace of Q

9.2.6.5 Eigenvectors and eigenvalues

For the eigenvector |q> of normal operator Q holds

$$|Q\ q> = |q\ q> = |q>\cdot q \tag{1}$$

$$<q\ Q^{\dagger}| = <q\ q^{*}| = q^{*}\cdot<q| \tag{2}$$

$$\forall_{|f> \in H}\ [\{< f|Q\ q >\}_q = \{< f|q > q\}_q = \{< q\ Q^{\dagger}|f >^{*}\}_q \tag{3}$$
$$= \{q^{*} < q|f >^{*}\}_q]$$

The eigenvalues of 2^n-on normal operator are 2^n-ons. For Hilbert spaces the eigenvalues are restricted to elements of a division ring.

$$Q = \sum_{j=0}^{n-1} I_j Q_i \tag{4}$$

The Q_j are self-adjoint operators.

9.2.6.6 Generalized Trotter formula

For bounded operators $\{A_j\}$ hold:

$$\lim_{n\to\infty}\left(\prod_{j=1}^{p} e^{A_j/n}\right)^n = \exp\left(\sum_{j=1}^{p} A_j\right) \tag{1}$$

$$= \lim_{n\to\infty}\left(1 + \frac{\sum_{j=1}^{p} A_j}{n}\right)^n$$

In general

$$\exp\left(\sum_{j=1}^{p} A_j\right) \neq \prod_{j=1}^{p} e^{A_j} \tag{2}$$

In the realm of quaternionic notion the Trotter formula is confusing.

9.2.6.7 Unitary operators

For unitary operators holds:

$$U^\dagger = U^{-1} \tag{1}$$

Thus

$$U \cdot U^\dagger = U^\dagger \cdot U = 1 \tag{2}$$

Suppose $U = I + C$ where U is unitary and C is compact. The equations $U\,U^* = U^*U = I$ and $C = U - I$ show that C is normal. The spectrum of C contains 0, and possibly, a finite set or a sequence

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tending to 0. Since $U = I + C$, the spectrum of U is obtained by shifting the spectrum of C by 1.

The unitary transform can be expressed as:

$$U = \exp(\underline{\tilde{\mathbf{I}}} \cdot \Phi / \hbar) \tag{3}$$

$$\hbar = h / (2 \cdot \pi) \tag{4}$$

Φ is Hermitian. The constant h refers to the granularity of the eigenspace.

Unitary operators have eigenvalues that are located in the unity sphere of the 2^n-ons field.

The eigenvalues have the form:

$$u = \exp(\underline{\mathbf{i}} \cdot \varphi / \hbar) \tag{5}$$

φ is real. $\underline{\mathbf{i}}$ is a unit length imaginary number in 2^n-on space. It represents a direction.

u spans a sphere in 2^n-on space. For constant $\underline{\mathbf{i}}$, u spans a circle in a complex subspace.

9.2.6.7.1 Polar decomposition

Normal operators N can be split into a real operator A and a unitary operator U. U and A have the same set of eigenvectors as N.

$$N = \|N\| \cdot U = A \cdot U \tag{1}$$

$$N = A \cdot U = U \cdot A \tag{2}$$

$$= A \cdot \exp(\underline{\tilde{\mathbf{I}}} \cdot \Phi) / \hbar)$$

$$= \exp{(\Phi_r + \underline{\tilde{\mathbf{I}}} \cdot \Phi)/\hbar}$$

Φ_r is a positive normal operator.

9.2.6.8   Ladder operator

9.2.6.8.1 General formulation

Suppose that two operators X and N have the com-mutation relation:

$$[N, X] = c \cdot X \tag{1}$$

for some scalar c. If |n> is an eigenstate of N with eigenvalue equation,

$$|N\,\text{n}> = |\text{n}> \cdot \text{n} \tag{2}$$

then the operator X acts on |n> in such a way as to shift the eigen-value by c:

$$
\begin{aligned}
|N \cdot X\,\text{n}> &= |(X \cdot N + [N, X])\,\text{n}> \\
&= |(X \cdot N + c \cdot X)\,\text{n}> \\
&= |X \cdot N\,\text{n}> + |X\,\text{n}> \cdot c \\
&= |X\,\text{n}> \cdot \text{n} + |X\,\text{n}> \cdot c \\
&= |X\,\text{n}> \cdot (\text{n}+c)
\end{aligned}
\tag{3}
$$

In other words, if |n> is an eigenstate of N with eigenvalue n then |X n> is an eigenstate of N with eigenvalue $n + c$.

The operator X is a *raising operator* for N if c is real and positive, and a *lowering operator* for N if c is real and negative.

If N is a Hermitian operator then c must be real and the Hermitian adjoint of X obeys the commutation relation:

$$[N, X^\dagger] = - c \cdot X^\dagger \tag{4}$$

In particular, if X is a lowering operator for N then $X^\dagger$ is a raising operator for N and vice-versa.

9.2.7   Unit sphere of Ħ

The ket vectors in **Ħ** that have their norm equal to one form together the **unit sphere** Θ of **Ħ**.

Base vectors are all member of the unit sphere. The eigenvectors of a normal operator are all member of the unit sphere.

The end points of the eigenvectors of a normal operator form a **grid** on the unit sphere Θ of **Ħ**.

9.2.8   Bra-ket in four dimensional space

The Bra-ket formulation can also be used in transformations of the four dimensional curved spaces.

The bra $\langle f$ is then a covariant vector and the ket $g \rangle$ is a contravariant vector. The inner product acts as a metric.

$$s = \langle f | g \rangle \tag{1}$$

The effect of a linear transformation L is then given by

$$s_L = \langle f | L g \rangle \tag{2}$$

The effect of a the transpose transformation $L^\dagger$ is then given by

$$\langle f L^\dagger \,|g\rangle = \langle f|Lg\rangle \tag{3}$$

For a unitary transformation U holds:

$$\langle Uf|Ug\rangle = \langle f|g\rangle \tag{4}$$

These definitions work for curved spaces with a Euclidian signature as well as for curved spaces with a Minkowski signature.

$$\langle \nabla f|\nabla g\rangle = \langle f|\nabla^2 g\rangle = \langle f|\Box g\rangle \tag{5}$$

9.2.9   Closure

The closure of $\mathbf{H}$ means that converging rows of vectors converge to a vector of $\mathbf{H}$.

In general converging rows of eigenvalues of Q do not converge to an eigenvalue of Q.

Thus, the set of eigenvalues of Q is open.

At best the density of the coverage of the set of eigenvalues is comparable with the set of 2^n-ons that have rational numbers as coordinate values.

With other words, compared to the set of real numbers the eigenvalue spectrum of Q has holes.

The set of eigenvalues of operator Q includes 0. This means that Q does not have an inverse.

The rigged Hilbert space $\mathbf{H}$ can offer a solution, but then the direct relation with quantum logic is lost.

9.2.10   Canonical conjugate operator P

The existence of a canonical conjugate represents a stronger requirement on the continuity of the eigenvalues of canonical eigenvalues.

Q has eigenvectors $\{|q>\}_q$ and eigenvalues q.

P has eigenvectors $\{|p>\}_p$ and eigenvalues p.

For each eigenvector |q> of Q we define an eigenvector |p> and eigenvalues p of P such that:

$$< q|p > = < p|q >^* = exp\,(\hat{\i} \cdot p \cdot q/\hbar) \tag{1}$$

$\hbar = h/(2\pi)$ is a scaling factor. $< q|p >$ is a quaternion. $\hat{\i}$ is a unit length imaginary quaternion.

9.2.11   Displacement generators

Variance of the scalar product gives:

$$\boldsymbol{i}\,\hbar\,\delta < q|p > = -p < q|p > \delta q \tag{1}$$

$$\boldsymbol{i}\,\hbar\,\delta < p|q > = -q < p|q > \delta p \tag{2}$$

In the rigged Hilbert space **H** the variance can be replaced by differentiation.

Partial differentiation of the function <q|p> gives:

$$\boldsymbol{i}\,\hbar\,\partial/\partial q_s < q|p > = -p_s < q|p > \tag{3}$$

$$\boldsymbol{i}\,\hbar\,\frac{\partial}{\partial p_s} < p|q > = -q_s < p|q > \tag{4}$$

9.3   Quaternionic L² space

The space of quaternionic measurable functions is a separable quaternionic Hilbert space. For example quaternionic probability amplitude distributions are measurable.[130]

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[130] http://en.wikipedia.org/wiki/Lp_space#Lp_spaces

This space is spanned by an orthonormal basis of quaternionic measurable functions. The shared affine-like versions of the parameter space of these functions is called **Palestra**[131]. When the Palestra is non-curved, then this base has a canonical conjugate, which is the quaternionic Fourier transform of the original base.

As soon as curvature of the Palestra arises, this relation is disturbed.

With other words: "In advance the Palestra has a virgin state."

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[131] The name Palestra is suggested by Henning Dekant's wive Sarah. It is a name from Greek antiquity. It is a public place for training or exercise in wrestling or athletics

10 **Gelfand triple**

The separable Hilbert space only supports countable orthonormal bases and countable eigenspaces. The rigged Hilbert space $\mathcal{H}$ that belongs to a separable Hilbert space $\mathcal{H}$ is a Gelfand triple. It supports non-countable orthonormal bases and continuum eigenspaces.

A rigged Hilbert space is a pair $(\mathcal{H}, \Phi)$ with $\mathcal{H}$ a Hilbert space, Φ a dense subspace, such that Φ is given a [topological vector space](#) structure for which the [inclusion map](#) i is continuous. Its name is not correct, because it is not a Hilbert space.

Identifying $\mathcal{H}$ with its dual space $\mathcal{H}^*$, the adjoint to i is the map

$$i^*: \mathcal{H} = \mathcal{H}^* \rightarrow \Phi^* \tag{1}$$

The duality pairing between Φ and Φ^* has to be compatible with the inner product on $\mathcal{H}$, in the sense that:

$$\langle u, v \rangle_{\Phi \times \Phi^*} = (u, v)_{\mathcal{H}} \tag{2}$$

whenever $u \in \Phi \subset \mathcal{H}$ and $v \in \mathcal{H} = \mathcal{H}^* \subset \Phi^*$.

The specific triple $(\Phi \subset \mathcal{H} \subset \Phi^*)$ is often named after the mathematician [Israel Gelfand](#)).

Note that even though Φ is isomorphic to Φ^* if Φ is a Hilbert space in its own right, this isomorphism is *not* the

same as the composition of the inclusion i with its adjoint i^*

$$i^*i\colon \Phi \subset \text{Ħ} = \text{Ħ}^* \to \Phi^* \tag{3}$$

10.1 Understanding the Gelfand triple

The Gelfand triple of a real separable Hilbert space can be understood via the enumeration model of the real separable Hilbert space. This enumeration is obtained by taking the set of eigenvectors of a normal operator that has rational numbers as its eigenvalues. Let the smallest enumeration value of the rational enumerators approach zero. Even when zero is reached, then still the set of enumerators is countable. Now add all limits of converging rows of rational enumerators to the enumeration set. After this operation the enumeration set has become a continuum and has the same cardinality as the set of the real numbers. This operation converts the Hilbert space into its Gelfand triple and it converts the normal operator in a new operator that has the real numbers as its eigenspace. It means that the orthonormal base of the Gelfand triple that is formed by the eigenvectors of the new normal operator has the cardinality of the real numbers. It also means that linear operators in this Gelfand triple have eigenspaces that are continuums and have the cardinality of the real numbers[132]. The same reasoning holds for complex number based Hilbert spaces and quaternionic Hilbert spaces and their respective Gelfand triples.

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[132] This story also applies to the complex and the quaternionic Hilbert spaces and their Gelfand triples.

11 Fourier transform

The Fourier transformation is a linear operator. This transform transfers functions to another parameter space. As a consequence the Fourier transform has no eigenvalues, but the Fourier transform knows functions that are invariant under Fourier transformation.

The Fourier transform cannot cope with functions that have curved parameter spaces. However, it is possible to reduce the parameter space to a domain in which the Fourier transform keeps acceptable accuracy. Another possibility is that the target function is flattened, such that its parameter space becomes flat.

The Fourier transform transfer a orthonormal set of base functions into a new a orthonormal set such that each member of the new set can be written as a linear combination of members of the old set such that none of the coefficients is zero. In fact all coefficients have the same norm.

The Fourier transform converts the nabla operator into an operator that does not differentiate but multiplies the converted function with a factor. That operator will be called a momentum operator.

The Fourier transform has an inverse. It turns the momentum operator into the nabla operator.

The Fourier transform converts convolution of two functions into the multiplication of the two functions and vice versa.

In order to simplify the discussion we restrict it to the case that the parameter spaces of the functions are not curved.

11.1 Fourier transform properties

11.1.1 Linearity

The Fourier transform is a linear operator

$$\mathcal{F}\big(g(q)\big) = \tilde{g}(p) \tag{1}$$

$$\mathcal{F}(a\,g(q) + b\,h(q)) = a\,\tilde{g}(p) + b\,\tilde{h}(p) \tag{2}$$

11.1.2 Differentiation

Fourier transformation converts differentiation into multiplication with the canonical conjugated coordinate.

$$g(q) = \nabla f(q) \tag{1}$$

$$\tilde{g}(p) = p\tilde{f}(p) \tag{2}$$

$$g(q) = \nabla f(q) = \nabla_0 f_0(q) \mp \langle \boldsymbol{\nabla}, \boldsymbol{f}(q)\rangle \pm \nabla_0 \boldsymbol{f}(q) \\ + \boldsymbol{\nabla} f_0(q) \pm \left(\pm \boldsymbol{\nabla} \times \boldsymbol{f}(q)\right) \tag{3}$$

$$\tilde{g}(k) = k\tilde{f}(k) = k_0\widetilde{f_0}(k) \mp \langle \mathbf{k}, \tilde{\boldsymbol{f}}(k)\rangle \pm k_0\tilde{\boldsymbol{f}}(k) \\ + \mathbf{k}\tilde{f}_0(k) \pm \left(\pm\mathbf{k} \times \tilde{\boldsymbol{f}}(k)\right) \tag{4}$$

For the imaginary parts holds:

$$\mathbf{g}(q) = \pm\nabla_0\boldsymbol{f}(q) + \boldsymbol{\nabla}f_0(q) \pm \left(\pm\boldsymbol{\nabla} \times \boldsymbol{f}(q)\right) \tag{5}$$

$$\tilde{\mathbf{g}}(k) = \pm k_0\tilde{\boldsymbol{f}}(k) + \mathbf{k}\tilde{f}_0(k) \pm \left(\pm\mathbf{k} \times \tilde{\boldsymbol{f}}(k)\right) \tag{6}$$

By using

$$\boldsymbol{\nabla} \times \boldsymbol{\nabla} f_0(q) = \mathbf{0} \tag{7}$$

and

$$\langle \boldsymbol{\nabla}, \boldsymbol{\nabla} \times \boldsymbol{f}(q) \rangle = 0 \tag{8}$$

It can be seen that for the static part ($\nabla_0 f(q) = 0$) holds:

$$\mathbf{g}(q) = \boldsymbol{\nabla} f_0(q) \pm \left(\pm \boldsymbol{\nabla} \times \boldsymbol{f}(q) \right) \tag{9}$$

$$\tilde{\mathbf{g}}(k) = \mathbf{k}\tilde{f}_0(k) \pm \left(\pm \mathbf{k} \times \tilde{\boldsymbol{f}}(k) \right) \tag{10}$$

11.1.3 Parseval's theorem

Parseval's theorem runs:

$$\int f^*(q) \cdot g(q) \cdot dV_q = \int \tilde{f}^*(p) \cdot \tilde{g}(p) \cdot dV_p \tag{1}$$

This leads to

$$\int |f(q)|^2 \cdot dV_q = \int \left| \tilde{f}(p) \right|^2 \cdot dV_p \tag{2}$$

11.1.4 Convolution

Through Fourier transformation a convolution changes into a simple product and vice versa.

$$\mathcal{F}(f(q) \circ g(q)) = \tilde{f}(p) \cdot \tilde{g}(p) \tag{1}$$

11.2 Helmholtz decomposition

The Helmholtz decomposition splits the **static** vector field $\boldsymbol{F}$ in a (transversal) divergence free part $\boldsymbol{F}_t$ and a (one dimensional longitudinal) rotation free part $\boldsymbol{F}_l$.

$$\boldsymbol{F} = \boldsymbol{F}_t + \boldsymbol{F}_l = \boldsymbol{\nabla} \times \boldsymbol{f} - \boldsymbol{\nabla} f_0 \tag{1}$$

Here f_0 is a scalar field and $\boldsymbol{f}$ is a vector field. In quaternionic terms f_0 and $\boldsymbol{f}$ are the real and the imaginary part of a quaternionic field f. $\boldsymbol{F}$ is an imaginary quaternionic distribution.

The significance of the terms "longitudinal" and "transversal" can be understood by computing the local three-dimensional Fourier transform of the vector field $\boldsymbol{F}$, which we call $\widetilde{\boldsymbol{F}}$. Next decompose this field, at each point $\boldsymbol{k}$, into two components, one of which points longitudinally, i.e. parallel to $\boldsymbol{k}$, the other of which points in the transverse direction, i.e. perpendicular to $\boldsymbol{k}$.

$$\widetilde{\boldsymbol{F}}(\boldsymbol{k}) = \widetilde{\boldsymbol{F}}_l(\boldsymbol{k}) + \widetilde{\boldsymbol{F}}_t(\boldsymbol{k}) \tag{2}$$

$$\langle \boldsymbol{k}, \widetilde{\boldsymbol{F}}_t(\boldsymbol{k}) \rangle = 0 \tag{3}$$

$$\boldsymbol{k} \times \widetilde{\boldsymbol{F}}_l(\boldsymbol{k}) = \boldsymbol{0} \tag{4}$$

The Fourier transform converts gradient into multiplication and vice versa. Due to these properties the inverse Fourier transform gives:

$$\boldsymbol{F} = \boldsymbol{F}_l + \boldsymbol{F}_t \tag{5}$$

$$\langle \boldsymbol{\nabla}, \boldsymbol{F}_t \rangle = 0 \tag{6}$$

$$\boldsymbol{\nabla} \times \boldsymbol{F}_l = \boldsymbol{0} \tag{7}$$

So, this split indeed conforms to the Helmholtz decomposition.

This interpretation relies on idealized circumstance in which the decomposition runs along straight lines. This idealized condition is

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not provided in a curved parameter space. In curved parameter space the decomposition and the interpretation via Fourier transformation only work locally and with reduced accuracy.

11.2.1 Quaternionic Fourier transform split

The longitudinal Fourier transform represents only part of the full quaternionic Fourier transform. It depends on the selection of a radial line $\boldsymbol{k}(q)$ in p space that under ideal conditions runs along a straight line.

$$\mathcal{F}_{\boldsymbol{k}}\big(g(q)\big) = \mathcal{F}(g(q), \boldsymbol{k}(q)) \tag{1}$$

Or

$$\mathcal{F}_{\parallel}\big(g(q)\big) \overset{\text{def}}{=} \mathcal{F}\left(g_{\parallel}(q)\right) \tag{2}$$

It relates to the full quaternionic Fourier transform $\mathcal{F}$

$$\mathcal{F}\big(g(q)\big) = \tilde{g}(p) \tag{3}$$

The inverse Fourier transform runs:

$$\mathcal{F}^{-1}(\tilde{g}(p)) = g(q) \tag{4}$$

The split in longitudinal and transverse Fourier transforms corresponds to a corresponding split in the multi-dimensional Dirac delta function.

11.3 Fourier integral

For the bra-ket inner product holds:

$$< q|\breve{P} f > = \hbar \cdot \nabla_q < q|f > = \hbar \cdot \nabla_q f^*(q) = g(q) \tag{1}$$

$$= \int_p < q|p > \cdot < p|g >$$

The static imaginary part is

$$< q|\bar{P} f > = \hbar \cdot \boldsymbol{\nabla}_q < q|f > = \hbar \cdot \boldsymbol{\nabla}_q \boldsymbol{f}^*(q) = \boldsymbol{g}(q) \tag{2}$$

$$= Im\left(\int_p < q|p > \cdot < p|\boldsymbol{g} >\right)$$

$$= \int_p Im(< q|p > \cdot < p|\boldsymbol{g} >)$$

$$= \int_p Im(< q|p > \cdot < p|\boldsymbol{g}_l >)$$

$$+ \int_p Im(< q|p > \cdot < p|\boldsymbol{g}_t$$
$$>)$$

$$= \int_p Im(< q|p > \cdot \tilde{\boldsymbol{g}}_l(p))$$

$$+ \int_p Im(< q|p > \cdot \tilde{\boldsymbol{g}}_t(p))$$

The left part is the longitudinal inverse Fourier transform of field $\widetilde{\boldsymbol{g}}(p)$.

The right part is the transverse inverse Fourier transform of field $\widetilde{\boldsymbol{g}}(p)$.

For the Fourier transform of $\mathbf{g}(q)$ holds the split:

$$
\widetilde{\boldsymbol{g}}(p) = \int\limits_{q} Im(<p|q>\cdot \boldsymbol{g_l}(q)) \tag{3}
$$
$$
+ \int\limits_{p} Im(<p|q>\cdot \boldsymbol{g_t}(q))
$$
$$
= \int\limits_{q} Im(<p|q>\cdot \boldsymbol{g}(q))
$$

The longitudinal direction is a one dimensional (radial) space. The corresponding transverse direction is tangent to a sphere in 3D. Its direction depends on the field $\mathbf{g}(q)$ or alternatively on the combination of field f and the selected (ideal) coordinate system $\check{Q}$.

For a weakly curved coordinate system $\breve{Q}$ the formulas hold with a restricted accuracy and within a restricted region.

11.3.1 Alternative formulation

The reference S. Thangavelu[133] provides an alternative specification of the multidimensional Fourier transform .

11.4 Functions invariant under Fourier transform
In this section we confine to a complex part of the Hilbert space. See http://en.wikipedia.org/wiki/Hermite_polynomials.

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[133] http://www.math.iitb.ac.in/atm/faha1/veluma.pdf

There exist two types of Hermite polynomials:

1. The probalist's Hermite polynomials:

$$H_n^{prob}(z) = (-1)^n \exp(\tfrac{1}{2}z^2)\, \frac{d^n}{dz^n} \exp(-\tfrac{1}{2}z^2).$$

2. The physicist's Hermite polynomials

$$H_n^{phys}(z) = (-1)^n \exp(z^2)\frac{d^n}{dx^n}\exp(-z^2)$$
$$= \exp(\tfrac{1}{2}z^2)\left(z - \frac{d}{dz}\right)\exp(-\tfrac{1}{2}z^2)$$

These two definitions are *not* exactly equivalent; either is a rescaling of the other:

$$H_n^{phys}(z) = 2^{n/2}\, H_n^{prob}(z\sqrt{2}) \tag{3}$$

In the following we focus on the physicist's Hermite polynomials.

The Gaussian function $\varphi(z)$ defined by

$$\varphi(x) = exp(-\pi\, z^2) \tag{4}$$

is an eigenfunction of F. It means that its Fourier transform has the same form.
As $\mathcal{F}^4 = I$ any λ in its spectrum $\sigma(\mathcal{F})$ satisfies $\lambda^4 = 1$: Hence,

$$\sigma(\mathcal{F}) = \{1; -1; i; -i\}. \tag{5}$$

We take the Fourier transform of the expansion:

$$exp(-\tfrac{1}{2} z^2 + 2 z c - c^2) \tag{6}$$

$$= \sum_{n=0}^{\infty} exp(-\tfrac{1}{2} z^2) H_n(z) c^n$$

$$/n!$$

First we take the Fourier transform of the left hand side:

$$\frac{1}{\sqrt{2\pi}} \int_{z=-\infty}^{\infty} exp(-\mathbf{k} z p_z) \, exp(-\tfrac{1}{2} z^2 + 2 z c - c^2) \, dz \tag{7}$$

$$= exp(-\tfrac{1}{2} p_z^2 - 2 \, \mathbf{k} \, p_z \, c + c^2)$$

$$= \sum_{n=0}^{\infty} exp(-\tfrac{1}{2} p_z^2) H_n(p_z) (-\mathbf{k} \, c)^n / n!$$

The Fourier transform of the right hand side is given by

$$\frac{1}{\sqrt{2\pi}} \sum_{n=0}^{\infty} \int_{z=-\infty}^{\infty} exp(-\mathbf{k} z p_z) \tag{8}$$

$$\cdot \, exp(-\tfrac{1}{2} z^2) H_n(z) c^n / n! \, dz$$

Equating like powers of c in the transformed versions of the left- and right-hand sides gives

$$\frac{1}{\sqrt{2\pi}} \int_{z=-\infty}^{\infty} \exp(-\boldsymbol{k}\, z\, p_z) \cdot$$
$$exp(-\tfrac{1}{2}\, z^2)\, H_n(z)\, c^n/n!\ dz \tag{9}$$

$$= (-\boldsymbol{k})^n$$
$$\cdot \exp(-\tfrac{1}{2}\, p_z^2)\, H_n(p_z)\, \frac{c^n}{n!}$$

Let us define the Hermite functions $\psi_n(z)$

$$\psi_n(z) \stackrel{\text{def}}{=} \ < z|\psi_n > = c_n \exp(-\tfrac{1}{2}\, z^2)\, H_n(z) \tag{10}$$

$$|\mathcal{F}\, \psi_n > = |\psi_n > (-\boldsymbol{k})^n \tag{11}$$

with suitably chosen c_n so as to make

$$\|\psi_n\|^2 \ = \ 1 \tag{12}$$

$$c_n = \frac{1}{\sqrt{2^n n!\, \sqrt{\pi}}} \tag{13}$$

The importance of the Hermite functions lie in the following theorem.

"The Hermite functions ψ_n; $n \in \mathsf{N}$ form an orthonormal basis for $L^2(\mathsf{R})$"

Consider the operator

$$H = -\tfrac{1}{2}\frac{d^2}{dz^2} + \tfrac{1}{2}\,z^2 \qquad (14)$$

Apply this to $\psi_n(z)$:

$$H \cdot \psi_n(z) = (\tfrac{1}{2} + n)\,\psi_n(z) \qquad (15)$$

Thus, ψ_n is an eigenfunction of H.

Let $f = \psi_{4k+j}$ be any of the Hermite functions. Then we have

$$\sum_{n=-\infty}^{\infty} f(y + n) \cdot \exp\left(-2\,\pi\,k\,x\,(y+n)\right) \qquad (16)$$

$$= (-k)^j \sum_{n=-\infty}^{\infty} f(x + n)\,\exp(2\,\pi\,k\,n\,y)$$

The vectors $|\psi_n\rangle$ are eigenvectors of the Fourier transform operator with eigenvalues $(-k)^n$. The eigenfunctions $\psi_n(x)$ represent eigenvectors $|\psi_n\rangle$ that span the complex Hilbert space $\mathbf{H}_k$.

For higher n the central parts of $\psi_n(x)$ and $|\psi_n(x)|^2$ become a sinusoidal form.
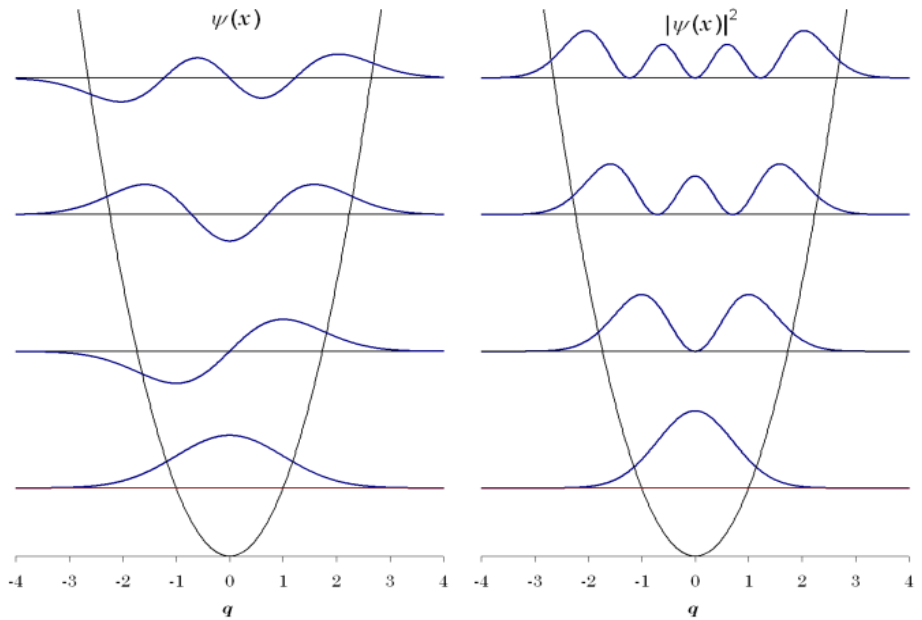
Figure 4

A **coherent state**[134] is a specific kind of state[135] of the quantum harmonic oscillator whose dynamics most closely resemble the oscillating behavior of a classical harmonic oscillator system. The ground state is a squeezed coherent state[136].

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[134] http://en.wikipedia.org/wiki/Coherent_state
[135] States
[136] Canonical conjugate: Heisenberg's uncertainty

11.5   Special Fourier transform pairs

Functions that keep the same form through Fourier transformation are:

$$f(q) = \exp(-|q|^2) \tag{1}$$

$$f(q) = \frac{1}{|q|} \tag{2}$$

$$f(q) = comb(q) \tag{3}$$

The comb function consists of a set of equidistant Dirac delta functions.

Other examples of functions that are invariant under Fourier transformation are the linear and spherical harmonic oscillators and the solutions of the Laplace equation.

11.6   Complex Fourier transform invariance properties

Each even function $f(q) \iff \tilde{f}(p)$ induces a Fourier invariant:

$$h(q) = \sqrt{2\pi}\, f(q) + \tilde{f}(q). \tag{1}$$

$$\tilde{h}(q) = \sqrt{2\pi}\, h(q) \tag{2}$$

Each odd function $f(q) \iff \tilde{f}(p)$ induces a Fourier invariant:

$$h(q) = \sqrt{2\pi}\, f(q) - \tilde{f}(q). \tag{3}$$

A function $f(q)$ is invariant under Fourier transformation *if and only if* the function f satisfies the differential equation

$$\frac{\partial^2 f(q)}{\partial q^2} - t^2 f(q) = \alpha\, f(q), \text{ for some scalar } \alpha \in C.$$ (4)

The Fourier transform invariant functions are fixed apart from a scale factor. That scale factor can be 1, *k*, -1 or –*k*. *k* is an imaginary base number in the longitudinal direction.

Fourier-invariant functions show iso-resolution, that is, $\Delta_p = \Delta_q$ in the Heisenberg's uncertainty relation.

For proves see: http://www2.ee.ufpe.br/codec/isoresolution_vf.pdf.

12 Quaternionic probability amplitude distributions

Continuous quaternionic distributions contain a scalar field in their real part and an associated vector field in their imaginary part. In a quaternionic probability amplitude distribution (QPAD), the scalar field can be interpreted as a distribution of the density of property carriers. The associated vector field can be interpreted as a distribution of the current density of these carriers. The squared modulus of the value of the QPAD can be interpreted as the probability density of the presence of the carrier of the charge at the location that is specified by the parameter. The charge can be any property of the carrier or it stands for the ensemble of the properties of the carrier. The QPAD inherits the sign flavor of the quaternionic distribution that defines the curvature of its parameter space.

If a QPAD is an E-type quaternionic distribution, then a continuous quaternionic function defines the curvature of the parameter space of the QPAD. The carriers can be interpreted as the function values of this allocation function. In this case the carriers are tiny patches of the parameter space of the QPAD. Their charge is formed by the discrete symmetry set (sign flavor) of the QPAD. This type of QPAD is suitable for application in quantum fluid dynamics.

If a QPAD is a D-type quaternionic distribution, then a continuous quaternionic function defines the curvature of the parameter space of the QPAD. The carriers can be interpreted as elements of a medium like a gas or a fluid. This type of QPAD is suitable for application in conventional fluid dynamics.

12.1 Potential functions

Each charge carrier corresponds to a potential function. In combination the charge carriers correspond to an integral potential. If the

charge carrier distribution is sufficiently localized, then the integral potential function approaches the form of the single carrier potential function.

12.2 Dynamic potential

If the charge carrier distribution is generated in a rate of one temporary element per progression step, then the potential of the single carriers is transmitted at that same rate. This transmission is performed by spherical waves that extend in the embedding continuum. The waves slightly fold the continuum. An integration of these effects over a series of progression steps will then show the static integral potential function.

12.3 Differential equation

For QPAD's the equation for the differential can be interpreted as a differential continuity equation. Another name for continuity equation is balance equation. The differential continuity equation is paired by an integral continuity equation. The differential equation runs:

$$g(q) = g_0(q) + \boldsymbol{g}(q) = \nabla f(q) \tag{1}$$

$$= \nabla_0 f_0(q) \mp \langle \boldsymbol{\nabla}, \boldsymbol{f}(q) \rangle$$

$$\pm \nabla_0 \boldsymbol{f}(q) + \boldsymbol{\nabla} f_0(q)$$
$$\pm \left(\pm \boldsymbol{\nabla} \right.$$
$$\left. \times \boldsymbol{f}(q) \right)$$

12.4 Continuity equation

Let us approach the balance equation from the integral variety of the balance equation.

When $\rho_0(q)$ is interpreted as a charge density distribution, then the conservation of the corresponding charge[137] is given by the continuity equation:

$$\text{Total change within } V = \text{flow into } V + \text{production in-} \tag{1}$$
side V

In formula this means:

$$\frac{d}{d\tau} \int_V \rho_0 \, dV = \oint_S \widehat{\boldsymbol{n}} \rho_0 \frac{\boldsymbol{v}}{c} \, dS + \int_V s_0 \, dV$$

$$\int_V \nabla_0 \rho_0 \, dV = \int_V \langle \boldsymbol{\nabla}, \boldsymbol{\rho} \rangle \, dV + \int_V s_0 \, dV \tag{3}$$

The conversion from formula (2) to formula (3) uses the Gauss theorem[138]. Here $\widehat{\boldsymbol{n}}$ is the normal vector pointing outward the surrounding surface S, $\boldsymbol{v}(\tau, \boldsymbol{q})$ is the velocity at which the charge density $\rho_0(\tau, \boldsymbol{q})$ enters volume V and s_0 is the source density inside V. In the above formula $\boldsymbol{\rho}$ stands for

$$\boldsymbol{\rho} = \rho_0 \boldsymbol{v}/c \tag{4}$$

It is the flux (flow per unit area and unit time) of ρ_0 .

The combination of $\rho_0(\tau, \boldsymbol{q})$ and $\boldsymbol{\rho}(\tau, \boldsymbol{q})$ is a quaternionic skew field $\rho(\tau, \boldsymbol{q})$ and can be seen as a probability amplitude distribution (QPAD).

$$\rho \stackrel{\text{def}}{=} \rho_0 + \boldsymbol{\rho} \tag{5}$$

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[137] Also see Noether's laws: http://en.wikipedia.org/wiki/Noether%27s_theorem
[138] http://en.wikipedia.org/wiki/Divergence_theorem

$\rho(\tau, \boldsymbol{q})\rho^*(\tau, \boldsymbol{q})$ can be seen as an overall probability density distribution of the presence of the carrier of the charge. $\rho_0(\tau, \boldsymbol{q})$ is a charge density distribution. $\boldsymbol{\rho}(\tau, \boldsymbol{q})$ is the current density distribution.

This results in the law of charge conservation:

$$s_0(\tau, \boldsymbol{q}) = \nabla_0 \rho_0(\tau, \boldsymbol{q})$$
$$\mp \langle \boldsymbol{\nabla}, (\rho_0(\tau, \boldsymbol{q})\boldsymbol{v}(\tau, \boldsymbol{q}) + \boldsymbol{\nabla} \times \boldsymbol{a}(\tau, \boldsymbol{q})) \rangle \tag{6}$$

$$= \nabla_0 \rho_0(\tau, \boldsymbol{q}) \mp \langle \boldsymbol{\nabla}, \boldsymbol{\rho}(\tau, \boldsymbol{q}) + \boldsymbol{A}(\tau, \boldsymbol{q}) \rangle$$

$$= \nabla_0 \rho_0(\tau, \boldsymbol{q}) \mp \langle \boldsymbol{v}(\tau, \boldsymbol{q}), \nabla \rho_0(\tau, \boldsymbol{q}) \rangle$$
$$\mp \langle \boldsymbol{\nabla}, \boldsymbol{v}(\tau, \boldsymbol{q}) \rangle \rho_0(\tau, \boldsymbol{q})$$

$$\mp \langle \boldsymbol{\nabla}, \boldsymbol{A}(\tau, \boldsymbol{q}) \rangle$$

The blue colored $\pm$ indicates quaternionic sign selection through conjugation of the field $\rho(\tau, \boldsymbol{q})$. The field $\boldsymbol{a}(\tau, \boldsymbol{q})$ is an arbitrary differentiable vector function.

$$\langle \boldsymbol{\nabla}, \boldsymbol{\nabla} \times \boldsymbol{a}(\tau, \boldsymbol{q}) \rangle = 0 \tag{7}$$

$\boldsymbol{A}(\tau, \boldsymbol{q}) \stackrel{\text{def}}{=} \boldsymbol{\nabla} \times \boldsymbol{a}(\tau, \boldsymbol{q})$ is always divergence free. In the following we will neglect $\boldsymbol{A}(\tau, \boldsymbol{q})$.

Equation (6) represents a balance equation for charge density. What this charge actually is, will be left in the middle. It can be one of the properties of the carrier or it can represent the full ensemble of the properties of the carrier.

Up to this point the investigation only treats the real part of the full equation. The full continuity equation runs:

$$s(\tau, \boldsymbol{q}) = \nabla \rho(\tau, \boldsymbol{q}) = s_0(\tau, \boldsymbol{q}) + \boldsymbol{s}(\tau, \boldsymbol{q}) \tag{8}$$

$$= \nabla_0 \rho_0(\tau, \boldsymbol{q}) \mp \langle \nabla, \boldsymbol{\rho}(\tau, \boldsymbol{q}) \rangle \pm \nabla_0 \boldsymbol{\rho}(\tau, \boldsymbol{q})$$
$$+ \nabla \rho_0(\tau, \boldsymbol{q})$$
$$\pm \left(\pm \nabla \times \boldsymbol{\rho}(\tau, \boldsymbol{q}) \right)$$

$$= \nabla_0 \rho_0(\tau, \boldsymbol{q}) \mp \langle \boldsymbol{v}(\tau, \boldsymbol{q}), \nabla \rho_0(\tau, \boldsymbol{q}) \rangle$$
$$\mp \langle \nabla, \boldsymbol{v}(\tau, \boldsymbol{q}) \rangle \rho_0(\tau, \boldsymbol{q})$$

$$\pm \nabla_0 \boldsymbol{v}(\tau, \boldsymbol{q}) + \nabla_0 \rho_0(\tau, \boldsymbol{q})$$
$$+ \nabla \rho_0(\tau, \boldsymbol{q})$$

$$\pm \left(\pm (\rho_0(\tau, \boldsymbol{q}) \nabla \times \boldsymbol{v}(\tau, \boldsymbol{q}) \right.$$
$$- \boldsymbol{v}(\tau, \boldsymbol{q})$$
$$\left. \times \nabla \rho_0(\tau, \boldsymbol{q}) \right)$$

$$s_0(\tau, \boldsymbol{q}) = 2\nabla_0 \rho_0(\tau, \boldsymbol{q}) \mp \langle \boldsymbol{v}(q), \nabla \rho_0(\tau, \boldsymbol{q}) \rangle \tag{9}$$
$$\mp \langle \nabla, \boldsymbol{v}(\tau, \boldsymbol{q}) \rangle \rho_0(\tau, \boldsymbol{q})$$

$$\boldsymbol{s}(\tau, \boldsymbol{q}) = \pm \nabla_0 \boldsymbol{v}(\tau, \boldsymbol{q}) \pm \nabla \rho_0(\tau, \boldsymbol{q}) \tag{10}$$

$$\pm \left(\pm (\rho_0(\tau, \boldsymbol{q}) \nabla \times \boldsymbol{v}(\tau, \boldsymbol{q}) - \boldsymbol{v}(\tau, \boldsymbol{q}) \right.$$
$$\left. \times \nabla \rho_0(\tau, \boldsymbol{q})) \right)$$

The red sign selection indicates a change of handedness by changing the sign of one of the imaginary base vectors. Conjugation also

causes a switch of handedness. It changes the sign of all three imaginary base vectors.

In its simplest form the full continuity equation runs:

$$s(\boldsymbol{q}, \tau) = \nabla \rho(\boldsymbol{q}, \tau)$$

Thus the full continuity equation specifies a quaternionic distribution s as a flat differential $\nabla \rho$.

When we go back to the integral balance equation, then holds for the imaginary parts:

$$\frac{d}{d\tau} \int_V \boldsymbol{\rho} \, dV = -\oint_S \hat{\boldsymbol{n}} \rho_0 \, dS - \oint_S \hat{\boldsymbol{n}} \times \boldsymbol{\rho} \, dS + \int_V \boldsymbol{s} \, dV \tag{4}$$

$$\int_V \nabla_0 \, \boldsymbol{\rho} \, dV = -\int_V \nabla \rho_0 \, dV - \int_V \nabla \times \boldsymbol{\rho} \, dV + \int_V \boldsymbol{s} \, dV \tag{5}$$

For the full integral equation holds:

$$\frac{d}{d\tau} \int_V \rho \, dV + \oint_S \hat{\boldsymbol{n}} \rho \, dS = \int_V s \, dV \tag{6}$$

$$\int_V \nabla \rho \, dV = \int_V s \, dV \tag{7}$$

Here $\hat{\boldsymbol{n}}$ is the normal vector pointing outward the surrounding surface S, $\boldsymbol{v}(\tau, \boldsymbol{q})$ is the velocity at which the charge density $\rho_0(\tau, \boldsymbol{q})$ enters volume V and s_0 is the source density inside V. In the above formula ρ stands for

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$$\rho = \rho_0 + \boldsymbol{\rho} = \rho_0 + \frac{\rho_0 \boldsymbol{v}}{c} \tag{8}$$

It is the flux (flow per unit of area and per unit of progression) of ρ_0. t stands for progression (not coordinate time).

12.5 Fluid dynamics

The quaternionic continuity equation is the foundation of quaternionic fluid dynamics. Depending on the nature of the streaming medium, this branch of physics exists in two forms.

- In conventional fluid dynamics the streaming charge carriers are elements of a gas or a liquid.

- In quantum fluid dynamics the streaming charge carriers are tiny patches of the parameter space of the QPAD. They correspond to the target values of an E-type quaternionic allocation function $\wp(x)$. This function has a flat parameter space that is spanned by the rational quaternions.

It means that in quantum fluid dynamics the coupling of QPAD's can affect the local curvature.

12.5.1 Coupling equation

In its simplest form the continuity equation runs:

$$\nabla\psi = \varphi$$

The continuity equation couples the local distribution ψ to a source φ.

The coupling strength can be made explicit. This results in the coupling equation.

$$\nabla\psi = m\,\phi$$

Here m is the coupling factor and ϕ is the adapted source.

13 Conservation laws

The following holds for all QPAD's!!!
Only the interpretation tells whether the QPAD concerns a quantum state function, a photon, a gluon or the field of a single charge, a field of a set of charges or a field corresponding to the density distribution of eventually moving charge carriers.

13.1 Differential potential equations

Let $\phi(q)$ define a quaternionic potential. The potential corresponds to a charge density distribution $\phi_0(q)$ and a current density distribution $\boldsymbol{\phi}(q)$.

Note: This means that the following holds for any QPAD!

$$\phi(q) = \rho_0(q) + \boldsymbol{\rho}(q) = \rho_0(q) + \rho_0(q)\boldsymbol{v}(q) \tag{1}$$

The gradient and curl of $\phi(q)$ are related. In configuration space holds:

$$\mathfrak{F}(q) \stackrel{\text{def}}{=} \nabla\phi(q) = \nabla_0\phi_0(q) \mp \langle \nabla, \boldsymbol{\phi}(q) \rangle \pm \nabla_0\boldsymbol{\phi}(q) \tag{2}$$
$$\pm \nabla\phi_0(q) \pm \left(\pm \nabla \times \boldsymbol{\phi}(q) \right)$$

$$\mathfrak{E}(q) \stackrel{\text{def}}{=} -\nabla\phi_0(q) \tag{3}$$

$$\mathfrak{B}(q) \stackrel{\text{def}}{=} \nabla \times \boldsymbol{\phi}(q) \tag{4}$$

$$\mathfrak{F}(q) \stackrel{\text{def}}{=} \nabla\phi(q) = \mathfrak{F}_0(q) + \mathfrak{F}(q) \tag{5}$$

$$\mathfrak{F}_0(q) = \nabla_0\phi_0(q) \mp \langle \nabla, \boldsymbol{\phi}(q) \rangle \tag{6}$$

$$\mathfrak{F}(q) = \mp\mathfrak{E}(q) \pm \mathfrak{B}(q) \pm \nabla_0\boldsymbol{\phi}(q) \tag{7}$$

Note: When the velocity $\boldsymbol{v}$ in $\boldsymbol{\phi}$ changes, then an extra term $\nabla_0\boldsymbol{\phi}(q)$ is added to equation (7).

13.1.1 Maxwell

In Maxwell equations, the electric field $\boldsymbol{E}(\boldsymbol{r}, t)$ is defined as:

$$E(r,t) \equiv -\nabla\phi_0(r,t) - \frac{\partial\phi(r,t)}{\partial t} = \mathfrak{E}(r,t) - \frac{\partial\phi(r,t)}{\partial t} \tag{1}$$

This is a remarkable decision, because $\dot{\boldsymbol{\phi}}$ can have components along $\mathfrak{E}$ and components along $\mathfrak{B}$, while $\mathfrak{E}$ and $\mathfrak{B}$ are mutually perpendicular.

Further:

$$\langle\nabla, E(r,t)\rangle = -\nabla^2\phi_0(r,t) - \frac{\partial\langle\nabla, \phi(r,t)\rangle}{\partial t} \tag{2}$$

$$= \frac{\rho_0(r,t)}{\varepsilon_0} - \frac{\partial\langle\nabla, \phi(r,t)\rangle}{\partial t}$$

In Maxwell equations, $\boldsymbol{B}(\boldsymbol{r})$ is defined as:

$$B(r,t) \equiv \nabla \times \phi(r,t) = \mathfrak{B}(r,t) \tag{3}$$

Further:

$$\nabla \times E(r,t) = -\frac{\partial B(r,t)}{\partial t} \tag{4}$$

$$\langle\nabla, B(r,t)\rangle = 0 \tag{5}$$

$$\nabla \times B(r, t) = \mu_0 (\rho + \varepsilon_0 \frac{\partial E}{\partial t}) \tag{6}$$

13.2 Gravity and electrostatics

Gravity and electrostatics can be treated by the same equations.

Description	Gravity	Electrostatics
Field	$g = -\nabla \varphi$	$E = -\nabla \varphi$
Force	$F = mg$	$F = QE$
Gauss law	$\langle \nabla, g \rangle = -4\pi G \rho$	$\langle \nabla, E \rangle = \dfrac{\rho}{\varepsilon}$
Poisson law $\Delta \varphi = \langle \nabla, \nabla \varphi \rangle$	$\Delta \varphi = 4\pi G \rho$	$\Delta \varphi = -\dfrac{\rho}{\varepsilon}$
Greens function	$\dfrac{-1}{\lvert r \rvert}$	$\dfrac{1}{\lvert r \rvert}$
Single charge potential	$\varphi = -\dfrac{4\pi G m}{\lvert r \rvert}$	$\varphi = \dfrac{Q}{4\pi\varepsilon \lvert r \rvert}$
Single charge field	$g = -\dfrac{4\pi G m}{\lvert r \rvert^2} r$	$E = \dfrac{Q}{4\pi\varepsilon \lvert r \rvert^2} r$
Two charge force	$F = -\dfrac{4\pi G m_1 m_2}{\lvert r \rvert^3} r$	$F = \dfrac{Q_1 Q_2}{4\pi\varepsilon \lvert r \rvert^3} r$
Mode	**attracting**	**repelling**

13.3 Flux vector

The longitudinal direction **k** of field $\mathfrak{E}(q)$ and the direction **i** of field $\mathfrak{B}(q)$ fix two mutual perpendicular directions. This generates curiosity to the significance of the direction **k** × **i.** With other words what happens with $\mathfrak{E}(q) \times \mathfrak{B}(q)$.

The **flux vector** $\mathfrak{S}(q)$ is defined as:

$$\mathfrak{S}(q) \stackrel{\text{def}}{=} \mathfrak{E}(q) \times \mathfrak{B}(q) \tag{1}$$

13.4   Conservation of energy

$$\langle \boldsymbol{\nabla}, \mathfrak{S}(q) \rangle = \langle \boldsymbol{\mathfrak{B}}(q), \boldsymbol{\nabla} \times \boldsymbol{\mathfrak{E}}(q) \rangle - \langle \boldsymbol{\mathfrak{E}}(q), \boldsymbol{\nabla} \times \boldsymbol{\mathfrak{B}}(q) \rangle \tag{1}$$

$$= -\langle \boldsymbol{\mathfrak{B}}(q), \nabla_0 \boldsymbol{\mathfrak{B}}(q) \rangle - \langle \boldsymbol{\mathfrak{E}}(q), \boldsymbol{\phi}(q) \rangle \\ - \langle \boldsymbol{\mathfrak{E}}(q), \nabla_0 \boldsymbol{\mathfrak{B}}(q) \rangle$$

$$= -\tfrac{1}{2}\nabla_0 (\langle \boldsymbol{\mathfrak{B}}(q), \boldsymbol{\mathfrak{B}}(q) \rangle + \langle \boldsymbol{\mathfrak{E}}(q), \boldsymbol{\mathfrak{E}}(q) \rangle) \\ - \langle \boldsymbol{\mathfrak{E}}(q), \boldsymbol{\phi}(q) \rangle$$

The **field energy density** is defined as:

$$u_{field}(q) = \tfrac{1}{2}(\langle \boldsymbol{\mathfrak{B}}(q), \boldsymbol{\mathfrak{B}}(q) \rangle + \langle \boldsymbol{\mathfrak{E}}(q), \boldsymbol{\mathfrak{E}}(q) \rangle) \tag{2}$$
$$= u_{\mathfrak{B}}(q) + u_{\mathfrak{E}}(q)$$

$\mathfrak{S}(q)$ can be interpreted as the **field energy current density**.
The continuity equation for field energy density is given by:

$$\nabla_0 u_{field}(q) + \langle \boldsymbol{\nabla}, \mathfrak{S}(q) \rangle = -\langle \boldsymbol{\mathfrak{E}}(q), \boldsymbol{\phi}(q) \rangle \tag{3}$$
$$= -\phi_0(q)\langle \boldsymbol{\mathfrak{E}}(q), \boldsymbol{v}(q) \rangle$$

This means that $\langle \boldsymbol{\mathfrak{E}}(q), \boldsymbol{\phi}(q) \rangle$ can be interpreted as a source term.

13.4.1   Interpretation in physics

Despite the fact that the above equations hold for any QPAD, we give here the physical interpretations when $\boldsymbol{\mathfrak{E}}$ is the electric field and $\boldsymbol{\mathfrak{B}}$ is the magnetic field.

$\phi_0(q)\boldsymbol{\mathfrak{E}}(q)$ represents **force** per unit volume.

$\phi_0(q)\langle \boldsymbol{\mathfrak{E}}(q), \boldsymbol{v}(q) \rangle$ represents **work** per unit volume, or, in other words, the power density. It is known as the Lorentz power

density and is equivalent to the time rate of change of the mechanical energy density of the charged particles that form the current $\boldsymbol{\phi}(q)$.

$$\nabla_0 u_{field}(q) + \langle \boldsymbol{\nabla}, \mathfrak{S}(q) \rangle = -\nabla_0 u_{mechanical}(q) \tag{1}$$

$$\nabla_0 u_{mechanical} = \langle \mathfrak{E}(q), \boldsymbol{\phi}(q) \rangle = \phi_0(q) \langle \mathfrak{E}(q), \boldsymbol{v}(q) \rangle \tag{2}$$

$$\nabla_0 \left(u_{field}(q) + u_{mechanical}(q) \right) = -\langle \boldsymbol{\nabla}, \mathfrak{S}(q) \rangle \tag{3}$$

Total change within V = flow into V + production inside V $\qquad(4)$

$$u(q) = u_{field}(q) + u_{mechanical}(q) \tag{5}$$
$$= u_B(q) + u_E(q) + u_{mechanical}(q)$$

$$U = U_{field} + U_{mechanical} = U_B + U_E + U_{mechanical} \tag{6}$$
$$= \int_V u \, dV$$

$$\frac{d}{dt} \int_V u \, dV = \oint_S \langle \hat{\boldsymbol{n}}, \mathfrak{S} \rangle dS + \int_V s_0 \, dV \tag{7}$$

Here the source s_0 is zero.

13.4.2   How to interpret $U_{mechanical}$

$U_{mechanical}$ is the energy of the private field (state function) of the involved particle(s).

13.5 Conservation of linear momentum

$\mathfrak{S}(q)$ can also be interpreted as the **field linear momentum density**. The time rate change of the field linear momentum density is:

$$\nabla_0 \mathfrak{S}(q) = \boldsymbol{g}_{field}(q) = \nabla_0\, \mathfrak{E}(q) \times \mathfrak{B}(q) + \mathfrak{E}(q) \times \nabla_0 \mathfrak{B}(q) \tag{1}$$

$$= \big(\nabla \times \mathfrak{B}(q) - \boldsymbol{\rho}(q)\big) \times \mathfrak{B}(q) - \mathfrak{E}(q) \times \nabla \times \mathfrak{E}(q) \tag{2}$$

$$\boldsymbol{G}(\mathfrak{E}) = \mathfrak{E} \times (\nabla \times \mathfrak{E}) = \langle \nabla\mathfrak{E}, \mathfrak{E} \rangle - \langle \mathfrak{E}, \mathfrak{E} \rangle \tag{3}$$
$$= \tfrac{1}{2}\nabla\langle \mathfrak{E}, \mathfrak{E} \rangle - \langle \mathfrak{E}, \mathfrak{E} \rangle$$

$$= -\nabla(\mathfrak{E}\mathfrak{E}) + \tfrac{1}{2}\nabla\langle \mathfrak{E}, \mathfrak{E} \rangle + \langle \nabla, \mathfrak{E} \rangle\mathfrak{E}$$

$$= -\nabla(\mathfrak{E}\mathfrak{E} + \tfrac{1}{2}\mathbf{1}_3\langle \mathfrak{E}, \mathfrak{E} \rangle) + \langle \nabla, \mathfrak{E} \rangle\mathfrak{E}$$

$$\boldsymbol{G}(\mathfrak{B}) = \mathfrak{B} \times (\nabla \times \mathfrak{B}) \tag{4}$$
$$= -\nabla(\mathfrak{B}\mathfrak{B} + \tfrac{1}{2}\mathbf{1}_3\langle \mathfrak{B}, \mathfrak{B} \rangle) + \langle \nabla, \mathfrak{B} \rangle\mathfrak{B}$$

$$\boldsymbol{H}(\mathfrak{B}) = -\nabla(\mathfrak{B}\mathfrak{B} + \tfrac{1}{2}\mathbf{1}_3\langle \mathfrak{B}, \mathfrak{B} \rangle) \tag{5}$$

$$\nabla_0 \mathfrak{S}(q) = \boldsymbol{G}(\mathfrak{B}) + \boldsymbol{G}(\mathfrak{E}) - \boldsymbol{\rho}(q) \times \mathfrak{B}(q) \tag{6}$$

$$= \boldsymbol{H}(\mathfrak{E}) + \boldsymbol{H}(\mathfrak{B}) - \boldsymbol{\rho}(q) \times \mathfrak{B}(q) + \langle \nabla, \mathfrak{B} \rangle\mathfrak{B} + \langle \nabla, \mathfrak{E} \rangle\mathfrak{E}$$

$$= \boldsymbol{H}(\mathfrak{E}) + \boldsymbol{H}(\mathfrak{B}) - \boldsymbol{\rho}(q) \times \mathfrak{B}(q) - \rho_0(q)\, \mathfrak{E}(q)$$

$$= \boldsymbol{H}(\mathfrak{E}) + \boldsymbol{H}(\mathfrak{E}) - \boldsymbol{f}(q) = \boldsymbol{\mathcal{T}}(q) - \boldsymbol{f}(q)$$

$\mathcal{T}(q)$ is the linear momentum flux tensor.

The linear momentum of the field contained in volume V surrounded by surface S is:

$$\boldsymbol{P}_{field} = \int_V \boldsymbol{g}_{field} \, dV \tag{7}$$

$$= \int_V \rho_0 \boldsymbol{\phi} \, dV$$

$$+ \int_V \langle \nabla\boldsymbol{\phi}, \mathfrak{E} \rangle \, dV + \oint_S \langle \hat{\boldsymbol{n}}, \mathfrak{E}\boldsymbol{A} \rangle dS$$

$$\boldsymbol{f}(q) = \boldsymbol{\rho}(q) \times \mathfrak{B}(q) + \rho_0(q) \, \mathfrak{E}(q) \tag{8}$$

Physically, $\boldsymbol{f}(q)$ is the Lorentz force density. It equals the time rate change of the mechanical linear momentum density $\boldsymbol{g}_{mechanical}$.

$$\boldsymbol{g}_{mechanical}(q) = \rho_{0m}(q)\boldsymbol{v}(q) \tag{9}$$

The force acted upon a single particle that is contained in a volume V is:

$$\boldsymbol{F} = \int_V \boldsymbol{f} \, dV = \int_V (\boldsymbol{\rho} \times \mathfrak{B} + \rho_0 \, \mathfrak{E}) \, dV \tag{10}$$

Brought together this gives:

$$\nabla_0 \left(\boldsymbol{g}_{field}(q) + \boldsymbol{g}_{mechanical}(q) \right) = -\langle \boldsymbol{\nabla}, \mathcal{T}(q) \rangle \tag{11}$$

This is the continuity equation for linear momentum.

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The component $\mathcal{T}_{ij}$ is the linear momentum in the i-th direction that passes a surface element in the j-th direction per unit time, per unit area.

$$\text{Total change within } V = \text{flow into } V + \text{production inside } V \tag{12}$$

$$\boldsymbol{g}(q) = \boldsymbol{g}_{field}(q) + \boldsymbol{g}_{mechanical}(q) \tag{13}$$

$$\boldsymbol{P} = \boldsymbol{P}_{field} + \boldsymbol{P}_{mechanical} = \int_V \boldsymbol{g}\, dV \tag{14}$$

$$\frac{d}{dt} \int_V \boldsymbol{g}\, dV = \oint_S \langle \hat{\boldsymbol{n}}, \boldsymbol{\mathcal{T}} \rangle dS + \int_V \boldsymbol{s}_g\, dV \tag{15}$$

Here the source $\boldsymbol{s}_g = 0$.

13.6   Conservation of angular momentum

13.6.1   *Field angular momentum*

The angular momentum relates to the linear momentum.

$$\boldsymbol{h}(\boldsymbol{q}_c) = (\boldsymbol{q} - \boldsymbol{q}_c) \times \boldsymbol{g}(q) \tag{1}$$

$$\boldsymbol{h}_{field}(\boldsymbol{q}_c) = (\boldsymbol{q} - \boldsymbol{q}_c) \times \boldsymbol{g}_{field}(q) \tag{2}$$

$$\boldsymbol{h}_{mechanical}(q) = (\boldsymbol{q} - \boldsymbol{q}_c) \times \boldsymbol{g}_{mechanical}(q) \tag{3}$$

$$\boldsymbol{\mathcal{K}}(\boldsymbol{q}_c) = (\boldsymbol{q} - \boldsymbol{q}_c) \times \boldsymbol{\mathcal{T}}(q) \tag{4}$$

This enables the balance equation for angular momentum:

$$\nabla_0 \left(\boldsymbol{h}_{field}(\boldsymbol{q}_c) + \boldsymbol{h}_{mechanical}(\boldsymbol{q}_c) \right) = -\langle \nabla, \mathcal{K}(\boldsymbol{q}_c) \rangle \tag{5}$$

Total change within V = flow into V + production inside V

$$\boldsymbol{J} = \boldsymbol{J}_{field} + \boldsymbol{J}_{mechanical} = \int_V \boldsymbol{h} \, dV \tag{6}$$

$$\frac{d}{dt} \int_V \boldsymbol{h} \, dV = \oint_S \langle \hat{\boldsymbol{n}}, \mathcal{K} \rangle dS + \int_V \boldsymbol{s}_{\boldsymbol{h}} \, dV \tag{7}$$

Here the source $\boldsymbol{s_h} = 0$.

For a localized charge density contained within a volume V holds for the mechanical torsion:

$$\tau(\boldsymbol{q}_c) = \int_V (\boldsymbol{q'} - \boldsymbol{q}_c) \times \boldsymbol{f}(q') dV \tag{8}$$

$$= \int_V (\boldsymbol{q'} - \boldsymbol{q}_c) \times \left(\rho_0(q') \mathfrak{E}(q') + \boldsymbol{j}(q') \right.$$
$$\left. \times \mathfrak{B}(q') \right) dV$$

$$= Q(\boldsymbol{q} - \boldsymbol{q}_c) \times (\mathfrak{E}(q) + \boldsymbol{v}(q) \times \mathfrak{B}(q))$$

$$\boldsymbol{J}_{field}(\boldsymbol{q}_c) = \boldsymbol{J}_{field}(\boldsymbol{0}) + \boldsymbol{q}_c \times \boldsymbol{P}(q) \tag{9}$$

Using

$$\langle \nabla a, b \rangle = n_\nu \frac{\partial a_\mu}{\partial q_\nu} b_\mu \tag{10}$$

$$\langle b, \nabla a \rangle = n_\mu \frac{\partial a_\mu}{\partial q_\nu} b_\mu \tag{11}$$

holds

$$\begin{aligned}
\mathbf{J}_{field}(\mathbf{0}) &= \int_V \mathbf{q}' \times \mathfrak{S}(q') dV \tag{12} \\
&= \int_V \mathbf{q}' \times \mathfrak{E}(q') \times \nabla \times \boldsymbol{\phi}(q') \, dV \\
&= \int_V (\mathbf{q}' \times \langle (\nabla \boldsymbol{\phi}), \mathfrak{E} \rangle - \langle \mathbf{q}' \times \mathfrak{E}, (\nabla \boldsymbol{\phi}) \rangle) \, dV \\
&= \int_V \mathbf{q}' \times \langle (\nabla \boldsymbol{\phi}), \mathfrak{E} \rangle dV \\
&\quad + \int_V \mathfrak{E} \times \boldsymbol{\phi} \, dV - \int_V \langle \nabla, \mathfrak{E} \mathbf{q}' \\
&\qquad\qquad \times \boldsymbol{\phi} \rangle dV \\
&\qquad\qquad + \int_V (\mathbf{q}' \\
&\qquad\qquad \times \boldsymbol{\phi}) \langle \nabla, \mathfrak{E} \rangle dV
\end{aligned}$$

13.6.2 Spin

Define the non-local spin term, *which does not depend on q′* as:

$$\boldsymbol{\Sigma}_{field} = \int_V \mathfrak{E}(q) \times \boldsymbol{\phi}(q) dV \tag{1}$$

Notice

$$\boldsymbol{\phi}(q) \times \boldsymbol{\nabla}\phi_0(q) = \phi_0 \boldsymbol{\nabla} \times \boldsymbol{\phi}(q) + \boldsymbol{\nabla} \times \big(\phi_0(q)\boldsymbol{\phi}(q)\big)$$

And

$$\boldsymbol{L}_{field}(\mathbf{0}) = \int_V \boldsymbol{q}' \times \langle (\boldsymbol{\nabla}\boldsymbol{\phi}), \mathfrak{E} \rangle dV + \int_V \boldsymbol{q}' \times \rho_0 \boldsymbol{\phi} dV \tag{2}$$

Using Gauss:

$$\int_V \langle \boldsymbol{\nabla}, \boldsymbol{a} \rangle dV = \oint_S \langle \widehat{\boldsymbol{n}}, \boldsymbol{a} \rangle dS \tag{3}$$

And

$$\rho_0 = \langle \boldsymbol{\nabla}, \mathfrak{E} \rangle \tag{4}$$

Leads to:

$$\boldsymbol{J}_{field}(\mathbf{0}) = \boldsymbol{\Sigma}_{field} + \boldsymbol{L}_{field}(\mathbf{0}) + \oint_S \langle \widehat{\boldsymbol{n}}, \mathfrak{E}\boldsymbol{q}' \times \boldsymbol{\phi} \rangle dS \tag{5}$$

13.6.3 Spin discussion

The spin term is defined by:

$$\boldsymbol{\Sigma}_{field} = \int_V \mathfrak{E}(q) \times \boldsymbol{\phi}(q) dV \tag{1}$$

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In free space the charge density ρ_0 vanishes and the scalar potential ϕ_0 shows no variance. Only the vector potential $\boldsymbol{\phi}$ may vary with q_0. Thus:

$$\mathfrak{E} = \boldsymbol{\nabla}\phi_0 - \nabla_0\boldsymbol{\phi} \approx -\nabla_0\boldsymbol{\phi} \tag{2}$$

$$\boldsymbol{\Sigma}_{field} \approx \int_V (\nabla_0\boldsymbol{\phi}(q)) \times \boldsymbol{\phi}(q)dV \tag{3}$$

Depending on the selected field $\boldsymbol{\Sigma}_{field}$ has two versions that differ in their sign. These versions can be combined in a single operator:

$$\boldsymbol{\Sigma}_{field} = \begin{bmatrix} \boldsymbol{\Sigma}^+_{field} \\ \boldsymbol{\Sigma}^-_{field} \end{bmatrix} \tag{4}$$

If $\frac{\boldsymbol{\phi}(q)}{|\boldsymbol{\phi}(q)|}$ can be interpreted as tantrix (q_0)) and $\frac{\nabla_0\boldsymbol{\phi}(q)}{|\nabla_0\boldsymbol{\phi}(q)|}$ can be interpreted as the principle normal $\boldsymbol{N}(q_0)$, then $\frac{(\nabla_0\boldsymbol{\phi}(q))\times\boldsymbol{\phi}(q)}{|(\nabla_0\boldsymbol{\phi}(q))\times\boldsymbol{\phi}(q)|}$ can be interpreted as the binormal $\mathfrak{B}(q_0)$.

From these quantities the curvature and the torsion[139] can be derived.

$$\begin{bmatrix} \dot{\boldsymbol{T}}(t) \\ \dot{\boldsymbol{N}}(t) \\ \dot{\boldsymbol{B}}(t) \end{bmatrix} = \begin{bmatrix} 0 & \kappa(t) & 0 \\ -\kappa(t) & 0 & \tau(t) \\ 0 & -\tau(t) & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{T}(t) \\ \boldsymbol{N}(t) \\ \boldsymbol{B}(t) \end{bmatrix} \tag{5}$$

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[139]Path characteristics