# Mapping the $^{10}B$, $^{9}Be$, $^{10}Be$, $^{11}B$, $^{11}C$, $^{12}C$ and $^{14}N$ Binding Energies with High Precision based Exclusively on the Up and Down Quark Masses

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June 27, 2013

*We extend the results of two recent letters by expressing the $^{10}B$, $^{9}Be$, $^{10}Be$, $^{11}B$, $^{11}C$, $^{12}C$ and $^{14}N$ binding energies, each independently and each to about parts-per-million or small parts-per-100,000 accuracy in AMU, exclusively as a function of the up and down current quark masses.*

PACS: 21.10.Dr; 27.10.+h; 14.65.Bt; 14.20.Dh; 27.40.+z; 14.60.Cd; 26.20.Cd

## 1. Introduction

This letter in a continuation of two very recent letters [1] and [2] which explain how nuclear biding and fusion energies can be mapped exclusively as the function of the up and down quark masses, to accuracy on the order of small parts per 100,000 or parts per million AMU based on Koide-type matrices applied to three quark masses inside the proton and neutron. The earlier letter [2] reported on $^{2}$H, $^{3}$H, $^{3}$He and $^{4}$He as well as the neutron minus proton mass difference and a relationship among the up, down and electron masses. The later letter [1] went on to report on $^{6}$Li, $^{7}$Li, $^{7}$Be and $^{8}$Be. Here we continue where [1] left off, and make a similar report as to all of $^{10}$B, $^{9}$Be, $^{10}$Be, $^{11}$B, $^{11}$C, $^{12}$C and $^{14}$N. For economy, the results in [1] and [2] will not be repeated here, except as directly necessary to support the derivations here, nor will the references used in those two letters be repeated here.

## 2. Mass/Energy Relation between $^{10}$B and $^{8}$Be, and $^{12}$C and $^{14}$N

We begin this letter by considering the $^{10}$B nuclide. For $^{6}$Li we considered the fusion reaction ${}^{4}_{2}He+2p\rightarrow {}^{6}_{3}Li+e^{+}+\nu+\text{Energy}$. We follow a similar route and consider the fusion reaction ${}^{8}_{4}Be+2p\rightarrow {}^{10}_{5}B+e^{+}+\nu+\text{Energy}$. The energy released during such a fusion event is:

$$\text{Energy}={}^{8}_{4}M+2M_{p}-{}^{10}_{5}M-m_{e}=0.006921034\text{ u}, \tag{2.1}$$

using empirical data ${}^{8}_{4}M=8.003110780\text{ u}$, ${}^{10}_{5}M=10.010194100\text{ u}$, $M_{p}=1.007276466812\text{ u}$ and $m_{e}=0.000548579909\text{ u}$. We recall from (2.2) of [1] that the energy released during ${}^{4}_{2}He+2p\rightarrow {}^{6}_{3}Li+e^{+}+\nu+\text{Energy}$ was given by $9\sqrt{m_{u}m_{d}}/(2\pi)^{1.5}$ to about 7 parts per million. Because $^{6}$Li has $A=Z+N=6$ nucleons and so has $9=3\times A/2$ up / down *quark pairs*, we interpreted this as indicating that each of the nine quark pairs gave up one $\sqrt{m_{u}m_{d}}/(2\pi)^{1.5}$ energy dose during this fusion. Following suit, we observe that $^{10}$B has $A=Z+N=10$ nucleons, and so contains $15=3\times A/2$ up / down quark pairs. Expecting some consistency, we construct the factor $15\sqrt{m_{u}m_{d}}/(2\pi)^{1.5}$ and subtract this from the empirical energy in (2.1) to obtain:

$$0.006921034\text{ u} - 15\sqrt{m_u m_d}/(2\pi)^{1.5} = 0.003543707\text{ u} \cong \sqrt{m_u m_d}\ . \tag{2.2}$$

So apparently there is still some energy that is unaccounted for when we open up the 2p shell with $^{10}$B. However, is the easily seen that the energy calculated in (2.2) differs from $\sqrt{m_u m_d}$ by $2.3983\times10^{-6}$ u i.e., by just over two parts per million AMU, as is also shown above. So we use (2.2) together with (2.1) to conclude that:

$$\begin{aligned}&\text{Energy}\left({}^{8}_{4}Be + 2p \rightarrow {}^{10}_{5}B + e^{+} + \nu + \text{Energy}\right)\\ &\qquad = {}^{8}_{4}M + 2M_p - {}^{10}_{5}M - m_e = \sqrt{m_u m_d} + 15\sqrt{m_u m_d}/(2\pi)^{1.5} = 0.006923432\text{ u}\end{aligned}\ . \tag{2.3}$$

This differs from the empirical value (2.1) by the same $2.3983\times10^{-6}$ u, or just over two parts per million. So when the stable nuclide $^{10}$B is created by fusing $^{8}$Be with two protons, apparently each up / down quark pair in the target $^{10}$B nuclide contributes one energy does of $\sqrt{m_u m_d}/(2\pi)^{1.5}$. But in addition, there is an overall energy dose of $\sqrt{m_u m_d}$ as well. Noting that in the 2s shell, the orbital angular momentum is *l*=0, but that 2p is the first shell in which nucleons have a non-zero *l*=1, it makes sense, at least preliminarily, to regard this extra $\sqrt{m_u m_d}$ dose that did not appear when we built $^{6}$Li, as being required to provide the energy needed to sustain one proton and one neutron in *n*=2, *l*=1, *m*=0 states. So we regard the $(3\times A/2)\cdot\sqrt{m_u m_d}/(2\pi)^{1.5}$ energy doses as pairwise contributions by the up and down quarks to sustain binding, and the overall $\sqrt{m_u m_d}$ dose as a contribution to sustain angular momentum.

Rather than stay inside the *n*=2, *l*=1, *m*=0 states of the 2p shell, let us see if we can strike further into the nuclear binding table by building the $^{14}$N in a similar way. Here, for the first time, we will have protons and neutrons in *n*=2, *l*=1, *m*=±1 states, i.e., with non-zero *m* magnetic quantum number states. The analogous reaction we wish to consider here, is ${}^{12}_{6}C + 2p \rightarrow {}^{14}_{7}N + e^{+} + \nu + \text{Energy}$. The energy released is:

$$\text{Energy} = {}^{12}_{6}M + 2M_p - {}^{14}_{7}M - m_e = 0.011478929\text{ u}\ . \tag{2.4}$$

This uses the empirical data ${}^{12}_{6}M = 11.996708521$ u, ${}^{14}_{7}N = 13.999233945$ and the proton and electron masses. Noting that these elements are both along the *Z*=*N* nuclide diagonal and have equal numbers of up and down quarks and that we have thus far utilized a $\sqrt{m_u m_d} = 0.003546105$ u construct which is $u \leftrightarrow d$ symmetric, let us also bring the similarly-symmetric $(m_u + m_d)/2 = 0.003827326$ u construct into play. This is about 8% larger than $\sqrt{m_u m_d}$, but has the appropriate symmetry and so should also be considered especially when working on the *Z*=*N* diagonal. Very interestingly, the above energy (2.4) differs from $3\times(m_u + m_d)/2$ by a mere $3.0490\times10^{-6}$ u. We therefore make the association:

$$\begin{aligned}&\text{Energy}\left({}^{12}_{6}C + 2p \rightarrow {}^{14}_{7}N + e^{+} + \nu + \text{Energy}\right)\\&\qquad = {}^{12}_{6}M + 2\cdot M_p - {}^{14}_{7}M - m_e = 3\times\left(m_u + m_d\right)/2 = 0.011481978 \text{ u}\end{aligned} . \tag{2.5}$$

Apparently, once we start to construct nuclides for which $m\neq 0$, nature replaces $\sqrt{m_u m_d}$ , and simply employs three "doses" of $\left(m_u + m_d\right)/2$ to construct $^{14}$N. Perhaps the number "3" representing these doses may be ascribed to the three complete shell levels 1s, 2s and 2p$^0$ (where the superscript "0" indicates $m$=0) upon which the proton and neutron to create $^{14}$N are overlaid.

## 3. Mass/Energy Relations for $^{9}$Be, $^{10}$Be, $^{11}$B and $^{11}$C

Having obtained the relationship (2.3) for $^{10}$B, which is a stable nuclide, let us see if we can branch out from here. First, we work over to $^{10}$B's lighter isotone $^{9}$Be. The reaction we shall consider is ${}^{9}_{4}Be + p \rightarrow {}^{10}_{5}B + \text{Energy}$, fusing a proton with $^{9}$Be to produce $^{10}$B for which the binding energy is now known in principle via (2.3). (See section 4 of [1] which shows how the deduction is done once the nuclear weight is established, and see section 4 below in which we shall explicitly calculate this binding energy.) The fusion energy relation is:

$$\text{Energy} = {}^{9}_{4}M + M_p - {}^{10}_{5}M = 0.007070247 \text{ u} , \tag{3.1}$$

using the empirical values ${}^{9}_{4}M = 9.009987880 \text{ u}$, ${}^{10}_{5}M = 10.010194100 \text{ u}$ and the proton mass. This differs from $2\sqrt{m_\mu m_d}$ by $2.19637\times 10^{-5}$ u or just over 2 parts per 100,000 AMU, which is within the ranges we have previously taken to be physically meaningful. So we now establish the close relationship:

$$\text{Energy}\left({}^{9}_{4}Be + p \rightarrow {}^{10}_{5}B + \text{Energy}\right) = {}^{9}_{4}M + M_p - {}^{10}_{5}M = 2\sqrt{m_\mu m_d} = 0.007092210 \text{ u} , \tag{3.2}$$

This binding energy for $^{9}$Be can now be deduced from this, and will be in section 4.

The next nuclide we consider branching to from $^{10}$B is the comparatively stable $^{10}$Be, which has a half-life of $1.39\times 10^{6}$ years before it decays through $\beta^-$ decay into its isotope $^{10}$B for which we deduced the fusion energy (2.3). Here the reaction is ${}^{10}_{4}Be \rightarrow {}^{10}_{5}B + e + \overline{\nu} + \text{Energy}$ and so the energy relationships are:

$$\text{Energy} = {}^{10}_{4}M - {}^{10}_{5}M - m_e = 0.000596800 \text{ u} . \tag{3.3}$$

Above, we use the empirical ${}^{10}_{4}M = 10.011339480 \text{ u}$, ${}^{10}_{5}M = 10.010194100 \text{ u}$ and the electron mass. In trying to fit this result, we recall from eq. [15] of [2] that the binding energy of $^{3}$He is retrodicted to under four parts per 100,000 to be $B\left({}^{3}He\right) = \sqrt{m_u}\left(\sqrt{m_d} + 2\sqrt{m_u}\right) = 2m_u + \sqrt{m_u m_d}$ . Keeping this in mind, we form three similar mass combinations

$\sqrt{m_d}\left(\sqrt{m_d}+2\sqrt{m_u}\right)=m_d+2\sqrt{m_u m_d}$, $\sqrt{m_d}\left(\sqrt{m_u}+2\sqrt{m_d}\right)=2m_d+\sqrt{m_u m_d}$ and $\sqrt{m_u}\left(\sqrt{m_u}+2\sqrt{m_d}\right)=m_u+2\sqrt{m_u m_d}$, as well as the foregoing divided by $(2\pi)^{1.5}$. All of these are readily constructed from the square root of an up or down quark mass times the trace of a Koide matrix for the proton or neutron, see, e.g., (15) of [2]. It turns out that the value in (3.3) differs from the final expression $\left(m_u+2\sqrt{m_u m_d}\right)/(2\pi)^{1.5}$ by $-5.0911\times10^{-6}$ u, that is, by five parts per million. We take this to be a meaningful relationship, and so write (3.3) as:

$$\begin{aligned}&\text{Energy}\left({}^{10}_{4}Be\rightarrow{}^{10}_{5}B+e+\overline{\nu}+\text{Energy}\right)\\&\quad={}^{10}_{4}M-{}^{10}_{5}M-m_e=\left(m_u+2\sqrt{m_u m_d}\right)/(2\pi)^{1.5}=0.000601891\text{ u}\end{aligned}\qquad(3.4)$$

Now we branch up to $^{11}$B via ${}^{10}_{5}B+p+e\rightarrow{}^{11}_{5}B+\nu+\text{Energy}$. The energies are:

$$\text{Energy}={}^{10}_{5}M+M_p+m_e-{}^{11}_{5}M=0.011456647\text{ u}.\qquad(3.5)$$

Above, we use ${}^{10}_{5}M=10.010194100$ u, ${}^{11}_{5}M=11.006562500$ and the proton and electron masses. It turns out that the above differs from $3\cdot(m_u+m_d)/2$ by $2.53311\times10^{-5}$ u, or under 3 parts per 100,000. We take this as a meaningful relationship, and so write (3.5) as:

$$\begin{aligned}&\text{Energy}\left({}^{10}_{5}B+p+e\rightarrow{}^{11}_{5}B+\nu+\text{Energy}\right)\\&\quad={}^{10}_{5}M+M_p+m_e-{}^{11}_{5}M=3\cdot(m_u+m_d)/2=0.011481978\text{ u}\end{aligned}\qquad(3.6)$$

So as a respective result of (2.3), (3.2), (3.4) and (3.6), it becomes possible to deduce the binding energies of four new nuclides: $^{10}$B, $^{9}$Be, $^{10}$Be and $^{11}$B. Before we explicitly deduce these four binding energies, let us also look at one final branch, this time from $^{11}$B to $^{11}$C. Carbon-11, which is used to label molecules in PET scans, has a half-life of 20.334(24) min before it $\beta^+$ decays into $^{11}$B which we have just uncovered in (3.6) above. This reaction is ${}^{11}_{6}C+e\rightarrow{}^{11}_{5}B+\nu+\text{Energy}$, which is represented as:

$$\text{Energy}={}^{11}_{6}M+m_e-{}^{11}_{5}M=0.002128200\text{ u}.\qquad(3.7)$$

Here we have used ${}^{11}_{5}M=11.006562500$ and ${}^{11}_{6}M=11.008142121$ u. Comparing to the usual constructs, we see that $4\left(2m_u+\sqrt{m_u m_d}\right)/(2\pi)^{1.5}$ differs by $-1.49327\times10^{-5}$ u, less than 2 parts in 100,000. So we take this to be meaningful, and rewrite (3.7) as:

$$\begin{aligned}&\text{Energy}\left({}^{11}_{6}C+e\rightarrow{}^{11}_{5}B+\nu+\text{Energy}\right)\\&\quad={}^{11}_{6}M+m_e-{}^{11}_{5}M=8m_u/(2\pi)^{1.5}+4\sqrt{m_u m_d}/(2\pi)^{1.5}=0.002113267\text{ u}\end{aligned}\qquad(3.8)$$

Now we shall explicitly decide the binding energies for all of $^{10}$B, $^{9}$Be, $^{10}$Be, $^{11}$B and $^{11}$C, before we turn separately to $^{12}$C which completes the $2p^0$ subshell (0 representing *m*=0).

## 4. Deduction of Binding Energies for $^{10}$B, $^{9}$Be, $^{10}$Be, $^{11}$B and $^{11}$C

As we are reminded in section 4 of [1], for a nuclide with *Z* protons and *N* neutrons hence *A*=*Z*+*N* nucleons, the binding energy ${}_{Z}^{A}B$ is related to its atomic weight ${}_{Z}^{A}M$ according to:

$$ {}_{Z}^{A}B = Z \cdot M_P + N \cdot M_N - {}_{Z}^{A}M \ . \tag{4.1} $$

So for the $^{10}$B, $^{9}$Be, $^{10}$Be, $^{11}$B and $^{11}$C binding energies, we need to find:

$$ \begin{aligned} {}_{5}^{10}B &= 5 \cdot M_P + 5 \cdot M_N - {}_{5}^{10}M \\ {}_{4}^{9}B &= 4 \cdot M_P + 5 \cdot M_N - {}_{4}^{9}M \\ {}_{4}^{10}B &= 4 \cdot M_P + 6 \cdot M_N - {}_{4}^{10}M \ . \\ {}_{5}^{11}B &= 5 \cdot M_P + 6 \cdot M_N - {}_{5}^{11}M \\ {}_{6}^{11}B &= 6 \cdot M_P + 5 \cdot M_N - {}_{6}^{11}M \end{aligned} \tag{4.2} $$

We begin by substituting (2.3), (3.2), (3.4), (3.6) and (3.8) into the above, rearranged so that the nuclear masses on the very right of each of the above may be replaced. This yields:

$$ \begin{aligned} {}_{5}^{10}B &= 3 \cdot M_P + 5 \cdot M_N - {}_{4}^{8}M + \sqrt{m_u m_d} + 15\sqrt{m_u m_d} \,/ \left(2\pi\right)^{1.5} + m_e \\ {}_{4}^{9}B &= 5 \cdot M_P + 5 \cdot M_N - {}_{5}^{10}M - 2\sqrt{m_\mu m_d} \\ {}_{4}^{10}B &= 4 \cdot M_P + 6 \cdot M_N - {}_{5}^{10}M - m_u \,/ \left(2\pi\right)^{1.5} - 2\sqrt{m_u m_d} \,/ \left(2\pi\right)^{1.5} - m_e \ . \\ {}_{5}^{11}B &= 4 \cdot M_P + 6 \cdot M_N - {}_{5}^{10}M + 3 \cdot \left(m_u + m_d\right) / 2 - m_e \\ {}_{6}^{11}B &= 6 \cdot M_P + 5 \cdot M_N - {}_{5}^{11}M - 8 m_u \,/ \left(2\pi\right)^{1.5} - 4\sqrt{m_u m_d} \,/ \left(2\pi\right)^{1.5} + m_e \end{aligned} \tag{4.3} $$

Next we substitute for ${}_{5}^{10}M$ in the second through fourth expressions, and for ${}_{5}^{11}M$ and again for ${}_{5}^{10}M$ in the final expression. This brings us to:

$$ \begin{aligned} {}_{5}^{10}B &= 3 \cdot M_P + 5 \cdot M_N - {}_{4}^{8}M + \sqrt{m_u m_d} + 15\sqrt{m_u m_d} \,/ \left(2\pi\right)^{1.5} + m_e \\ {}_{4}^{9}B &= 3 \cdot M_P + 5 \cdot M_N - {}_{4}^{8}M - \sqrt{m_u m_d} + 15\sqrt{m_u m_d} \,/ \left(2\pi\right)^{1.5} + m_e \\ {}_{4}^{10}B &= 2 \cdot M_P + 6 \cdot M_N - {}_{4}^{8}M + \sqrt{m_u m_d} + 13\sqrt{m_u m_d} \,/ \left(2\pi\right)^{1.5} - m_u \,/ \left(2\pi\right)^{1.5} \\ {}_{5}^{11}B &= 2 \cdot M_P + 6 \cdot M_N - {}_{4}^{8}M + 3 \cdot \left(m_u + m_d\right) / 2 + \sqrt{m_u m_d} + 15\sqrt{m_u m_d} \,/ \left(2\pi\right)^{1.5} \\ {}_{6}^{11}B &= 3 \cdot M_P + 5 \cdot M_N - {}_{4}^{8}M + 3 \cdot \left(m_u + m_d\right) / 2 + \sqrt{m_u m_d} - 8 m_u \,/ \left(2\pi\right)^{1.5} + 11\sqrt{m_u m_d} \,/ \left(2\pi\right)^{1.5} + m_e \end{aligned} \quad .(4.4) $$

Now the foregoing all contain the nuclear weight ${}^{8}_{4}M$ of ${}^{8}$Be. So now we invert (4.1) specifically for ${}^{8}$Be, to write:

$$ {}^{8}_{4}M = 4\cdot M_P + 4\cdot M_N - {}^{8}_{4}B \,. \tag{4.5} $$

Substituting this into all of (4.4) and reducing, next yields:

$$ \begin{aligned} {}^{10}_{5}B &= \quad \left(M_N - M_P\right) + {}^{8}_{4}B + \sqrt{m_u m_d} + 15\sqrt{m_u m_d} \,/\left(2\pi\right)^{1.5} + m_e \\ {}^{9}_{4}B &= \quad \left(M_N - M_P\right) + {}^{8}_{4}B - \sqrt{m_u m_d} + 15\sqrt{m_u m_d} \,/\left(2\pi\right)^{1.5} + m_e \\ {}^{10}_{4}B &= 2\left(M_N - M_P\right) + {}^{8}_{4}B + \sqrt{m_u m_d} + 13\sqrt{m_u m_d} \,/\left(2\pi\right)^{1.5} - m_u \,/\left(2\pi\right)^{1.5} \\ {}^{11}_{5}B &= 2\left(M_N - M_P\right) + {}^{8}_{4}B + 3\cdot\left(m_u + m_d\right)/2 + \sqrt{m_u m_d} + 15\sqrt{m_u m_d} \,/\left(2\pi\right)^{1.5} \\ {}^{11}_{6}B &= \quad \left(M_N - M_P\right) + {}^{8}_{4}B + 3\cdot\left(m_u + m_d\right)/2 + \sqrt{m_u m_d} - 8m_u \,/\left(2\pi\right)^{1.5} + 11\sqrt{m_u m_d} \,/\left(2\pi\right)^{1.5} + m_e \end{aligned} \,. \tag{4.6} $$

Now we just need to make three final substitutions and reduce: From [1.10] of [1]:

$$ M_N - M_P = m_u - \left(3m_d + 2\sqrt{m_\mu m_d} - 3m_u\right)/\left(2\pi\right)^{\frac{3}{2}} \,. \tag{4.7} $$

From [4.5] through [4.7] of [1]:

$$ {}^{8}_{4}B = 12m_u + 12m_d - 2\sqrt{m_u m_d} - \left(20m_d + 64\sqrt{m_u m_d} + 20m_u\right)/\left(2\pi\right)^{1.5} \,. \tag{4.8} $$

And from [1.11] of [1]:

$$ m_e = 3\left(m_d - m_u\right)/\left(2\pi\right)^{1.5} \,. \tag{4.9} $$

Making the substitutions (4.7) through (4.9) into all of (4.6), reducing, and evaluating using the quark masses from [1.12] and [1.13] of [1], namely:

$$ m_u = 0.002387339327 \text{ u} \,, \tag{4.10} $$
$$ m_d = 0.005267312526 \text{ u} \,, \tag{4.11} $$

finally yields for ${}^{10}$B, ${}^{9}$Be, ${}^{10}$Be, ${}^{11}$B and ${}^{11}$C, respectively:

$$
\begin{aligned}
{}^{10}_{5}B &= 13m_u + 12m_d - \sqrt{m_u m_d} - \left(20m_u + 20m_d + 51\sqrt{m_u m_d}\right)/\left(2\pi\right)^{1.5} = 0.0694937119 \text{ u} \\
{}^{9}_{4}B &= 13m_u + 12m_d - 3\sqrt{m_u m_d} - \left(20m_u + 20m_d + 51\sqrt{m_u m_d}\right)/\left(2\pi\right)^{1.5} = 0.0624015014 \text{ u} \\
{}^{10}_{4}B &= 14m_u + 12m_d - \sqrt{m_u m_d} - \left(15m_u + 26m_d + 55\sqrt{m_\mu m_d}\right)/\left(2\pi\right)^{1.5} = 0.0697316901 \text{ u} \quad . \\
{}^{11}_{5}B &= \frac{31}{2}m_u + \frac{27}{2}m_d - \sqrt{m_u m_d} - \left(14m_u + 26m_d + 53\sqrt{m_\mu m_d}\right)/\left(2\pi\right)^{1.5} = 0.0818155590 \text{ u} \\
{}^{11}_{6}B &= \frac{29}{2}m_u + \frac{27}{2}m_d - \sqrt{m_u m_d} - \left(28m_u + 20m_d + 55\sqrt{m_\mu m_d}\right)/\left(2\pi\right)^{1.5} = 0.0788624224 \text{ u}
\end{aligned}
\tag{4.12}
$$

Respective empirical values for the above are 0.0695128136 u ($\Delta = -1.910169\times10^{-5}$ u); 0.0624425669 u ($\Delta = -4.106544\times10^{-5}$ u); 0.0697558829 u ($\Delta = -2.419278\times10^{-5}$ u); 0.0818093296 u ($\Delta = 6.22936\times10^{-6}$ u); and finally, 0.0788412603 u ($\Delta = 2.116207\times10^{-5}$ u).

## 5. Binding Energy for $^{12}$C

Carbon-12 has $Z$=$A$=6 and fully fills the 2p$^0$ subshell for both protons and neutrons. It contains 18 up and down quarks alike. Like $^4$He and $^8$Be, we expect that the binding energy for $^{12}$C will be symmetric under $u \leftrightarrow d$ interchange. Therefore, we expect that the only admissible numbers will be $\sqrt{m_u m_d}$ and $\frac{1}{2}\left(m_u + m_d\right)$ and multiples and combinations thereof.

Using the proton and neutron "energy numbers" from (1.6) and (1.7) of [1]

$$\Delta E_P = m_d + 2m_u - \left(m_d + 4\sqrt{m_u m_d} + 4m_u\right)/\left(2\pi\right)^{1.5}, \tag{5.1}$$

$$\Delta E_N = m_u + 2m_d - \left(m_u + 4\sqrt{m_u m_d} + 4m_d\right)/\left(2\pi\right)^{1.5}, \tag{5.2}$$

(1.2) of [1] reported that the $^4$He alpha particle binding energy is:

$${}^{4}_{2}B = 2\cdot\Delta E_P + 2\cdot\Delta E_N - 2\sqrt{m_u m_d} \tag{5.3}$$

to under 3 parts per million AMU. Similarly, in (3.3) of [1] we found that the $^8$Be binding energy is (see the fully-expanded expression (4.8) above):

$${}^{8}_{4}B = 4\cdot\Delta E_P + 4\cdot\Delta E_N - 2\sqrt{m_u m_d} - 32\sqrt{m_u m_d}\ /\left(2\pi\right)^{1.5}, \tag{5.4}$$

to about 2 parts per 100,000 AMU. If we define an energy "dosage" $D_1 \equiv \frac{1}{2}\sqrt{m_u m_d}$, then we may write (5.3) in terms of $A$=$Z$+$N$ as:

$${}^{4}_{2}B = Z\cdot\Delta E_P + N\cdot\Delta E_N - A\cdot D_1 \tag{5.5}$$

Using this same dosage, (5.4) may be written as:

$$ {}_{4}^{8}B = Z \cdot \Delta E_P + N \cdot \Delta E_N - \frac{A}{2} D_1 - \frac{A}{2}\left(16D_1 / \left(2\pi\right)^{1.5}\right), \tag{5.6}$$

recalling that in obtaining (5.6), we took advantage of $16 \cong \left(2\pi\right)^{1.5} = 15.7496099457$, see [1] between [3.1]and [3.2]. This is was what accounted for the almost immediate alpha-decay of one $^{8}$Be nucleus into two $^{4}$H nuclei.

It turns out after some trial and error fitting based on the foregoing, that the $^{12}$C binding energy may be specified, not using $\sqrt{m_u m_d}$ , but rather, the other $u \leftrightarrow d$ symmetric construct $\frac{1}{2}\left(m_u + m_d\right)$ which differs from $\sqrt{m_u m_d}$ by about 8%, and which has previously appeared in (2.5) for $^{14}$N and (3.6) for $^{11}$B. Specifically, it may be calculated that a $^{12}$C binding energy defined in terms of quark masses as:

$$ {}_{6}^{12}B = 6 \cdot \Delta E_P + 6 \cdot \Delta E_N - \left(m_u + m_d\right) - 12\left(m_u + m_d\right) / \left(2\pi\right)^{1.5} = 0.0989087255 \text{ u} \tag{5.7}$$

will differ from the empirical energy 0.0989397763 u by $-3.10508 \times 10^{-5}$ u .

To obtain an "apples-to-apples" comparison with (5.5) and (5.6) to help discern the overall pattern of full-shell *Z*=*N*=even elements such as $^{4}$He, $^{8}$Be, $^{12}$C, $^{16}$O, $^{20}$Ne, $^{24}$Mg, etc., which as we have seen in section 3 here appear to form a "backbone" from which it then becomes possible to branch out to close isotones, isobars and isotopes, let us define another dosage number $D_2 \equiv \frac{1}{4}\left(m_u + m_d\right)$. Using this in (5.7) allows us to write:

$$ {}_{6}^{12}B = Z \cdot \Delta E_P + N \cdot \Delta E_N - \frac{A}{3} D_2 - \frac{A}{4} \cdot \left(16D_2 / \left(2\pi\right)^{1.5}\right). \tag{5.8}$$

While it is not yet clear what the overall formulation is for ${}_{Z}^{A}B$ in general for the *Z*=*N*=even backbone, (5.5), (5.6) and (5.8) start to give us a sense of what to be looking for. Trying to further fit $^{16}$O, $^{20}$Ne and $^{24}$Mg, the next three backbone nuclides, may provide a better view of how to propagate this backbone all the way through the nuclide table, and provide the "tree trunk" for then branching out as in section 3 above, in order to "map" the complete "nuclear genome" as a function of up and down quark masses to low parts per 100,000 or parts per million AMU.

## 6. Derivation of the $^{14}$N binding Energy

Finally, with one more data point on the nuclear "backbone" identified in (5.7), let us make us of (2.5) and (5.7) to deduce the $^{14}$N binding energy. This is the first element we are considering in the 2p$^{\pm 1}$ subshell. As in section 4, we start with (4.1) which tells us that:

$$ {}_{7}^{14}B = 7 \cdot M_P + 7 \cdot M_N - {}_{7}^{14}M \ . \tag{6.1}$$

We next rearrange (2.5) to separate ${}^{14}_{7}M$ and use this in (6.1), thus:

$$ {}^{14}_{7}B = 5\cdot M_P + 7\cdot M_N + 3\times\left(m_u + m_d\right)/2 - {}^{12}_{6}M + m_e . \tag{6.2}$$

Then using (4.1) in the inverted form ${}^{12}_{6}M = 6\cdot M_P + 6\cdot M_N - {}^{12}_{6}B$, we rewrite (6.2) as:

$$ {}^{14}_{7}B = \left(M_N - M_P\right) + 3\times\left(m_u + m_d\right)/2 + {}^{12}_{6}B + m_e . \tag{6.3}$$

Now, we simply use (4.7), (5.7), (5.1), (5.2) and (4.9) in the above and reduce. Using the quark masses (4.10), (4.11), we finally obtain:

$$ {}^{14}_{7}B = \frac{39}{2}m_u + \frac{37}{2}m_d - \left(42m_u + 42m_d + 50\sqrt{m_u m_d}\right)/\left(2\pi\right)^{1.5} = 0.1123277324 \text{ u} . \tag{6.4}$$

The empirical binding energy is 0.1123557343 u, which differs by $2.800186\times10^{-5}$ u . This is our first nuclide which contains protons and neutrons for which $m\neq0$.

The incremental approach of deducing binding energies by "weaving" from one nuclide to other nearby nuclides through the close consideration of fusion and data decay reactions as first elaborated in [1] appears to be very much re-validated by the results obtained here as well. Additionally this sort of approach gives us confidence that our overall expressions for binding energies are correct, because they are incrementally constructed in this manner, brick by brick or stitch by stitch so to speak, enhancing the probability that the relationships obtained are meaningful, and are not random fortuitous coincidences.

## 7. Conclusion

Deep inelastic scattering is the tool most widely used to probe the quark structure inside of protons and neutrons, But the European Muon Collaboration as well as the long-recognized existence of mass defects in the nuclear table, make it clear that the structure of the quarks inside of individual nucleons will be materially affected by whether those nucleons are free, or are bound together as part of a composite nucleus. This also appears to depend even upon the particular shell within which a particular nucleon resides. Therefore, it seems that one very good way to understand quark structure is to examine various nuclei and how the quark structure changes depending upon the particular nucleus and nuclear shell in question.

What the results detailed here and in the two prior letters [1] and [2] demonstrate very clearly, is that the nuclear weights of the various nuclides themselves, converted into fusion release and binding energies, are in fact telling us a great deal about what is going on inside of those nucleons in relation to the nuclei and shells within which they sit, even without resort to deep scattering. In other words, the well-characterized mass defects long observed in the nuclear table are the best, most precise signals and evidence we have about what is actually going on with the quarks inside of various nuclei, and we don't need to smash particles together in order to acquire this information. But, it now becomes very important to decipher this signal evidence in order to understand what it is truly telling us about the behavior of quarks inside of nucleons and nuclei and nuclear shells.

The results in this letter as well as the two recent letters [1] and [2] tell us in very exact terms what is happening to the energies inside of nuclei as a direct function of the quark masses, as well as to the quark energy structure itself, on a shell by-shell and nucleon-by-nucleon basis. Further extension of these results, as well as their careful deciphering, may finally begin to inform us at a very detailed and granular level, what is really happening with the quarks inside of protons and neutrons, and with the protons and neutrons inside atomic nuclei.

In the same way that Feynman diagrams are developed term-by-term from invariant amplitude expressions to inform us about the nature of particle interactions, it may well be that nuclear models can be similarly constructed term-by-term from expressions such as (4.12) and (6.4) and the backbones in section 5, to help us understand how atomic nuclei are put together and how they are structured.  All of this may in turn shed some long-needed light on how matter really binds together to form the material world we observe and inhabit.

## References

[1] J. Yablon, *Fitting the $^{6}Li$, $^{7}Li$, $^{7}Be$ and $^{8}Be$ Binding Energies to High Precision based Exclusively on the Up and Down Quark Masses* (2013) (preprint: http://vixra.org/abs/1306.0207)

[2] J. Yablon, *Fitting the $^{2}H$, $^{3}H$, $^{3}He$, $^{4}He$ Binding Energies and the Neutron minus Proton Mass Difference to Parts-Per-Million based Exclusively on the Up and Down Quark Masses* (2013) (preprint: http://vixra.org/abs/1306.0203)