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## Proofs that theory of special relativity is false and my Diophantine equations solutions

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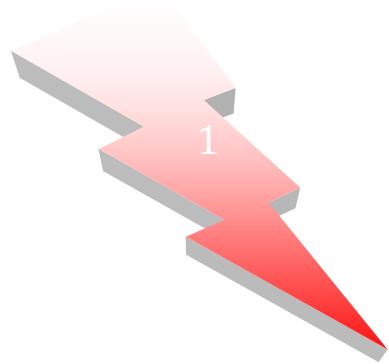
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## Abstract // Streszczenie

This article contains proofs that theory of special relativity is false and my Diophantine equations solutions. I am presenting this mathematical work mainly to attract attention to my proof that special relativity is false.

I have worked on diophantine solutions for more than two years. I can prove that my work is completely independent from the work of others and that two years ago I had solution to (as I call it) *general case* for solutions without little Fermat theorem and *simple case* with little Fermat theorem, which is much more than others achieved, but I didn't want to publish it until it would be complete. I sent it to the Polish professors of mathematics and to myself so I really can prove and document that I had it two years ago. I sent it for example on 10/26/2011 to polish full professor PhD. Edmund Puczyłowski (<http://www.mimuw.edu.pl/wydział/organizacja/pracownicy/edmund.puczyłowski.xml>) from University of Warsaw and I can prove it with my correspondence with him (I gave full content of this document that I sent to him in *Appendix 1*). I sent also some diophantine solutions (the simplest case with use of little Fermat theorem) to full professor PhD. Jerzy Tiuryn from University of Warsaw (<http://www.mimuw.edu.pl/wydział/organizacja/pracownicy/jerzy.tiuryn.xml>) on 02/23/2011 and I can prove it too.

I've searched the Internet and found very little work on this matter:

- 1.) Wolfram – nothing.
- 2.) Wikipedia: Fermat Last Theorem/Diophantine equations – single special case;
- 3.) <http://cp4space.files.wordpress.com/2012/10/moda-ch12.pdf> – that does not define all solutions

But what I've seen is that:

- 1.) There is given really very little solutions in comparison to my solutions,
- 2.) There are not all solutions of (as I call it) “*general*” or at least “*simple*” case of presented equations for the cases like for example:  $ua^x + wb^y = vc^z$
- 3.) There is not proof that presented solutions are all such (which I call “*complex not derived*”) solutions for any case, like for example:  $ua^x + wb^y = vc^z$ ,
- 4.) There is not proof when there exist such (*complex not derived*) solutions,
- 5.) There are not solutions for simultaneous equations,
- 6.) There are not solutions for rational exponents,
- 7.) As I know work of others contains only case of solution when

$$\sum_{i=1}^n \frac{c_i}{d} a_i^{x_i} = b^z = \left( \sum_{i=1}^n \frac{c_i}{d} l_i^{x_i} \right)^{t * lcm(x) + 1}$$

or even only  $\sum_{i=1}^n a_i^{x_i} = b^z = \left( \sum_{i=1}^n l_i^{x_i} \right)^{t * lcm(x) + 1}$

which is very little. And does not show how to solve equation without solving  $qz = t * lcm(x) + 1$ , so this algorithm to solve equation has not complexity  $O(1)$  while my has  $O(1)$ .

- 8.) There is no solution given for any case (especially for general case) to equations that has coefficient not equal to 1 on the right side.

Which all and much more I've done in this article.

If my Diophantine equation solutions are not enough I also give a inverse function to  $Li(n)$  function. I think it should be enough.

I named this kind of Diophantine equation that I've described in this article after my surname, because I need to refere to them in this article.

Finally I can present part of my work. Thanks for reading. I have more and I will publish it in my book that should come out next year.

**Please, give me an endorsement on arxiv (on physics, math), If you can. My username on arxiv: Zbigniew\_Plotnicki**

(and let me know at my e-mail address: [Zbigniew.Plotnicki.proofs@hotmail.com](mailto:Zbigniew.Plotnicki.proofs@hotmail.com))

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## Introduction // Wstęp

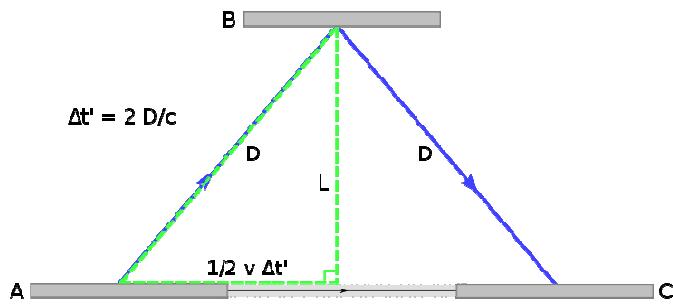
First of all special relativity applies to any point of inertial frame of reference (that has speed  $v$ ), and not only just to material objects that belong to it. It results from Lorentz transformation, because it concerns everything that has a speed.

// Szczególna teoria względności dotyczy dowolnego punktu inercjalnego układu odniesienia (poruszającego się z prędkością  $v$ ), a nie tylko materialnych obiektów, które do niego należą. Wynika to z transformacji Lorentza, ponieważ dotyczy ona wszystkiego co się porusza.

Firstly, proof that nothing can have speed equal to  $c$ .

The truth is that we can not divide by 0( $= \sqrt{1 - \left(\frac{c}{c}\right)^2}$ ), so we have for  $v = c$ :

// Prawdą jest, że nie możemy dzielić przez 0( $= \sqrt{1 - \left(\frac{c}{c}\right)^2}$ ), więc dla  $v = c$  mamy:



$$D^2 = \left(\frac{c\Delta t'}{2}\right)^2 + (L)^2$$

$$D = \sqrt{\left(\frac{c\Delta t'}{2}\right)^2 + (L)^2} = \sqrt{\left(\frac{c\Delta t'}{2}\right)^2 + \left(\frac{c\Delta t}{2}\right)^2}$$

$$\Delta t' = \frac{2D}{c} = \sqrt{\left(\frac{c\Delta t'}{c}\right)^2 + (\Delta t)^2}$$

$$(\Delta t')^2 = \left(\frac{c\Delta t'}{c}\right)^2 + (\Delta t)^2$$

$$(\Delta t')^2 \left(1 - \frac{c}{c}\right) = (\Delta t)^2 \Leftrightarrow \Delta t = 0$$

So ray of light is stuck in time and can not do nothing (for example can not move) and nothing can happen to it, because any activity takes place in time, however, it changes the position in time, so it have to have in moving frame infinite speed to move at any distance in zero time. But light has finite speed, so this is the proof that if special relativity is correct, then light never can have speed  $c$ , but this is the speed of light in vacuum, so there is no vacuum or special relativity is false.

// Więc promień światła jest zatrzymany w czasie i nie może nic zrobić (np. poruszać się) ani nic nie może się z nim działać, ponieważ każda czynność ma miejsce w czasie, jednak zmienia pozycję, więc musi mieć w poruszającym się układzie odniesienia nieskończoną prędkość, żeby przemieścić się w dowolne miejsce w zerowym czasie. Ale światło ma skończoną prędkość, więc jest to dowód, że STW jest fałszywa, więc jest to dowód, że jeśli STW jest prawidłowa, to światło nie może nigdy mieć prędkości  $c$ , ale to jest prędkość światła w próżni, więc albo STW jest fałszywa, albo prózna nie istnieje.

What is more it is impossible to have the same time interval equal zero at one frame and greater than zero in second frame, because there is no injective function from set of interval of real numbers to the set of single number, so uncountable set of events have not equivalents in this first frame. So in general there is impossible in special relativity to have speed of light in vacuum. So this is the second proof that, if special relativity is correct, then light never can have speed  $c$ , but light has this speed in vacuum, so there does not exist vacuum or special relativity is false.

// Co więcej niemożliwe jest, aby w jednym układzie odniesienia ten sam interwał czasu był zerowy, a w innym niezerowy, ponieważ nie istnieje różnowartościowa funkcja z rzeczywistego interwału na pojedynczą liczbę, dlatego nieprzeliczalna liczba zdarzeń nie ma swoich odpowiedników w pierwszym układzie odniesienia. Tak więc niemożliwe jest w STW, aby cokolwiek poruszało się z prędkością światła, jednak światło w próżni ma taką prędkość, więc prózna nie istnieje, albo STW jest fałszywe.

At the other hand there can not be found any inertial frame, unless you found absolute frame of reference or a speed of this absolute frame in frame that experiment take place, so to this moment theory of relativity, even if it were true, would be additionally completely useless and couldn't be applied nowhere.

// Z drugiej strony nie da się wyznaczyć inercjalnego układu odniesienia, aż nie wyznaczy się absolutnego układu odniesienia lub prędkości w tym układzie układu, w którym ma miejsce eksperyment. Tak więc STW, nawet gdyby była prawdziwa, do momentu znalezienia absolutnego układu byłaby całkowicie bezużyteczna i nie mogłaby być zastosowana nigdzie.

The Lorentz transformation is false for example for this reason that even if no object can achieve greater speed than the speed of light in relation to the background, then still frame of reference, which is only a vector of speed, can have any speed, making any object be able to reach any speed, and therefore the Lorentz transformation is simply false. It is based on the fact, that where there is no speed limit, because you can always add more meters, by moving the frame of reference, so the maximum speed is a plus infinity, there is introduced an artificial constraint, by introducing in place of the infinity constant  $c$ . It is known that only infinite speed, so such one that without the passage of time moves an object from any point to any other, does not depend on the frame of reference and you can't add any  $\frac{m}{s}$  to it, because the  $\Delta t = 0$ . For any finite speed, you can add any number of  $\frac{m}{s}$ , just by moving frame of reference at any high speed – no one can forbid it, what Lorentz and Einstein just did not understand. Alleged time dilation or length contraction is implicated from that – the assumption that infinite speed value is finite, and any relativistic effects follow this error.

// Transformacja Lorentza jest fałszywa choćby z tego powodu, że jeśli nawet żaden obiekt nie może osiągnąć względem swojego tła prędkości większej od prędkości światła, to układ odniesienia, który jest jedynie wektorem prędkości może mieć prędkość dowolną, sprawiając, że dowolny obiekt może

osiągnąć w nim dowolną prędkość, a więc transformacja Lorentza jest po prostu fałszywa. Bazuje ona na tym, że tam, gdzie nie ma ograniczenia – mowa o prędkości – bo zawsze można dodawać kolejne metry przesuwając odpowiednio układ odniesienia, czyli maksymalna prędkość jest plus nieskończona, wprowadzono sztuczne ograniczenie, wprowadzając na miejsce nieskończoności stałą  $c$ . Wiadomo, że tylko prędkość nieskończona, czyli taka, która bez upływu czasu przenosi obiekt z dowolnego punktu w dowolny, nie zależy od układu odniesienia i nie można do niej dodać już  $\frac{m}{s}$ , ponieważ  $\Delta t = 0$ . Do dowolnej skończonej prędkości można dodać dowolną liczbę  $\frac{m}{s}$  po prostu poruszając z dowolnie dużą prędkością układ odniesienia – nikt tego nigdy nikomu nie może zabronić, czego Lorentz i Einstein po prostu nie rozumiał. Stąd też wynika rzekoma dylatacja, czy kontrakcja, to jest z przyjęcia, że nieskończona prędkość ma wartość skończoną  $c$ , i wszelkie wzory relatywistyczne są jedynie skutkiem tego błędu.

Special relativity follows a very naive idea to allow variability in time to bend everything to the assumption that the speed of light does not depend on the frame of reference and, therefore, leads to so absurd proposals like this one, that the speed of frame of reference may not exceed the speed of light, even when such movement is only theoretical.

// Teoria relatywistyczna wynika z bardzo naiwnego pomysłu, aby dopuścić zmienność czasu, żeby nagiąć wszystko do założenia, że prędkość światła nie zależy od układu odniesienia, dlatego prowadzi do tak absurdalnych wniosków, jak ten, że prędkość układu odniesienia nie może przekroczyć prędkości światła, nawet podczas gdy ruch takiego układu jest jedynie teoretyczny.

Anyway, special relativity assumes something that it does not prove: that there is no length contraction across the direction of movement, and so simply is not proved.

// Tak w ogóle STW zakłada coś, czego nie udowodnia: że nie ma kontrakcji w poprzek ruchu, a więc zwyczajnie nie jest udowodniona.

The truth about time is that time cannot slow down, because this is an action, and so would have to slow down in time, and that time could not slow down in itself, so it must slow down at different time, so the same point would have had two conflicting times, which is impossible. If time slowed down in itself, there would not be any point of reference for this change, and so if hypothetically assuming that hour last longer, then there would be no way to determine or specify, or do it, and therefore it would not be able to last longer. This means that in contrary to appearances, there is no speed of time, which could increase or decrease, but time has its immutable natural course. Time has not tempo because tempo is measured in time, and time is not in another time, because time is the only one, because two different times are always in conflict with each other. The only measure for a time is a period, and the speed is always a measure of something divided by the period, hence there is no speed of time. It is interesting to know that the slowdown of tempo of everything does not mean that time slowed down. And it is not true that if time slows it necessarily slow down every phenomenon, or that if the every phenomenon slow down it is necessarily the slowdown of the time.

// Prawda o czasie jest natomiast taka, że czas nie może zwolnić, bo zwolnienie to czynność, a więc musiałby zwolnić w czasie, a że czas nie może zwolnić sam w sobie, a więc musi zwolnić w innym czasie, więc w tym samym punkcie obowiązywałby dwa sprzeczne czasy, co jest niemożliwe. Gdyby czas miał zwolnić sam w sobie, to nie byłoby żadnego punktu odniesienia dla tej zmiany, a więc gdyby hipotetycznie zakładając godzina miała trwać dłużej, to nie byłoby sposobu, aby to stwierdzić, ani

określić, ani wykonać, a zatem nie mogła by trwać dłużej. Oznacza to, że czas wbrew pozorom nie ma żadnej prędkości, którą można by zwiększać, czy zmniejszać, ale ma swój niezmienny naturalny bieg. Czas nie ma tempa, bo tempo mierzone jest w czasie, a czas nie jest w innym czasie, bo czas jest tylko jeden, bo dwa różne czasy są zawsze ze sobą sprzeczne. Jedyna miara dotycząca czasu to okres, a że prędkość to zawsze miara czegoś podzielona przez okres, stąd czas nie ma żadnej prędkości. Warto wiedzieć, że zwolnienie tempa zachodzenia zjawisk nie jest równoważne ze zmianą tempa upływu czasu. I nie jest prawdą ani, że jeśli czas zwalnia to koniecznie zwalniają zjawiska, ani, że jeśli zwalniają zjawiska to koniecznie czas zwalnia.

Of course, it is possible that the speed of light in the “unmoving field” is the same in every direction and is like an impulse derived from the point regardless of the direction of movement of this point, and perhaps the gravity holds this ray using its fields. However, it's easy to see that if the speed will be the same in all directions within e.g.: gravitational field of the Earth, it will be different in the gravity of the solar system, even more in the galaxies, clusters of galaxies, etc.

// Oczywiście możliwe, że prędkość światła w danym „nieruchomym polu” jest taka sama w każdym kierunku i jest jakby impulsem wyprowadzonym z punktu niezależnie od kierunku ruchu tego punktu, i być może grawitacja przytrzymuje za pomocą swego pola ten promień. Jednak łatwo zauważyć, że jeśli prędkość będzie taka sama we wszystkich kierunkach w obrębie np.: pola grawitacyjnego Ziemi, to będzie inna w polu grawitacyjnym układu słonecznego, jeszcze bardziej w polu galaktyki, gromady galaktyk, itd..

So both special relativity postulates are wrong.

1. First of all, for every movement there is always mass field related “main frame” of reference – real background of movement that is the resultant in given point of gravitational interactions of all masses. It is possible that there exist absolute frame (exactly one), that mean that every point of this field has absolute zero speed, but every other “main frame” is not absolute. So every point of space has its own “main frame”, and every points close to each other has a slightly different “main frame”, because has slightly different position to the all masses.
2. Second: speed of light depends on the frame of reference and is constant in all directions only in this “main frame”, since the “main frame” gravitational field keeps the waves of light, because if mass rotates for example mass of galaxy, then its “main frame” also rotates, because everything in reach of this mass is “kept” by this mass.
3. Speed of any object depends of the frame of reference.

// Tak więc obydwa postulaty szczególnej teorii względności są błędne.

1. Po pierwsze istnieje zawsze wyróżniony układ związany z polem masy, który jest wypadkową w danym punkcie oddziaływań wszystkich mas. Możliwe, że istnieje absolutny układ odniesienia (dokładnie jeden), ale każdy inny wyróżniony układ odniesienia nie jest absolutny. Tak więc każdy punkt ma swój wyróżniony układ, i wszystkie bliskie sobie punkty mają trochę inny wyróżniony układ odniesienia, ponieważ mają trochę inną pozycję względem wszystkich mas.
2. Po drugie prędkość światła zależy od układu odniesienia i w wyróżnionym układzie jest stała we wszystkich kierunkach, gdyż w nim pole masy utrzymuje fale światła, ponieważ jeśli na



przykład masa galaktyki rotuje wtedy wyróżniony układ także rotuje, ponieważ wszystko w zasięgu tej masy jest „trzymane” przez masę.

3. Prędkość dowolnego obiektu zawsze zależy od układu odniesienia.

Gravity is the result of accumulation of an ordinary influence of atomic interactions in a larger distance. Therefore, the more atoms the greater the mass. A similar accumulation on larger gravitational distances can make the galaxies be more attracted than it was due to the sum of their masses, and that is the fifth interaction (super-gravity). It is probably infinitely many degrees of such impact, if there are infinitely many degrees of clusters (body, galaxy, galaxies cluster, etc.). And successive derivations of mass interactions holds the light in their field.

// Grawitacja jest rezultatem nawarstwienia zwykłego oddziaływanie atomowego na większej odległości. Dlatego im więcej atomów tym większa masa. Podobne nawarstwienie oddziaływanie grawitacyjnego na większych odległościach może sprawiać, że galaktyki przyciągają się bardziej niż by to wynikało z sumy ich mas – jest to piąte oddziaływanie (supergrawitacja). Prawdopodobnie jest nieskończoność dużo stopni takiego oddziaływanie, jeśli jest nieskończoność dużo stopni gromad (ciało, galaktyka, gromada, itd.). I to właśnie kolejne pochodne oddziaływanie mas trzymają światło w swoim polu.

## How did Einstein prove that $\mathbf{1} = \mathbf{0}$

Consider two objects, one moves at the speed of  $\frac{1}{4}c$ , and the second at a speed of  $\frac{1}{2}c$ .

// Rozważmy dwa obiekty, jeden porusza się z prędkością  $\frac{1}{4}c$ , a drugi z prędkością  $\frac{1}{2}c$ .

So we have three times:  $t_0, t_1, t_2$ , and:

// Mamy zatem trzy upływy czasu:  $t_0, t_1, t_2$ , i:

$$t_0 = \frac{t_1}{\sqrt{1 - \frac{(\frac{1}{4}c)^2}{c^2}}} = \frac{4}{\sqrt{15}} t_1$$

$$t_0 = \frac{t_2}{\sqrt{1 - \frac{(\frac{1}{2}c)^2}{c^2}}} = \frac{2}{\sqrt{3}} t_2$$

$$\frac{4}{\sqrt{15}} t_1 = \frac{2}{\sqrt{3}} t_2 \Leftrightarrow t_1 = \frac{\sqrt{5}}{2} t_2$$

Now let's look at it from the perspective of the first object considering the formula for the composition of velocities:

// Teraz popatrzmy na to z perspektywy pierwszego obiektu uwzględniając wzór na kompozycję prędkości:

$$v' = \frac{\left(\frac{1}{2}c - \frac{1}{4}c\right)}{1 - \frac{\frac{1}{2}c * \frac{1}{4}c}{c^2}} = \frac{1}{4}c * \frac{8}{7} = \frac{2}{7}c$$

$$t_1 = \frac{t_2}{\sqrt{1 - \frac{(\frac{2}{7}c)^2}{c^2}}} = \frac{t_2}{\sqrt{1 - \frac{4}{49}}} = \frac{t_2}{\sqrt{\frac{45}{49}}} = \frac{t_2}{\frac{3\sqrt{5}}{7}} = \frac{7\sqrt{5}}{15} t_2 = \frac{\sqrt{5}}{2} t_2 \Leftrightarrow 14 = 15 \Leftrightarrow \mathbf{0} = \mathbf{1}$$

And not considering this formula:

// I nie uwzględniając tego wzoru:

$$t_1 = \frac{t_2}{\sqrt{1 - \frac{(\frac{1}{2}c - \frac{1}{4}c)^2}{c^2}}} = \frac{4}{\sqrt{15}} t_2 = \frac{\sqrt{5}}{2} t_2 \Leftrightarrow 8 = 5\sqrt{3} \Leftrightarrow 64 = 75 \Leftrightarrow 0 = 11 \Leftrightarrow \mathbf{0} = \mathbf{1}$$

**QED.**

This is not needed to prove that special relativity is false, but if you want to look at it more generally, here it is:

Consider two objects, one moves at the speed of  $v_1$ , and the second at a speed of  $v_2$ .

// Rozważmy dwa obiekty, jeden porusza się z prędkością  $v_1$ , a drugi z prędkością  $v_2$ .

So we have three times:  $t_0, t_1, t_2$ , and:

// Mamy zatem trzy upływy czasu:  $t_0, t_1, t_2$ , i:

$$t_0 = \frac{t_1}{\sqrt{1 - \frac{v_1^2}{c^2}}}$$

$$t_0 = \frac{t_2}{\sqrt{1 - \frac{v_2^2}{c^2}}}$$

$$\frac{t_1}{\sqrt{1 - \frac{v_1^2}{c^2}}} = \frac{t_2}{\sqrt{1 - \frac{v_2^2}{c^2}}} \Leftrightarrow t_1 = t_2 \sqrt{\frac{\left(1 - \frac{v_1^2}{c^2}\right)}{\left(1 - \frac{v_2^2}{c^2}\right)}}$$

Now let's look at it from the perspective of the first object considering the formula for the composition of velocities:

// Teraz spójrzmy na to z perspektywy pierwszego obiektu, uwzględniając wzór na kompozycję prędkości:

$$v' = \frac{v_2 - v_1}{1 - \frac{v_1 v_2}{c^2}}$$

$$t_1 = \frac{t_2}{\sqrt{1 - \frac{\left(\frac{v_2 - v_1}{1 - \frac{v_1 v_2}{c^2}}\right)^2}{c^2}}} = t_2 \sqrt{\frac{\left(1 - \frac{v_1^2}{c^2}\right)}{\left(1 - \frac{v_2^2}{c^2}\right)}}$$

$$1 - \frac{\left(\frac{v_2 - v_1}{1 - \frac{v_1 v_2}{c^2}}\right)^2}{c^2} = \frac{\left(1 - \frac{v_2^2}{c^2}\right)}{\left(1 - \frac{v_1^2}{c^2}\right)} = \frac{c^2 - v_2^2}{c^2 - v_1^2}$$

$$\frac{\left(\frac{v_2 - v_1}{1 - \frac{v_1 v_2}{c^2}}\right)^2}{c^2} = \frac{v_2^2 - v_1^2}{c^2 - v_1^2} = \frac{(v_2 - v_1)(v_2 + v_1)}{c^2 - v_1^2}$$

$$\frac{\frac{v_2 - v_1}{\left(1 - \frac{v_1 v_2}{c^2}\right)^2}}{c^2} = \frac{v_2 + v_1}{c^2 - v_1^2}$$

$$\frac{c^2(v_2 + v_1)\left(1 - \frac{v_1 v_2}{c^2}\right)}{c^2 - v_1^2} = \frac{v_2 - v_1}{1 - \frac{v_1 v_2}{c^2}} = v'$$

$$\frac{(v_2 + v_1)(c^2 - v_1 v_2)}{c^2 - v_1^2} = \frac{c^2(v_2 - v_1)}{c^2 - v_1 v_2} = v'$$

$$(v_2 + v_1)(c^2 - v_1 v_2)^2 - c^2(v_2 - v_1)(c^2 - v_1^2) = 0$$

As the polynomial of degree 3 it has at most 3 solutions, so this equation in general is of course false.

// Jak wielomian stopnia 3 to równanie ma conajwyżej 3 rozwiązania, więc ogólnie jest fałszywe.

Now we are sure that composition of velocities or time dilation is incorrect, but they are both implicated by the same assumption that speed of light is constant, so this assumption is false.

// Teraz mamy pewność że wzór na kompozycję prędkości lub dylatację czasu jest nieprawidłowy, ale oba implikowane są przez założenie, że prędkość światła nie zależy od układu odniesienia, a więc to założenie jest fałszywe.

**QED.**

Confused?

// Zmieszany?

So let's calculate time dilation for two points moving with the same speed in opposite directions. They have of course the same dilation of time ( $\Delta t'_1 = \Delta t'_2$ ), but let's see it from the perspective of one of them:

// Więc policzmy dylatację czasu dla dwóch punktów poruszających się z tą samą prędkością w tym samym kierunku, ale o przeciwnym zwrocie. Mają oczywiście tą samą dylatację ( $\Delta t'_1 = \Delta t'_2$ ), ale spójrzmy na to z perspektywy dowolnego z nich:

$$v' = \frac{v + v}{1 + \frac{v^2}{c^2}} = 2 \frac{v}{1 + \left(\frac{v}{c}\right)^2} > 0$$

$$\Delta t'_2 = \frac{\Delta t'_1}{\sqrt{1 - \left(\frac{v'}{c}\right)^2}} > \Delta t'_1$$

What does it mean? That means that special relativity implicates that **every frame of reference has infinitely many tempos of time**. Yes, it is not a mistake. Every frame has so many tempos of time how many other frames of reference there exist (infinitely many), because for each such frame this frame has other time dilation. This is of course impossible, because in the same frame time can not run with even two different tempos – they simply would contradict each other.

// Co to onacza? Oznacza to, że STW implikuje, że każdy układ odniesienia ma niskoźczenie wiele temp upływu czasu. Tak, to nie jest pomyłka. Każdy układ odniesienia ma tyle temp czasu ile istnieje różnych od niego układów odniesienia (niskoźczenie wiele), ponieważ dla każdego takiego układu odniesienia ten układ ma inną dylatację czasu. To jest oczywiście niemożliwe, ponieważ w tym samym układzie odniesienia czas nie może biec w nawet dwóch różnych tempach – byłby one po prostu w sprzeczności ze sobą.

**QED.**

Can't you believe yet that so famous and acknowledged, hundred years old theory can be so not wise?  
Yeap, so read so much more... 😊

// Nie możesz jeszcze uwierzyć, że tak sława, uznana, stuletnia teoria może być tak niemądra? Taak, więc czytaj dalej dużo więcej...

When you'll be more initiated then you will understand perfectly well that Einstein simply put constant speed  $c$  (the speed of light in a vacuum) in place of the infinite speed and bend (literally) all the rest to fit the assumption that infinite speed is equal to the speed of light, because only the infinite speed, what should know any graduate, does not depend on the speed of the frame of reference, because it moves any object from any place to any place in zero time.

// Jak już będziesz bardziej wtajemniczony zrozumiesz doskonale, że Einstein po prostu wstawił w miejsce nieskończonej prędkości stałą  $c$  (prędkość światła w próżni) i nagiął (dosłownie) całą resztę, żeby pasowało do założenia, że nieskończona prędkość równa jest prędkością światła, bo tylko nieskończona prędkość, o czym powinien wiedzieć każdy maturzysta, nie zależy od prędkości układu odniesienia, bo przenosi dowolny obiekt z dowolnego miejsca w dowolne miejsce w zerowym czasie.

**Proofs that theory of special relativity is false (almost every paragraph in two languages: English//Polish)**

PACS 2010 index number:

1. Lorentz transformation, 03.30.+p
2. Relativity: special relativity, 03.30.+p
3. Relativity: classical, 04.20.-q

## Closed chain

Imagine that we have polygon of which every edge is a path that some point traveled with a speed proportional to the proportion of length of this edge to the length of the shortest edge in this polygon. Next edge to every edge is moving with endpoint of this edge according to direction of its movement. So polygon is simply scaled from size zero to some size with speed of movement of the endpoint of smallest edge.

This could be any polygon, for example square.

Note that in frame of every endpoint of every edge next endpoint of next edge is moving with certain speed inertially, because edge of this endpoint is moving with frame of previous endpoint of previous edge. And every endpoint moves inertially in regard to the start point of the smallest edge. So, according to the special relativity, for every edge this is true, that time in frame of next edge slows down. But as this is closed chain of edges, so this is true that time in frame of every endpoint of every edge runs slower than it runs. And that is impossible.

// Proszę sobie wyobrazić wielokąt, którego każda krawędź to ścieżka, wzdłuż której pewien punkt podróże z prędkością proporcjonalną do proporcji długości tej krawędzi do długości najkrótszej krawędzi w tym wielokącie. Każda następna krawędź porusza się z tym punktem (końcowym) poprzedniej krawędzi wzdłuż kierunku jaki wyznacza ta poprzednia krawędź. Tak więc wielokąt skaluje się od rozmiaru zerwego z prędkością punktu końcowego najkrótszej krawędzi.

To może być dowolny zamknięty wielokąt, np.: kwadrat.

Proszę zauważyć, że w układzie związanym z punktem końcowym każdej krawędzi punkt końcowy następnej krawędzi porusza się inercjalnie z określona prędkością, bo ta następna krawędź porusza się wraz z punktem końcowym poprzedniej krawędzi. I każdy punkt końcowy porusza się inercjalnie względem początku najkrótszej krawędzi. Więc dla każdej krawędzi jest prawdą, że czas w układzie związanym z następną krawędzią zwalnia. Ale to jest zamknięty wielokąt, więc wychodzi na to, że każdy taki punkt porusza się wolniej od samego siebie. To jest niemożliwe.

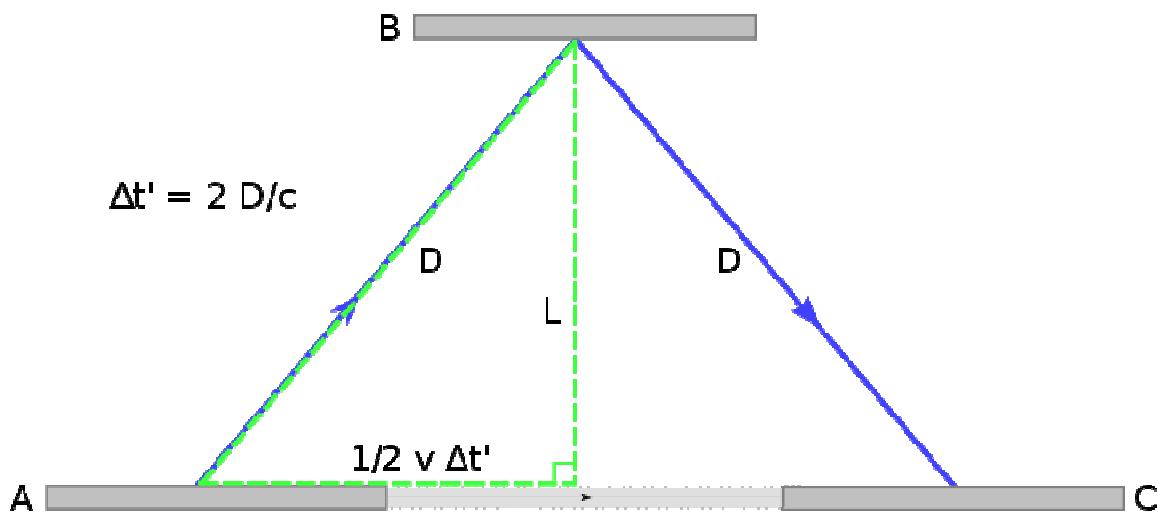
Let's look. As we have inertial frame of endpoint of  $i$ -th edge and we look at this edge at such angle that next edge is horizontal, we have classic example of inertial frame seen from the point of view of other inertial frame, so there applies time dilation formula:

$$\begin{aligned}
 t_i &= \frac{t_{i+1}}{\sqrt{1 - \left(\frac{v_i}{c}\right)^2}} \Leftrightarrow t_{i+1} = t_i \sqrt{1 - \left(\frac{v_i}{c}\right)^2} \\
 t_{i+1} &= \frac{t_{i+2}}{\sqrt{1 - \left(\frac{v_{i+1}}{c}\right)^2}} \Leftrightarrow t_{i+2} = t_{i+1} \sqrt{1 - \left(\frac{v_{i+1}}{c}\right)^2} = t_i \sqrt{1 - \left(\frac{v_i}{c}\right)^2} \sqrt{1 - \left(\frac{v_{i+1}}{c}\right)^2} \\
 &\dots \\
 t_i &= t_i \sqrt{1 - \left(\frac{v_i}{c}\right)^2} * \dots * \sqrt{1 - \left(\frac{v_{(i+n) \bmod n}}{c}\right)^2} \Leftrightarrow v_1 = \dots = v_n = 0
 \end{aligned}$$

Note: You may think that I've made mistake for example by not considering or considering formula of composition of velocities, but as I proved in the introduction to this article both ways are wrong, so I did not assume anything about speed, and got this result ( $v_1 = \dots = v_n = 0$ ).

Of course, the resultant speed will be quite different, however, special relativity allows you to look at each edge as a separated frame of reference (with separated tempo of time) in which:

// Oczywiście wypadkowa prędkości będzie zupełnie inna, jednak STW pozwala patrzeć na każdą krawędź jako na odrębny układ odniesienia, w którym zachodzi:



Where  $L$  is chosen so that the ray of light will return to the source at the time in which endpoint of the edge will travel certain distance.

// Gdzie  $L$  jest tak dobrane, żeby promień powrócił do źródła w czasie, w którym punkt końcowy krawędzi przebędzie pewien dystans.

So:

// Skąd:

$$\Delta t' = \frac{\Delta t}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \geq \Delta t$$

And as a frame of reference may not have two different tempos of time so such chain of edges leads to the conclusion that the speed of all endpoints are equal to zero, because otherwise all these points would run slower than they run.

// I jako że dany układ odniesienia nie może mieć dwóch różnych upływów czasu taki łańcuch prowadzi do wniosku, że prędkości wszystkich punktów końcowych są równe zero, bo inaczej każdy z tych punktów poruszałby się wolniej od samego siebie.

**QED.**

In the regular polygon we can also start from any edge with the same result (we can simply watch regular polygon from every side getting the same situation, so  $v_i = \dots = v_n = v$ ):

For example:

$$t_i = \frac{t_{i+1}}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \Leftrightarrow t_{i+1} = t_i \sqrt{1 - \left(\frac{v}{c}\right)^2}$$

$$t_{i+1} = \frac{t_{i+2}}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \Leftrightarrow t_{i+2} = t_{i+1} \sqrt{1 - \left(\frac{v}{c}\right)^2} = t_i \left( \sqrt{1 - \left(\frac{v}{c}\right)^2} \right)^2$$

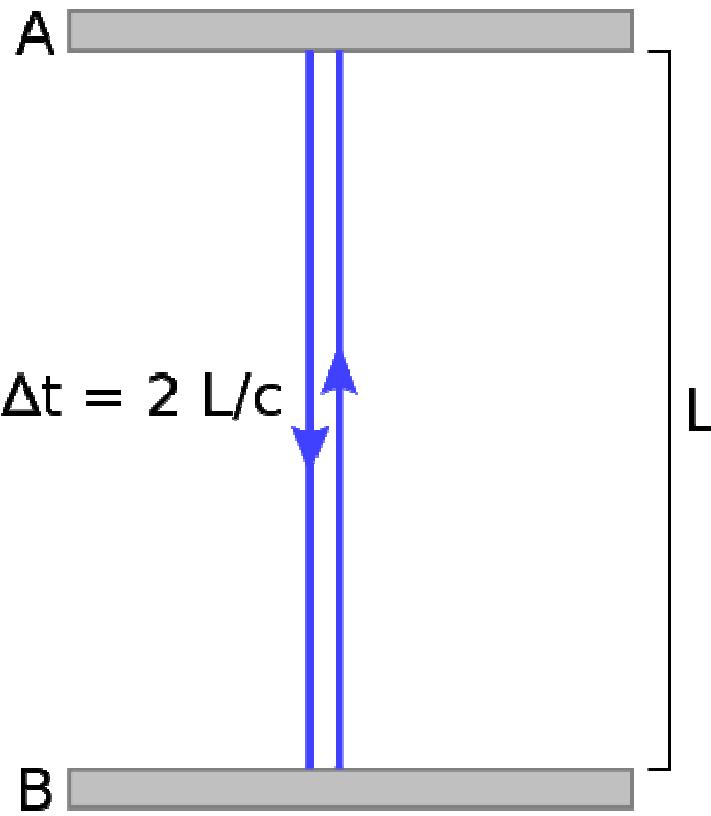
...

$$t_i = t_i \left( \sqrt{1 - \left(\frac{v}{c}\right)^2} \right)^n \Leftrightarrow v = 0$$

so  $t_0 = \dots = t_{n-1}$ , so there is no time dilation.

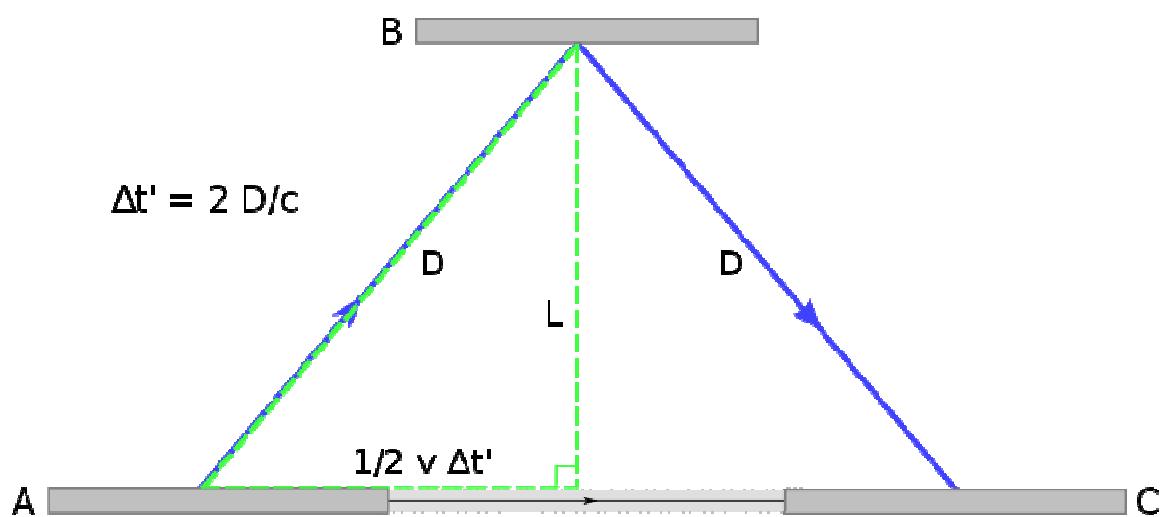
**QED.**

## ϑ Płotnicki's factor



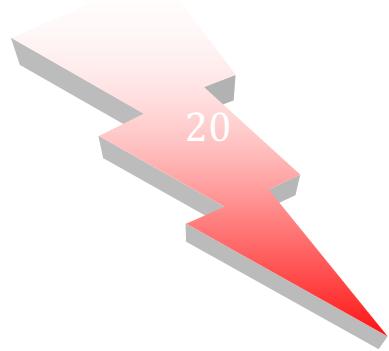
The situation at object // Sytuacja w obiekcie

$$\Delta t = \frac{2L}{c}$$



The situation on the outside // Sytuacja na zewnątrz

First of all let's calculate it like Einstein did it:



// W pierwszej kolejności obliczmy to tak jak Einstein:

$$D^2 = \left(\frac{v\Delta t'}{2}\right)^2 + (L)^2$$

$$D = \sqrt{\left(\frac{v\Delta t'}{2}\right)^2 + (L)^2} = \sqrt{\left(\frac{v\Delta t'}{2}\right)^2 + \left(\frac{c\Delta t}{2}\right)^2}$$

$$\Delta t' = \frac{2D}{c} = \sqrt{\left(\frac{v\Delta t'}{c}\right)^2 + (\Delta t)^2}$$

$$(\Delta t')^2 = \left(\frac{v\Delta t'}{c}\right)^2 + (\Delta t)^2$$

$$\Delta t' = \frac{\Delta t}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

Now let's look:

$$\Delta t' = \frac{2D}{c}$$

$$\Delta t' = \mu \Delta t = \left(\frac{\mu}{\gamma}\right) \gamma \Delta t = \Delta t \vartheta \gamma = \vartheta \frac{\Delta t}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

$$(\Delta t')^2 = \left(\frac{v\Delta t'}{c}\right)^2 + (\vartheta \Delta t)^2$$

$$\Delta t' = \frac{2D}{c} = \sqrt{\left(\frac{v\Delta t'}{c}\right)^2 + (\vartheta \Delta t)^2}$$

$$D = \sqrt{\left(\frac{v\Delta t'}{2}\right)^2 + \left(\vartheta \frac{c\Delta t}{2}\right)^2} = \sqrt{\left(\frac{v\Delta t'}{2}\right)^2 + (\vartheta L)^2}$$

This is the proof that there could be perpendicular length contraction ( $\vartheta$ ), which Lorentz and Einstein simply did not take into consideration, so they only considered zero height objects, that in reality simply do not exist.

I will calculate later in this article time dilation for the ray of light moving at an angle  $\alpha$  with respect to the direction of movement of the object. So no one can reproach that prove does not contain some special case.

// To jest dowód, że może istnieć pionowa kontrakcja, której Lorentz i Einstein zwyczajnie nie uwzględnili.

Dalej w artykule obliczyłem, wykorzystując dokładnie taką samą metodę, której Lorentz użył, dylatację czasu dla promienia światła poruszającego się pod kątem  $\alpha$  w stosunku do kierunku ruchu obiektu. Tak więc nikt nie może zarzucić, że dowód nie obejmuje jakiegoś specjalnego przypadku.

In all cases time dilation is like that:

// We wszystkich przypadkach dylatacja czasu jest taka:

$$\Delta t' = \mu \Delta t = \left(\frac{\mu}{\gamma}\right) \gamma \Delta t = \Delta t \vartheta \gamma = \vartheta \frac{\Delta t}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

Where could be  $\vartheta = \sqrt{1 - \left(\frac{v}{c}\right)^2}$ , so first of all it is not at all possible to prove that there is time dilation. So just as well there could be perpendicular contraction instead of time dilation – the same light will have constant velocity. Let's assume that there is no time dilation just like Einstein assumed that there is not perpendicular length contraction. Then:

// Gdzie możliwe jest, że  $\vartheta = \sqrt{1 - \left(\frac{v}{c}\right)^2}$ , więc po pierwsze nie jest w ogóle możliwe udowodnić, że istnieje dylatacja czasu. Więc równie dobrze może istnieć kontrakcja prostopadła do kierunku ruchu zamiast dylatacji czasu – tak samo czas będzie miał stałą prędkość. Założymy więc, że nie ma dylatacji czasu, tak samo jak Einstein założył, że nie ma prostopadłej kontrakcji. Wtedy:

$$D = \sqrt{\left(\frac{v \Delta t'}{2}\right)^2 + (L')^2} = \sqrt{\left(\frac{v \Delta t'}{2}\right)^2 + \left(\frac{L'}{L} L\right)^2} = \sqrt{\left(\frac{v \Delta t'}{2}\right)^2 + \left(\frac{L' c \Delta t}{L} \frac{2}{2}\right)^2}$$

$$\Delta t = \frac{2D}{c} = \sqrt{\left(\frac{v \Delta t'}{c}\right)^2 + \left(\frac{L'}{L} \Delta t\right)^2}$$

$$(\Delta t)^2 = \left(\frac{v \Delta t'}{c}\right)^2 + \left(\frac{L'}{L} \Delta t\right)^2$$

$$\Delta t = \frac{L'}{L} \frac{\Delta t}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

$$1 = \frac{\frac{L'}{L}}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

$$\frac{L'}{L} = \sqrt{1 - \left(\frac{v}{c}\right)^2} = \vartheta$$

So above we've just "proved" that there is perpendicular length contraction exactly the same as Einstein "proved" that there is time dilation. Of course both (perpendicular contraction and time

dilation) are not wise conclusions, but you have to take into consideration their existence when you can not prove the opposite.

// Więc powyżej “udowodniliśmy” właśnie, że istnieje prostopadła kontrakcja długości identycznie jak Einstein “udowodnił”, że istnieje dylatacja czasu. Oczywiście obie konkluzje nie są mądro, ale trzeba wziąć pod uwagę ich istnienie, jeśli nie potrafi się udowodnić, że nie istnieją.

This is why Płotnicki's factor is so important. Because otherwise we simply can not prove anything, just like Einstein did not prove anything.

// To jest powód, dla którego współczynnik Płotnickiego jest taki ważny. Ponieważ w przeciwnym wypadku (bez jego założenia) po prostu nie możemy udowodnić niczego, dokładnie tak samo jak Einstein nie udowodnił niczego.

Note: Płotnicki's factor is given to achieve completeness of proof. You can assume  $\vartheta = 1$  and whole “proof” will be similar, but at the other hand it won't be a proof anymore.

// Uwaga: Współczynnik płotnickiego jest wprowadzony dla osiągnięcia kompletności dowodu. Możesz przyjąć  $\vartheta = 1$  i cały “dowód” będzie podobny, ale z drugiej strony nie będzie już dowodem.

We don't have to ensure that  $\frac{\Delta t'}{\Delta t} = \mu$  is independent from all variables used in the experiment except  $v$ , because of course we do not want to prove that there is time dilation or perpendicular length contraction, so for us:  $\frac{\Delta t'}{\Delta t} = \mu(v, L) = \vartheta(v, L) * \gamma(v)$ ,  $\frac{L'}{L} = \vartheta(v, L)$ .

// Nie musimy zapewniać, że  $\frac{\Delta t'}{\Delta t} = \mu$  jest niezależne od wszystkich zmiennych użytych w eksperymencie z wyjątkiem  $v$ , ponieważ nie chcemy udowodnić dylatacji czasu, ani prostopadłej kontrakcji, więc dla nas:  $\frac{\Delta t'}{\Delta t} = \mu(v, L) = \vartheta(v, L) * \gamma(v)$ ,  $\frac{L'}{L} = \vartheta(v, L)$ .

You have to understand that time dilation is a real change in tempo of time, and not only relative observation, due to the special relativity principle that speed of light is always  $c$ . Because if it was only relative observation, the speed of light would be different in laboratory frame of reference. The same is for length contraction.

// Trzeba zrozumieć, że dylatacja czasu jest prawdziwą zmianą tempa upływu czasu, a nie tylko relatywną obserwacją, z powodu pryncypialnej zasady szczególnej teorii względności, zgodnie z którą prędkość światła jest stała. Ponieważ gdyby to była tylko relatywna obserwacja, prędkość światła w laboratorium byłaby większa niż w układzie związanym z poruszającym się obiektem. To samo dotyczy kontrakcji długości.

PWN (Polish Scientific Publisher, [http://aneksy.pwn.pl/podstawy\\_fizyki/?id=802](http://aneksy.pwn.pl/podstawy_fizyki/?id=802)) argues that there can not be perpendicular contraction, because if people passing each other stick out their hands to the same height in a direction perpendicular to the motion, than one of them would observe that in the second frame the size decreases, and the second would observe that at first frame it increases, and this is apparently in contradiction with assumption that the laws of physics in all inertial frames are the same and none frame is preferable. First of all it is not true, because to do so one observer

must inform the second of what he sees. Otherwise each of them will observe the same – shorter or longer length. Secondly the same concern time dilation.

// PWN (Polskie Wydawnictwo Naukowe, [http://aneksy.pwn.pl/podstawy\\_fizyki/?id=802](http://aneksy.pwn.pl/podstawy_fizyki/?id=802)) argumentuje, że nie może być prostopadłej zmiany wielkości, ponieważ gdyby mijający się ludzie wysunęli na tę samą wysokość dlonie w kierunku prostopadłym do ich ruchu, to, gdyby istniała taka kontrakcja, jeden z nich zaobserwowałby że w drugim układzie wielkość się zmniejsza, a drugi, że w pierwszym się zwiększa, a to jest niby w sprzeczności z założeniem, że prawa fizyki we wszystkich układach inercjalnych są takie same i żaden układ nie jest wyróżniony. Po pierwsze to nie jest prawda, bo aby tak było, jeden obserwator musi poinformować drugiego, co widzi. Inaczej każdy ze swego układu zaobserwuje to samo – skrócenie lub wydłużenie długości. Po drugie to samo dotyczy dylatacji czasu.

Let us take an example: Suppose that next to the John, who stands in place, goes Anna and both show watches to each other and inform each other what are they seeing. Then Jack sees that Anna's time runs slower, and Anna knows that John's time runs faster. Therefore we have the same violation of the same law. Special relativity simply assumes that each of them can read the time only from the perspective of its own frame, which is a simple nonsense, because they can inform each other. So if they can read each other's times, then looking from the perspective of John, he notes that Anna time runs slower, and, at the other hand, looking from the perspective of Anna, as none of the frames is preferable, John notes that Anna time runs faster, because Anna simply informs John what she sees. This is a simple contradiction. And there is no information barrier between them, as you may think, because even if they use light to communication they can be in any close distance to each other at the moment of measurement that it does not matter that there is very little delay in communication, and what's more it can be recalculated using knowledge of movement of Anna's frame, about which standing John can be also informed, to get accurate measurement.

Imagine that John and Anna are moving relatively to each other with a constant speed in a straight line and on this path clocks are placed, on both sides of John and Anna are all synchronized respectively with the watch of John and Anna. Then John and Anna pass each other at some point in the space and send exactly at this point only one message: "I can see that your time runs slower/faster.". In view of the fact that the distance is 0, the time needed to send this message is also 0. Additionally information has one bit, so size of channel is 0, because it is enough to check binary state of some point. Special relativity argues that both send a message "slower", which is a contradiction, because they both know at this point two things that are in contradiction:

1. "My time runs faster"
2. "My time runs slower"

So first of all this is already another proof that special relativity is false, but we can prove it also other ways that I showed farther in this article.

This leads to the conclusion, that slowdown of time, if at all, have to be only a subjective observation, and then the same applies to the constant speed of light, so the speed of light depends on the frame of reference.

The idea that time may run at different tempos gently speaking is already not wise, but the idea, that it objectively can run more slowly, where it objectively runs faster, is gently speaking an extremely not wise.

This is really such simple: time objectively runs slower/faster like in this formula

$$\Delta t' = \vartheta \frac{\Delta t}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

then and only then when speed of light does not depend on the frame of reference, because from such assumption this formula is implicated and this formula implicates such assumption. Simply speaking these are equivalent statements that there is time dilation like in this formula and that speed of light does not depend on the frame of reference. So if this formula does not give objective time dilation, then speed of light depend on the frame of reference. Dot.

QED.

// Weźmy przykład: Założmy, że obok Janka, który stoi w miejscu przelatuje Anna i oboje pokazują sobie zegarki i mówią sobie, co widzą na zegarku drugiej osoby. Co się okazuje, że Janek widzi, że czas u Anny biegnie wolniej, i Anna widzi, że czas Janka biegnie szybciej. Zatem mamy identyczne złamanie tej samej zasady. STW zakłada zwyczajnie, że każde z nich może odczytać czas tylko z perspektywy swojego układu, co jest zwykłym nonsensem, bo mogą informować się nawzajem o tym, co widzą. Więc skoro oboje mogą odczytać nawzajem swoje czasy, to raz patrząc z perspektywy Janka zauważa on, że czas u Anny biegnie wolniej, a drugi raz patrząc z perspektywy układu Anny, jako, że żaden układ nie jest wyróżniony, Janek zauważa, że czas u Anny biegnie szybciej, bo Anna zwyczajnie informuje go o tym, co widzi. To jest prosta sprzeczność. I nie ma żadnej bariery informacyjnej, jak czytelnik może pomyśleć, ponieważ pomiary mogą być zrobione w chwili dowolnie dużego zbliżenia obserwatorów, czyli przy bardzo małym opóźnieniu informacyjnym, a samo opóźnienie może być skorygowane dzięki również przesyłanej informacji o ruchu układu.

Wyobraźmy sobie, że Jan i Anna poruszają się względem siebie z pewną stałą prędkością po linii prostej i na tej drodze rozstawione są zegary, po stronie Jana i Anny wszystkie są zsynchronizowane odpowiednio z zegarkiem Jana i Anny. Jan i Anna mijają się w pewnym punkcie przestrzeni i wysyłają sobie dokładnie w tym momencie tylko jeden komunikat: „widzę, że twój czas płynie wolniej/szybciej”. W związku z tym, że odległość jest 0, czas potrzebny na jej wysłanie również jest 0. Dodatkowo informacja ma jeden bit, więc wielkość kanału jest 0, ponieważ wystarczy sprawdzić binarny stan pewnego punktu. STW twierdzi, że oboje wyśle sobie komunikat „wolniej”, co jest sprzeczną, ponieważ oboje wiedzą w tym punkcie dwie rzeczy, które są w sprzeczności:

1. „Mój czas biegnie szybciej”
2. „Mój czas biegnie wolniej”

Tak więc po pierwsze to już jest kolejny dowód, że STW jest fałszywa, ale można udowodnić to również na inne sposoby, co pokazałem dalej w tym artykule.

Co prowadzi do wniosku, że zwolnienie czasu obserwane w drugim układzie, jeśli w ogóle, jest tylko subiektywną obserwacją, a nie faktem, i wówczas to samo dotyczy stałej prędkości światła, więc prędkość światła zależy od układu odniesienia.

Pomysł, że czas może biec w różnym tempie już delikatnie mówiąc nie jest mądry, ale, że może obiektywnie biec wolniej tam, gdzie obiektywnie biegnie szybciej, jest delikatnie mówiąc już extremalnie niemądre.

To jest naprawdę tak proste: czas obiektywnie biegnie wolniej/szybciej tak jak w tym wzorze:

$$\Delta t' = \vartheta \frac{\Delta t}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

Wtedy i tylko wtedy, gdy prędkość światła nie zależy od układu odniesienia, bo z takiego założenia ten wzór jest wyprowadzony. Tak więc jeśli ten wzór nie daje obiektywnej miary dylatacji czasu, to prędkość światła zależy od układu odniesienia. Kropka.

QED.

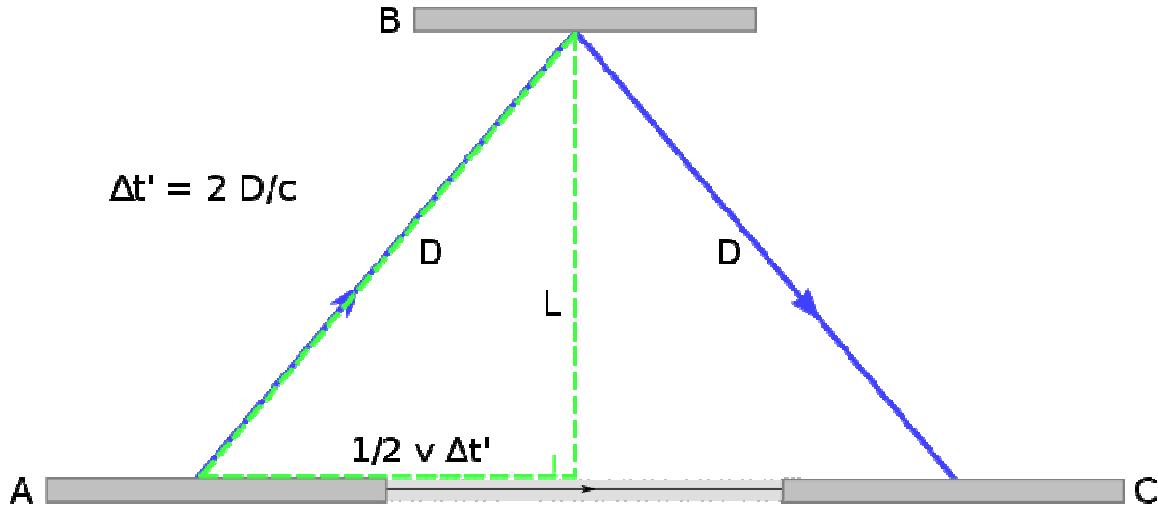
So either there can't be time dilation or there may be perpendicular length contraction. And of course there can't be time dilation and perpendicular length contraction, as I proved above.

// Tak więc albo nie może istnieć dylatacja czasu, albo może istnieć prostopadła kontrakcja długości. Oczywiście nie może istnieć dylatacja czasu, ani prostopadła kontrakcja długości, jak dowiodłem powyżej.

But let's see in the next chapter what will happen when we allow both: time dilation, and perpendicular length contraction.

// Ale zobaczymy w następnych rozdziałach, co będzie, jeśli dopuścimy możliwość istnienia obu: dylatacji czasu, i prostopadłej kontrakcji długości.

## The simplest closed chain – simple case // Najprostszy zamknięty łańcuch – prosty przypadek



There are only two possibilities: time runs slower for moving object or it only looks like running slower. In second case velocity of light is not the same for a different frames of reference, but only looks as being the same. In first case we have:

$$\Delta t' = \Delta t \vartheta \gamma$$

Of course there is only one tempo of time for a concrete frame of reference, because two different tempos of time would be in conflict with each other, which is impossible. If there would be different tempo of time than  $\Delta t' = \Delta t \vartheta \gamma$ , then the **speed of light** wouldn't be the same regardless of the frame of reference, because it is **constant regardless of the frame of reference then and only then when  $\Delta t' = \Delta t \vartheta \gamma$** .

If we move point of view with speed  $v$  in direction opposite to current movement of the moving frame, then we have analogous situation, so:

$$v' = \frac{0 + v}{1 + \frac{0 * v}{c^2}} = v$$

$$v'' = \frac{v - v}{1 + \frac{v * v}{c^2}} = 0$$

$$\Delta t = \Delta t' \vartheta \gamma = \Delta t (\vartheta \gamma)^2 \Leftrightarrow \vartheta \gamma = 1$$

Second proof:

$$(\Delta t \geq \Delta t' = \Delta t \vartheta \gamma \geq \Delta t \text{ or } \Delta t \leq \Delta t' = \Delta t \vartheta \gamma \leq \Delta t) \Leftrightarrow \Delta t = \Delta t \vartheta \gamma \Leftrightarrow \vartheta \gamma = 1$$

So there is no time dilation and no length contraction.

$$\text{So } \Delta t' = \Delta t = \frac{2D}{c} = \frac{2L}{c} \Leftrightarrow D = L \Leftrightarrow v = 0 \Leftrightarrow \gamma = 1.$$



And we know that  $\vartheta\gamma = 1$ , so  $\vartheta = 1$ , so we have no perpendicular length contraction too.

So for  $v > 0$  we have  $\Delta t' = \Delta t = \frac{2D}{c_2} = \frac{2L}{c_1}$ , but  $D > L \Leftrightarrow c_2 > c_1$ . So speed of light is not constant regardless of the frame of reference.

**QED.**

## The simplest closed chain – general case

// Najprostszy zamknięty łańcuch – ogólny przypadek

To ensure that noone ever and for any special case won't prove time dilation or that the speed of light is constant regardless of the frame of reference, here below is a general case.

// Aby zapewnić, że nikt nigdy dla żadnego specjalnego przypadku nie udowodni, że istnieje dylatacja czasu lub że prędkość światła nie zależy od układu odniesienia, oto poniżej jest ogólny przypadek.

So let's calculate time dilation for ray moving in other direction than perpendicular. Of course, the speed of light is independent of the direction of movement in the frame of reference.

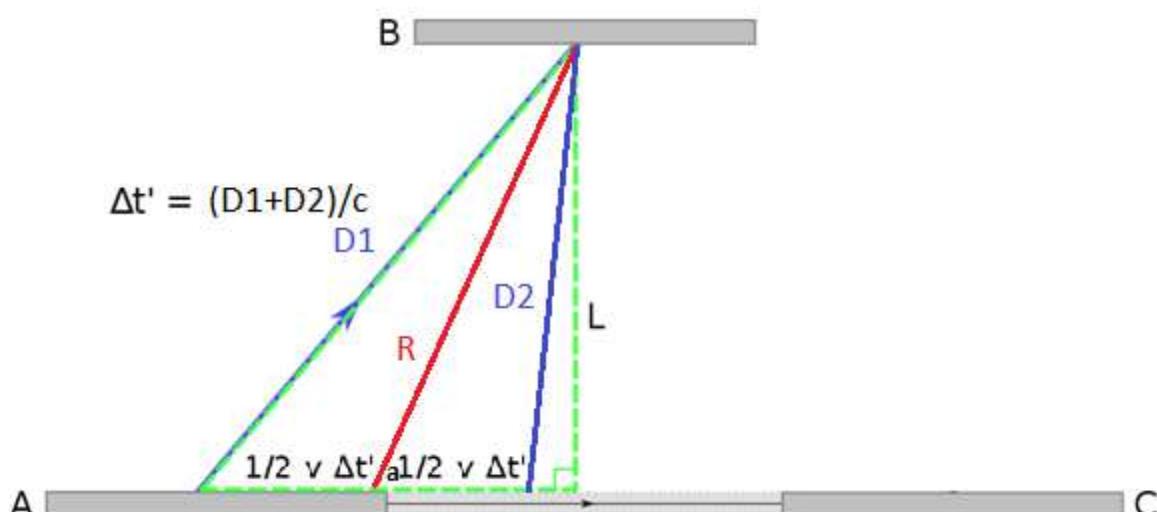
// Zbadajmy dylatację czasu dla promienia poruszającego się w innym kierunku niż pionowy. Oczywiście prędkość światła jest niezależna także od kierunku ruchu w układzie.

Suppose that in the laboratory frame of reference moves object A (frame of reference A) at the speed of  $v$ . Object A has length  $D$ . Object B is a point. At some moment from the beginning of object A starts ray of light (object B) that reflects from the mirror that is placed perpendicular to the direction of movement of B at the end of object A, so reflected ray comes back to the source.

// Założmy, że w układzie laboratoryjnym porusza się obiekt A(układ A) z prędkością  $v$ . Obiekt A ma długość  $D$ . Obiekt B jest punktowy. W pewnym momencie z początku obiektu A startuje promień świetlny (obiekt B), który odbija się od lustra umieszczonego pod kątem prostym do kierunku ruchu B na końcu obiektu A, a zatem promień odbity powraca do źródła.

Proof for general case: for the ray of light moving at an angle  $\alpha$  with respect to the direction of movement of the object we have:

// Dowód przypadku ogólnego: dla promienia światła poruszającego się pod kątem  $\alpha$  względem kierunku ruchu obiektu mamy:



$$\Delta t = \frac{2R}{c}$$

$$\frac{L}{R} = \sin \alpha$$

Where:

$\alpha$  is for  $\alpha$ .

$\vartheta$  is for perpendicular contraction.

$\mu$  is for contraction of distance between beginning and end of movement, and we assume that it is unknown.

$R$  is a distance that light travels in time  $\frac{\Delta t}{2}$  in a frame of reference of moving object.

$L = R \sin \alpha \vartheta$  is a vertical scalar component of the way that light travels in time  $\frac{\Delta t}{2}$ , and as it is perpendicular to the direction of movement of the object, so it is perpendicularly contracted.

$R \cos \alpha \mu$  is a horizontal scalar component of the way that light travels in time  $\frac{\Delta t}{2}$ , measured in laboratory frame of reference, so length is contracted, because this component is parallel to the direction of movement of the object.

$\frac{v\Delta t'}{2}$  is a distance that object travels in time  $\frac{\Delta t'}{2}$  in laboratory frame of reference.

$R \cos \alpha \mu + \frac{v\Delta t'}{2}$  is a horizontal scalar component of the way that light travels in first phase in time  $\frac{\Delta t'}{2}$  in laboratory frame of reference.

$R \cos \alpha \mu - \frac{v\Delta t'}{2}$  is a horizontal scalar component of the way that light travels in second phase in time  $\frac{\Delta t'}{2}$  in laboratory frame of reference.

$D_1$  is a distance that light travels in first phase in time  $\frac{\Delta t'}{2}$  in laboratory frame of reference.

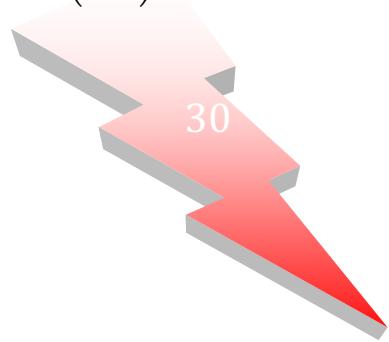
$D_2$  is a distance that light travels in second phase in time  $\frac{\Delta t'}{2}$  in laboratory frame of reference.

The same and identical, as in Lorentz calculation of time dilation, we have right triangles:

$$\begin{aligned} \left( R \cos \alpha \mu + \frac{v\Delta t'}{2} \right)^2 + (R \sin \alpha \vartheta)^2 &= R^2((\cos \alpha \mu)^2 + (\sin \alpha \vartheta)^2) + v\Delta t' R \cos \alpha \vartheta + \left( \frac{v\Delta t'}{2} \right)^2 \\ &= D_1^2 \end{aligned}$$

$$\begin{aligned} \left( R \cos \alpha \mu - \frac{v\Delta t'}{2} \right)^2 + (R \sin \alpha \vartheta)^2 &= R^2((\cos \alpha \mu)^2 + (\sin \alpha \vartheta)^2) - v\Delta t' R \cos \alpha \vartheta + \left( \frac{v\Delta t'}{2} \right)^2 \\ &= D_2^2 \end{aligned}$$

$$\Delta t' = \frac{D_1 + D_2}{c}$$



$$2v\Delta t' R \cos \alpha \mu = (D_1 - D_2)(D_1 + D_2) = (D_1 - D_2)\Delta t' c$$

$$2\frac{v}{c}R \cos \alpha \mu = (D_1 - D_2)$$

$$2R^2((\cos \alpha \mu)^2 + (\sin \alpha \vartheta)^2) + 2\left(\frac{v\Delta t'}{2}\right)^2 = D_1^2 + D_2^2$$

$$4\left(\frac{v}{c}\right)^2 R^2 \cos^2 \alpha \mu^2 = (D_1^2 + D_2^2) - 2D_1 D_2$$

$$2D_1 D_2 = 2R^2((\cos \alpha \mu)^2 + (\sin \alpha \vartheta)^2) + 2\left(\frac{v\Delta t'}{2}\right)^2 - 4\left(\frac{v}{c}\right)^2 R^2 \cos^2 \alpha \mu^2$$

$$D_1(-D_2) = -R^2((\cos \alpha \mu)^2 + (\sin \alpha \vartheta)^2) - \left(\frac{v\Delta t'}{2}\right)^2 + 2\left(\frac{v}{c}\right)^2 R^2 \cos^2 \alpha \mu^2 = z$$

$$2\frac{v}{c}R \cos \alpha \mu = (D_1 + (-D_2)) = -y$$

$$\begin{aligned} \left\{ \begin{aligned} delta &= y^2 - 4z \\ &= 4\left(\frac{v}{c}\right)^2 R^2 \cos^2 \alpha \mu^2 \\ &\quad + 4\left(R^2((\cos \alpha \mu)^2 + (\sin \alpha \vartheta)^2) + \left(\frac{v\Delta t'}{2}\right)^2 - 2\left(\frac{v}{c}\right)^2 R^2 \cos^2 \alpha \vartheta^2\right) \\ &= 4\left(R^2((\cos \alpha \mu)^2 + (\sin \alpha \vartheta)^2) + \left(\frac{v\Delta t'}{2}\right)^2 - \left(\frac{v}{c}\right)^2 R^2 \cos^2 \alpha \vartheta^2\right) \end{aligned} \right\} \end{aligned}$$

$$X_{1,2} = \frac{v}{c}R \cos \alpha \mu \pm \frac{\sqrt{delta}}{2}$$

$$D_1 = \frac{v}{c}R \cos \alpha \mu + \frac{\sqrt{delta}}{2} > 0$$

$$D_2 = -\frac{v}{c}R \cos \alpha \mu + \frac{\sqrt{delta}}{2} > 0$$

$$\begin{aligned} \Delta t' &= \frac{D_1 + D_2}{c} = \frac{\sqrt{delta}}{c} = 2\sqrt{\left(\frac{R}{c}\right)^2 ((\cos \alpha \mu)^2 + (\sin \alpha \vartheta)^2) + \left(\frac{v\Delta t'}{2}\right)^2 - \left(\frac{v}{c}\right)^2 \left(\frac{R}{c}\right)^2 \cos^2 \alpha \mu^2} \\ &= 2\sqrt{\left(\frac{\Delta t}{2}\right)^2 ((\cos \alpha \mu)^2 + (\sin \alpha \vartheta)^2) + \left(\frac{\frac{v}{c}\Delta t'}{2}\right)^2 - \left(\frac{v}{c}\right)^2 \left(\frac{\Delta t}{2}\right)^2 \cos^2 \alpha \mu^2} \end{aligned}$$

$$\Delta t' = \sqrt{(\Delta t)^2 ((\cos \alpha \mu)^2 + (\sin \alpha \vartheta)^2) + \left(\frac{v}{c}\Delta t'\right)^2 - \left(\frac{v}{c}\right)^2 (\Delta t)^2 \cos^2 \alpha \mu^2}$$

$$(\Delta t')^2 \left(1 - \left(\frac{v}{c}\right)^2\right) = (\Delta t)^2 ((\cos \alpha \mu)^2 + (\sin \alpha \vartheta)^2) - \left(\frac{v}{c}\right)^2 (\Delta t)^2 \cos^2 \alpha \mu^2$$

$$\Delta t' = \Delta t \gamma \sqrt{(\sin \alpha \vartheta)^2 + \left(1 - \left(\frac{v}{c}\right)^2\right) \cos^2 \alpha \mu^2}$$

$$\Delta t' = \Delta t \gamma \sqrt{(\sin \alpha \vartheta)^2 + \cos^2 \alpha \left(\frac{\mu}{\gamma}\right)^2}$$

Where beginning and end of movement are co-local in frame of object ( $O$ ) moving at the speed  $v$ , so:

$$\Delta t' = \Delta t \gamma \sqrt{(\sin \alpha \vartheta)^2 + \cos^2 \alpha \left(\frac{\mu}{\gamma}\right)^2} = \Delta t \gamma \vartheta$$

$$\Leftrightarrow \sqrt{(\sin \alpha \vartheta)^2 + \cos^2 \alpha \left(\frac{\mu}{\gamma}\right)^2} = \vartheta$$

$$\cos^2 \alpha \left(\frac{\mu}{\gamma}\right)^2 = \vartheta^2 (1 - \sin^2 \alpha) = \vartheta^2 \cos^2 \alpha \Leftrightarrow \mu = \vartheta \gamma$$

So correctness of relativistic effects does not depend on the angle between movement and ray of light that is taken into consideration then and only then when  $\mu = \vartheta \gamma$ . And correctness of relativistic effects must not depend on the angle between movement and ray of light that is taken into consideration, simply because ray of light can move in any direction, so  $\mu = \vartheta \gamma$ . So we have time dilation formula correct for every angle of movement of ray of light:

$$\Delta t' = \Delta t \gamma \vartheta$$

At the other hand we can prove that correctness of relativistic effects does not depend on the angle between movement and ray of light that is taken into consideration then and only then when  $\mu = \vartheta \gamma$ . Let's check it, but only without taking into consideration perpendicular length contraction, because otherwise there are of course other transformation formulas:

$$x_1 = \gamma(x'_1 - vt'_1), x_2 = \gamma(x'_2 - vt'_2), D = x_2 - x_1$$

$$x'_1 = \gamma(x_1 + vt_1), x'_2 = \gamma(x_2 + vt_2), D' = x'_2 - x'_1$$

$$D = \gamma(D' - S')$$

$$D' = \gamma(D + S)$$

$$D + S = \left(\frac{D}{\gamma} + S'\right) * \frac{1}{\gamma} = \frac{D}{(\gamma)^2} + \frac{S'}{\gamma}$$

$$S = \frac{S'}{\gamma} + \frac{D}{(\gamma)^2} - D = \frac{S'}{\gamma} \Leftrightarrow D = 0 \Leftrightarrow D' = S'$$

And  $D = 0$ , so:

$$S' = S \gamma \Leftrightarrow \mu = \gamma$$

So we've proved that correctness of relativistic effects, when we do not take into consideration perpendicular length contractin, does not depend on the angle between movement and ray of light that is taken into consideration.

Now, as we assume that time really slows down, we have:

$$\Delta t' = \Delta t \vartheta \gamma$$

Of course there is only one tempo of time for a concrete frame of reference, because two different tempos of time would be in conflict with each other, which is impossible. If there would be different tempo of time than  $\Delta t' = \Delta t \vartheta \gamma$ , then the speed of light wouldn't be the same regardless of the frame of reference, because it is constant regardless of the frame of reference then and only then when  $\Delta t' = \Delta t \vartheta \gamma$ .

If we move point of view with speed  $v$  in direction opposite to current movement of the moving frame, then we have analogous situation, so:

$$v' = \frac{0 + v}{1 + \frac{0 * v}{c^2}} = v$$

$$v'' = \frac{v - v}{1 + \frac{v * v}{c^2}} = 0$$

$$\Delta t = \Delta t' \vartheta \gamma = \Delta t (\vartheta \gamma)^2 \Leftrightarrow \vartheta \gamma = 1$$

Second proof:

$$(\Delta t \geq \Delta t' = \Delta t \vartheta \gamma \geq \Delta t \text{ or } \Delta t \leq \Delta t' = \Delta t \vartheta \gamma \leq \Delta t) \Leftrightarrow \Delta t = \Delta t \vartheta \gamma \Leftrightarrow \vartheta \gamma = 1$$

So there is no time dilation and no length contraction.

From the triangles we have (using law of cosines):

$$R = \frac{1}{2} \sqrt{2D_1^2 + 2D_2^2 - (v\Delta t')^2}$$

$$(2R)^2 = 2D_1^2 + 2D_2^2 - (v\Delta t')^2 \leq (D_1 + D_2)^2$$

$$D_1^2 + D_2^2 - 2D_1 D_2 \leq (v\Delta t')^2$$

$$(D_1 - D_2)^2 \leq (v\Delta t')^2$$

$$D_1 \leq v\Delta t' + D_2$$

Which is triangle inequality.

$$\text{So } \Delta t' = \Delta t = \frac{D_1 + D_2}{c} = \frac{2R}{c} \Leftrightarrow D_1 + D_2 = 2R \Leftrightarrow D_1 = D_2 = R \Leftrightarrow v = 0 \Leftrightarrow \gamma = 1.$$

And we know that  $\vartheta \gamma = 1$ , so  $\vartheta = 1$ , so we have no perpendicular length contraction too.

$$\text{So for } v > 0 \text{ we have } \Delta t' = \Delta t = \frac{D_1 + D_2}{c_2} = \frac{2R}{c_1}, \text{ but } D_1 + D_2 > 2R \Leftrightarrow c_2 > c_1,$$

So  $c_2 > c_1$ . So speed of light is not constant regardless of the frame of reference.

**QED.**

## Summary

As the assumption, that speed of light is constant, implicates negation of itself, so this assumption is false.

This is a proof that the speed of any object may not be constant regardless of the frame of reference, and that there is no speed limit, that is, that the speed limit is a plus infinite, for which formulas transform in the Galilean transformation.

// To jest to dowód, że prędkość jakiegokolwiek obiektu nie może być stała bez względu na układ odniesienia oraz, że nie istnieje prędkość graniczna, czyli, że prędkość graniczna jest plus nieskończona, dla której wzory przekształcają się w transformację Galileusza.

The same thing has happened with the similarly calculated length contraction and mass in special relativity.

// Identycznie rzecz ma się z analogicznie obliczaną kontrakcją przestrzeni i masą relatywistyczną.

$$S' = \frac{S}{\vartheta\gamma}$$

$$m' = m\vartheta\gamma$$

So Lorentz among other things made a mistake, assuming that the speed of light is constant and not considering perpendicular contraction, while Einstein was suggested by the incorrect transformation.

// Tak więc Lorentz popełnił błąd, rozpatrując tylko jeden kierunek ruchu światła i nie uwzględniając pionowej kontrakcji, natomiast Einstein zasugerował się tym błędnym przekształceniem.

Of course, theory of relativity is based on the Lorentz transformation, so in this way the theory of relativity has been disproved.

// Oczywiście teoria względności oparta jest na transformacji Lorentza, zatem w ten oto sposób teoria względności została obalona.

So equation:

// Tak więc równanie:

$$E_k = m'c^2 - mc^2 = mc^2(\vartheta\gamma - 1) = 0$$

So Einstein's  $E = 0$ .

// Więc Einsteinowskie  $E = 0$

## Composition of velocities

How long it takes for two rays of light that are undisturbed, independent, and go one to another, and started from points  $A$  and  $B$  that are in two light-years distance one from another to meet? Does every of them travels one light-year in one year? Yes. So will they meet at the middle of the way between  $A$  and  $B$ ? Yes. So does they travel 2 light year in just one year? Yes. So what is the speed of approaching one another? Is it two light-years divided by one year? So let's now calculate:

// Po upływie ilu sekund spotkają się dwa niezakłócone, niezależne, biegnące ku sobie promienie światła, startujące z punktów  $A$  i  $B$  odległych od siebie o 600 tysięcy kilometrów? Czy każdy z nich pokona w ciągu sekundy 300 tysięcy kilometrów. Tak. Czy więc w czasie 1 sekundy się spotkają po środku drogi? Tak. Czy więc w czasie 1 sekundy pokonają w sumie 600 tysięcy kilometrów? Tak. A więc jaka będzie prędkość zbliżania? 600 tysięcy kilometrów na 1 sekundę. A teraz policzmy:

$$\frac{c + c}{1 + \frac{c * c}{c^2}} = c$$

How long it takes to travel two light-years at speed  $c$ ? Two years. All right, so what will happen in two years period? Will every ray independently one from another travel two light-years? Yes. So after that ray from the source  $A$  will be at point of source  $B$ , and ray from source  $B$  will be at point of source  $A$ ? Yes. So what will be a distance between them? Two light-years. And what was a distance between them at the beginning of movement? Two light-years. Is speed a distance traveled divided by time of this travel? Yes. So will it be  $\frac{\text{two light\_years} + \text{two light\_years}}{\text{two years}} = 2c$ ? Yes. Didn't we calculate

before that they should meet one with another at this time (Two years)? Did they pass one another one year earlier? Yes. So is the speed of approaching one another greater than  $c$ ? Yes. Did distance between  $A$  and  $B$  get longer to four light-years? No. Or did a time in a unmoving frame of reference slow down? No. So is the composition of velocities in special relativity correct? No. So is it true that  $a \Rightarrow b$ , where  $a$  = "speed of light is constant" and  $b$  = "meeting should occur after two light-years"?

Yes. As  $b$  is false, so is it true that  $(a \Rightarrow \text{false}) = \text{true}$ ? Yes. So then  $a = \text{false}$ ? Yes. So is theory of relativity correct? No. **QED.**

// W ciągu ilu sekund zostanie pokonana odległość 600 tysięcy km z prędkością  $c$ ? W czasie 2 sekund. Ok, co się stanie w ciągu dwóch sekund? Czy każdy promień z osobna pokona 600 tysięcy kilometrów? Tak. A zatem promień  $A$  dotrze do źródła  $B$  i promień  $B$  dotrze do źródła  $A$ ? Tak. Więc w jakiej odległości będą od siebie? 600 tysięcy kilometrów. Czy wzór na prędkość to droga przebyta przez czas, w którym została przebyta? Tak. Czy więc wzajemną prędkość można wyliczyć dzieląc odległość przebytą przez promień światła względem siebie przez czas, w którym ją przebyły? Tak. Czy będzie to  $\frac{600 \text{ tysięcy km} + 600 \text{ tysięcy km}}{2s} = 600 \text{ tysięcy } \frac{\text{km}}{\text{s}}$ ? Tak. Czy nie wyliczyliśmy, że powinny się w tym czasie spotkać? Tak. Czy sekundę wcześniej minęły się? Tak. Czy więc prędkość zbliżania jest większa od  $c$ ? Tak. Czy może dystans między  $A$  i  $B$  wydłużył się do 1.2 miliona km, podczas gdy jesteśmy nieruchomi wobec punktów  $A, B$ ? Nie. A może czas w nieruchomym układzie odniesienia zwolnił? Nie. Czy więc wzór na składanie prędkości, który wynika z założenia o stałej prędkości światła jest dobry? Nie. Czy więc prawdą jest, że  $a \Rightarrow b$ , gdzie  $a$  = „prędkość światła jest stała”,  $b$  = „spotkanie powinno nastąpić w ciągu dwóch sekund”? Tak. Skoro  $b$  okazało się fałszywe to prawdą

jest, że  $(a \Rightarrow \text{false}) = \text{true}$ ? Tak. Czy wtedy  $a = \text{false}$ ? Tak. Zatem teoria względności jest fałszywa? Tak. **QED.**

Of course there is relativity of perception, but only optical relativity due to the fact that the ray of light reflected from any object have to travel certain distance in certain time before it will fall into eyes, and such relativity is easy to calculate.

// Oczywiście istnieje względność postrzegania, ale wyłącznie optyczna, związana z tym, że promień musi przebyć określoną drogę w określonym czasie, zanim wpadnie do oka, i można ją dość prosto wyliczyć.

## Fundamental rules of composition of velocities

If we have formulas  $C_+$ ,  $C_-$  for composition of velocities  $a$  and  $b$ , where  $a$  and  $b$  are vectors only in space, because in inertial frames velocity is constant in time, then:

// Jeśli mamy wzory  $C_+$ ,  $C_-$  na kompozycję prędkości  $a$  i  $b$ , gdzie  $a$  i  $b$  są wektorami tylko w przestrzeni, ponieważ w inercjalnych układach odniesienia prędkość jest stała w czasie, wtedy:

1.  $C_-(\mathbf{a}, \mathbf{b}) + C_-(\mathbf{b}, \mathbf{a}) = \mathbf{0}$
2.  $C_-(\mathbf{a}, -\mathbf{b}) + C_(-\mathbf{b}, \mathbf{a}) = C_-(\mathbf{a}, -\mathbf{b}) - C_-(\mathbf{b}, -\mathbf{a}) = \mathbf{0} \Leftrightarrow C_-(\mathbf{a}, -\mathbf{b}) = C_-(\mathbf{b}, -\mathbf{a}) = C_+(\mathbf{a}, \mathbf{b})$

Commutative property of  $C_+$ :

// Własność przemienności operacji  $C_+$ :

3.  $C_+(\mathbf{a}, \mathbf{b}) = C_+(\mathbf{b}, \mathbf{a})$
4.  $C_+(\mathbf{a}, \mathbf{b}, \mathbf{c}) = C_+(\mathbf{a}, C_+(\mathbf{b}, \mathbf{c})) = C_+(\mathbf{b}, C_+(\mathbf{a}, \mathbf{c})) = C_+(\mathbf{c}, C_+(\mathbf{a}, \mathbf{b}))$

etc.

// itd.

And for every  $x$  there must be suficed these rules:

// I dla każdego  $x$  musi być spełnione:

5.  $C_-(\mathbf{a}, \mathbf{b}) = C_-(C_-(\mathbf{a}, \mathbf{x}), C_-(\mathbf{b}, \mathbf{x}))$

Composition does not depend on the frame of reference. That is the fundamental rule of composition of velocities. It is true, because:

// Kompozycja prędkości nie zależy od układu odniesienia. To jest fundamentalna reguła kompozycji prędkości. Jest prawdziwa, ponieważ:

a.) velocity is always traveled distance divided by time of this travel, so relative velocity is always distance between two points of space at the same ending moment of movement minus distance between two points of space at the same beginning moment of movement divided by time interval between these moments,

// prędkość to zawsze droga przebyta podzielona przez czas, w którym została przebyta, więc względna prędkość to zawsze odległość między dwoma punktami w przestrzeni w tym samym końcowym momencie ruchu minus odległość między dwoma punktami w przestrzeni w tym samym początkowym momencie ruchu podzielona przez przedział czasu między tymi momentami,

b.) distance between two points of space at the same moment does not depend on the frame of reference.

// odległość dwóch punktów w przestrzeni w tym samym momencie nie zależy od układu odniesienia.

These are fundamental laws, and special relativity simply does not comply with them, therefore special relativity is false.

// To są fundamentalne prawa, i STW po prostu się do nich nie stosuje, dlatego jest fałszywa.

And the rule of preservation of speed as a composition of two speeds in relation to intermediate speed, that is derived from rule 5:

// I reguła zachowania kompozycji dwóch prędkości względem prędkości pośredniej, która jest wyprowadzona z reguły 5:

$$6. \quad C_-(a, b) = C_-(C_-(a, x), C_-(b, x)) = C_-(C_-(a, x), -C_-(x, b)) = C_+(C_-(a, x), C_-(x, b))$$

Otherwise  $C$  is incorrect, because then when you calculate the same in many different ways, for example when you change frame of reference or composite two speeds using intermediate speed, you get different result.

// W innym wypadku  $C$  jest nieprawidłowe, ponieważ wtedy gdy liczysz to samo na wiele różnych sposobów, na przykład jeśli zmieniasz układ odniesienia lub gdy liczysz kompozycje dwóch prędkości używając prędkości pośredniej, otrzymujesz inny wynik.

These rules are of course sufficed in the Galilean transformation. These postulates probably also implicates Galilean transformation. In Galilean transformation we have:

// Te reguły są oczywiście spełnione przez transformację Galileusza. Te postulaty prawdopodobnie implikują transformację Galileusza. W transformacji Galileusza mamy:

$$C_+(a, b) = a + b$$

$$C_-(a, b) = a - b$$

As I showed below these rules are not sufficed in special relativity.

// Jak pokazałem poniżej te reguły nie są spełnione w STW.

Imagine three objects moving at the same straight path with speeds:  $v_1 = \frac{1}{4}c$ ,  $v_2 = \frac{2}{4}c$ ,  $v_3 = \frac{3}{4}c$

// Wyobraź sobie trzy obiekty poruszające się wzdłuż tej samej prostej ścieżki z prędkościami:

$$v_1 = \frac{1}{4}c, v_2 = \frac{2}{4}c, v_3 = \frac{3}{4}c$$

Let's calculate composition of velocities for  $v_2, v_3$ :

// Policzymy kompozycję prędkości  $v_2, v_3$ :

$$v_{2,3} = \frac{v_3 - v_2}{1 - \frac{v_3 v_2}{c^2}} = \frac{\frac{3}{4}c - \frac{2}{4}c}{1 - \frac{\frac{3}{4}c * \frac{2}{4}c}{c^2}} = \frac{\frac{1}{4}c}{1 - \frac{\frac{3}{8}c^2}{c^2}} = \frac{\frac{1}{4}c}{\frac{5}{8}c} = \frac{2}{5}c$$

Now let's calculate  $v_2$  and  $v_3$  from the point of view of first object:

// Teraz policzymy  $v_2$  i  $v_3$  z perspektywy pierwszego obiektu:

$$v_{1,2} = \frac{v_2 - v_1}{1 - \frac{v_2 v_1}{c^2}} = \frac{\frac{2}{4}c - \frac{1}{4}c}{1 - \frac{\frac{2}{4}c * \frac{1}{4}c}{c^2}} = \frac{\frac{1}{4}c}{1 - \frac{\frac{1}{8}c^2}{c^2}} = \frac{\frac{1}{4}c}{\frac{7}{8}c} = \frac{4}{14}c$$

$$v_{1,3} = \frac{v_3 - v_1}{1 - \frac{v_3 v_1}{c^2}} = \frac{\frac{3}{4}c - \frac{1}{4}c}{1 - \frac{\frac{3}{4}c * \frac{1}{4}c}{c^2}} = \frac{\frac{2}{4}c}{1 - \frac{\frac{3}{16}c^2}{c^2}} = \frac{\frac{2}{4}c}{\frac{13}{16}c} = \frac{8}{13}c$$

And now calculate composition of velocities:

// I wyliczmy kompozycję prędkości:

$$v_{1,2,3} = \frac{v_{1,3} - v_{1,2}}{1 - \frac{v_{1,2} v_{1,3}}{c^2}} = \frac{\frac{8}{13}c - \frac{4}{14}c}{1 - \frac{\frac{8}{13}c * \frac{4}{14}c}{c^2}} = \frac{\frac{112 - 52}{182}c}{1 - \frac{\frac{32}{182}c^2}{c^2}} = \frac{\frac{60}{182}c}{1 - \frac{32}{182}c} = \frac{60}{150}c = \frac{2}{3}c \neq \frac{2}{5}c = v_{2,3}$$

So composition of velocities depends on the frame of reference in which it is measured. So there is nothing like one relative speed, because all depends on the frame of reference.

// Tak więc kompozycja prędkości zależy od układu odniesienia w którym jest mierzona. Tak więc nie ma jednej relatywnej prędkości między dwoma obiektami, ponieważ wszystko zależy od układu odniesienia.

So we have here time dilation that depends not only from the single relative velocity between two frames of reference but also from third frame. Our formula was wrong:

// Tak więc mamy tutaj dylatację czasu, która zależy nie tylko od pojedyńczej relatywnej prędkości między dwoma układami odniesienia, ale również od trzeciego układu odniesienia. Nasz wzór okazał się zły:

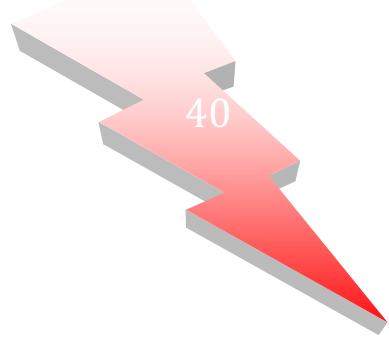
$$\Delta t_2 = \frac{\Delta t_3}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

Because tell me which relative velocity we should choose and why... ? Is there any preferable frame of reference?

// Bo powiedz mi która relatywna prędkość powinna być użyta i dlaczego... ? Czy istnieje jakiś wyróżniony układ odniesienia?

And that is not all because relative velocity between object 1 and object 3 should be a composition of relative velocities between object 1 and object 2 and between object 2 and object 3:

// I to nie jest wszystko, ponieważ relatywna prędkość pomiędzy obiektami 1 i 2 powinna być kompozycją relatywnych prędkości między obiektami 1 i 2 i między obiektami 2 i 3:



$$v_{(1,2),(2,3)} = \frac{v_{1,2} + v_{2,3}}{1 + \frac{v_{1,2}v_{2,3}}{c^2}} = \frac{\frac{4}{14}c + \frac{2}{5}c}{1 + \frac{\frac{4}{14}c * \frac{2}{5}c}{c^2}} = \frac{20 + 28}{70}c * \frac{70}{70 - 8} = \frac{58}{62}c = \frac{29}{31}c$$

But:

// Ale:

$$v_{1,3} = \frac{8}{13}c$$

**QED.**

## Optical relativity

$O = (o_x, o_y, o_z)$  is an observer point,

$P_1 = (x_1, y_1, z_1)$  is a begining of movement as it is seen,

$P_2 = (x_2, y_2, z_2) = (x_1 + tv_x, y_1 + tv_y, z_1 + tv_z)$  is and end of movement as it is seen,

$v' = (v'_x, v'_y, v'_z)$  is real velocity of movement,

Relativistic (as it is seen) time (where  $t'$  is real time):

$$t = t' + \frac{OP_2}{c} - \frac{OP_1}{c}$$

Real coordinates of object:

$$x'_1 = x_1 + v'_x \frac{OP_1}{c}$$

$$x'_2 = x_2 + v'_x \frac{OP_2}{c}$$

Relativistic (as it is seen) velocity of movement:

$$\begin{aligned} v_x &= \frac{x_2 - x_1}{t} = \frac{x_2 - x_1}{\frac{OP_2}{c} - \frac{OP_1}{c} + \frac{ct'}{c}} = \frac{c(x_2 - x_1)}{OP_2 - OP_1 + ct'} \\ v &= \frac{c}{OP_2 - OP_1 + ct'} (x_2 - x_1, y_2 - y_1, z_2 - z_1) \end{aligned}$$

## Albert Einstein's achievement

Albert Einstein simply like a bull in a china shop broke with impunity fundamental law of physics, so:

1. First of all and the greatest “sin”: finite speed can be not dependent of the frame of reference, so of course there is speed limit that can be taken as a constance in place of infinity
2. Secondly, time and distance are of a gum, so there are no simultaneous events at all and there does not exist constant distance between two points of space – all depends on the velocity.
3. And it is not all, because the same event can be in different place and time.
4. Velocity is no more traveled distance divided by time of travel, does someone know why? I know why... because there is constant speed of light taken as plus infinity.
5. There is infinitely many tempos of time in every frame of reference
6. Mass is no longer amount of matter ☺, and so on...

And the only justification to all this was some not wise and incorrect explanation to the fact that in field of Earth gravity light travels with the same velocity in all directions, which of course for every conscious person, knowing that finite speed always depends on the frame of reference, implicates that speed of such a ray is different for example in frame of reference of Solar System, because Earth is moving in it. No one thought...?

// Albert Einstein zwyczajnie jak słoń w składzie porcelany bezkarnie złamał podstawowe prawa fizyki, więc:

1. Po pierwsze i największy „grzech”: skończona prędkość może nie zależeć od układu odniesienia, tak więc oczywiście istnieje limit prędkości, który może być wzięty jako stała w miejscu nieskończoności
2. Po drugie, czas i odległość są z gumy, więc nie ma równoczesnych zdarzeń w ogóle i nie istnieje stała odległość między dwoma punktami przestrzeni – wszystko zależy od prędkości
3. I to jeszcze nie wszystko, ponieważ to samo wydarzenie może być w różnych miejscach i czasie.
4. Prędkość nie jest już drogą podzieloną przez czas podróży, ktoś wie dlaczego? Ja wiem... ponieważ ustalono stałą prędkość światła jako plus nieskończoną.
5. Istnieje nieskończenie wiele temp czasu w każdym układzie odniesienia
6. Masa nie jest już dłużej miarą ilości materii☺, itd.

I jedynym usprawiedliwieniem dla tego wszystkiego było pewne niezbyt mądre i nieprawidłowe wyjaśnienie faktu, że w polu grawitacyjnym Ziemi światło porusza się z tą samą prędkością we wszystkich kierunkach, co dla osoby świadomej, że skończona prędkość zawsze zależy od układu odniesienia, oznacza, że prędkość takiego promienia światła na przykład w układzie odniesienia Układu Słonecznego jest inna, ponieważ Ziemia porusza się w nim. Nikt nie pomyślał?

## Appendix A - Plotnicki's equations – part I

## Theorem 1 – Płotnicki's equation with use of little Fermat theorem – the simplest case

**Important note:** where there is not stated otherwise, there variables with the same name but different indexes are different variables. Often for example  $a$  is a set of variables  $a_i$  for every  $i$  or set of variables  $a_{i,j}$  for every  $i, j$ , but only in these cases when it is stated so. Sometimes there is used a variable with name  $x_i$ , where there is comma after  $i$ , which means that it is set of  $x_{i,j}$  for every  $j$ . The same is for case  $x_{i,j}$  where comma is before  $i$ , which means that it is set of  $x_{j,i}$  for every  $j$ . And that is all – there is no other rules in variable names reading and identification. You will see that it is very clear notation when it comes to more complicated cases.

**Theorem:** There is infinitely many solutions for equation like this:

$$\sum_{i=1}^n c_i a_i^{x_i} = d b^z$$

where  $\gcd(\prod_{i=1}^n x_i, z) = 1$

where for every  $i$ :  $c_i, a_i, d, b$  are rationals and  $n, x_i, z$  are integers.

### Proof

First of all we can use little Fermat's theorem:

When  $z$  is prime and  $\gcd(\prod_{i=1}^n x_i, z) = 1$  then we can use little Fermat's theorem:

$$\left( p(r_i * \text{lcm}(x_1, \dots, x_n))^{z-1} \bmod z \right) = p, \quad \gcd(r_i * \text{lcm}(x_1, \dots, x_n), z) = 1 \quad , \quad \text{then } z \text{ divides } (qz - k)(r_i * \text{lcm}(x_1, \dots, x_n))^{z-1} + k$$

So we have infinitely many solutions in form:

$$\sum_{i=1}^n c_i \left( \left( \sum_{i=1}^n c_i l_i^{x_i} \right)^{\frac{(qz-k)*(r*\text{lcm}(x_1, \dots, x_n))^{z-1}}{k}} * l_i \right)^{x_i} = d \left( \left( \sum_{i=1}^n c_i l_i^{x_i} \right)^{\frac{1}{k}} \right)^{(qz-k)(r*\text{lcm}(x_1, \dots, x_n))^{z-1}+k}$$

For any integer  $r$  such that  $\gcd(r, z) = 1$ .

For any rationals  $c_i, d, l_i$ .

And for any integer  $k, q$  such that  $k < qz$  and  $k$  is prime or 1 and  $\sum_{i=1}^n c_i l_i^{x_i} = n^k$  then this equation could be solved the same way for  $k > 1$  and could be any  $l_i$  for  $k = 1$ .

In general we have rational solutions above and when  $\frac{\sum_{i=1}^n c_i l_i^{x_i}}{d}$  and for every  $i$ :  $c_i, l_i$  are integers,

then we have integer solutions.

QED.

**Example:**

$$wa^x + vb^y = c^z$$

We have:

$$\begin{aligned} w \left( (wl^x + vm^y)^{\frac{(qz-k)(xy)^{z-2}y}{k}} * l \right)^x + v \left( (wl^x + vm^y)^{\frac{(qz-k)(xy)^{z-2}x}{k}} * m \right)^y \\ = (wl^x + vm^y) \left( (wl^x + vm^y)^{\frac{1}{k}} \right)^{(qz-k)(xy)^{z-1}} \\ = \left( (wl^x + vm^y)^{\frac{1}{k}} \right)^k \left( (wl^x + vm^y)^{\frac{1}{k}} \right)^{(qz-k)(xy)^{z-1}} \\ = \left( (wl^x + vm^y)^{\frac{1}{k}} \right)^{(qz-k)(xy)^{z-1}+k} = \left( \left( (wl^x + vm^y)^{\frac{1}{k}} \right)^p \right)^z \end{aligned}$$

For  $w = v = 1$ :

$$\begin{aligned} \left( (l^x + m^y)^{\frac{(qz-k)(xy)^{z-2}y}{k}} * l \right)^x + \left( (l^x + m^y)^{\frac{(qz-k)(xy)^{z-2}x}{k}} * m \right)^y \\ = \left( (l^x + m^y)^{\frac{1}{k}} \right)^{(qz-k)(xy)^{z-1}+k} = \left( \left( (l^x + m^y)^{\frac{1}{k}} \right)^p \right)^z \end{aligned}$$

**Example**

$$2x^2 + 3x^3 = x^5$$

$$l = 2, m = 1$$

$$2l^2 + 3m^3 = 11$$

$$\begin{aligned} 2 \left( 11^{(5-1)(2*3)^{(5-2)*3}} * 2 \right)^2 + 3 \left( 11^{(5-1)(2*3)^{(5-2)*2}} * 2 \right)^3 &= 11^{(5-1)(2*3)^{5-1}} (11) = 11^{4*6^4+1} \\ &= 11^{5185} = (11^{1037})^5 \end{aligned}$$

## Theorem 2 – Płotnicki's equation with use of little Fermat theorem – simple case

Theorem : there is infinitely many solutions for equation like this:

$$\sum_{i=1}^n c_i a_i^{x_i} = d b^z$$

where  $\gcd(\prod_{i=1}^n x_i, z) = 1$

where for every  $i$ :  $c_i, a_i, d, b$  are rationals and  $n, x_i, z$  are integers.

for every  $i$ : for every rational  $l_i$  and for every  $j$ : for every rational  $p_j, t$  and every integer  $q_j, f$  that suffices equation:

$$\sum_{i=1}^n c_i l_i^{x_i} = d t^{f \cdot z} \sum_{j=1}^m p_j^{q_j}$$

where  $f$  could be 0, for every  $j$ :  $\gcd(q_j, z) = 1$ , we have infinitely many solutions:

$$\begin{aligned} \sum_{i=1}^n c_i \left( \prod_{j=1}^m p_j^{(t_j z - q_j) * \frac{(r_j * \text{lcm}(x_1, \dots, x_n))^{z-1}}{x_i}} * y^{\frac{\text{lcm}(x_1, \dots, x_n, z)}{x_i} * l_i} \right)^{x_i} &= \\ = \sum_{i=1}^n c_i l_i^{x_i} \prod_{j=1}^m p_j^{(t_j z - q_j) * \frac{(r_j * \text{lcm}(x_1, \dots, x_n))^{z-1}}{x_i}} * y^{\text{lcm}(x, z)} & \\ = d t^{f \cdot z} * \prod_{j=1}^m p_j^{(t_j z - q_j) * \frac{(r_j * \text{lcm}(x_1, \dots, x_n))^{z-1}}{x_i} + q_j} * y^{\text{lcm}(x, z)} &= d c^z \end{aligned}$$

Where for every  $j$ :  $\gcd(r_j, z) = 1$ .

Where  $c_i, d, l_i$  are any rationals and for every  $j$ :  $q_j < t_j z$ , where  $q_j, t_j$  are any integers.

In general we have rational solutions above, and when  $\frac{\sum_{i=1}^n c_i l_i^{x_i}}{d}$  and for every  $i$ :  $c_i, l_i$  are integers, then we have integer solutions.

Example

$$4x^5 + 2y^3 = x^2$$

For  $l_1 = 1, l_2 = 2$ :

$$4 * 1^5 + 2 * 2^3 = 4 + 2 * 8 = 20 = 2^2 * 5$$

$$4 \left( (2^2)^{(2-1) * \frac{(15)^{2-1}}{5}} (5)^{(2*2-1) * \frac{(15)^{2-1}}{5}} * 1 \right)^5 + 2 \left( (2^2)^{(2-1) * \frac{(15)^{2-1}}{3}} (5)^{(4-1) * \frac{(15)^{2-1}}{3}} * 2 \right)^3$$

$$= (2^2)^{(2-1)*(15)^{2-1}} (5)^{(4-1)*(15)^{2-1}} (4 * 1^5 + 2 * 2^3) = (2^2)^{15+1} (5)^{3*15+1} = ((2^2)^8 (5)^{23})^2$$

### Theorem 3 - Płotnicki's equation with use of little Fermat theorem - general case

Theorem : there is infinitely many solutions for equation like this:

$$\sum_{i=1}^n c_i \prod_{j=1}^{m_i} a_{i,j}^{x_{i,j}} = d \prod_{j=1}^{m_0} b_j^{z_j}$$

where  $\gcd\left(\prod_{i=1}^n \prod_{j=1}^{m_i} x_{i,j}, \prod_{j=1}^{m_0} z_j\right) = 1$

where for every  $i, j$ :  $c_i, a_{i,j}, d, b_j$  are rationals and  $n, m_i, x_{i,j}, z_j$  are integers.

for every  $i, j$ : for every rational  $l_{i,j}$  and every rational  $p_{i,j}, t_i$  and every integer  $q_{i,j}, f_i$  that suffices equation:

$$\sum_{i=1}^n c_i \prod_{j=1}^{m_i} l_{i,j}^{x_{i,j}} = d \prod_{i=1}^u \left( t_i^{f_i * z_i} \prod_{j=1}^{v_i} p_{i,j}^{q_{i,j}} \right)$$

where for every  $i$ :  $f_i$  could be 0, for every  $i, j$ :  $\gcd(q_{i,j}, z_i) = 1$ , we have infinitely many solutions:

$$\begin{aligned} & \sum_{i=1}^n c_i \prod_{j=1}^{m_i} \left( \prod_{s=1}^u \left( \prod_{k \in S_{i,j,s}} p_{s,k}^{\frac{(r_{s,k} z_s - q_{s,k}) * (r_{s,k} * \text{lcm}(x))^{z_s-1}}{x_{i,j}}} * \prod_{k \in T_{i,j,s}} y_{s,k}^{\frac{\text{lcm}(x, z_s)}{x_{i,j}}} \right) * l_{i,j} \right)^{x_{i,j}} \\ &= d \prod_{i=1}^u \left( t_i^{f_i * z_i} \prod_{j=1}^{v_i} p_{i,j}^{(t_{i,j} z_i - q_{i,j}) * (r_{i,j} * \text{lcm}(x))^{z_i-1} + q_{i,j}} \prod_{j=1}^{w_i} y_{i,j}^{\text{lcm}(x, z_i)} \right) \\ &= d \prod_{i=1}^u \left( t_i^{f_i} \prod_{j=1}^{v_i} p_{i,j}^{h_{i,j}} \prod_{j=1}^{w_i} y_{i,j}^{\frac{\text{lcm}(x, z_i)}{z_i}} \right)^{z_i} \end{aligned}$$

Where for every  $i, j$ :  $\gcd(r_{i,j}, z_i) = 1$ .

Where for every  $i, s$ :  $\bigcup_{j=1}^{m_i} S_{i,j,s} = \{1, \dots, v_i\}$ ,  $\bigcup_{j=1}^{m_i} T_{i,j,s} = \{1, \dots, w_i\}$ ,

for every  $i, j, k, s$  where  $j \neq k$ :  $S_{i,j,s} \cap S_{i,k,s} = \emptyset, T_{i,j,s} \cap T_{i,k,s} = \emptyset$ ,

$x$  is a set of all  $x_{i,j}$ ,  $z$  is a set of all  $z_i$ .

Where  $c_i, d, l_i$  are any rationals and for every  $s, k$ :  $q_{s,k} < t_{s,k} z_s$ , where  $q_{s,k}, t_{s,k}$  are any integers.

In general we have rational solutions above and when  $\frac{\sum_{i=1}^n c_i \prod_{j=1}^{m_i} l_{i,j}^{x_{i,j}}}{d}$  and for every  $i, j$ :  $c_i, l_{i,j}$  are integers, then we have integer solutions.

More generally:

$$\begin{aligned}
& \sum_{i=1}^n c_i \prod_{j=1}^{m_i} \left( \prod_{s=1}^u \left( \prod_{k \in U_{i,j,s}} p_{s,k}^{u_{i,j,s,k} * \frac{lcm(x)}{x_{i,j}}} * \prod_{k \in T_{i,j,s}} y_{s,k}^{\frac{lcm(x,z_s)}{x_{i,j}}} \right) * l_{i,j} \right)^{x_{i,j}} \\
& = d \prod_{i=1}^u \left( t_i^{f_i * z_i} \prod_{j=1}^{v_i} p_{i,j}^{(t_{i,j}z_i - q_{i,j}) * (r_{i,j} * lcm(x))^{z_i-1} + q_{i,j}} \prod_{j=1}^{w_i} y_{i,j}^{lcm(x,z_i)} \right) \\
& = d \prod_{i=1}^u \left( t_i^{f_i} \prod_{j=1}^{v_i} p_{i,j}^{h_{i,j}} \prod_{j=1}^{w_i} y_{i,j}^{\frac{lcm(x,z_i)}{z_i}} \right)^{z_i}
\end{aligned}$$

for every  $i, s$ :  $\bigcup_{j=1}^{m_i} U_{i,j,s} = \{1, \dots, v_s\}$ ,  $\bigcup_{j=1}^{m_i} T_{i,j,s} = \{1, \dots, w_s\}$ ,

for every  $i, j, k, s$  where  $j \neq k$ :  $T_{i,j,s} \cap T_{i,k,s} = \emptyset$ ,

for every  $i, j$ :  $z_i \mid ((t_{i,j}z_i - q_{i,j}) * lcm(x)^{z_i-1} + q_{i,j})$  {little Fermat theorem},

$x$  is a set of all  $x_{i,j}$ ,  $z$  is a set of all  $z_i$ .

Where for every  $i, s, k$ :  $\sum_{j=1}^{m_i} u_{i,j,s,k} = (t_{s,k}z_s - q_{s,k}) * (r_{s,k})^{z_s-1} * lcm(x)^{z_s-2}$

## **Appendix B – Plotnicki's equations – part II**

### Theorem 1 – useful theorem

Theorem:  $ab = t \prod_{i=1}^n c_i + x$ , has integer solution for every  $a$  for given  $c_i$ , and given  $t$  (2.) or  $x$  (1.), where  $\gcd(a, \prod_{i=1}^n c_i) = 1$  and  $\gcd(x, \prod_{i=1}^n c_i) = 1$ .

- 1.) So for every  $x, y$ , where  $\gcd(x, y) = 1$ :  $nx \bmod y = k, n \leq y$  has solution for every  $k$ :  
 $0 \leq k < y$ , in sequence  $p = \text{abs}(x - y)$ :  
 $p \bmod y, 2p \bmod y, \dots, yp \bmod y \Leftrightarrow nx \bmod y == (n + y)x \bmod y \Leftrightarrow$  every value of rest repeats every  $y * x$ , so between  $n$  and  $n + y$  there does not repeat any rest so every  $k$  is there. QED.
- 2.) So for every  $x, y$ , where  $\gcd(x, y) = 1$ :  $x \bmod y = k, n \leq y$  has solution for every  $k$ :  
 $0 \leq k < y$ , in sequence  $p = \text{abs}(x - y)$ :  
 $p \bmod y, 2p \bmod y, \dots, yp \bmod y \Leftrightarrow x \bmod y == (x + 1) \bmod y + 1 \Leftrightarrow$  every value of rest appears between  $x$  and  $x + y$ . QED.

## The simplest Diophantine equation and how to deal with $d$ (part I)

$$wa^x = vb^y$$

Where  $\gcd(x, y) = 1$ .

First of all we can divide equation by  $\gcd(w, v)$ , so we can assume  $\gcd(w, v) = 1$

$$w \left( v^p * w^k * u^{\frac{\text{lcm}(x,y)}{x}} \right)^x = v \left( w^q * v^l * u^{\frac{\text{lcm}(x,y)}{y}} \right)^y$$

Now we can solve  $qy = xk + 1, px = yl + 1$  {see *Theorem 1*}

$$w \left( v^p * w^{\frac{qy-1}{x}} * u^{\frac{\text{lcm}(x,y)}{x}} \right)^x = v \left( w^q * v^{\frac{px-1}{y}} * u^{\frac{\text{lcm}(x,y)}{y}} \right)^y$$

And these are all solutions when  $w$  and  $v$  are primes.

All solutions for:

$$\begin{aligned} w &= \prod_{i=1}^m w_i^{q_i} \\ v &= \prod_{i=1}^m v_i^{p_i} \\ w \left( \prod_{i=1}^m v_i^{\frac{\text{lcm}(p_i, p, x)}{x}} * \prod_{i=1}^m w_i^{\frac{\text{lcm}(q_i, q, y) - q_i}{x}} * u^{\frac{\text{lcm}(x,y)}{x}} \right)^x \\ &= v \left( \prod_{i=1}^m w_i^{\frac{\text{lcm}(q_i, q, y)}{y}} * \prod_{i=1}^m v_i^{\frac{\text{lcm}(p_i, p, x) - p_i}{y}} * u^{\frac{\text{lcm}(x,y)}{y}} \right)^y \end{aligned}$$

For three (where  $\gcd(x, y) = \gcd(x, z) = \gcd(y, z) = 1$ ):

$$\begin{aligned} w \left( v^p * f^{r_1} * w^{\frac{qy-1}{x}} * u^{\frac{\text{lcm}(x,y,z)}{x}} \right)^x &= v \left( w^q * f^{r_2} * v^{\frac{px-1}{y}} * u^{\frac{\text{lcm}(x,y,z)}{y}} \right)^y \\ &= f \left( w^{\frac{qy}{z}} * v^{\frac{px}{z}} * f^{\frac{r_1x-1}{z}} * u^{\frac{\text{lcm}(x,y,z)}{z}} \right)^z \\ r_2y = r_1x \Rightarrow r_1 &= h \frac{\text{lcm}(x, y)}{x}, r_2 = h \frac{\text{lcm}(x, y)}{y} \\ w \left( v^{p \frac{\text{lcm}(x,z)}{x}} * f^{h \frac{\text{lcm}(x,y)}{x}} * w^{\frac{q * \text{lcm}(y,z) - 1}{x}} * u^{\frac{\text{lcm}(x,y,z)}{x}} \right)^x &= v \left( w^{q \frac{\text{lcm}(y,z)}{y}} * f^{h \frac{\text{lcm}(x,y)}{y}} * v^{\frac{p * \text{lcm}(x,z) - 1}{y}} * u^{\frac{\text{lcm}(x,y,z)}{y}} \right)^y \\ &= f \left( w^{q \frac{\text{lcm}(y,z)}{z}} * v^{p \frac{\text{lcm}(x,z)}{z}} * f^{\frac{h * \text{lcm}(x,y) - 1}{z}} * u^{\frac{\text{lcm}(x,y,z)}{z}} \right)^z \end{aligned}$$

And there is solution for general case (where for every different  $i, j$ :  $\gcd(x_i, x_j) = 1$ ):

$$c_1 a_1^{x_1} = \cdots = c_n a_n^{x_n}$$

$$c_k \left( \prod_{i=1}^{k-1} c_i^{p_i \frac{lcm(x_1, \dots, x_{i-1}, x_{i+1}, \dots, x_n)}{x_k}} \prod_{i=k+1}^n c_i^{p_i \frac{lcm(x_1, \dots, x_{i-1}, x_{i+1}, \dots, x_n)}{x_k}} * c_k^{\frac{p_k * lcm(x_1, \dots, x_{k-1}, x_{k+1}, \dots, x_n) - 1}{x_k}} \right)^{x_k} * u^{\frac{lcm(x_1, \dots, x_n)}{x_k}}$$

And all solutions for:

$$c_i = \prod_{j=1}^{m_i} c_{i,j}^{p_{i,j}}$$

$$c_k \left( \prod_{i=1}^{k-1} \prod_{j=1}^{m_i} c_{i,j}^{\frac{lcm(p_i p_{i,j}, x_k)}{x_k}} * \prod_{i=k+1}^n \prod_{j=1}^{m_i} c_{i,j}^{\frac{lcm(p_i p_{i,j}, x_k)}{x_k}} * \prod_{j=1}^{m_k} c_{k,j}^{\frac{lcm(p_k p_{k,j}, x_k)}{x_k}} * lcm(x_1, \dots, x_{k-1}, x_{k+1}, \dots, x_n) - p_{k,j} * u^{\frac{lcm(x_1, \dots, x_n)}{x_k}} \right)^{x_k}$$

So first of all when  $x$  or  $y$  is odd we can solve:

$$wr_1^x + vr_2^y = 0$$

So we can solve for every  $f$  and  $\gcd(k, l) = 1$ :

$$w(gk)^x + v(gl)^y = fc^z$$

Using analogous method we can solve for every  $d$  and  $\gcd\left(\prod_{j=1}^{m_1} a_{1,j}^{x_{1,j}}, \prod_{j=1}^{m_2} a_{2,j}^{x_{2,j}}\right) = 1$

$$\sum_{i=1}^2 c_i \prod_{j=1}^{m_i} a_{i,j}^{x_{i,j}} = d \prod_{j=1}^{m_0} b_j^{z_j}$$

And we can easily find infinitely many solutions for:

$$\sum_{i=1}^n c_i a_i^{x_i} = ndb^z$$

$$\sum_{i=1}^n \sum_{j=1}^{m_i} c_j a_j^{x_j} = \sum_{i=1}^n m_i db^{z_i}$$

And for example:

$$\sum_{i=1}^n c_i a_i^{x_i} = \sum_{i=n+1}^{2n} c_i a_i^{x_i}$$

So we can find infinitely many solutions if at least half of factors of sum of equation has odd power or negative coefficient. What's more in such a case we can solve:

$$\sum_{i=1}^{2n} c_i r_i^{x_i} = 0$$

So we can solve for any  $d$  and  $\gcd(l_1, \dots, l_{2n}) = 1$ :

$$\sum_{i=1}^n c_i a_i^{x_i} = d b^z$$

So equation:

$$\sum_{i=1}^n c_i a_i^{x_i} = d b^z$$

that has at least half of factors with odd power or negative coefficient, can be solved always:

a.) When it has even number of factors of sum, then it can be solved with:

$$\sum_{i=1}^n c_{p(i)} a_{p(i)}^{x_{p(i)}} = \sum_{i=n+1}^{2n} -c_{p(i)} a_{p(i)}^{x_{p(i)}}$$

where  $p$  is some permutation of  $1 \dots n$ .

b.) When it has odd number of factors of sum, then it can be solved firstly with:

$$\sum_{i=1}^{2n} c_i r_i^{x_i} = 0$$

and then:

$$\sum_{i=1}^n c_{p(i)} r_{p(i)}^{x_{p(i)}} = \sum_{i=n+1}^{2n} -c_{p(i)} r_{p(i)}^{x_{p(i)}}$$

where  $p$  is some permutation of  $1 \dots n$ .

There is of course also a generalization:

$$\prod_{i=1}^{n_1} w_{1,i} a_{1,i}^{x_{1,i}} = \dots = \prod_{i=1}^{n_k} v_{k,i} b_{k,i}^{k,y}$$

So all that is said above applies also for general case of Płotnicki's equation.

So for example it can be used to solve for every  $d$  and  $\gcd(l) = 1$ :

$$\sum_{i=1}^2 c_i \prod_{j=1}^{m_i} a_{i,j}^{x_{i,j}} = d \prod_{j=1}^{m_0} b_j^{z_j}$$

but it is not for this article. I will probably write about it in my book that will come out next year.

And that all is not all. The same easy we can find solutions for:

$$\sum_{i=1}^{m_1} d_{1,i} b_{1,i}^{x_{1,i}} = \dots = \sum_{i=1}^{m_k} d_{k,i} b_{k,i}^{x_{k,i}} = c_1 a_1^{x_1} = \dots = c_n a_n^{x_n}$$

Where:

for every  $i, j, k, l$  where  $(i, j) <> (k, l)$ :  $\gcd(x_{i,j}, x_{k,l}) = 1$ .

for every  $i, j, k$ :  $\gcd(x_{i,j}, x_k) = 1$ .

for every  $i, j$  where  $i <> j$ :  $\gcd(x_i, x_j) = 1$ .

To find solutions it is enough to treat value of every  $\sum_{l=1}^{m_j} d_{j,i} l_{j,i}^{x_{j,i}}$  for every  $l_{j,i}$  for any  $i, j$  as a coefficient in equation. Then we have from equation above simply the same kind of equation for any  $l_{i,j}$  for any  $i, j$ :

$$a_{-1}^{\text{lcm}(x_1)} \sum_{i=1}^{m_1} d_{1,i} l_{1,i}^{x_{1,i}} = \dots = a_{-k}^{\text{lcm}(x_k)} \sum_{i=1}^{m_k} d_{k,i} l_{k,i}^{x_{k,i}} = c_1 a_1^{x_1} = \dots = c_n a_n^{x_n}$$

The same is possible for general case of Plotnicki's equations.

## Theorem 2 – Płotnicki's equation – simple case

Theorem: there is infinitely many solutions for equation like this:

$$\sum_{i=1}^n c_i a_i^{x_i} = d b^z$$

where  $\gcd(\prod_{i=1}^n x_i, z) = 1$

where for every  $i$ :  $c_i, a_i, d, b$  are rationals and  $n, x_i, z$  are integers.

for every  $i$ : for every rational  $l_i$  and for every  $j$ : for every rational  $p_j, t$  and every integer  $q_j, f$  that suffices equation:

$$\sum_{i=1}^n c_i l_i^{x_i} = d t^{f * z} \prod_{j=1}^m p_j^{q_j}$$

where  $f$  could be 0, for every  $j$ :  $\gcd(q_j, z) = 1$ , we have infinitely many solutions:

$$\begin{aligned} \sum_{i=1}^n c_i \left( \prod_{j=1}^m p_j^{(t_j + f_j * z) * \frac{r_j * \text{lcm}(x_1, \dots, x_n)}{x_i}} * y^{\frac{\text{lcm}(x_1, \dots, x_n, z)}{x_i}} * l_i \right)^{x_i} &= \\ = \sum_{i=1}^n c_i l_i^{x_i} \prod_{j=1}^m p_j^{(t_j + f_j * z) * r_j * \text{lcm}(x_1, \dots, x_n)} * y^{\text{lcm}(x_1, \dots, x_n, z)} & \\ = d t^{f * z} * \prod_{j=1}^m p_j^{(t_j + f_j * z) * r_j * \text{lcm}(x_1, \dots, x_n) + q_j} * y^{\text{lcm}(x_1, \dots, x_n, z)} &= d c^z \end{aligned}$$

Where for every  $i$ :  $r_i$  is any integer such that  $\gcd(r_i, z) = 1$

Where for every  $j$ :  $t_j$  is any integer such that  $z | (t_j * r_j * \text{lcm}(x_1, \dots, x_n) + q_j)$  {for details see: *Theorem 1*}

Where for every  $j$ :  $f_j$  is any integer.

In general we have rational solutions above and when  $\frac{\sum_{i=1}^n c_i l_i^{x_i}}{d}$ , and for every  $j$ :  $p_j, t_j, y$  are integers we have integer solutions.

And these are the only solutions for  $\gcd(a) > 1$  for most cases, where  $a$  is set of variables, which is proved for case  $va^x \pm wb^y = uc^z$ .

So for every  $l_i$  we have as much subclasses of solutions as much "images of divisibility" of given  $\sum_{i=1}^n \frac{c_i}{d} l_i^{x_i}$  exists in form:

$$t^{f * z} \prod_{j=1}^m p_j^{q_j}$$

So for given  $\gcd(a_1, \dots, a_n) = t^{f_z} \prod_{j=1}^m p_j^{t_j \operatorname{lcm}(x_1, \dots, x_n)}$

there is only one image of divisibility  $t^{f_z} \prod_{j=1}^m p_j^{q_j}$  for which are constant numbers of  $l_i$  such that  $\sum_{i=1}^n \frac{c_i}{d} l_i^{x_i} = t^{f_z} \prod_{j=1}^m p_j^{q_j}$ , which has only above solutions.

And if equation has one solution :  $\sum_{i=1}^n \frac{c_i}{d} l_i^{x_i} = b^z$ , then it has infinitely many solutions:

$$\sum_{i=1}^n \frac{c_i}{d} \left( g^{t * \frac{\operatorname{lcm}(x_1, \dots, x_n, z)}{x_i}} * l_i \right)^{x_i} = \sum_{i=1}^n \frac{c_i}{d} l_i^{x_i} g^{t * \operatorname{lcm}(x_1, \dots, x_n, z)} = \left( b * g^{t * \frac{\operatorname{lcm}(x_1, \dots, x_n, z)}{z}} \right)^z$$

for every g,t

And those are all solutions that can be derived from  $\sum_{i=1}^n \frac{c_i}{d} l_i^{x_i} = b^z$ .

Derivation also works when  $\gcd(z, \prod_{i=1}^n x_i) > 1$ .

**Definitons:**

When  $\gcd(a_1, \dots, a_n) = 1$  then it is not complex solution.

When  $\gcd(a_1, \dots, a_n) > 1$  then it is complex solution.

Where  $a$  is variables set.

And those are all solutions (derived from all not complex solutions) when there are not complex not derived solutions (when  $\gcd(\prod_{i=1}^n x_i, z) > 1$ ).

So putting both together, when we know all not complex solutions (that the amount of is constant number or zero and such  $a$  is small), we know all solutions of Diophantine equation.

### Theorem 3 – Płotnicki's equation – general case

Theorem: there is infinitely many solutions for equation like this:

$$\sum_{i=1}^n c_i \prod_{j=1}^{m_i} a_{i,j}^{x_{i,j}} = d \prod_{j=1}^{m_0} b_j^{z_j}$$

where  $\gcd\left(\prod_{i=1}^n \prod_{j=1}^{m_i} x_{i,j}, \prod_{j=1}^{m_0} z_j\right) = 1$

where for every  $i, j$ :  $c_i, a_{i,j}, d, b_j$  are rationals and  $n, m_i, x_{i,j}, z_j$  are integers.

for every  $i, j$ : for every rational  $l_{i,j}$  and every rational  $p_{i,j}, t_i$  and every integer  $q_{i,j}, f_i$  that suffices equation:

$$\sum_{i=1}^n c_i \prod_{j=1}^{m_i} l_{i,j}^{x_{i,j}} = d \prod_{i=1}^u \left( t_i^{f_i * z_i} \prod_{j=1}^{v_i} p_{i,j}^{q_{i,j}} \right)$$

where for every  $i$ :  $f_i$  could be 0, for every  $i, j$ :  $\gcd(q_{i,j}, z_i) = 1$ , we have infinitely many solutions:

$$\begin{aligned} & \sum_{i=1}^n c_i \prod_{j=1}^{m_i} \left( \prod_{s=1}^u \left( \prod_{k \in S_{i,j,s}} p_{s,k}^{(t_{s,k} + f_{s,k} * z_s) * \frac{r_{s,k} * lcm(x)}{x_{i,j}}} * \prod_{k \in T_{i,j,s}} y_{s,k}^{\frac{lcm(x, z_s)}{x_{i,j}}} \right) * l_{i,j} \right)^{x_{i,j}} \\ &= d \prod_{i=1}^u \left( t_i^{f_i * z_i} \prod_{j=1}^{v_i} p_{i,j}^{(t_{i,j} + f_{i,j} * z_i) * r_{i,j} * lcm(x) + q_{i,j}} \prod_{j=1}^{w_i} y_{i,j}^{\frac{lcm(x, z_i)}{z_i}} \right) \\ &= d \prod_{i=1}^u \left( t_i^{f_i} \prod_{j=1}^{v_i} p_{i,j}^{h_{i,j}} \prod_{j=1}^{w_i} y_{i,j}^{\frac{lcm(x, z_i)}{z_i}} \right)^{z_i} \end{aligned}$$

Where

for every  $i, s$ :  $\bigcup_{j=1}^{m_i} S_{i,j,s} = \{1, \dots, v_s\}$ ,  $\bigcup_{j=1}^{m_i} T_{i,j,s} = \{1, \dots, w_s\}$ ,

for every  $i, j, k, s$  where  $j \neq k$ :  $S_{i,j,s} \cap S_{i,k,s} = \emptyset$ ,  $T_{i,j,s} \cap T_{i,k,s} = \emptyset$ ,

for every  $i, j$ :  $t_{i,j}$  is any integer such that:  $z_i | ((t_{i,j}) * r_{i,j} * lcm(x) + q_{i,j})$  {for details see: *Theorem 1*},

for every  $i, j$ :  $f_{i,j}$  is any integer,

for every  $i, j$ :  $r_{i,j}$  is any integer such that  $\gcd(r_{i,j}, z_i) = 1$ ,

$x$  is a set of all  $x_{i,j}$ ,  $z$  is a set of all  $z_i$ .

In general we have rational solutions above and when  $\frac{\sum_{i=1}^n c_i \prod_{j=1}^{m_i} l_{i,j}^{x_{i,j}}}{d}$ ,  $p_{i,j}, t_i, y_k$  are integers we have integer solutions.

More generally:

$$\begin{aligned}
& \sum_{i=1}^n c_i \prod_{j=1}^{m_i} \left( \prod_{s=1}^u \left( \prod_{k \in U_{i,j,s}} p_{s,k}^{u_{i,j,s,k} * \frac{lcm(x)}{x_{i,j}}} * \prod_{k \in T_{i,j,s}} y_{s,k}^{\frac{lcm(x,z_i)}{x_{i,j}}} \right) * l_{i,j} \right)^{x_{i,j}} \\
& = d \prod_{i=1}^u \left( t_i^{f_i * z_i} \prod_{j=1}^{v_i} p_{i,j}^{(t_{i,j} + f_{i,j} * z_i) * r_{i,j} * lcm(x) + q_{i,j}} \prod_{j=1}^{w_i} y_{i,j}^{lcm(x,z_i)} \right) \\
& = d \prod_{i=1}^u \left( t_i^{f_i} \prod_{j=1}^{v_i} p_{i,j}^{h_{i,j}} \prod_{j=1}^{w_i} y_{i,j}^{\frac{lcm(x,z_i)}{z_i}} \right)^{z_i}
\end{aligned}$$

for every  $i, s$ :  $\bigcup_{j=1}^{m_i} U_{i,j,s} = \{1, \dots, v_s\}$ ,  $\bigcup_{j=1}^{m_i} T_{i,j,s} = \{1, \dots, w_s\}$ ,

for every  $i, j, k, s$  where  $j \neq k$ :  $T_{i,j,s} \cap T_{i,k,s} = \emptyset$ ,

for every  $i, j$ :  $z_i \mid ((t_{i,j}) * r_{i,j} * lcm(x) + q_{i,j})$  {for details see: *Theorem I*},

for every  $i, j$ :  $t_{i,j}$  is any integer such that:  $z_i \mid ((t_{i,j}) * r_{i,j} * lcm(x) + q_{i,j})$  {for details see: *Theorem I*},

for every  $i, j$ :  $f_{i,j}$  is any integer ( $f_i$  is completely other integer with other meaning),

for every  $i, j$ :  $r_{i,j}$  is any integer such that  $\gcd(r_{i,j}, z_i) = 1$ ,

$x$  is a set of all  $x_{i,j}$ ,  $z$  is a set of all  $z_i$ .

Where for every  $i, s, k$ :  $\sum_{j=1}^{m_i} u_{i,j,s,k} = (t_{s,k} + f_{s,k} * z_s) * r_{s,k}$

### Example 1

$$a^x + b^y c^z = d^w, \text{ where } \gcd(xyz, w) = 1$$

$$k^x + l^y m^z = p_1^{q_1} * \dots * p_m^{q_m} * t^{f * w}$$

Any divisor  $p_i^{(t_i + f_i * z) * r_i}$  below can be divided between variables  $b$  and  $c$  like this:  $p_i^{(t_i + f_i * z) * r_i} = p_i^{u_1 + u_2}$ , where  $p_i^{u_1}$  is for  $b$  and  $p_i^{u_2}$  is for  $c$ , where  $u_1$  or  $u_2$  can be 0. For example:

$$\begin{aligned}
& \left( \prod_{i=1}^m p_i^{(t_i+f_i*z)*\frac{r_i*lcm(x,y,z)}{x}} * \prod_{i=1}^k y_i^{\frac{lcm(x,y,z,w)}{x}} * k \right)^x \\
& + \left( \prod_{i \in P_1} p_i^{(t_i+f_i*z)*\frac{r_i*lcm(x,y,z)}{x}} * \prod_{i \in Q_1} y_i^{\frac{lcm(x,y,z,w)}{x}} * l \right)^y \\
& * \left( \prod_{i \in P_2} p_i^{(t_i+f_i*z)*\frac{r_i*lcm(x,y,z)}{x}} * \prod_{i \in Q_2} y_i^{\frac{lcm(x,y,z,w)}{x}} * m \right)^z \\
= & \prod_{i \in (P_1+P_2)} p_i^{(t_i+f_i*z)*\frac{r_i*lcm(x,y,z)}{x}+q_i} * t^{f*w} * \prod_{i \in (Q_1+Q_2)} y_i^{lcm(x,y,z,w)} = d^w
\end{aligned}$$

And simplier:

$$\begin{aligned}
& \left( \prod_{i=1}^m p_i^{(t_i+f_i*z)*\frac{r_i*lcm(x,y,z)}{x}} * (y_1 y_2)^{\frac{lcm(x,y,z,w)}{x}} * k \right)^x \\
& + \left( \prod_{i \in P_1} p_i^{(t_i+f_i*z)*\frac{r_i*lcm(x,y,z)}{x}} * (y_1)^{\frac{lcm(x,y,z,w)}{y}} * l \right)^y \\
& * \left( \prod_{i \in P_2} p_i^{(t_i+f_i*z)*\frac{r_i*lcm(x,y,z)}{x}} * (y_2)^{\frac{lcm(x,y,z,w)}{z}} * m \right)^z \\
= & \prod_{i \in (P_1+P_2)} p_i^{(t_i+f_i*z)*\frac{r_i*lcm(x,y,z)}{x}+q_i} * t^{f*w} * (y_1 y_2)^{lcm(x,y,z,w)} = d^w
\end{aligned}$$

Where  $P_1 + P_2 = \{1, \dots, m\}$ ,  $Q_1 + Q_2 = \{1, \dots, k\}$ ,  $P_1 \cap P_2 = \emptyset$ ,  $Q_1 \cap Q_2 = \emptyset$

For example:

$$a^2 + b^3 c^5 = d^7$$

$$2^2 + 2^3 * 2^5 = 260 = 26 * 10$$

$$t1 * (2 * 3 * 5) + 1 = 7q1$$

$$t2 * (2 * 3 * 5) + 1 = 7q2$$

$$t1 = 3, t2 = 3 + 7 = 10$$

so:

$$\begin{aligned}
& (26^{3*3*5} * 10^{10*3*5} * 2)^2 + (26^{3*2*3} * 2)^3 * (10^{10*2*3} * 2)^5 = (26)^{3*30+1} * 10^{10*30+1} \\
& = (26^{13} * 10^{43})^7
\end{aligned}$$

For  $d^w$  it will give all complex solutions.

Example 2

$b^y c^z$  can be calculated as  $f^{y+z}$ , but it will not give all possible solutions, but there still is a way to calculate them:

$$d^w - a^x = b^y c^z, \text{ where } \gcd(wx, yz) = 1$$

$$k^w - l^x = p_1^{q_1} * \dots * p_m^{q_m} * t_b^{f*y} * t_c^{g*z}$$

So  $p_i$  have to be selected such a way to construct  $b^y c^z$ .

For example :

$$d^7 - a^2 = b^3 c^5$$

$$2^7 - 2^2 = 124 = 2^2 * 31$$

$$2 * 7 * t1 + 2 = 3q1$$

$$2 * 7 * t2 + 1 = 5q2$$

$$t1 = 2, t2 = 1$$

$$\begin{aligned} ((61^{1*2} * 2^{2*2}) * 2)^7 - ((61^{1*7} * 2^{2*7}) * 2)^2 &= (2^7 - 2^2) * (2^{14} * 61^{14}) \\ &= (2^2 * 61) * (2^{28} * 61^{14}) = 2^{30} * 61^{15} = (2^{10})^3 * (61^3)^5 \end{aligned}$$

The same is for derivation:

$$\begin{aligned} &\left( g^{t_1 * \frac{\text{lcm}(x,y,z,w)}{w}} * h^{t_2 * \frac{\text{lcm}(x,y,z,w)}{w}} k \right)^w - \left( g^{t_1 * \frac{\text{lcm}(x,y,z,w)}{x}} h^{t_2 * \frac{\text{lcm}(x,y,z,w)}{x}} * l \right)^x \\ &= (k^w - l^x) \left( g^{t_1 * \frac{\text{lcm}(x,y,z,w)}{y}} \right)^y * \left( h^{t_1 * \frac{\text{lcm}(x,y,z,w)}{z}} \right)^z \end{aligned}$$

And the same is for combinations when there exist partial solved solution:

$$d^7 - a^3 = b^3 c^5$$

$$2^7 - 2^3 = 120 = 2^3 * (3 * 5)$$

There is always infinitely many complex not derived solutions only when  $\gcd(x, z) = 1$ , where  $x$  is multiplication of all powers except those that are at some position ( $z$ ); or there exists combination (there exist partially solved solution, eg.:  $d^7 - a^3 = b^3 c^5, 2^7 - 2^3 = 120 = 2^3 * (3 * 5)$ ), where the condition should be sufficed only for those  $x_{i,j}$  that are not solved; of course for example for  $d^{11} - a^2 = b^3 c^5$  even for partially solved solution  $(2^3) * (14^1)$  divisibilities could be exchanged  $3 \rightarrow 5, 1 \rightarrow 3$ ; and there exist always infinitely many complex derived solutions if there exist at least one solution – proved.

So in general this is the way to calculate all rational complex solutions of Diophantine equations where there exist such  $j$  that  $\gcd(x, z) = 1$ , where  $z$  is a multiplication of powers at some position in equation, eg.:  $2x^3 + 3y^5v^3 = 5z^7w^2$ , etc.



## How to deal with $d$ - part II

When we have solution for:

$$\sum_{i=1}^n c_i a_i^{x_i} = b^z$$

Where for every  $i$ :  $\gcd(x_i, z) = 1$ .

Then we can multiply both sides for example by  $d^{pz+1} = d^{q*lcm(x)}$ :

$$d^{q*lcm(x)} \sum_{i=1}^n c_i a_i^{x_i} = \sum_{i=1}^n c_i \left( d^{q \frac{lcm(x)}{x_i}} a_i \right)^{x_i} = d^{pz+1} b^z = d(d^p b)^z$$

Where  $x$  is a set of  $x_i$  for every  $i$ .

For every  $i$ : for every rational  $l_i$  and for every  $j$ : for every rational  $p_j, d_j, t$  and every integer  $q_j, v_j, u_j, f$  that suffices equation:

$$\sum_{i=1}^n c_i l_i^{x_i} = \prod_{j=1}^o d_j^{v_j} t^{f*z} \prod_{j=1}^m p_j^{q_j}, \text{ where } d = \prod_{j=1}^o d_j^{u_j}$$

where  $f$  could be 0, for every  $j$ :  $\gcd(q_j, z) = 1$ , we have infinitely many solutions:

$$\begin{aligned} \sum_{i=1}^n c_i \left( \prod_{j=1}^m p_j^{(t_j + f_j * z) * \frac{r_j * lcm(x_1, \dots, x_n)}{x_i}} * \prod_{j=1}^o d_j^{s_j * \frac{lcm(x)}{x_i}} * y^{\frac{lcm(x_1, \dots, x_n, z)}{x_i}} * l_i \right)^{x_i} &= \\ &= \sum_{i=1}^n c_i l_i^{x_i} \prod_{j=1}^m p_j^{(t_j + f_j * z) * r_j * lcm(x_1, \dots, x_n)} * \prod_{j=1}^o d_j^{s_j * lcm(x)} * y^{lcm(x_1, \dots, x_n, z)} \\ &= \prod_{j=1}^o d_j^{v_j} t^{f*z} * \prod_{j=1}^m p_j^{(t_j + f_j * z) * r_j * lcm(x_1, \dots, x_n) + q_j} * \prod_{j=1}^o d_j^{w_j * z + (u_j - v_j)} \\ &* y^{lcm(x_1, \dots, x_n, z)} = d c^z \end{aligned}$$

Where for every  $i$ :  $r_i$  is any integer such that  $\gcd(r_i, z) = 1$

Where for every  $j$ :  $t_j$  is any integer such that  $z|(t_j * r_j * lcm(x_1, \dots, x_n) + q_j)$  {for details see: *Theorem 1*}

Where for every  $j$ :  $w_j$  is any integer such that  $s_j * lcm(x) = w_j * z + (u_j - v_j)$  {for details see: *Theorem 1*}

Where for every  $j$ :  $f_j$  is any integer.

In general we have rational solutions above and when for every  $j$ :  $p_j, t_j, d_j, y$  are integers we have integer solutions.

And for  $n = 2$  that are all complex not derived solutions.

The same is for Płotnicki's equation with use of little Fermat theorem.

**The same is for:**

$$\sum_{i=1}^n c_i \prod_{j=1}^{m_i} a_{i,j}^{x_{ij}} = d \prod_{j=1}^{m_0} b_j^{z_j}$$

**Where we have simply just more possible places to place  $d^{q * lcm(x)}$**

Using Chinese remainder theorem we could also find solutions for:

$$\sum_{i=1}^n c_i a_i^{x_i} = d_1 b_1^{z_1} = \dots = d_k b_k^{z_k}$$

Where for every  $i, j$ :  $\gcd(x_i, z_j) = 1$ .

For every  $i$ : for every rational  $l_i$  and for every  $j$ : for every rational  $p_j, d_j$  and every integer  $q_j, v_j, u_j, f$  that suffices equation:

$$\sum_{i=1}^n c_i l_i^{x_i} = \prod_{j=1}^o d_j^{v_j} \prod_{j=1}^m p_j^{q_j}, \text{ where } d = \prod_{j=1}^o d_j^{u_j}$$

we have infinitely many solutions:

We have to find solution of:

for every  $i = 1, \dots, k, j = 1, \dots, m$ :

$$Q_j = -q_j \pmod{z_i}$$

$$Q_j = 0 \pmod{\text{lcm}(x)}$$

for every  $i = 1, \dots, k, j = 1, \dots, o$ :

$$V_j = u_j - v_j \pmod{z_i}$$

$$V_j = 0 \pmod{\text{lcm}(x)}$$

Then we have solutions in form:

$$\begin{aligned}
\sum_{i=1}^n c_i \left( \prod_{j=1}^m p_j^{x_i} * \prod_{j=1}^o d_j^{x_i} * y^{\frac{lcm(x,z)}{x_i}} * l_i \right)^{x_i} &= \sum_{i=1}^n c_i l_i^{x_i} \prod_{j=1}^m p_j^{q_j} * \prod_{j=1}^o d_j^{v_j} * y^{lcm(x,z)} \\
&= \prod_{j=1}^m p_j^{q_j+q_j} * \prod_{j=1}^o d_j^{v_j+v_j+(u_j-u_j)} * y^{lcm(x,z)} \\
&= \prod_{j=1}^m p_j^{q_j+q_j} * \prod_{j=1}^o d_j^{v_j-(u_j-v_j)+u_j} * y^{lcm(x,z)} = d_1 b_1^{z_1} = \dots = d_k b_k^{z_k}
\end{aligned}$$

Where  $x$  is a set of  $x_j$  for every  $j$ .

Where  $z$  is a set of  $z_j$  for every  $j$ .

Analogous solutions exist of course also for general case of Plotnicki's equations.

Of course I could use Chinese remainder theorem everywhere, but in general case of Plotnicki's equation this is not enough to give all solutions or it would be necessary to divide every such solution in two parts, which would not be elegant. So I decided not to use this theorem, especially from that reason that everywhere else it is enough to use single equation, so Chinese theorem is not needed. Of course results are the same.

**There is also a way to find solutions for:**

$$\sum_{i=1}^{m_1} c_{1,i} a_{1,i}^{x_{1,i}} = \dots = \sum_{i=1}^{m_k} c_{k,i} a_{k,i}^{x_{k,i}} = d_1 b_1^{z_1} = \dots = d_m b_m^{z_m}$$

**when we have:**

$$\sum_{i=1}^{m_1} c_{1,i} a_{1,i}^{x_{1,i}} = \dots = \sum_{i=1}^{m_k} c_{k,i} a_{k,i}^{x_{k,i}} = b_1^{z_1} = \dots = b_m^{z_m}$$

That I will probably describe in details in my coming next year book.

Here is simplified example for simple case of Plotnicki's equation:

$$d_1^{p_1 z_1 + 1} = d_1^{q_1 * \frac{lcm(x,z)}{z_1}}$$

...

$$d_m^{p_m z_m + 1} = d_m^{q_m * \frac{lcm(x,z)}{z_m}}$$

Where  $x$  is a set of  $x_{i,j}$  for every  $i, j$ .

Where  $z$  is a set of  $z_i$  for every  $i$ .

$$\prod_{i=1}^m d_i^{q_i * \frac{lcm(x,z)}{z_i}} \sum_{i=1}^{m_1} c_{1,i} a_{1,i}^{x_{1,i}} = \sum_{i=1}^{m_1} c_{1,i} \left( \prod_{j=1}^m d_j^{q_j * \frac{lcm(x,z)}{x_{1,j} z_j}} a_{1,i} \right)^{x_{1,i}} = \dots =$$

$$= \prod_{i=1}^m d_i^{q_i * \frac{lcm(x,z)}{z_i}} \sum_{i=1}^{m_k} c_{k,i} a_{k,i}^{x_{k,i}} = \sum_{i=1}^{m_k} c_{k,i} \left( \prod_{j=1}^m d_j^{q_j * \frac{lcm(x,z)}{x_{k,j} z_j}} a_{k,i} \right)^{x_{k,i}} =$$

$$= d_1^{p_1 z_1 + 1} \prod_{i=2}^m d_i^{q_i * \frac{lcm(x,z)}{z_i}} b_1^{z_1} = d_1 \left( d_1^{p_1} \prod_{i=2}^m d_i^{q_i * \frac{lcm(x,z)}{z_1 z_i}} b_1 \right)^{z_1} = \dots =$$

$$= d_1^{p_1 z_1 + 1} \prod_{i=1}^m d_i^{q_i * \frac{lcm(x,z)}{z_i}} b_1^{z_1} = d_m \left( d_m^{p_m} \prod_{i=1}^{m-1} d_i^{q_i * \frac{lcm(x,z)}{z_m z_i}} b_m \right)^{z_m}$$

## How to deal with $d$ - part III

...in:

$$\sum_{i=1}^n c_i \prod_{j=1}^{m_i} a_{i,j}^{x_{i,j}} = d \prod_{j=1}^{m_0} b_j^{z_j}$$

where  $\gcd\left(\prod_{i=1}^n \prod_{j=1}^{m_i} x_{i,j}, \prod_{j=1}^{m_0} z_j\right) = 1$

For this example

$$2x^3 + 3y^5v^3 = 5z^7w^2$$

it is enough to find such  $2k^3 + 3l^5m^3$  that is divisible by 5, which in this example is really very simple (eg:  $k = l = m = 1$ ) or solve in rational numbers without such a requirement. When  $c_1 = \dots = c_{n=2k} = c$  and at least half of  $x_i$  are odd it is simple to find such  $l_i$  that  $d$  divides  $\sum_{i=1}^{2k=n} c_i (du_i + (-1)^{g(x_i)})^{x_i}$ .

In general infinitely many complex not derived solution exist when  $\sum_{i=1}^n c_i \prod_{j=1}^{m_i} r_{i,j}^{x_{i,j}} = 0$  has a solution (which can be solved often with the same method and so on). Because then for any  $k_{i,j}$  for every  $i$  and  $j$ :  $d$  divides  $\sum_{i=1}^n c_i \prod_{j=1}^{m_i} (dk_{i,j} + r_{i,j})^{x_{i,j}}$

Imagine that we have for example equation  $\sum_{i=1}^n prime_i a_i^{prime_i} = prime_{n+1} b^{prime_{n+1}}$

Then we need to solve

$$\sum_{i=1}^n prime_i a_i^{prime_i} = 0$$

So for:

$$\sum_{i=1}^{n-1} i a_i^{prime_i} = i(-a_n)^{prime_n}$$

We use the same method and so on...

Then we go to the equation:

$$2x^2 + 3y^3 + 5z^5 = 7(-w)^7$$

Where we need to solve

$$2x^2 + 3y^3 = 5(-z)^5$$

And here we need to solve (see *The simplest Diophantine equation*):

$$2l_1^2 + 3l_2^3 = 0 \Leftrightarrow 2l_1^2 = 3(-l_2)^3 \Leftrightarrow 2(2 * 3^2 * k^3)^2 = 3(2 * 3 * k^2)^3 \Leftrightarrow$$

$$l_1 = (5 * l'_1 + 2 * 3^2 * k^3), l_2 = (5 * l'_2 - 2 * 3 * k^2)$$

For  $k = 1, l'_1 = 1, l'_2 = 2$ :

$$2(18 + 5)^2 + 3(10 - 6)^3 = 1058 + 192 = 1250 = 5 * 250$$

As we have  $l_1, l_2$  we can solve:

$$2x^2 + 3v^3 = 5(-z)^5$$

When we solve this, we can solve:

$$2x^2 + 3v^3 + 5z^5 = 7w^7$$

And so on... to the equation:

$$\sum_{i=1}^n prime_i a_i^{prime_i} = prime_{n+1} b^{prime_{n+1}}$$

That we can solve now.

The last method is to select all  $l_{i,j}$  divisible by  $d$  or select some subset of  $l_{i,j}$  to be divisible by  $d$  and calculate rest with this method that is showed above, for example for:

$$2x^2 + 3v^2 + 5z^3 = 7w^7$$

you could put  $7k$  to  $l_x$  and find solution to

$$3r_v^2 + 5r_z^3 = 0$$

Of course it is very simple (see *The simplest Diophantine equation*).

To find all solutions use a computer. Complexity of such an algorithm is  $O(d^n)$ .

## Theorem 4 – how equations can be simplified

Theorem: every equation that can be simplified using:

$$Q(x) * R(x) = q * R(x)$$

$$\frac{R(x)}{Q(x)} = \frac{R(x)}{q}$$

$$R(x)^{Q(x)} = R(x)^q$$

where  $R(X)$  is acceptable polynomial and  $Q(x)$  is every function that could give rational (in first and second rule) or integer (in third rule) result,

to the form of acceptable polynomial:

$$\sum_{i=1}^n c_i \prod_{j=1}^{m_i} a_{i,j}^{x_{i,j}} = d \prod_{j=1}^{m_0} b_j^{z_j}$$

where  $\gcd\left(\prod_{i=1}^n \prod_{j=1}^{m_i} x_{i,j}, \prod_{j=1}^{m_0} z_j\right) = 1$ , has infinitely many complex not derived solutions.

So there are two simple rules in a formulation of Plotnicki equation:

1. Use every variable always in the same power or in expression where it could be simplified to the constance.
2. Reduce, if you want, everything that does not introduce alone standing constance to expression.

So acceptable equation suffices mainly three conditions:

- a.) does not evaluate to expression that have some variable two times with different exponents or this variable can have set the same value in all places
- b.) does not evaluate to expression that have alone standing constance.
- c.)  $\gcd\left(\prod_{i=1}^n \prod_{j=1}^{m_i} x_{i,j}, \prod_{j=1}^{m_0} z_j\right) = 1$

So for example you may think that there is no such solution to the:

$$a^3 + b^5 c^7 = d^7$$

But you would be wrong, because you can put any number to  $c$  and get for example:

$$a^3 + 128b^5 = d^7$$

Other example is:

$$\frac{x+1}{x-1} (a^y + (b^z)^{c^2-d^3}) = e^u$$

Which can be simplified for example to ( $x = 3, c = 3, d = 2$ ):

$$2a^y + 2b^z = e^v$$

You can also solve equation like this:

$$a^x + b^y(c^z + d^w) = e^v, \text{ where } \gcd(xyzw, v) = 1$$

So any  $x^y$  can occur any number of time but under condition that all those occurrences can be simplified to  $x^y Q$ , where  $Q$  is every acceptable polynomial that haven't got  $x$  and any variable from outer expression or this variable can be set to the same value.

And by the way there is a simple rule that every QR can be always simplified, when  $Q$  or  $R$  is acceptable polynomial, by putting any number to every variable that  $R$  use (when  $Q$  is acceptable polynomial) or  $Q$  use (when  $R$  is acceptable polynomial). Then simply  $x^y Q = qx^y$ , where  $q$  is a constant or  $x^y Q = pQ$ , where  $p$  is a constant. So for example:

$$a^x(c^z - d^w) + b^y(c^z + d^w) = e^v$$

Could be very easily solved:

$$p(c^z - d^w) + q(c^z + d^w) = e^v$$

Or:

$$pa^x + qb^y = e^v$$

The same is for  $\frac{Q}{R}$  where  $Q$  is acceptable polynomial:

$$\frac{a^x + b^y}{(e^g - f^h)} + \frac{c^z}{(e^g + f^h)} - \frac{d^w}{(e^g - f^h)(e^g + f^h)} = e^v$$

Could be easily solved:

$$qa^x + qb^y + pc^z - pqe^v = d^w$$

There is of course a possibility to solve using the same metod equation like this:

$$(x^a + y^b z^c)(w^d - v^e) = p^r q^s$$

or:

$$(x^a + y^b z^c)(w^d - v^e) = (p^r)(q^s)$$

or:

$$\frac{(x^a + y^b z^c)}{(w^d - v^e)} = \frac{p^r q^s}{k^m l^n}$$

And that is not all, because you can solve equations like this:

$$(x^a + y^b z^c - p^r q^s)(w^d - v^e + f^g) = 0$$

Etc.

In the end you could think that you can not solve equation like this:

$$x^{10} + y^9 + z^6 + w^5 + v^3 + h^2 = 0$$

Because there is not such power  $f$  that  $\gcd\left(\frac{10*9*6*5*3*2}{f}, f\right) = 1$ , but you would be wrong, because you can solve it for example this way:

$$x^{10} + y^9 + z^6 + w^5 + v^3 + h^2 = 0$$

$$-w^5 = y^9 + z^6$$

$$-v^3 = x^{10} + h^2$$

$$(y^9 + z^6 + w^5) + (x^{10} + v^3 + h^2) = 0 + 0 = 0$$

The same easy you can solve:

$$7\sqrt{x^3 + y^5} = 2z^7$$

## Proof that there are not other complex not derived solutions

Proof for the case:

// Polish: Dowód dla przypadku

$$wa^x + vb^y = fc^z$$

$$w(g^p k)^x + v(g^q l)^y = f(g^r m)^z$$

Of course we can assume that  $\gcd(wk^x, vl^y, wk^x + vl^y) = s = 1$ , because when we align power of divisors of  $s$  to  $z$  then equation can be divided by these divisors which does not applies for other divisors of  $wk^x + vl^y$ .

Secondly, when we assume that  $\gcd(f, wk^x + vl^y) = \gcd(w, fm^z - vl^y) = \gcd(v, fm^z - wk^x) = 1$ , then coefficients  $w, v, f$  can be always choosed, because they do not depend on the  $k, l, m$ .

As you will notice, if  $\gcd(a, b, c) = g > 1$ ,  $\gcd(wk, vl) = 1$ , then at least two factors of sum must have  $g$  in the same power, so they must be aligned. In addition, you must ensure that all divisors of  $wk^x + vl^y$  had the power divisible by  $z$  at the right side of the equation. If some prime factor of  $g$  is aligned for the sum of the two factors and will be in power  $z$  for the third, then another prime factor can not be aligned for another pair of factors of sum in the equation, because it will lost alignment of this firstly aligned prime factor. What leads to the template solution presented in this document.

If we align divisors for concrete two factors of sum in eqation then we assume some  $l$  and  $k$ , which implicates what we need to align on the right side, so before we align some of them, there is no (we don't know any)  $m$  for  $fc^z$ , and so the alignment of two other factors of the sum in equation is not possible. If we tried to define in some moment such  $m$  on the basis of aligned to  $z$  dividers of  $wk^x + vl^y$ , then if we wanted to keep the  $\gcd(m, k') = \gcd(m, l') = 1$ , then it means that:

1' when  $m$  has all prime divisors of  $wk^x + vl^y$ :

$$k' = g_k \frac{k}{t_k}, l' = g_l \frac{l}{t_l}$$

Where  $t_k$  is eventually divisor of  $k$ ,  $g_k$  is eventually divisor of  $g^{\frac{\text{lcm}(x,y)}{x}}$ , and  $g_l$  is eventually divisor of  $g^{\frac{\text{lcm}(x,y)}{y}}$ .

Then  $\frac{g^{\text{lcm}(x,y)}(wk^x + vl^y)}{fm^z} = \frac{w\left(g^{\frac{\text{lcm}(x,y)}{x}}k\right)^x}{w(k')^x} = \frac{\left(g^{\frac{\text{lcm}(x,y)}{x}}k\right)^x}{g_k^x\left(\frac{k}{t_k}\right)^x} = t_k^x \left(\frac{g^{\frac{\text{lcm}(x,y)}{x}}}{g_k}\right)^x \Leftrightarrow g_k^x(wk^x + vl^y) =$

$fm^z t_k^x$ , but  $\gcd(wk^x + vl^y, ft_k^x) = 1$ , so  $ft_k^x p = g_k^x$ , but then  $p(wk^x + vl^y) = m^z$ , and that means that  $m$  has all divisors of  $c^z$  aligned, so  $m$  divides  $c$ , what is possible only at the end, when all divisors of  $c$  are aligned to  $z$ , so there is nothing to be aligned.

2' when  $m$  has not all prime divisors of  $wk^x + vl^y$ :

$$k' = g_k s_k \frac{k}{t_k}, l' = g_l s_l \frac{l}{t_l}, \gcd(s_k, t_k m) = \gcd(s_l, t_l m) = 1$$

Where  $t_k$  is eventually divisor of  $k$ ,  $w'$  is eventually divisor of  $w$ ,  $g_k$  is eventually divisor of  $g^{\frac{\text{lcm}(x,y)}{x}}$ , and  $g_l$  is eventually divisor of  $g^{\frac{\text{lcm}(x,y)}{y}}$ ,  $s_k, s_l$  are divisible at most by these prime divisors of  $wk^x + vl^y$  (in some powers), that does not divide  $m$ .

$$\frac{g^{\text{lcm}(x,y)}(wk^x + vl^y)}{fm^z} = \frac{w \left( g^{\frac{\text{lcm}(x,y)}{x}} k \right)^x}{w(k')^x} = \frac{\left( g^{\frac{\text{lcm}(x,y)}{x}} k \right)^x}{\left( g_k s_k \frac{k}{t_k} \right)^x} = \frac{t_k^x g^{\text{lcm}(x,y)}}{g_l^x s_k^x} \Leftrightarrow g_l^x s_k^x (wk^x + vl^y) = fm^z t_k^x , \text{ but}$$

$\gcd(wk^x + vl^y, ft_k^x) = 1$ , so  $ft_k^x p = g_k^x s_k^x$ , then  $p(wk^x + vl^y) = m^z$ , so  $m$  has all divisors of  $wk^x + vl^y$ . Contradiction.

**Proof for the case:**

// Dowód dla:

Secondly, when we assume that:

$$\begin{aligned} \gcd \left( d, \sum_{i=1}^2 c_i \prod_{j=1}^{m_i} l_{i,j}^{x_{i,j}} \right) &= \gcd \left( c_1, d \prod_{j=1}^{m_0} l_{b,j}^{z_j} - c_2 \prod_{j=1}^{m_2} l_{2,j}^{x_{2,j}} \right) \\ &= \gcd \left( c_2, d \prod_{j=1}^{m_0} l_{b,j}^{z_j} - c_1 \prod_{j=1}^{m_1} l_{1,j}^{x_{1,j}} \right) = 1 \end{aligned}$$

then coefficients  $c_1, c_2, d$  can be always choosed, because they do not depend on the  $l_{i,j}, l_{b,j}$ .

$$\sum_{i=1}^2 c_i \prod_{j=1}^{m_i} a_{i,j}^{x_{i,j}} = d \prod_{j=1}^{m_0} b_j^{z_j}$$

If we align divisors for concrete two factors of sum in equation then we assume some  $l_{i,j}$ , which implicates what we need to align on the right side, so before we align some of them, there is no (we don't know any)  $m_j$  for  $d \prod_{j=1}^{m_0} b_j^{z_j}$ , and so the alignment of two other factors of the sum in equation is not possible. If we tried to define in some moment such  $m_j$  on the basis of aligned to  $z_j$  dividers of  $\sum_{i=1}^2 c_i \prod_{j=1}^{m_i} l_{i,j}^{x_{i,j}}$ , then if we wanted to keep the  $\gcd(m_j, l_{i,j}') = 1$ , then it means that:

1'  $m_j$  has not all prime divisors  $\sum_{i=1}^2 c_i \prod_{j=1}^{m_i} l_{i,j}^{x_{i,j}}$ . Then

$$l_{i,j}' = g_{i,j} s_{i,j} \frac{l_{i,j}}{t_{i,j}}$$

$$\gcd(s_{i,j}, t_{i,j} m) = 1$$

$$\frac{g^{\text{lcm}(x)} \sum_{i=1}^2 c_i \prod_{j=1}^{m_i} l_{i,j}^{x_{i,j}}}{d \prod_{j=1}^{m_0} m_j^{z_j}} = \frac{c_i g^{\text{lcm}(x)} \prod_{j=1}^{m_i} (l_{i,j})^{x_{i,j}}}{c_i \prod_{j=1}^{m_i} (l'_{i,j})^{x_{i,j}}} = \frac{g^{\text{lcm}(x)} \prod_{j=1}^{m_i} (l_{i,j})^{x_{i,j}}}{\prod_{j=1}^{m_i} \left( g_{i,j} s_{i,j} \frac{l_{i,j}}{t_{i,j}} \right)^{x_{i,j}}} = \frac{\prod_{j=1}^{m_i} t_{i,j}^{x_{i,j}} g^{\text{lcm}(x)}}{\prod_{j=1}^{m_i} (g_{i,j} s_{i,j})^{x_{i,j}}}$$

$\Leftrightarrow \prod_{j=1}^{m_i} (g_{i,j} s_{i,j})^{x_{i,j}} \sum_{i=1}^2 \left( c_i \prod_{j=1}^{m_i} l_{i,j}^{x_{i,j}} \right) = \prod_{j=1}^{m_i} t_{i,j}^{x_{i,j}} d \prod_{j=1}^{m_0} m_j^{z_j}$ , but  
 $\gcd \left( \sum_{i=1}^2 \left( c_i \prod_{j=1}^{m_i} l_{i,j}^{x_{i,j}} \right), d \prod_{j=1}^{m_i} t_{i,j}^{x_{i,j}} \right) = 1$ , so  $d \prod_{j=1}^{m_i} t_{i,j}^{x_{i,j}} p = \prod_{j=1}^{m_i} (g_{i,j} s_{i,j})^{x_{i,j}}$ , then  
 $p \left( \sum_{i=1}^2 \left( c_i \prod_{j=1}^{m_i} l_{i,j}^{x_{i,j}} \right) \right) = \prod_{j=1}^{m_0} m_j^{z_j}$ , so  $m_j$  has all divisors of  $\sum_{i=1}^2 \left( c_i \prod_{j=1}^{m_i} l_{i,j}^{x_{i,j}} \right)$ . Contradiction.

So for every  $j$ :  $m_j$  has all prime divisors of  $\sum_{i=1}^2 c_i \prod_{j=1}^{m_i} l_{i,j}^{x_{i,j}}$ .

2'  $m_j$  has all prime divisors of  $\sum_{i=1}^2 c_i \prod_{j=1}^{m_i} l_{i,j}^{x_{i,j}}$ . Then

$$l'_{i,j} = g_{i,j} \frac{l_{i,j}}{t_{i,j}}$$

and:

$$\frac{g^{\text{lcm}(x)} \sum_{i=1}^2 c_i \prod_{j=1}^{m_i} l_{i,j}^{x_{i,j}}}{d \prod_{j=1}^{m_0} m_j^{z_j}} = \frac{c_i g^{\text{lcm}(x)} \prod_{j=1}^{m_i} (l_{i,j})^{x_{i,j}}}{c_i \prod_{j=1}^{m_i} (l'_{i,j})^{x_{i,j}}} = \frac{g^{\text{lcm}(x)} \prod_{j=1}^{m_i} (l_{i,j})^{x_{i,j}}}{\prod_{j=1}^{m_i} \left( c'_{i,j} \frac{l_{i,j}}{t_{i,j}} \right)^{x_{i,j}}} = \frac{\prod_{j=1}^{m_i} t_{i,j}^{x_{i,j}} g^{\text{lcm}(x)}}{\prod_{j=1}^{m_i} (g_{i,j})^{x_{i,j}}}$$

$$\Leftrightarrow \prod_{j=1}^{m_i} (g_{i,j})^{x_{i,j}} \sum_{i=1}^2 c_i \prod_{j=1}^{m_i} l_{i,j}^{x_{i,j}} = \prod_{j=1}^{m_i} t_{i,j}^{x_{i,j}} d \prod_{j=1}^{m_0} m_j^{z_j}$$

but  $\gcd \left( \sum_{i=1}^2 c_i \prod_{j=1}^{m_i} l_{i,j}^{x_{i,j}}, d \prod_{j=1}^{m_i} t_{i,j}^{x_{i,j}} \right) = 1$ , so  $d \prod_{j=1}^{m_i} t_{i,j}^{x_{i,j}} p = \prod_{j=1}^{m_i} (g_{i,j})^{x_{i,j}}$ , but then  
 $p \sum_{i=1}^2 c_i \prod_{j=1}^{m_i} l_{i,j}^{x_{i,j}} = \prod_{j=1}^{m_0} m_j^{z_j}$ . That means that for every  $j$ :  $m_j$  has all divisors of  $b_j^{z_j}$  aligned, so  
 $m_j$  divides  $b_j$ , what is possible only at the end, when all divisors of  $b_j$  are aligned to  $z_j$ , so there is  
nothing to be aligned.

It can be probably proved also for more complex equations, but it is much more complicated.  
Probably for most, if not all, equations presented solutions are all solutions for  $\gcd(a) > 1$ .

// Polish:

Po drugie, kiedy założymy, że  $\gcd(f, wk^x + vl^y) = \gcd(w, fm^z - vl^y) = \gcd(v, fm^z - wk^x) = 1$ , wtedy współczynniki  $w, v, f$  mogą być zawsze dobrane, ponieważ nie zależą od  $k, l, m$ .

Jak łatwo zauważyc, jeśli  $\gcd(a, b, c) = g > 1$ ,  $\gcd(wk, vl) = 1$ , to przynajmniej dwa czynniki sumy muszą mieć  $g$  w tej samej potędze, czyli muszą być wyrównane. Dodatkowo trzeba zadbać o to, żeby wszystkie podzielniki  $wk^x + vl^y$  miały potęgę podzielną przez  $z$  po prawej stronie równania. Jeśli jakiś czynnik pierwszy  $g$  zostanie wyrównany dla danych dwóch czynników sumy i będzie w potędze  $z$  dla trzeciego czynnika, to inny czynnik pierwszy  $g$  nie może być wyrównany dla innej pary czynników sumy równania, bo zostanie utracone wyrównanie do  $z$  tego pierwszego czynnika. Co już prowadzi wprost do szablonu rozwiązania przedstawionego w tym dokumencie.

Jeśli wyrównujemy podzielniki dla dwóch czynników dodawania w wyrażeniu to zakładamy jakieś  $l$  i  $k$ , z których wynika jakie podzielniki musimy wyrównać do  $z$  po prawej stronie, a więc zanim nie wyrównamy pewnych podzielników nie istnieje żadne (nie znamy żadnego)  $m$  dla  $fc^z$ , a więc wyrównanie dwóch innych czynników równania nie jest możliwe. Gdybyśmy próbowali określić w pewnym momencie takie  $m$  na podstawie wyrównanych do  $z$  podzielników  $wk^x + vl^y$ , to gdybyśmy chcieli zachować  $\gcd(m, k') = \gcd(m, l') = 1$ , to okazałoby się, że:

1'  $m$  ma wszystkie pierwsze podzielniki  $wk^x + vl^y$

$$k' = g_k \frac{k}{t_k}, l' = g_l \frac{l}{t_l}$$

Gdzie  $t_k$  to ewentualny podzielnik  $k$ , a  $g_k$  i  $g_l$  to ewentualne podzielniki odpowiednio  $g^{\frac{\text{lcm}(x,y)}{x}}, g^{\frac{\text{lcm}(x,y)}{y}}$ .

$$\text{i że } \frac{g^{\text{lcm}(x,y)}(wk^x + vl^y)}{fm^z} = \frac{w \left( g^{\frac{\text{lcm}(x,y)}{x}} k \right)^x}{wk'^x} = \frac{\left( g^{\frac{\text{lcm}(x,y)}{x}} k \right)^x}{\left( g_k \frac{k}{t_k} \right)^x} = \frac{t_k^x g^{\text{lcm}(x,y)}}{g_k^x} \Leftrightarrow g_k^x (wk^x + vl^y) = fm^z t_k^x, \text{ ale}$$

$\gcd(wk^x + vl^y, ft_k^x) = 1$ , więc  $ft_k^x p = g_k^x$ , ale wtedy  $p(wk^x + vl^y) = m^z$ , co by oznaczało, że  $m$  ma wszystkie wyrównane podzielniki  $c^z$ , więc  $m$  dzieli  $c$ , co jest możliwe tylko na samym końcu, gdy wszystkie podzielniki  $c$  są już wyrównane do  $z$ , więc nie ma co wyrównywać.

2'  $m$  nie ma wszystkich podzielników  $wk^x + vl^y$

$$k' = g_k s_k \frac{k}{t_k}, l' = g_l s_l \frac{l}{t_l}, \gcd(s_k, t_k m) = \gcd(s_l, t_l m) = 1$$

Gdzie  $t_k$  to ewentualny podzielnik  $k$ , a  $w'$  to ewentualny podzielnik  $w$ ,  $g_k$  i  $g_l$  to ewentualne podzielniki odpowiednio  $g^{\frac{\text{lcm}(x,y)}{x}}, g^{\frac{\text{lcm}(x,y)}{y}}$ , a  $s_k$  jest podzielne tylko conajwyżej przez te podzielniki pierwsze  $wk^x + vl^y$  (w pewnych potęgach), przez które nie jest podzielne  $m$ .

$$\frac{g^{\text{lcm}(x,y)}(wk^x + vl^y)}{fm^z} = \frac{w \left( g^{\frac{\text{lcm}(x,y)}{x}} k \right)^x}{w(k')^x} = \frac{\left( g^{\frac{\text{lcm}(x,y)}{x}} k \right)^x}{\left( g_k s_k \frac{k}{t_k} \right)^x} = \frac{t_k^x g^{\text{lcm}(x,y)}}{g_k^x s_k^x} \Leftrightarrow g_k^x s_k^x (wk^x + vl^y) = fm^z t_k^x, \text{ ale}$$

$\gcd(wk^x + vl^y, ft_k^x) = 1$ , więc  $ft_k^x p = g_k^x s_k^x$ , wtedy  $p(wk^x + vl^y) = m^z$ , więc  $m$  ma wszystkie podzielniki  $wk^x + vl^y$ . Sprzeczność.

Proof for the case:

// Dowód dla:

Po drugie, jeśli założymy, że:

$$\gcd \left( d, \sum_{i=1}^2 c_i \prod_{j=1}^{m_i} l_{i,j}^{x_{i,j}} \right) = \gcd \left( c_1, d \prod_{j=1}^{m_0} l_{b,j}^{z_j} - c_2 \prod_{j=1}^{m_2} l_{2,j}^{x_{2,j}} \right)$$

$$= \gcd \left( c_2, d \prod_{j=1}^{m_0} l_{b,j}^{z_j} - c_1 \prod_{j=1}^{m_1} l_{1,j}^{x_{1,j}} \right) = 1$$

wtedy współczynniki  $c_1, c_2, d$  can moga być zawsze dobrane, ponieważ nie zależy od  $l_{i,j}, l_{b_j}$ .

$$\sum_{i=1}^2 c_i \prod_{j=1}^{m_i} a_{i,j}^{x_{i,j}} = d \prod_{j=1}^{m_0} b_j^{z_j}$$

Jeśli wyrównujemy podzielniki dla dwóch czynników dodawania w wyrażeniu to zakładamy jakieś  $l_{i,j}$ , z których wynika jakie podzielniki musimy wyrównać do  $z_i$  po prawej stronie, a więc zanim nie wyrównamy pewnych podzielników nie istnieje żadne (nie znamy żadnego)  $m$  dla  $d \prod_{j=1}^{m_0} b_j^{z_j}$ , a więc wyrównanie dwóch innych czynników równania nie jest możliwe. Gdybyśmy próbowali określić w pewnym momencie takie  $m$  na podstawie wyrównanych do  $z_i$  podzielników  $\sum_{i=1}^2 c_i \prod_{j=1}^{m_i} l_{i,j}^{x_{i,j}}$ , to gdybyśmy chcieli zachować  $\gcd(m, l'_{i,j}) = 1$ , to okazałoby się, że:

1'  $m_j$  ma wszystkie pierwsze podzielniki  $\sum_{i=1}^2 c_i \prod_{j=1}^{m_i} l_{i,j}^{x_{i,j}}$

$$l'_{i,j} = g_{i,j} \frac{l_{i,j}}{t_{i,j}}$$

i że:

$$\begin{aligned} \frac{g^{\text{lcm}(x)} \sum_{i=1}^2 c_i \prod_{j=1}^{m_i} l_{i,j}^{x_{i,j}}}{d \prod_{j=1}^{m_0} m_j^{z_j}} &= \frac{c_i g^{\text{lcm}(x)} \prod_{j=1}^{m_i} (l_{i,j})^{x_{i,j}}}{c_i \prod_{j=1}^{m_i} (l'_{i,j})^{x_{i,j}}} = \frac{g^{\text{lcm}(x)} \prod_{j=1}^{m_i} (l_{i,j})^{x_{i,j}}}{\prod_{j=1}^{m_i} \left( g_{i,j} \frac{l_{i,j}}{t_{i,j}} \right)^{x_{i,j}}} = \frac{\prod_{j=1}^{m_i} t_{i,j}^{x_{i,j}} g^{\text{lcm}(x)}}{\prod_{j=1}^{m_i} (g_{i,j})^{x_{i,j}}} \\ &\Leftrightarrow \prod_{j=1}^{m_i} (g_{i,j})^{x_{i,j}} \sum_{i=1}^2 c_i \prod_{j=1}^{m_i} l_{i,j}^{x_{i,j}} = d c_i \prod_{j=1}^{m_i} t_{i,j}^{x_{i,j}} \prod_{j=1}^{m_0} m_j^{z_j} \end{aligned}$$

ale  $\gcd(\sum_{i=1}^2 c_i \prod_{j=1}^{m_i} l_{i,j}^{x_{i,j}}, d \prod_{j=1}^{m_i} t_{i,j}^{x_{i,j}}) = 1$ , więc  $d \prod_{j=1}^{m_i} t_{i,j}^{x_{i,j}} p = \prod_{j=1}^{m_i} (g_{i,j})^{x_{i,j}}$ , ale wtedy  $p \sum_{i=1}^2 c_i \prod_{j=1}^{m_i} l_{i,j}^{x_{i,j}} = \prod_{j=1}^{m_0} m_j^{z_j}$ , co by oznaczało, że  $m_j$  ma wszystkie wyrównane podzielniki  $b_j^{z_j}$ , więc  $m_j$  dzieli  $b_j$ , co jest możliwe tylko na samym końcu, gdy wszystkie podzielniki  $b_j$  są już wyrównane do  $z_j$ , więc nie ma co wyrównywać.

2'  $m_j$  nie ma wszystkich podzielników  $\sum_{i=1}^2 c_i \prod_{j=1}^{m_i} l_{i,j}^{x_{i,j}}$

$$l'_{i,j} = g_{i,j} s_{i,j} \frac{l_{i,j}}{t_{i,j}}$$

$$\gcd(s_{i,j}, t_{i,j} m) = 1$$

$$\frac{g^{\text{lcm}(x)} \sum_{i=1}^2 c_i \prod_{j=1}^{m_i} l_{i,j}^{x_{i,j}}}{d \prod_{j=1}^{m_0} m_j^{z_j}} = \frac{c_i g^{\text{lcm}(x)} \prod_{j=1}^{m_i} (l_{i,j})^{x_{i,j}}}{c_i \prod_{j=1}^{m_i} (l'_{i,j})^{x_{i,j}}} = \frac{g^{\text{lcm}(x)} \prod_{j=1}^{m_i} (l_{i,j})^{x_{i,j}}}{\prod_{j=1}^{m_i} \left( g_{i,j} s_{i,j} \frac{l_{i,j}}{t_{i,j}} \right)^{x_{i,j}}} = \frac{\prod_{j=1}^{m_i} t_{i,j}^{x_{i,j}} g^{\text{lcm}(x)}}{\prod_{j=1}^{m_i} (g_{i,j} s_{i,j})^{x_{i,j}}}$$

$$\Leftrightarrow \prod_{j=1}^{m_i} (g_{i,j} s_{i,j})^{x_{i,j}} \sum_{i=1}^2 \left( c_i \prod_{j=1}^{m_i} l_{i,j}^{x_{i,j}} \right) = d c_i \prod_{j=1}^{m_i} t_{i,j}^{x_{i,j}} \prod_{j=1}^{m_0} m_j^{z_j}$$

ale  $\gcd\left(\sum_{i=1}^2 \left(c_i \prod_{j=1}^{m_i} l_{i,j}^{x_{i,j}}\right), d \prod_{j=1}^{m_i} t_{i,j}^{x_{i,j}}\right) = 1$ , więc  $d \prod_{j=1}^{m_i} t_{i,j}^{x_{i,j}} p = \prod_{j=1}^{m_i} (g_{i,j} s_{i,j})^{x_{i,j}}$ , wtedy  $p \left(\sum_{i=1}^2 \left(c_i \prod_{j=1}^{m_i} l_{i,j}^{x_{i,j}}\right)\right) = \prod_{j=1}^{m_0} m_j^{z_j}$ , więc  $m_j$  ma wszystkie podzielniki  $\sum_{i=1}^2 \left(c_i \prod_{j=1}^{m_i} l_{i,j}^{x_{i,j}}\right)$ . Sprzeczność.

Dowód da się prawdopodobnie przeprowadzić także dla bardziej złożonych równań, jednak jest to o wiele bardziej skomplikowane. Prawdopodobnie dla większości, jeśli nie wszystkich, równań przedstawione rozwiązania są wszystkimi rozwiązaniami dla  $\gcd(a) > 1$ .

## Proof – when there are complex not derived solutions

There are complex not derived solutions only when

$$\gcd\left(\prod_{i=1}^n \prod_{j=1}^{m_i} x_{i,j}, \prod_{j=1}^{m_0} z_j\right) = 1$$

Proof for the case:

$$wa^x + vb^y = fc^z$$

If for each  $w = x$  or  $y$  or  $z$ :  $\gcd\left(\frac{xyz}{w}, w\right) > 1$ , it is impossible to align the powers by this method, so the only possible alignment is:

// Jeśli dla każdego  $w = x$  lub  $y$  lub  $z$ :  $\gcd\left(\frac{xyz}{w}, w\right) > 1$ , to nie da się wyrównać potęg tą metodą, więc jedyne możliwe wyrównanie to:

$$w \left( g^{\frac{\text{lcm}(x,y,z)}{x}} k \right)^x + v \left( g^{\frac{\text{lcm}(x,y,z)}{y}} l \right)^y = f \left( g^{\frac{\text{lcm}(x,y,z)}{z}} m \right)^z$$

$$\frac{w}{f} \left( g^{\frac{\text{lcm}(x,y,z)}{x}} k \right)^x + \frac{v}{f} \left( g^{\frac{\text{lcm}(x,y,z)}{y}} l \right)^y = \left( g^{\frac{\text{lcm}(x,y,z)}{z}} \right)^z \left( \frac{w}{f} k^x + \frac{v}{f} l^y \right) = c^z$$

Then, as can be seen  $\frac{w}{f} k^x + \frac{v}{f} l^y = m^z$ , so we have a solution. Hence the equation has complex not derived solution then and only then when for some  $w = x$  or  $y$  or  $z$ :  $\gcd\left(\frac{xyz}{w}, w\right) = 1$ , and has an infinite number of them.

// Wtedy jak widać  $\frac{w}{f} k^x + \frac{v}{f} l^y = m^z$ , czyli mamy rozwiązanie pochodne. Stąd równanie to ma złożone niepochodne rozwiązania wtedy i tylko wtedy gdy dla pewnego  $w = x$  lub  $y$  lub  $z$ :  $\gcd\left(\frac{xyz}{w}, w\right) = 1$ , i ma ich nieskończenie wiele.

More general proof

$$\sum_{i=1}^2 \frac{c_i}{d} \prod_{j=1}^{m_i} \left( \prod_{k=1}^s y_k^{u_{i,j,k} \frac{\text{lcm}(x,z)}{x_{i,j}}} * l_{i,j} \right)^{x_{i,j}} = \sum_{i=1}^2 \frac{c_i}{d} \prod_{j=1}^{m_i} l_{i,j}^{x_{i,j}} \prod_{i=1}^{m_0} \left( \prod_{k=1}^s y_k^{p_i \frac{\text{lcm}(x,z)}{z_i}} \right)^{z_i} = \prod_{j=1}^{m_0} b_j^{z_j}$$

Where for every  $i, k$ :  $\sum_{j=1}^{m_i} u_{i,j,k} = p_f$

Then:  $\sum_{i=1}^2 \frac{c_i}{d} \prod_{j=1}^{m_i} l_{i,j}^{x_{i,j}} = \prod_{j=1}^{m_0} B_j^{z_j}$ , so we have derived solution. So equation has complex not derived solutions then and only then when for some  $z = x_1$  or ... or  $x_2$ :  $\gcd(x, z) = 1$ .

Where  $x_i = \prod_{j=1}^{m_i} x_{i,j}$ ,  $x = \prod_{i=1}^n x_i$ .

## Simultaneous Plotnicki's equations

And if there is a solution for:  $\sum_{k=1}^n (c_{i,k} - c_{j,k}) \prod_{j=1}^{m_i} l_{k,j}^{x_{k,j}} = 0$ , and  $\gcd(a) > 1$  then and only then (because otherwise  $b_i \neq b_j$ ) simultaneous equation has infinitely many solutions:

$$\sum_{k=1}^n c_{i,k} \prod_{j=1}^{m_i} (P_{k,j} * l_{k,j})^{x_{k,j}} = b^z$$

where  $c_{i,j}$  are rationals.

So for two equations there is always a solution when there is at least one such  $x_k$  that  $\gcd(x \text{ without } x_k, x_k) = 1$ , because then it is Plotnicki's equation.

### Example 1

$$\begin{cases} x^2 + y^3 = z^5 \\ 2x^2 - 3y^3 = z^5 \end{cases}$$

There is the smallest  $l_i$  such that  $l_1^2 + l_2^3 = 2l_1^2 - 3l_2^3 \Leftrightarrow \left(\frac{l_1}{2}\right)^2 = l_2^3 : l_1 = 2x^3 = 2, l_2 = x^2 = 1, l_1^2 + l_2^3 = 5$ , so:

$$t_1 = 4 \Rightarrow 5|(2 * 3 * t_1 + 1)$$

$$\begin{cases} (5^{3*4} * 2)^2 + (5^{2*4} * 1)^3 = (2^2 + 1^3) * 5^{24} = 5 * 5^{24} = 5^{25} = (5^5)^5 \\ 2(5^{3*4} * 2)^2 - 3(5^{2*4} * 1)^2 = (2 * 2^2 - 3 * 1^3) * 5^{24} = 5 * 5^{24} = (5^5)^5 \end{cases}$$

So the smallest complex solution is:

$$(x, y, z) = (5^{3*4} * 2, 5^{2*4} * 1, 5^5) = (244140625, 390625, 3125)$$

### Example 2

$$\begin{cases} x^2 + y^3 = 2z^5 \\ x^2 - y^3 = z^5 \end{cases}$$

$$\begin{cases} \frac{x^2}{2} + \frac{y^3}{2} = z^5 \\ x^2 - y^3 = z^5 \end{cases}$$

There is the smallest  $l_i$  such that  $\frac{l_1^2}{2} + \frac{l_2^3}{2} = l_1^2 - l_2^3 \Leftrightarrow l_1^2 = 3l_2^3 : l_1 = 9x^3 = 9, l_2 = 3x^2 = 3, \frac{l_1^2}{2} + \frac{l_2^3}{2} = \frac{81+27}{2} = 54$ , so:

$$54 = 2 * 3^3$$

$$t_1 = 2 \Rightarrow 5|2t_1 + 1$$

$$t_2 = 4 \Rightarrow 5|1t_2 + 1$$

$$\begin{cases} \frac{(3^{3*2} * 2^{3*4} * 9)^2}{2} + \frac{(3^{2*2} * 2^{2*4} * 3)^3}{2} = \frac{(9^2 + 3^3)}{2} * 2^{24} * 3^{18} = 2 * 3^3 * 2^{24} * 3^{12} = 2^{25} * 3^{15} = (2^5 * 3^3)^5 \\ (3^{3*2} * 2^{3*4} * 9)^2 - (3^{2*2} * 2^{2*4} * 3)^3 = 2 * 3^3 * 2^{24} * 3^{12} = (2^5 * 3^3)^5 \end{cases}$$

So the smallest complex solution is:

$$(x, y, z) = (3^{3*2} * 2^{3*4} * 9, 3^{2*2} * 2^{2*4} * 3, (2^5 * 3^3)^5)$$

### Example 3

$$\begin{cases} 2x^2 - y^2 = w^7 \\ x^2 + z^2 = w^7 \end{cases}$$

$l_x^2 - l_y^2 - l_z^2 = 0$  then we have pitagorean triple  $(l_z, l_y, l_x) = (p^2 - q^2, 2pq, p^2 + q^2)$

The smallest pitagorean triple is (3,4,5):

$$2l_x^2 - l_y^2 = l_x^2 + l_z^2 = 25 + 9 = 34 = 2 * 17$$

$$t_1 = 3 \Rightarrow 7|(2 * t_1 + 1)$$

$$\begin{cases} 2(2^3 * 17^3 * 5)^2 - (2^3 * 17^3 * 4)^2 = 2^6 * 17^6 * (50 - 16) = 2^6 * 17^6 * (2 * 17) = (2 * 17)^7 \\ (2^3 * 17^3 * 5)^2 + (2^3 * 17^3 * 3)^2 = 2^6 * 17^6 * (25 + 9) = 2^6 * 17^6 * (2 * 17) = (2 * 17)^7 \end{cases}$$

So the smallest complex solution is:

$$(x, y, z, w) = (34^3 * 5, 34^3 * 4, 34^3 * 3, 34) = (39304 * 5, 39304 * 4, 39304 * 3, 34)$$

### Example 4

$$\begin{cases} 2x^4 - y^2 = w^7 \\ x^4 + z^2 = w^7 \end{cases}$$

$l_x^4 - l_y^2 - l_z^2 = 0$  then we have pitagorean triple  $(l_z, l_y, l_x^2) = (p^2 - q^2, 2pq, p^2 + q^2)$

The smallest  $l_x$  will be from pitagorean triple (3,4,5):  $l_x^2 = 3^2 + 4^2 = 25$

$$2l_x^4 - l_y^2 = l_x^4 + l_z^2 = 5^4 + (3 * 5)^2 = 850 = 2 * 5^2 * 17$$

$$t_1 = 5 \Rightarrow 7|(4 * t_1 + 1)$$

$$t_1 = 3 \Rightarrow 7|(4 * t_1 + 2)$$

$$\begin{cases} 2(2^5 * 5^3 * 17^5 * 5)^4 - (2^{5*2} * 5^{3*2} * 17^{5*2} * 5 * 4)^2 = 2^{20} * 5^{12} * 17^{20} * (850) = (2^3 * 5^2 * 17^3)^7 \\ (2^5 * 5^3 * 17^5 * 5)^4 + (2^{5*2} * 5^{3*2} * 17^{5*2} * 5 * 3)^2 = 2^{20} * 5^{12} * 17^{20} * (850) = (2^3 * 5^2 * 17^3)^7 \end{cases}$$

So the smallest complex solution is:

$$\begin{aligned} & (x, y, z, w) \\ &= (2^5 * 5^3 * 17^5 * 5, 2^{5*2} * 5^{3*2} * 17^{5*2} * 5 * 4, 2^{5*2} * 5^{3*2} * 17^{5*2} * 5 * 3, 2^3 * 5^2 * 17^3) \\ &= (28397140000, 645118048143680000000, 483838536107760000000, 982600) \end{aligned}$$

### Theorem 5 - complex solutions with alone standing constance

There is complex solution for equation like this:

$$\sum_{i=1}^n c_i \prod_{j=1}^{m_i} a_{i,j}^{x_{i,j}} + C = d \prod_{j=1}^{m_0} b_j^{z_j}$$

where  $\gcd\left(\prod_{i=1}^n \prod_{j=1}^{m_i} x_{i,j}, \prod_{j=1}^{m_0} z_j\right) = 1$ , then and only then when there is sufficed condition

$$C = \left( \prod_{i=1}^u \left( t_i^{f_i * z_i} \prod_{j=1}^{v_i} p_{i,j}^{(t_{i,j} + f_{i,j} * z_s) * r_{i,j} * lcm(x)} \prod_{j=1}^{w_i} y_{i,j}^{lcm(x, z_i)} \right) \right) * l_c$$

$$\sum_{i=1}^n c_i \prod_{j=1}^{m_i} l_{i,j}^{x_{i,j}} + l_c = \prod_{i=1}^u \left( t_i^{f_i * z_i} \prod_{j=1}^{v_i} p_{i,j}^{q_{i,j}} \right)$$

where  $l_c$  could be 1.

So alone standing constance have to be treated like new variable with exponent 1. And that is all. And then and only then when there is such solution that new variable is equal to  $C$ , there is complex not derived solution for the equation.

Example 1

$$x^2 + 25 = y^3$$

For  $l_c = 1, l_x = 2$ :

$$t = 1 \Rightarrow 3|2 * t + 1$$

$$(5^{1*1} * 2)^2 + 5^2 = (2^2 + 1) * (5^2) = 5^{2+1} = 5^3$$

So solution is  $(x, y) = (10, 5)$ .

Example 2

$$x^2 + 123 = y^3$$

$$123 = 3 * 41$$

So:  $l_c$  could be 1 or 3 or 41

$$1' l_c = 1$$

Then  $l_x^2 + 1 = 3 * 41 = 123 \Rightarrow l_x = \sqrt{122}$ , so there is no solutions.

$$2' l_c = 3$$

Then  $l_x^2 + 3 = 41 \Rightarrow l_x = 2\sqrt{7}$ , so there is no solutions.

$$3' l_c = 41$$

Then  $l_x^2 + 41 > 41 = 3$ , so there is no solutions.

**Conclusion:** There is not solutions of this equation for  $\gcd(x, y) > 1$ .

## Appendix C – Plotnicki's equations – part III

### Theorem 1 - useful theorem II

Theorem:  $\frac{a}{b}q = t \prod_{i=1}^n c_i + x$ , has integer solution for every  $a$  for given  $c_i$ , and given  $t$  (2.) or  $x$  (1.), where  $\gcd(a, b) = 1$ ,  $\gcd(a, \prod_{i=1}^n c_i) = 1$  and  $\gcd(x, \prod_{i=1}^n c_i) = 1$ .

It is enough to see that

$$\frac{a}{b}q = t \prod_{i=1}^n c_i + x \Leftrightarrow a|(t \prod_{i=1}^n c_i + x)$$

Example

$$\frac{3}{5}q = 2t + 1 \Leftrightarrow 3r = 2t + 1$$

$$t = 1 + 3k, r = 1 + 2k, q = (1 + 2k) * 5$$

## Theorem 2 - Płotnicki's equation with use of little Fermat theorem - general case - rational exponents

Theorem : there is infinitely many solutions for equation like this:

$$\sum_{i=1}^n c_i \prod_{j=1}^{m_i} a_{i,j}^{\frac{x_{i,j}}{y_{i,j}}} = d \prod_{j=1}^{m_0} b_j^{\frac{z_j}{z'_j}}$$

where  $\gcd\left(\prod_{i=1}^n \prod_{j=1}^{m_i} x_{i,j}, \prod_{j=1}^{m_0} z_j\right) = 1$

where for every  $i, j$ :  $c_i, a_{i,j}, d, b_j$  are rationals and  $n, m_i, x_{i,j}, y_{i,j}, z_j, z'_j$  are integers.

for every  $i, j$ : for every rational  $l_{i,j}$  and every rational  $p_{i,j}, t_i$  and every integer  $q_{i,j}, f_i$  that suffices equation:

$$\sum_{i=1}^n c_i \prod_{j=1}^{m_i} l_{i,j}^{\frac{x_{i,j}}{y_{i,j}}} = d \prod_{i=1}^u \left( t_i^{f_i * z_i} \prod_{j=1}^{v_i} p_{i,j}^{q_{i,j}} \right)$$

where for every  $i$ :  $f_i$  could be 0, for every  $i, j$ :  $\gcd(q_{i,j}, z_i) = 1$ , we have infinitely many solutions:

$$\begin{aligned} & \sum_{i=1}^n c_i \prod_{j=1}^{m_i} \left( \prod_{s=1}^u \left( \prod_{k \in S_{i,j,s}} p_{s,k}^{(t_{s,k} z_s - q_{s,k}) * y_{i,j} * \frac{(r_{s,k} * \text{lcm}(x))^{z_s-1}}{x_{i,j}}} * \prod_{k \in T_{i,j,s}} y_{s,k}^{\frac{\text{lcm}(x, z_s)}{x_{i,j}}} \right) * l_{i,j} \right)^{\frac{x_{i,j}}{y_{i,j}}} \\ &= d \prod_{i=1}^u \left( t_i^{f_i * z_i} \prod_{j=1}^{v_i} p_{i,j}^{(t_{i,j} z_i - q_{i,j}) * (r_{i,j} * \text{lcm}(x))^{z_i-1} + q_{i,j}} \prod_{j=1}^{w_i} y_{i,j}^{\text{lcm}(x, z_i)} \right) \\ &= d \prod_{i=1}^u \left( \left( t_i^{f_i} \prod_{j=1}^{v_i} p_{i,j}^{h_{i,j}} \prod_{j=1}^{w_i} y_{i,j}^{\frac{\text{lcm}(x, z_i)}{z_i}} \right)^{z'_i} \right)^{\frac{z_i}{z'_i}} \end{aligned}$$

Where for every  $i, j$ :  $\gcd(r_{i,j}, z_i) = 1$ .

Where for every  $i, s$ :  $\bigcup_{j=1}^{m_i} S_{i,j,s} = \{1, \dots, v_i\}$ ,  $\bigcup_{j=1}^{m_i} T_{i,j,s} = \{1, \dots, w_i\}$ ,

for every  $i, j, k, s$  where  $j \neq k$ :  $S_{i,j,s} \cap S_{i,k,s} = \emptyset, T_{i,j,s} \cap T_{i,k,s} = \emptyset$ ,

$x$  is a set of all  $x_{i,j}$ ,  $z$  is a set of all  $z_i$ .

Where  $c_i, d, l_i$  are any rationals and for every  $s, k$ :  $q_{s,k} < t_{s,k} z_s$ , where  $q_{s,k}, t_{s,k}$  are any integers.

In general we have rational solutions above and when  $\frac{\sum_{i=1}^n c_i \prod_{j=1}^{m_i} l_{i,j}^{\frac{x_{i,j}}{y_{i,j}}}}{d}$ , and for every  $i, j$ :  $c_i, l_{i,j}$  are integers, we have integer solutions.

More generally:

$$\begin{aligned}
& \sum_{i=1}^n c_i \prod_{j=1}^{m_i} \left( \prod_{s=1}^u \left( \prod_{k \in U_{i,j,s}} p_{s,k}^{u_{i,j,s,k} * y_{i,j} * \frac{lcm(x)}{x_{i,j}}} * \prod_{k \in T_{i,j,s}} y_{s,k}^{\frac{lcm(x,z_s)}{x_{i,j}}} \right) * l_{i,j} \right)^{\frac{x_{i,j}}{y_{i,j}}} \\
&= d \prod_{i=1}^u \left( t_i^{f_i * z_i} \prod_{j=1}^{v_i} p_{i,j}^{(t_{i,j}z_i - q_{i,j}) * (r_{i,j} * lcm(x))^{z_i-1} + q_{i,j}} \prod_{j=1}^{w_i} y_{i,j}^{lcm(x,z_i)} \right) \\
&= d \prod_{i=1}^u \left( \left( t_i^{f_i} \prod_{j=1}^{v_i} p_{i,j}^{h_{i,j}} \prod_{j=1}^{w_i} y_{i,j}^{\frac{lcm(x,z_i)}{z_i}} \right)^{z'_i} \right)^{\frac{z_i}{z'_i}}
\end{aligned}$$

for every  $i, s$ :  $\bigcup_{j=1}^{m_i} U_{i,j,s} = \{1, \dots, v_s\}$ ,  $\bigcup_{j=1}^{m_i} T_{i,j,s} = \{1, \dots, w_s\}$ ,

for every  $i, j, k, s$  where  $j \neq k$ :  $T_{i,j,s} \cap T_{i,k,s} = \emptyset$ ,

for every  $i, j$ :  $z_i \mid ((t_{i,j}z_i - q_{i,j}) * lcm(x)^{z_i-1} + q_{i,j})$  {little Fermat theorem},

$x$  is a set of all  $x_{i,j}$ ,  $z$  is a set of all  $z_i$ .

Where for every  $i, s, k$ :  $\sum_{j=1}^{m_i} u_{i,j,s,k} = (t_{s,k}z_s - q_{s,k}) * (r_{s,k})^{z_s-1} * lcm(x)^{z_s-2}$

### Theorem 3 – Płotnicki's equation – general case – rational exponents

Theorem: there is infinitely many solutions for equation like this:

$$\sum_{i=1}^n c_i \prod_{j=1}^{m_i} a_{i,j}^{\frac{x_{i,j}}{y_{i,j}}} = d \prod_{j=1}^{m_0} b_j^{\frac{z_j}{z'_j}}$$

where  $\gcd\left(\prod_{i=1}^n \prod_{j=1}^{m_i} x_{i,j}, \prod_{j=1}^{m_0} z_j\right) = 1$ .

where for every  $i, j$ :  $c_i, a_{i,j}, d, b_j$  are rationals and  $n, m_i, x_{i,j}, y_{i,j}, z_j, z'_j$  are integers.

for every  $i, j$ : for every rational  $l_{i,j}$  and every rational  $p_{i,j}, t_i$  and every integer  $q_{i,j}, f_i$  that suffices equation:

$$\sum_{i=1}^n c_i \prod_{j=1}^{m_i} l_{i,j}^{\frac{x_{i,j}}{y_{i,j}}} = d \prod_{i=1}^u \left( t_i^{f_i * z_i} \prod_{j=1}^{v_i} p_{i,j}^{q_{i,j}} \right)$$

where for every  $i$ :  $f_i$  could be 0, for every  $i, j$ :  $\gcd(q_{i,j}, z_i) = 1$ , we have infinitely many solutions:

$$\begin{aligned} & \sum_{i=1}^n c_i \prod_{j=1}^{m_i} \left( \prod_{s=1}^u \left( \prod_{k \in S_{i,j,s}} p_{s,k}^{\frac{(t_{s,k} + f_{s,k} * z_s) * r_{s,k} * y_{i,j} * \text{lcm}(x)}{x_{i,j}}} * \prod_{k \in T_{i,j,s}} y_{s,k}^{\frac{y_{i,j} * \text{lcm}(x, z_s)}{x_{i,j}}} \right) * l_{i,j}^{\frac{x_{i,j}}{y_{i,j}}} \right) \\ &= d \prod_{i=1}^u \left( t_i^{f_i * z_i} \prod_{j=1}^{v_i} p_{i,j}^{\frac{(t_{i,j} + f_{i,j} * z_i) * r_{i,j} * \text{lcm}(x) + q_{i,j}}{x_{i,j}}} \prod_{j=1}^{w_i} y_{i,j}^{\frac{\text{lcm}(x, z_i)}{x_{i,j}}} \right) \\ &= d \prod_{i=1}^u \left( \left( t_i^{f_i} \prod_{j=1}^{v_i} p_{i,j}^{h_{i,j}} \prod_{j=1}^{w_i} y_{i,j}^{\frac{\text{lcm}(x, z_i)}{z_i}} \right)^{\frac{z_i}{z'_i}} \right) \end{aligned}$$

Where

for every  $i, s$ :  $\bigcup_{j=1}^{m_i} S_{i,j,s} = \{1, \dots, v_s\}$ ,  $\bigcup_{j=1}^{m_i} T_{i,j,s} = \{1, \dots, w_s\}$ ,

for every  $i, j, k, s$  where  $j \neq k$ :  $S_{i,j,s} \cap S_{i,k,s} = \emptyset$ ,  $T_{i,j,s} \cap T_{i,k,s} = \emptyset$ ,

for every  $i, j$ :  $t_{i,j}$  is such integer that:  $z_i | (t_{i,j} * r_{i,j} * \text{lcm}(x) + q_{i,j})$ , {for details see: *Theorem 1*},

for every  $i, j$ :  $f_{i,j}$  is any integer,

for every  $i, j$ :  $r_{i,j}$  is any integer such that  $\gcd(r_{i,j}, z_i) = 1$ ,

$x$  is a set of all  $x_{i,j}$ ,  $z$  is a set of all  $z_i$ .

In general we have rational solutions above and when  $\frac{\sum_{i=1}^n c_i \prod_{j=1}^{m_i} l_{i,j}^{\frac{x_{i,j}}{y_{i,j}}}}{d}$ ,  $p_{i,j}, t_i, y_k$  are integers we 88

have integer solutions.

More generally:

$$\begin{aligned}
& \sum_{i=1}^n c_i \prod_{j=1}^{m_i} \left( \prod_{s=1}^u \left( \prod_{k \in U_{i,j,s}} p_{s,k}^{u_{i,j,s,k} * y_{i,j} * \frac{lcm(x)}{x_{i,j}}} * \prod_{k \in T_{i,j,s}} y_{s,k}^{y_{i,j} * \frac{lcm(x, z_i)}{x_{i,j}}} \right) * l_{i,j} \right)^{\frac{x_{i,j}}{y_{i,j}}} \\
& = d \prod_{i=1}^u \left( t_i^{f_i * z_i} \prod_{j=1}^{v_i} p_{i,j}^{(t_{i,j} + f_{i,j} * z_s) * r_{i,j} * lcm(x) + q_{i,j}} \prod_{j=1}^{w_i} y_{i,j}^{lcm(x, z_i)} \right) \\
& = d \prod_{i=1}^u \left( \left( t_i^{f_i} \prod_{j=1}^{v_i} p_{i,j}^{h_{i,j}} \prod_{j=1}^{w_i} y_{i,j}^{\frac{lcm(x, z_i)}{z_i}} \right)^{z'_i} \right)^{\frac{z_i}{z'_i}}
\end{aligned}$$

for every  $i, s$ :  $\bigcup_{j=1}^{m_i} U_{i,j,s} = \{1, \dots, v_s\}$ ,  $\bigcup_{j=1}^{m_i} T_{i,j,s} = \{1, \dots, w_s\}$ ,

for every  $i, j, k, s$  where  $j \neq k$ :  $T_{i,j,s} \cap T_{i,k,s} = \emptyset$ ,

for every  $i, j$ :  $t_{i,j}$  is such integer that:  $z_i | (t_{i,j} * r_{i,j} * lcm(x) + q_{i,j})$ , {for details see: *Theorem 1*},

for every  $i, j$ :  $f_{i,j}$  is any integer ( $f_i$  is completely other integer with other meaning),

for every  $i, j$ :  $r_{i,j}$  is any integer such that  $\gcd(r_{i,j}, z_i) = 1$ ,

$x$  is a set of all  $x_{i,j}$ ,  $z$  is a set of all  $z_i$ .

Where for every  $i, s, k$ :  $\sum_{j=1}^{m_i} u_{i,j,s,k} = (t_{s,k} + f_{s,k} * z_s) * r_{s,k}$

Example

$$5x^{\frac{2}{3}} + 3y^{\frac{3}{5}} = z^{\frac{5}{3}}$$

$$5 * 1^{\frac{2}{3}} + 3 * 1^{\frac{3}{5}} = 8 = 2^3$$

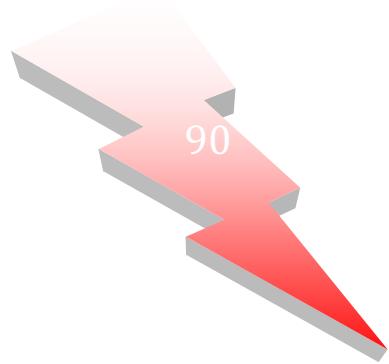
$$5q = 2 * 3 * t + 3 \Leftrightarrow 5q = 3 * (2t + 1)$$

$$t = 2 + 5k \Rightarrow 5|3 * (2t + 1)$$

$$5 * (2^{2*3*3} * 1)^{\frac{2}{3}} + 3 * (2^{2*2*5} * 1)^{\frac{3}{5}} = 2^{12} * (5 + 3) = 2^{12} * 2^3 = 2^{15} = (2^3)^5 = (2^9)^3$$

## **Proof that there are not other complex not derived solutions – rational exponents**

Is almost the same as for integer exponents, so proof is not worth to be rewritten.



## **Appendix D – Plotnicki's equations – part IV – What is next? – the unlimited field of Płotnicki's equations.**

## Simple case

When we have to calculate solution for:

$$\sum_{i=1}^k c_i v_i^{x_i} = \sum_{i=1}^l d_i w_i^{y_i}$$

where  $\gcd(x, y) = 1$ ,  $x$  is a set of  $x_i$ ,  $y$  is a set of  $y_i$ .

First we have to solve simultaneous equation in form:

$$\begin{cases} \sum_{i=1}^k c_i a_i^{x_i} = p^{\text{lcm}(y)} \\ \sum_{i=1}^l d_i b_i^{y_i} = q^{\text{lcm}(x)} \end{cases}$$

Then we can solve it from the equation:

$$\sum_{i=1}^k c_i a_i^{x_i} * \sum_{i=1}^l d_i b_i^{y_i} = \sum_{i=1}^l d_i b_i^{y_i} * \sum_{i=1}^k c_i a_i^{x_i}$$

We have solution in form:

$$\sum_{i=1}^k c_i \left( g^{\frac{\text{lcm}(x,y)}{x_i}} q^{\frac{\text{lcm}(x)}{x_i}} a_i \right)^{x_i} = \sum_{i=1}^l d_i \left( g^{\frac{\text{lcm}(x,y)}{y_i}} p^{\frac{\text{lcm}(y)}{y_i}} b_i \right)^{y_i}$$

For example:

$$w a^x + v b^y = u c^z$$

$$\begin{cases} w f^x + v g^y = p^z \\ u d^z = q^{\text{lcm}(x,y)} \end{cases}$$

There are given rules to solve both equations, so:

$$(w f^x + v g^y) u d^z = p^z u d^z = q^{\text{lcm}(x,y)} (w f^x + v g^y)$$

So here we have complex derived solution:

$$u(p d)^z = w \left( q^{\frac{\text{lcm}(x,y)}{x}} f \right)^x + v \left( q^{\frac{\text{lcm}(x,y)}{y}} g \right)^y$$

So this is the next method how to deal with “coefficient on the right side” and works always.

As we know how to solve:

$$\sum_{i=1}^k c_i v_i^{x_i} = \sum_{i=1}^l d_i w_i^{y_i}$$

We could solve for example:

$$\begin{cases} \sum_{i=1}^k c_i a_i^{x_i} = \sum_{i=1}^l g_i e_i^{y_i} \\ \sum_{i=1}^m d_i b_i^{z_i} = \sum_{i=1}^n h_i f_i^{w_i} \end{cases}$$

Where  $\gcd(x, y) = \gcd(z, w) = 1$ .

$x$  is a set of  $x_i$ ,  $y$  is a set of  $y_i$ ,  $z$  is a set of  $z_i$ ,  $w$  is a set of  $w_i$ .

And from there we have:

$$\sum_{i=1}^k c_i a_i^{x_i} \sum_{i=1}^n h_i f_i^{w_i} = \sum_{i=1}^m d_i b_i^{z_i} \sum_{i=1}^l g_i e_i^{y_i}$$

$$\sum_{i=1}^k c_i a_i^{x_i} \sum_{i=1}^m d_i b_i^{z_i} = \sum_{i=1}^l g_i e_i^{y_i} \sum_{i=1}^n h_i f_i^{w_i}$$

$$\frac{\sum_{i=1}^k c_i a_i^{x_i}}{\sum_{i=1}^n h_i f_i^{w_i}} = \frac{\sum_{i=1}^m d_i b_i^{z_i}}{\sum_{i=1}^l g_i e_i^{y_i}}$$

$$\frac{\sum_{i=1}^k c_i a_i^{x_i}}{\sum_{i=1}^m d_i b_i^{z_i}} = \frac{\sum_{i=1}^l g_i e_i^{y_i}}{\sum_{i=1}^n h_i f_i^{w_i}}$$

$$\sum_{i=1}^k c_i a_i^{x_i} \pm \sum_{i=1}^n h_i f_i^{w_i} = \sum_{i=1}^m d_i b_i^{z_i} \pm \sum_{i=1}^l g_i e_i^{y_i}$$

$$\sum_{i=1}^k c_i a_i^{x_i} \pm \sum_{i=1}^m d_i b_i^{z_i} = \sum_{i=1}^l g_i e_i^{y_i} \pm \sum_{i=1}^n h_i f_i^{w_i}$$

Etc.

And much more, eg.:

$$\prod_{j=1}^l \sum_{i=1}^{k_j} c_{j,i} a_{j,i}^{x_{j,i}} = \prod_{j=1}^n \sum_{i=1}^{m_j} d_{j,i} b_{j,i}^{y_{j,i}}$$

$$\prod_{j=1}^l \sum_{i=1}^{k_j} c_{j,i} a_{j,i}^{x_{j,i}} + \sum_{i=1}^{k_j} e_{j,i} g_{j,i}^{x_{j,i}} = \prod_{j=1}^n \sum_{i=1}^{m_j} d_{j,i} b_{j,i}^{y_{j,i}} + \sum_{i=1}^{k_j} f_{j,i} h_{j,i}^{x_{j,i}}$$

And so on...

## General simple case

$$\sum_{i=1}^{k_1} c_{1,i} a_{1,i}^{x_{1,i}} = \dots = \sum_{i=1}^{k_n} c_{n,i} a_{n,i}^{x_{n,i}}$$

where for every  $j$ :  $\gcd(x \text{ without } x_j, x_j) = 1$ ,  $x$  is a set of  $x_i$ ,  $x_i$  is a set of  $x_{i,j}$ .

We have for every  $j$ :

$$\left\{ \sum_{i=1}^{k_j} c_{j,i} a_{j,i}^{x_{j,i}} = p_j^{\operatorname{lcm}(x \text{ without } x_j)} \right.$$

And then solution in form:

$$\begin{aligned} \sum_{i=1}^{k_1} c_{1,i} \left( g^{\frac{\operatorname{lcm}(x)}{x_{1,i}}} \prod_{j=2}^n p_j^{\frac{\operatorname{lcm}(x \text{ without } x_j)}{x_{1,i}}} a_{1,i} \right)^{x_{1,i}} &= \dots \\ = \sum_{i=1}^{k_l} c_{l,i} \left( g^{\frac{\operatorname{lcm}(x)}{x_{l,i}}} \prod_{j=1}^{l-1} p_j^{\frac{\operatorname{lcm}(x \text{ without } x_j)}{x_{l,i}}} \prod_{j=l+1}^n p_j^{\frac{\operatorname{lcm}(x \text{ without } x_j)}{x_{l,i}}} a_{l,i} \right)^{x_{l,i}} &= \dots \\ = \sum_{i=1}^{k_n} c_{n,i} \left( g^{\frac{\operatorname{lcm}(x,y)}{x_{n,i}}} \prod_{j=1}^{n-1} p_j^{\frac{\operatorname{lcm}(x \text{ without } x_j)}{x_{n,i}}} a_{n,i} \right)^{x_{n,i}} \end{aligned}$$

## The most general case

The same is in the most general case:

$$\sum_{i=1}^{n_1} c_{1,i} \prod_{j=1}^{m_{1,i}} a_{1,i,j}^{x_{1,i,j}} = \dots = \sum_{i=1}^{n_k} c_{k,i} \prod_{j=1}^{m_{k,i}} a_{k,i,j}^{x_{k,i,j}}$$

where  $\gcd(x \text{ without } x_j, x_j) = 1$ ,  $x$  is a set of  $x_i$ ,  $x_i$  is a set of  $x_{i,j}$ .

For example:

$$\begin{aligned} \sum_{i=1}^l d_i \prod_{j=1}^{o_i} b_{i,j}^{y_{i,j}} * \sum_{i=1}^n c_i \prod_{j=1}^{m_i} a_{i,j}^{x_{i,j}} &= \sum_{i=1}^n c_i \prod_{j=1}^{m_i} a_{i,j}^{x_{i,j}} * \sum_{i=1}^l d_i \prod_{j=1}^{o_i} b_{i,j}^{y_{i,j}} \\ \sum_{i=1}^n c_i \prod_{j=1}^{m_i} a_{i,j}^{x_{i,j}} &= p^{\operatorname{lcm}(y_{i,j})} \\ \sum_{i=1}^l d_i \prod_{j=1}^{o_i} b_{i,j}^{y_{i,j}} &= q^{\operatorname{lcm}(x_{i,j})} \end{aligned}$$

Or:

$$\begin{aligned} \sum_{i=1}^n c_i \prod_{j=1}^{m_i} a_{i,j}^{x_{i,j}} &= \prod_{j=1}^{\max(o_i)} p_j^{\operatorname{lcm}(y_{j,j})} \\ \sum_{i=1}^l d_i \prod_{j=1}^{o_i} b_{i,j}^{y_{i,j}} &= \prod_{j=1}^{\max(m_i)} q_j^{\operatorname{lcm}(x_{j,j})} \end{aligned}$$

This method is not so difficult, but is too difficult to be elegantly showed. You could easily see it. The only difference here is that you have simply more possibilities to place  $p_j$  and  $q_j$  for every  $i$ .

So in general you can easily solve equations like this:

$$\prod_{j=1}^l \sum_{i=1}^{k_j} c_{j,i} \prod_{e=1}^{g_{j,i}} a_{j,i,e}^{x_{j,i,e}} = \prod_{j=1}^n \sum_{i=1}^{m_j} d_{j,i} \prod_{f=1}^{h_{j,i}} b_{j,i,f}^{y_{j,i,f}}$$

And from this point we can solve:

$$\prod_{j=1}^{l_1} \sum_{i=1}^{k_{1,j}} c_{1,j,i} \prod_{e=1}^{g_{1,j,i}} a_{1,j,i,e}^{x_{1,j,i,e}} = \dots = \prod_{j=1}^{l_n} \sum_{i=1}^{k_{n,j}} c_{n,j,i} \prod_{e=1}^{g_{n,j,i}} a_{n,j,i,e}^{x_{n,j,i,e}}$$

And so on... More about it you will find in my book.



## Rational exponents

The same is for rational exponents.

For example:

First we have to solve simultaneous equation in form:

$$\begin{cases} \sum_{i=1}^k c_i a_i^{\frac{x_i}{x'_i}} = p^{lcm(y)} \\ \sum_{i=1}^l d_i b_i^{\frac{y_i}{y'_i}} = q^{lcm(x)} \end{cases}$$

Then we can solve it from the equation:

$$\sum_{i=1}^k c_i a_i^{\frac{x_i}{x'_i}} * \sum_{i=1}^l d_i b_i^{\frac{y_i}{y'_i}} = \sum_{i=1}^l d_i b_i^{\frac{y_i}{y'_i}} * \sum_{i=1}^k c_i a_i^{\frac{x_i}{x'_i}}$$

We have solution in form:

$$\sum_{i=1}^k c_i \left( g^{\frac{x'_i * lcm(x,y)}{x_i}} q^{\frac{x'_i * lcm(x)}{x_i}} a_i \right)^{\frac{x_i}{x'_i}} = \sum_{i=1}^l d_i \left( g^{\frac{y'_i * lcm(x,y)}{y_i}} p^{\frac{y'_i * lcm(y)}{y_i}} b_i \right)^{\frac{y_i}{y'_i}}$$

And so on...

## Appendix 1 – Inverse function to Li(n)

```

int prime(int n)
{
    typedef double real_type;
    const int ilogsum_limit = 3;
    real_type* ilogsumt = new real_type[ilogsum_limit];
    for (int i = 0; i < ilogsum_limit; ++i) ilogsumt[i] = 0.0;
    for (int i = 2; i <= n; ++i)
    {
        ilogsumt[0] += log(real_type(i)*log(real_type(i)));
        for (int j = 1; j < ilogsum_limit; ++j)
            if (ilogsumt[j - 1] > 1.0) ilogsumt[j] += log(ilogsumt[j - 1]);
    }
    const int result = ilogsumt[ilogsum_limit-1];
    delete [] ilogsumt;
    return result;
}

```

$$prime(n) = f_\infty(n)$$

$$f_0(n) = \sum_{i=1}^n \ln(i)$$

$$f_k(0) = 0, f_k(n) = f_k(n-1) + \max(\ln(f_{k-1}(n)), 0)$$

or:

$$f_k(n) = \sum_{i=1}^n \max(\ln(f_{k-1}(i)), 0)$$

Function  $prime(n)$  runs in time  $O(n) = O\left(\frac{p_n}{\ln p_n}\right)$  and tends very quickly to  $\log(p_1, \dots, p_n)$  and  $p_n$ , where  $p_i$  is i-th prime number. The best performance can be obtained calculating  $prime(n)$  for all numbers in the range 1 ... n, or for a set of complexity  $O(n)$ , then the complexity of calculating each  $prime(i)$  is  $O(1)$ .

For  $ilogsum\_limit = 4$  with double precision (a higher value for this type causes already deterioration of result due to errors in floating point operations) it gets average percentage difference less than 1% for  $p_{1073}$  ( $p_{1096}$  for  $\ln(p_1 * \dots * p_n)$ ) 8623 (8803), and one promile for  $p_{18415}$  ( $p_{18491}$  for  $\ln(p_1 * \dots * p_n)$ ), which is the prime number 205417 (206273). Probably there is no better known approximation for  $p_i$  that does not use primes and it is very possible that in general it does not exist.

// Polish: Funkcja  $prime(n)$  działa w czasie  $O(n) = O\left(\frac{p_n}{\ln p_n}\right)$  i dąży bardzo szybko do  $\ln(p_1 * \dots * p_n)$  i  $p_n$ , gdzie  $p_i$  to i-ta liczba pierwsza. Najlepszą wydajność można uzyskać licząc  $prime(n)$  dla wszystkich liczb z przedziału 1 ... n, lub dla zbioru o złożoności  $O(n)$ , wtedy złożoność obliczenia każdego  $prime(i)$  jest  $O(1)$ .

Na marginesie: oczywiście definicja liczby pierwszej powinna brzmieć: „liczba podzielna tylko przez samą siebie i 1”, czyli powinna być nią również jedynka.

Już dla `ilogsum_limit==4` przy precyzyji `double` (większa wartość dla tego typu powoduje już pogorszenie wyniku ze względu na błędy operacji zmiennoprzecinkowych) uzyskuje średnią różnicę procentową mniejszą od 1% już przy  $p_{1073}$  ( $p_{1096}$  dla  $\ln(p_1 * \dots * p_n)$ ), czyli 8623 (8803), a jednopromilową różnicę przy  $p_{18415}$  ( $p_{18491}$  dla  $\ln(p_1 * \dots * p_n)$ ), czyli liczbie pierwszej 205417 (206273). Przy kilkumilionowej liczbie pierwszej schodzi do około jednomilionowej. Prawdopodobnie nie istnieje żadne lepsze znane przybliżenie  $p_i$  nie wykorzystujące liczb pierwszych i bardzo możliwe, że w ogóle nie istnieje. Algorytmowi można również bardzo łatwo podać największą znaną liczbę pierwszą  $p_i < p_n$  (wystarczy do  $i$  liczyć wszystkie poziomy poza ostatnim, dla  $i$  podać  $p_i$  na ostatnim poziomie i kontynuować obliczenia już dla wszystkich poziomów), zwiększając znacznie precyzyję obliczeń. Na podstawie dwóch kolejnych liczb  $\text{prime}(i)$  można osiągnąć precyzyję taką, jakby zaczęło się obliczenia od nich i można rozpoczęć obliczanie od nich, bo można na ich podstawie obliczyć całą tablicę `ilogsumt`. Tak więc przeznaczając na wcześniej obliczone pary sąsiednich liczb  $\text{prime}(i)$  pamięć  $O\left(p_n^{\frac{1}{3}}\right)$  tak jak to ma miejsce w najlepszym algorytmie Lagarias-Miller-Odlyzko dla  $\pi(n)$ ,

można użykać złożoność  $O\left(\frac{p_n}{\ln p_n}\right) = O\left(\frac{p_n^{\frac{2}{3}}}{\ln p_n}\right)$ . Ponadto mając zapamiętane `ilogsum_limit` liczb z tablicy `ilogsumt` dla  $\text{prime}(n)$  można odtworzyć  $\text{prime}(n - c)$  i  $\text{prime}(n + c)$  w czasie  $O(c)$ .

W czasie  $O\left(\frac{n}{\ln n}\right)$  da się zatem oszacować bardzo dokładnie  $\pi(i)$  dla wszystkich liczb z przedziału  $1 \dots n$ , nie znając żadnej liczby pierwszej. Jest to zatem algorytm niemal tak szybki jak Lehmera ( $O\left(\frac{n}{\ln^4 n}\right)$ , 1994r.) i Meissela ( $O\left(\frac{n}{\ln^3 n}\right)$ , 1985-1994r.), przy czym zużywa tylko  $O(1)$  pamięci, a nie  $O\left(\frac{n^{\frac{1}{3}}}{\ln n}\right)$  lub odpowiednio  $O\left(\frac{n^{\frac{1}{2}}}{\ln n}\right)$ , a więc nie ma ograniczenia pamięciowego na obliczenie wielkich wartości  $n$ , oraz dla zbioru liczb  $\pi(i)$  o złożoności  $O(n)$  złożoność obliczenia pojedynczej wartości to  $O(1)$ .

Oto algorytm:

```
int pi(int n)
{
    typedef double real_type;
    const int ilogsum_limit = 3;
    real_type* ilogsumt = new real_type[ilogsum_limit];
    for (int i = 0; i < ilogsum_limit; ++i) ilogsumt[i] = 0.0;
    for (int i = 2; i <= n; ++i)
    {
        ilogsumt[0] += log(real_type(i)*log(real_type(i)));
        for (int j = 1; j < ilogsum_limit; ++j)
            if (ilogsumt[j - 1] > 1.0) ilogsumt[j] += log(ilogsumt[j - 1]);
        if (ilogsumt[ilogsum_limit - 1] > n)
        {
            delete [] ilogsumt;
            return i - 2;
        }
    }
}
```

The accuracy of the algorithm  $pi(n)$  for `ilogsum_limit = 3` and double precision numbers is basically the same as  $Li(n)$  from the table from Wikipedia:  $pi(10^7) = 664919$ ,  $Li(10^7) = 664918$ ,  $\pi(10^7) = 664579$ ;  $pi(10^8) = 5762211$ ,  $Li(10^8) = 5762209$ ,  $\pi(10^8) = 5761455$ . For 9

numbers of greater precision you can probably get exactly the same result as  $Li(n)$ . Therefore it seems that the  $prime(n)$  is the inverse of the  $Li(n)$ , which gives a much smaller errors from the formula proposed in [www.mathworld.wolfram.com/PrimeFormulas.html](http://www.mathworld.wolfram.com/PrimeFormulas.html) (15). It also maintains the relation  $prime(n) < p_n$ .  $prime(n)$  algorithm also has much simpler form than this proposed there.

// Dokładność algorytmu  $pi(n)$  dla `ilogsum_limit = 3` i liczb dokładności `double` jest w zasadzie identyczna jak  $Li(n)$  z tabeli z wikipedii:  $pi(10^7) = 664919$  ,  $Li(10^7) = 664918$  ,  $\pi(10^7) = 664579$  ;  $pi(10^8) = 5762211$  ,  $Li(10^8) = 5762209$  ,  $\pi(10^8) = 5761455$  . Dla liczb większej precyzji prawdopodobnie można uzyskać wynik identyczny albo nawet lepszy niż  $Li(n)$ . **Wydaje się więc, że  $prime(n)$  jest funkcja odwrotna do  $Li(n)$** , przy czym daje o wiele mniejsze błędy od wzoru zaproponowanego w [www.mathworld.wolfram.com/PrimeFormulas.html](http://www.mathworld.wolfram.com/PrimeFormulas.html) (15) – zachowuje także relację  $prime(n) < p_n$ . Algorytm  $prime(n)$  ma też o wiele prostszą postać od zaproponowanego tam rozwinięcia. Dla  $n$  około 50 milionów dokładność  $pi(n)$  jest rzędu czterech pierwszych wiodących liczb. Precyzję tego algorytmu również można łatwo i znacznie podnieść podając największą znaną liczbę  $p_i < p_n$ .

## Appendix 2

Solutions of equation:

$$a^2 \pm b^2 = c^z$$

If  $z$  is not divisible by 2 then we have infinitely many non coprime solutions.

If  $z$  is divisible by 2 then we have the same problem

$$a = p_1^2 \mp q_1^2$$

$$b = 2p_1q_1$$

$$c^{\frac{z}{2}} = p_1^2 \pm q_1^2$$

So we always come to equation:

$$c^{\frac{z}{2^j}} = p_j^2 \pm q_j^2$$

Where  $\frac{z}{2^j}$  is odd. So we always has infinitely many solutions. And that's are all non coprime solutions.

Additionally there is always coprime solution for:

$$c^{\frac{z}{2^j}} = p_j^2 - q_j^2 = (p_j - q_j)(p_j + q_j)$$

$$p_j - q_j = e^{z_j}$$

$$p_j + q_j = f^{z_j}$$

$$p_j = \frac{e^{z_j} + f^{z_j}}{2}$$

$$q_j = \frac{e^{z_j} - f^{z_j}}{2}$$

And that are not all coprime solutions to the equation:

$$a^2 - b^2 = c^z$$

Because  $c^{z_j}$  can be  $2p_jq_j$  where  $\gcd(p_j, q_j) = 1$ .

For:

$$a^2 \pm b^2 = c^2$$

As  $\gcd(2,2,2) > 1$ , so there is no possibility to align common divisors to  $2z$  other than:

$$(gk)^2 \pm (gl)^2 = (gm)^2$$

which can be divided by  $g^2$ , to get

$$k^2 \pm l^2 = m^2$$

so all non coprime solutions are derived from coprime. Of course  $c$  may be  $d^z$  above.

So putting all together we have all solutions for:

$$\mathbf{a}^2 - \mathbf{b}^2 = \mathbf{c}^z$$

And all non coprime solutions for:

$$\mathbf{a}^2 + \mathbf{b}^2 = \mathbf{c}^z$$

And we knot that:

$$\mathbf{a}^2 \pm \mathbf{b}^2 = \mathbf{c}^2$$

has no non coprime not derived solutions.

QED.

## Appendix 3

Solution of equation:

$$a^{2^x} + b^{2^y} = c^{2^z}$$

As Fermat showed there is not solution for

$$a^4 \pm b^4 = c^2$$

So the only possible solutions are:

$$a^2 \pm b^2 = c^{2^z}$$

And there of course are infinitely many solutions, because:

$$a^2 + b^2 = c^2$$

has only solutions

$$a = p^2 - q^2, b = 2pq, c = p^2 + q^2$$

So  $a$  or  $c$  could be  $t^2$  that gives:

$$t^4 = c^2 - b^2$$

Where

$$t^2 = p^2 - q^2$$

Or:

$$a^2 + b^2 = t^4$$

Where

$$t^2 = p^2 + q^2$$

And so on.

QED.

## Appendix 4 – Content of email to the full professor in University of Warsaw Edmund Puczyłowski (10/26/2011)

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Theorem 1:  $ab = t^* \text{Mul}(c) + x$ , has integer solution for every  $a$  for given  $c$ , and given  $t$  (2) or  $x$  (1), where  $\text{gcd}(a, \text{Mul}(c))=1$  and  $\text{gcd}(x, \text{Mul}(c))=1$ .

- 1.) So for every  $x, y$ , where  $\text{gcd}(x, y)=1$ :  $nx \bmod y = k$ ,  $n \leq y$  has solution for every  $k$ :  $0 \leq k < y$ , in sequence  $p = \text{abs}(x-y)$ :  $p \bmod y, 2p \bmod y, \dots, yp \bmod y \Leftrightarrow nx \bmod y == (n+y)x \bmod y \Leftrightarrow$  every value of rest repeats every  $y*x$ , so between  $n$  and  $n+y$  there does not repeat any rest so every  $k$  is there.
- 2.) So for every  $x, y$ , where  $\text{gcd}(x, y)=1$ :  $x \bmod y = k$ ,  $n \leq y$  has solution for every  $k$ :  $0 \leq k < y$ , in sequence  $p = \text{abs}(x-y)$ :  $p \bmod y, 2p \bmod y, \dots, yp \bmod y \Leftrightarrow x \bmod y == (x+1) \bmod y+1 \Leftrightarrow$  every value of rest appears between  $x$  and  $x + y$ .

For:

$$a^x + b^y = c^z, \text{ where } \text{gcd}(xy, z) = 1$$

for every

$$d = k^x + l^y = p_1^{q_1} * \dots * p_n^{q_n} * m^{f_z}$$

where  $f$  could be 0, we have only solutions:

$$\begin{aligned} & (p_1^{(t_1+f_1*z)*y} * \dots * p_n^{(t_n+f_n*z)*y} * k)^x + (p_1^{(t_1+f_1*z)*x} * \dots * p_n^{(t_n+f_n*z)*x} * l)^y \\ &= p_1^{(t_1+f_1*z)*xy+q_1} * \dots * p_n^{(t_n+f_n*z)*xy+q_n} * m^{f_z} = c^z \end{aligned}$$

So for every  $k, l$  we have as much subclasses of solutions as much “images of divisibility” exists, in form:

$$p_1^{q_1} * \dots * p_n^{q_n} * m^{f_z} \text{ of given } k^x + l^y.$$

So for given  $\text{gcd}(a, b, c) = p_1^{t_1*xy} * \dots * p_n^{t_n*xy} * m^{f_z}$

there is only one image of divisibility  $p_1^{q_1} * \dots * p_n^{q_n} * m^{f_z}$  for which are constant numbers of  $k, l$  pairs such that  $k^x + l^y = p_1^{q_1} * \dots * p_n^{q_n} * m^{f_z}$ , which has only above solutions.

And if equation has one solution :  $k^x + l^y = m^z$ , then it has infinitely many solutions:

$$\left(g^{\frac{tzy}{fgcd(zy,x)}} * k\right)^x + \left(g^{\frac{txz}{fgcd(zy,x)}} * l\right)^y = (k^x + l^y) \left(g^{\frac{txy}{fgcd(zy,x)}}\right)^z, \text{ for every } g, t$$

$\text{fgcd}(zy, x)$  could be also  $\text{fgcd}(zx, y)$

where  $\text{fgcd}(x,y)$  could be every selected divisor of  $\text{gcd}(x,y)$

And those are all solutions that can be derived from  $k^x + l^y = m^z$ .

$\text{Gcd}(a)=1$  – not complex solutions

$\text{Gcd}(a)>1$  – complex solutions

Where  $a$  is variables set.

And those are all solutions (derived from all not complex solutions) when there are not complex not derived solutions (when for  $w=x,y,z : \text{gcd}(xyz/w,w) > 1$ ).

So putting both together, when we know all  $\text{gcd}(a)=1$  solutions (that the amount of is constant number or zero), we know all solutions of Diophantine equation.

And if there is a solution for:  $(n[i,1] - n[j,1])k^x + (n[i,2] - n[j,2])l^y = 0$ , and  $a!=b!=c$  and  $\text{gcd}(a,b,c)>1$  then and only then (because otherwise  $c[i]!=c[j]$ ) simultaneous equation has infinitely many solutions:

$$n[i,1](P1 * k)^x + n[i,2](P2 * l)^y = c^z, \text{ where } n[i,j] \text{ are rationals.}$$

There is also a solution for eg.:

$$a^x + b^y c^z = d^w, \text{ where } \text{gcd}(xyz, w) = 1$$

$$k^x + l^y m^z = p1^{q1} * ... * pn^{qn} * m^{f*w}$$

$$\begin{aligned} & (p1^{(t1+f1*z)*y*z} * ... * pn^{(tn+fn*z)*y*z} * k)^x \\ & + (\text{subset}(p1^{(t1+f1*z)*x*z} * ... * pn^{(tn+fn*z)*x*z}) * l)^y \\ & * (\text{subset}(p1^{(t1+f1*z)*x*y} * ... * pn^{(tn+fn*z)*x*y}) * m)^z \\ & = (p1)^{(t1+f1*z)*xyz+q1} * ... * pn^{(tn+fn*z)*xyz+qn} * m^{f*w} = c^z \end{aligned}$$

For example:

$$2^2 + 2^3 * 2^5 = 260 = 26 * 10$$

$$t1 * (2 * 3 * 5) + 1 = 7q1$$

$$t2 * (2 * 3 * 5) + 1 = 7q2$$

$$t1 = 3, t2 = 3 + 7 = 10$$

so:

$$\begin{aligned} & (26^{3*3*5} * 10^{10*3*5} * 2)^2 + (26^{3*2*3} * 2)^3 * (10^{10*2*3} * 2)^5 = (26)^{3*30+1} * 10^{10*30+1} \\ & = (26^{13} * 10^{43})^7 \end{aligned}$$

For  $d^w$  it will give all complex solutions.

$b^y c^z$  can be calculated as  $f^{y+z}$ , but it will not give all possible solutions, but there still is a way to calculate them:

$$d^w - a^x = b^y c^z, \text{ where } \gcd(wx, yz) = 1$$

$$k^w - l^x = p_1^{q_1} * \dots * p_n^{q_n} * m^{f*y} * n^{g*z}$$

So p, t have to be selected such a way to contruct  $b^y c^z$ .

For example :

$$d^7 - a^2 = b^3 c^5$$

$$2^7 - 2^2 = 124 = 2^2 * 31$$

$$2 * 7 * t1 + 2 = 3q1$$

$$2 * 7 * t2 + 1 = 5q2$$

$$t1 = 2, t2 = 1$$

$$\begin{aligned} ((61^{1*2} * 2^{2*2}) * 2)^7 - ((61^{1*7} * 2^{2*7}) * 2)^2 &= (2^7 - 2^2) * (2^{14} * 61^{14}) \\ &= (2^2 * 61) * (2^{28} * 61^{14}) = 2^{30} * 61^{15} = (2^{10})^3 * (61^3)^5 \end{aligned}$$

The same is for derivation:

$$\begin{aligned} &\left( g^{\frac{t1xyz}{\gcd(xyz,w)}} * h^{\frac{t2xyz}{\gcd(xyz,w)}} k \right)^w - \left( g^{\frac{t1yzw}{\gcd(xyz,w)}} h^{\frac{t2yzw}{\gcd(xyz,w)}} l \right)^x \\ &= (k^w - l^x) \left( g^{\frac{t1xzw}{\gcd(xyz,w)}} \right)^y * \left( h^{\frac{t2xyw}{\gcd(xyz,w)}} \right)^z \end{aligned}$$

And the same is for combinations when there exist partial solved solution:

$$d^7 - a^2 = b^2 c^5$$

$$2^7 - 2^2 = 124 = (2^2) * 31$$

So to calucalte all solutions you need to know only not complex solutions which are few or zero.

So there always is infinitely many complex not derived solutions only when  $\gcd\left(\frac{\text{Mul}(x)}{s[j]}, s[j]\right) = 1$ , where  $\text{Mul}(x)$  is multiplication of all powers, and  $s[j]$  is a multiplication of powers at position j; or there exists combination (there exist partial solved solution, eg.:  $d^7 - a^2 = b^2 c^5, 2^7 - 2^2 = 124 = (2^2) * (31)^1$ , where the condition should be sufficed only for those  $x[k]$  that are not solved; of course in this case  $(b^2 c^5, (3^2) * (14^1))$  divisibilities could be exchanged 2->5, 1->2) – proved; and there exist always infinitely many complex derived solutions if there exist at least one solution – proved.

So in general this is the way to calculate all rational complex solutions of Diophantine equations where there exist such  $j$  that  $\text{gcd}\left(\frac{\text{Mul}(x)}{x[j]}, x[j]\right) = 1$ , where  $x[j]$  is a multiplication of powers at position  $j$  in equation, eg.:  $2x^3 + 3y^5v^3 = 5z^7w^2$ , etc.