On the gravitational mass

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The paper presents a natural definition of gravitational mass without invoking new entities. The approach suggested expands the application field of the law of gravitational interaction between material objects located in a gravitating medium. The paper demonstrates the existence of a functional relationship between the inertial and gravitational masses, which has brought us to a conclusion that the condition under which the inertial and gravitational masses would be equal is rather speculative and not practically realizable. We give here real examples of existence of the negative gravitational mass, as well as natural cases of the gravitational repulsion. Some cases of gravitational dipole as a physical object existing in our natural environment are also presented.

1. Problem definition

Consider the problem of finding forces acting on a motionless material body $\mathcal{G}$ of a fixed volume and constant density, which is placed in a certain homogeneous static medium $\mathcal{F}$ (with uniformly distributed density of matter). The existence of density stipulates the existence of the gravity field generated both by body $\mathcal{G}$ and medium $\mathcal{F}$. Assume that material body $\mathcal{G}$ experiences gravitational force only from medium $\mathcal{F}$. In addition, assume for simplicity that spatial domain $\mathcal{F}$ contains a certain *gravitating medium*[1], e.g., zero-viscosity incompressible liquid in the form of a finite-radius sphere. In the scope of our problem definition, material body $\mathcal{G}$ plays the role of a foreign inclusion in domain ($\mathcal{F}$) under consideration; therefore, we will refer to it as *anomaly*[2]. Let us choose as anomaly $\mathcal{G}$ a homogeneous rigid sphere located at some fixed distance from the domain $\mathcal{F}$ center.

The conditions of the matter density constancy in domain $\mathcal{F}$ and of spherical shape of domains $\mathcal{G}$ and $\mathcal{F}$ are not mandatory, but allow us to simplify the problem and focus on its main aspect, namely, on deriving an expression for the forces acting upon the anomaly in the gravitating medium.

[1]Hereinafter, term gravitating medium means that this is a medium with a preset matter density distribution.

[2]In gravimetry, term "anomaly" is often used to define an inclusion (body) with the density different from that of the surrounding medium [1, 2].

Consider Cartesian coordinate system $\mathbf{O}xyz$ (Fig. 1). Let the coordinate system origin (point $\mathbf{O}$) be the gravitational attraction center of spherical domain $\mathcal{F}$.

Designate the force of gravitational interaction between anomaly $\mathcal{G}$ and domain $\mathcal{F}$ matter as $\underline{F}^{-}$. Due to the central symmetry of the problem about the domain geometry and matter distribution, this force is directed towards the medium's center of gravitational attraction (point $\mathbf{O}$). Gravity field of the domain $\mathcal{F}$ matter ensures the centrosymmetrical pressure distribution in the $\mathcal{F}$ medium. The presence of the pressure gradient in the medium surrounding anomaly $\mathcal{G}$ gives rise to the so-called buoyant force $\underline{F}^{+}$.

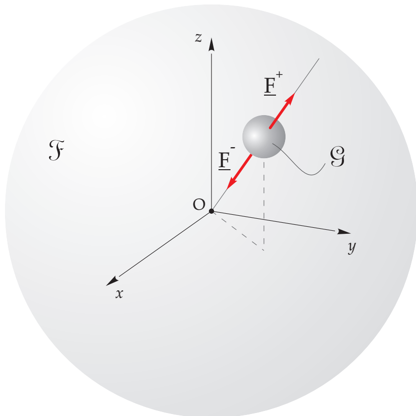


Figure 1: Gravitating medium $\mathcal{F}$ with spherical anomaly $\mathcal{G}$.

Therefore, anomaly $\mathcal{G}$ experiences two quasi-independent oppositely directed collinear forces. Evidently, this straight line is tangential to the gravity field line of domain $\mathcal{F}$ at the application point of the forces defined above. The sum of these forces is:

$$\underline{F} = \underline{F}^{+} + \underline{F}^{-} \ , \tag{1}$$

where $\underline{F}^{-}$ is the force of direct gravitational effect on anomaly $\mathcal{G}$ from the domain $\mathcal{F}$ matter, which acts towards point $\mathbf{O}$, and $\underline{F}^{+}$ is the buoyant force caused by the presence of the pressure gradient in medium $\mathcal{F}$.

The case considered may be regarded as a liquid drop with a pellet inside in weightlessness[3]. For more simplicity, neglect the external gravitational effects. Under these conditions, the pellet can "sink" towards the drop center or "emerge" to the surface depending on the ratio between its own density and that of the liquid.

Regardless of the triviality of our problem defined as above, let us consider each force component of equation (1).

[3]A spatial domain where gravitational forces are counterpoised by centrifugal or other forces, and the system is equilibrium on the average.

2. Direct action of the gravitating medium on the anomaly

Let point $\mathbf{O}'$ be the geometrical center of anomaly $\mathcal{G}$ under consideration. Since the central symmetry makes all the radial directions of the matter density distribution in sphere $\mathcal{F}$ equivalent, let us put point $\mathbf{O}'$ on axis $\mathbf{O}z$ at distance δ from point $\mathbf{O}$ that is the domain $\mathcal{F}$ geometrical center (Fig. 2).

The auxiliary rectangular coordinate system $\mathbf{O}'x'y'z$ related to anomaly $\mathcal{G}$ is oriented so that axis $\mathbf{O}'x'$ is parallel to axis $\mathbf{O}x$, while axis $\mathbf{O}'y'$ is parallel to axis $\mathbf{O}y$.

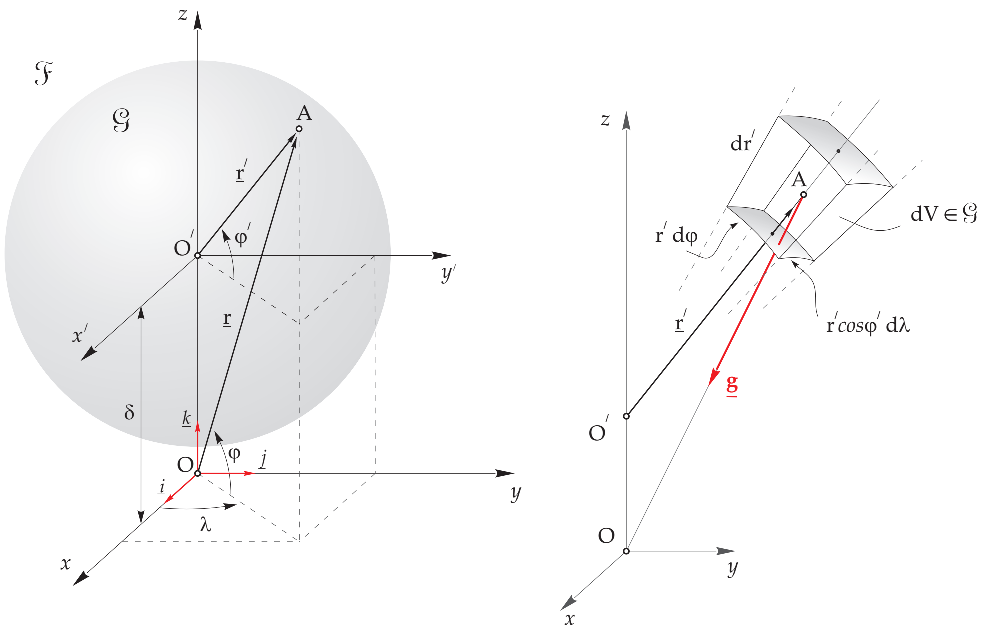


Figure 2:

The gravity field gradient $\underline{\mathbf{g}}_{\mathcal{F}}$ exists at each point of spherical domain $\mathcal{F}$ under consideration:

$$\underline{\mathbf{g}}_{\mathcal{F}} = -\mathbf{G}\frac{4}{3}\pi\rho_{\mathcal{F}}\,\underline{r}\ , \tag{2}$$

where $\underline{r}$ is the radius-vector of an arbitrary point $\mathbf{A}$ of domain $\mathcal{F}$; $\mathbf{G}$ is the gravitational constant. Detailed derivation of this formula is shown in, e.g., [1, 3]. As relation (2) shows, gravity field gradient $\underline{\mathbf{g}}_{\mathcal{F}}$ at the point with radius-vector $\underline{r}$ is always directed towards the domain $\mathcal{F}$ geometrical center; in this case, this is point $\mathbf{O}$.

Now we know the gravity field gradient at each point of spatial domain $\mathcal{F}$ and can estimate the gravitational force acting on anomaly $\mathcal{G}$. Since the gravitational interaction is additive, each elementary volume dV of the domain $\mathcal{G}$ matter experiences non-zero gravitational effect from the domain $\mathcal{F}$ matter. The resultant vector of gravitational action of medium $\mathcal{F}$ on anomaly $\mathcal{G}$ is the following integral:

$$\underline{F}^{-} = \int\limits_{V_{\mathcal{G}}} d\underline{F}^{-} \ , \quad d\underline{F}^{-} = \rho_{\mathcal{G}} dV \, \underline{\mathbf{g}}_{\mathcal{F}} \ . \tag{3}$$

Since our problem is symmetrical about axis $\mathbf{O}z$, it is evident that

$$F_x^{-} = \underline{i} \cdot \underline{F}^{-} = 0, \quad F_y^{-} = \underline{j} \cdot \underline{F}^{-} = 0, \quad F_z^{-} = \underline{k} \cdot \underline{F}^{-} = \int\limits_{V_{\mathcal{G}}} \underline{k} \cdot d\underline{F}^{-} \ .$$

Substitute (3) and (2):

$$F_z^{-} = \int\limits_{V_{\mathcal{G}}} \underline{k} \cdot d\underline{F}^{-} = \rho_{\mathcal{G}} \int\limits_{V_{\mathcal{G}}} \underline{k} \cdot \underline{\mathbf{g}}_{\mathcal{F}} \, dV = -\frac{4}{3}\pi \mathbf{G} \rho_{\mathcal{G}} \rho_{\mathcal{F}} \int\limits_{V_{\mathcal{G}}} \overbrace{\underline{k} \cdot \underline{r}}^{r \sin\varphi} \ dV \ . \tag{4}$$

Let us express product $r \sin\varphi$ and elementary volume dV created near point $\mathbf{A} \in \mathcal{G}$ based on the Fig. 2 geometry. Point $\mathbf{A}$ position in coordinate system $\mathbf{O}'x'y'z$ is specified by spherical coordinates (r', φ', λ). Hence, the necessary relations take the following form:

$$dV = r' d\varphi' \cdot r' \cos\varphi' \ d\lambda \cdot dr' = r'^2 \cos\varphi' \ dr' \ d\varphi' \ d\lambda \ . \tag{5}$$

and

$$r \sin\varphi = \delta + r' \sin\varphi' \ . \tag{6}$$

Substituting 5 and (6) into (4), obtain:

$$F_z^{-} = -\frac{4}{3}\pi \mathbf{G} \rho_{\mathcal{G}} \rho_{\mathcal{F}} \left(\delta V_{\mathcal{G}} + \overbrace{\int\limits_{V_{\mathcal{G}}} r' \sin\varphi' \ dV}^{\mathbf{I}} \right) , \tag{7}$$

where integral $\mathbf{I} = 0$ because

$$\mathbf{I} = \int\limits_{V_{\mathcal{G}}} r' \sin\varphi' \ \overbrace{r' d\varphi' r' \cos\varphi' d\lambda dr'}^{dV} = \int\limits_{0}^{2\pi} d\lambda \int\limits_{0}^{R_{\mathcal{G}}} r'^3 dr' \int\limits_{-\pi/2}^{\pi/2} \cancelto{0}{\sin\varphi' \cos\varphi' d\varphi'} = 0.$$

Grouping the factors in equation (7), obtain

$$F_z^- = \overbrace{-\frac{4}{3}\pi\mathbf{G}\rho_F\delta}^{\mathbf{g}_\mathcal{F}(\delta)}\ \rho_\mathcal{G}V_\mathcal{G}$$

or, in the vector form,

$$\underline{F}^- = \rho_\mathcal{G}V_\mathcal{G}\ \underline{\mathbf{g}}_\mathcal{F}(\delta)\ . \tag{8}$$

Thus, gravitational force $\underline{F}^-$ acting upon anomaly $\mathcal{G}$ is directed towards the domain $\mathcal{F}$ center of gravitational attraction. Relation (8) is true also in the case when the medium $\mathcal{F}$ gravitational attraction center is geometrically within anomaly $\mathcal{G}$, i.e., at $\delta < R_\mathcal{G}$.

3. Indirect action of the gravitating medium on the anomaly

Spherical domain $\mathcal{F}$ with uniformly distributed density $\rho_\mathcal{F}$ induces a centrosymmetrical gravity field with the gravitation attraction center at point **O**. In its turn, this field generates in the medium under consideration a centrosymmetrical pressure distribution with an appropriate radial gradient. Here we ignore creation of pressure gradients by all the factors other than gravitation. Hence, placing anomaly $\mathcal{G}$ into gravitating medium $\mathcal{F}$, we face the fact that the anomaly experiences a buoyant force. Since anomaly $\mathcal{G}$ is an object with a non-zero volume, integration of the pressure effect over its surface gives us a force tending to "push" $\mathcal{G}$ out into the domain of the medium $\mathcal{F}$ minimum pressure.

Let us see how the pressure is distributed in domain $\mathcal{F}$. For this purpose, select in it an elementary volume dV in the vicinity of point **A** with spherical coordinates r, φ and λ (Fig. 3).

$$dV = \overbrace{r d\varphi \cdot r d\lambda \cdot \cos\varphi}^{dS} \cdot dr$$

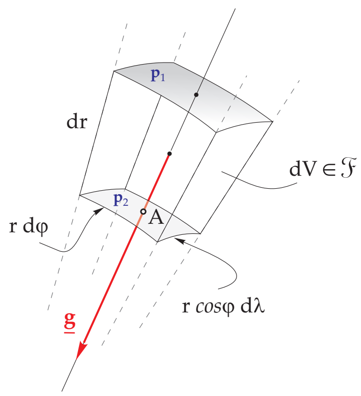


Figure 3: Elementary volume dV in domain $\mathcal{F}$.

The selected elementary volume of the domain $\mathcal{F}$ medium is in equilibrium. The lower surface (area dS) of elementary volume dV undergoes pressure p_2 while the upper one undergoes pressure p_1; in our case, $p_2 > p_1$. In addition, the elementary volume dV mass experiences gravitational force directed towards the medium $\mathcal{F}$

gravitational attraction center. Here we consider a stationary case; hence, the selected elementary volume $dV \in \mathcal{F}$ is in equilibrium. Taking into account all these facts, let us construct an equilibrium equation for elementary volume dV (Fig. 3):

$$p_2 dS - (p_1 dS + \rho_{\mathcal{F}} dV\, \mathbf{g}_{\mathcal{F}}) = 0\,, \quad \text{where} \quad dV = dS dr\,. \tag{9}$$

Therefore, pressure increment dp can be written as follows:

$$dp = p_2 - p_1 = \rho_{\mathcal{F}} dr\, \mathbf{g}_{\mathcal{F}}\,.$$

Substitute in this relation expression (2) for the gravity field gradient $\mathbf{g}_{\mathcal{F}}$ and integrate over r:

$$p(r) = -\frac{2}{3}\pi \mathbf{G} \rho_{\mathcal{F}}^2 r^2 + const\,.$$

The constant may be determined from the condition at the domain $\mathcal{F}$ boundary:

$$p\Big|_{r=R_{\mathcal{F}}} = p_{\mathcal{F}}\,,$$

where $p_{\mathcal{F}}$ is the external pressure at the domain $\mathcal{F}$ boundary. Let $p_{\mathcal{F}} = 0$; then the pressure at an arbitrary point of domain $\mathcal{F}$ may be expressed as

$$p(r) = -\frac{2}{3}\pi \mathbf{G} \rho_{\mathcal{F}}^2 \left(r^2 - R_{\mathcal{F}}^2\right)\,. \tag{10}$$

Now let us determine buoyant force $\underline{F}^{+}$ acting upon anomaly $\mathcal{G}$, which is caused by the presence of pressure gradient in medium $\mathcal{F}$:

$$\underline{F}^{+} = \int\limits_{S_{\mathcal{G}}} p(\underline{r}) \underline{dS}, \quad \text{where} \quad \underline{dS} = \underline{n}\, dS\,, \tag{11}$$

$p(\underline{r})$ is the medium $\mathcal{F}$ pressure on elementary area dS of the anomaly $\mathcal{G}$ surface, $\underline{n}$ is the normal to elementary area dS of the anomaly surface at point $\mathbf{A}(\underline{r})$. Fig. 3 shows that

$$dS = R_{\mathcal{G}} \cos\varphi' d\lambda\, r d\varphi'\,.$$

In calculating integral (15), we should take into account that, in the scope of our definition, the problem possesses geometrical and force symmetry about axis $\mathbf{O}z$; hence,

$$F_x^{+} = \underline{i} \cdot \underline{F}^{+} = 0\,, \quad F_y^{+} = \underline{j} \cdot \underline{F}^{+} = 0\,, \quad F_z^{+} = \underline{k} \cdot \underline{F}^{+} = \int\limits_{S_{\mathcal{G}}} p(\underline{r}) \overbrace{\underline{k} \cdot \underline{n}}^{\sin\varphi'} dS\,.$$

Therefrom, taking into account the dS expression, obtain

$$F_z^+ = \int\limits_{S_\mathcal{G}} p(r)\sin\varphi' dS = \int\limits_{-\pi/2}^{\pi/2} p(r)\sin\varphi' \overbrace{R_\mathcal{G}\cos\varphi' d\lambda\, r d\varphi'}^{dS} =$$

$$= R_\mathcal{G} \int\limits_0^{2\pi} d\lambda \int\limits_{-\pi/2}^{\pi/2} p(r) r \sin\varphi' \cos\varphi' d\varphi' \ .$$

Integrating this over λ and substituting relation (10) for $p(r)$, obtain

$$F_z^+ = \frac{4}{3}\pi R_\mathcal{G}^2\, \pi \mathbf{G} \rho_\mathcal{F}^2 \int\limits_{-\pi/2}^{\pi/2} \left(r^2 - R_\mathcal{F}^2\right) \sin\varphi' \cos\varphi' d\varphi' \ . \qquad (12)$$

Derive r^2 from the problem geometry (Fig. 2):

$$r^2 = \delta^2 + R_\mathcal{G}^2 + 2\delta R_\mathcal{G} \sin\varphi' \ , \qquad (13)$$

where δ is the shift of the anomaly $\mathcal{G}$ center; r is the distance between the domain $\mathcal{F}$ gravity attraction center and a point on the anomaly $\mathcal{G}$ surface. Substituting (13) into (12), obtain:

$$F_z^+ = \frac{4}{3}\pi R_\mathcal{G}^2\, \pi \mathbf{G} \rho_\mathcal{F}^2 \int\limits_{-\pi/2}^{\pi/2} \left(\overbrace{\delta^2 + R_\mathcal{G}^2 + 2\delta R_\mathcal{G} \sin\varphi'}^{r^2} - R_\mathcal{F}^2 \right) \sin\varphi' \cos\varphi' d\varphi' \ . \qquad (14)$$

Let us calculate the auxiliary integrals:

$$\mathbf{I}_1 = \int\limits_{-\pi/2}^{\pi/2} \sin\varphi' \cos\varphi' \, d\varphi' = \frac{1}{2} \int\limits_{-\pi/2}^{\pi/2} d \sin^2\varphi' = 0 \ ,$$

$$\mathbf{I}_2 = \int\limits_{-\pi/2}^{\pi/2} \sin^2\varphi' \cos\varphi' \, d\varphi' = \frac{1}{3} \int\limits_{-\pi/2}^{\pi/2} d \sin^3\varphi' = \frac{2}{3} \ .$$

Then, rewrite relation (14) taking into account $\mathbf{I}_1$ and $\mathbf{I}_2$:

$$F^+ = \frac{4}{3}\pi R_\mathcal{G}^2\, \pi \mathbf{G} \rho_\mathcal{F}^2 2\delta R_\mathcal{G} \cdot \mathbf{I}_2 = \overbrace{\frac{4}{3}\pi R_\mathcal{G}^3}^{V_\mathcal{G}} \rho_\mathcal{F} \overbrace{\frac{4}{3}\pi \mathbf{G} \rho_\mathcal{F} \delta}^{-\mathbf{g}_\mathcal{F}(\delta)}$$

Hence, anomaly $\mathcal{G}$ placed into domain $\mathcal{F}$ with a pressure gradient undergoes buoyant force F^{+}:

$$\underline{F}^{+} = -\rho_{\mathcal{F}} V_{\mathcal{G}} \cdot \underline{\mathbf{g}}_{\mathcal{F}}(\delta)\,. \tag{15}$$

Buoyant force $\underline{F}^{+}$ is always directed opposite to the gravity field gradient $\underline{\mathbf{g}}$ that has generated the pressure gradient in medium $\mathcal{F}$.

4. Sum of forces acting on the anomaly

Two forces are now defined: *attraction force* $\underline{F}^{-}$ and *buoyant force* $\underline{F}^{+}$; these forces act on anomaly $\mathcal{G}$ in gravitating medium $\mathcal{F}$ at distance δ from point $\mathbf{O}$. These two forces always lie on the same straight line tangential to the gravity field line at the preset point of domain $\mathcal{F}$. They are directed oppositely to each other and are of the same (gravitational) nature. All the above allows us to state that anomaly $\mathcal{G}$ placed into static gravitating medium $\mathcal{F}$ undergoes not two forces ($\underline{F}^{+}$and $\underline{F}^{-}$) but only one, namely, their sum:

$$\underline{F} = \underline{F}^{+} + \underline{F}^{-} = -\rho_{\mathcal{F}} V_{\mathcal{G}}\, \underline{\mathbf{g}}_{\mathcal{F}} + \rho_{\mathcal{G}} V_{\mathcal{G}}\, \underline{\mathbf{g}}_{\mathcal{F}} = \overbrace{\underbrace{\rho_{\mathcal{G}} V_{\mathcal{G}}}_{M_{\mathcal{G}}} \left(1 - \frac{\rho_{\mathcal{F}}}{\rho_{\mathcal{G}}}\right)}^{m_{\mathcal{G}}} \underline{\mathbf{g}}_{\mathcal{F}}\,.$$

Here $M_{\mathcal{G}}$ designates the anomaly $\mathcal{G}$ mass in its classical meaning, namely, as a product of the anomaly matter density $\rho_{\mathcal{G}}$ by its volume $V_{\mathcal{G}}$, while $m_{\mathcal{G}}$ designates the anomaly $\mathcal{G}$ mass that is involved in gravitational interaction with gravitating medium $\mathcal{F}$.

Historically, it happened so that those forces were discovered in different historical epochs and were always regarded as different force factors acting on the studied object of a certain mass. Thus, it has been found out that the only force acting upon anomaly $\mathcal{G}$ in gravitating medium $\mathcal{F}$ in the absence of other external impacts is

$$\underline{F} = m_{\mathcal{G}} \underline{\mathbf{g}}_{\mathcal{F}}\,, \quad \text{where} \quad m_{\mathcal{G}} = M_{\mathcal{G}} \left(1 - \frac{\rho_{\mathcal{F}}}{\rho_{\mathcal{G}}}\right)\,, \tag{16}$$

and $\underline{\mathbf{g}}_{\mathcal{F}}$ is the medium $\mathcal{F}$ gravity field gradient in the vicinity of anomaly $\mathcal{G}$. This force sign (direction) depends only on the ratio between densities of the anomaly and medium surrounding it. Relation (16) shows that gravitational interaction within the gravitating medium is determined by not the total anomaly mass but only by its portion; therefore, we introduce a concept of *the gravitational mass*[4] and designate it as $m_{\mathcal{G}}$. As for the total mass of anomaly $\mathcal{G}$ designated above as $M_{\mathcal{G}}$, we call it *inertial mass* and define as the product of the anomaly volume $V_{\mathcal{G}}$ by the anomaly density $\rho_{\mathcal{G}}$.

Therefore, we may assert that we have established a functional relationship between the gravitational (m) and inertial (M) masses of an anomaly in a gravitating medium:

$$m = M\left(1 - \frac{\rho_0}{\rho}\right) . \tag{17}$$

The anomaly gravitational mass is fully determined by the ratio between the densities of medium (ρ_0) and anomaly (ρ). Fig. 4 clearly shows that the inertial mass may be equal to the gravitational mass under only one condition: when the surrounding medium density is zero, which is purely speculative and exceptional case. In all other cases, gravitational mass m is **strictly** unequal to inertial mass M.

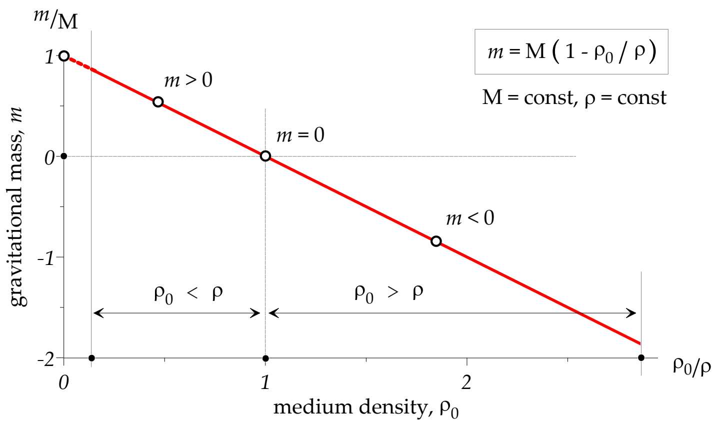


Figure 4: The ratio between the inertial (M) and gravitational (m) masses as a linear function of the gravitating medium density ρ_0.

We have introduced the expression for the gravitational mass (anomaly) in the paper devoted to studying the nature of the Earth's gravity center motion under the action of external gravitational forces [5]; later formula (17) was used in the paper on the Earth's core motion under the influence of the Moon's perigee mass [6] (rev.1).

[4]Literature, e.g., [4], presents many different definitions for the gravitational mass. The above-considered versions of gravitational interaction of material bodies enabled us to define the gravitational mass so as to fit best the experimental results and natural phenomena and embrace the known cause-and-effect relations: **gravitational mass is an anomaly of density in the surrounding medium**. The more pragmatic definition of the gravitational mass may look as follows: the body gravitational mass is that determines gravitational interaction with other bodies and is measurable by instruments.

Case 1. Body weight on the Earth's surface. The body weight F on the Earth's surface is the force of interaction between two gravitational masses: the Earth and an object resting on its surface, whose inertial mass is M and density is ρ

$$F = mg\,, \quad \text{where} \quad m = M\left(1 - \frac{\rho_0}{\rho}\right)\,, \quad g = \mathbf{G}\frac{M_\oplus}{R_\oplus^2}\left(1 - \frac{\rho_0}{\rho_\oplus}\right)\,, \tag{18}$$

Here $\rho_\oplus$ is the Earth's density; $R_\oplus$ is the Earth's radius; $M_\oplus$ is the Earth's inertial mass; ρ_0 is the density of the atmosphere layer closest to the Earth's surface; $\mathbf{G}$ is the gravitational constant.

Case 2. Pendulum oscillation equation. Let the pendulum inertial mass M be connected to hinge $\mathbf{O}$ with a weightless and rigid rod L in length (Fig. 5). Now let us write the equation for mathematical pendulum plane oscillations in the Earth's gravity field taking into account that the gravitational and inertial masses are different, i.e., there is some surrounding medium with density $\rho_0 \neq 0$ where the oscillations take place. Dissipation effects in the medium under consideration are neglected.

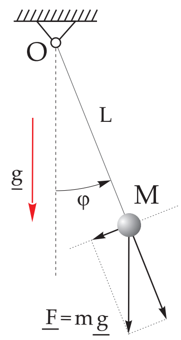


Figure 5:

$$J\ddot{\varphi} = -F\sin\varphi\, L\,,$$

where $J = ML^2$ is the mass M momentum of inertia with respect to the suspension point $\mathbf{O}$; $F = mg$ is the force of gravitational interaction between the Earth and pendulum gravitational masses; m is the pendulum gravitational mass. Hence, taking into account (17), the pendulum oscillation equation takes the following form:

$$\ddot{\varphi} + \left(1 - \frac{\rho_0}{\rho}\right)\frac{g}{L}\sin\varphi = 0\,, \tag{19}$$

where ρ is the pendulum material density.

5. Gravitational interaction of a pair of bodies in the gravitating medium

As shown earlier (17), the gravitational and inertial masses are connected by the following relation:

$$m = M\left(1 - \frac{\rho_0}{\rho}\right)\,,$$

where M is the inertial mass; ρ_0 is the density of surrounding gravitating medium $\mathcal{F}$; $\rho = {}^{M}\!/\!_{V}$ is the density of an inertial mass occupying volume V (the number of matter units per unit volume). Special emphasis should be made on that inertial mass M of the material body under consideration may be identical to its gravitational mass m only when the surrounding medium density is zero. The $\rho_0 = 0$ condition is absolutely speculative and not realizable physically. Based on the results obtained, let us reformulate the customary law of universal gravitation especially for the case of gravitational interaction between two anomalies $\mathcal{G}_1$ and $\mathcal{G}_2$ located at distance r from each other (Fig. 6) in homogeneous gravitating medium $\mathcal{F}$ with density ρ_0:

$$F = \mathbf{G}\frac{m_1 m_2}{r^2}, \quad \text{where} \quad m_1 = M_1\left(1 - \frac{\rho_0}{\rho_1}\right),\; m_2 = M_2\left(1 - \frac{\rho_0}{\rho_2}\right). \tag{20}$$

Here m_1 and m_2 are gravitational masses of the anomalies; $(M_1,\ \rho_1)$ and $(M_2,\ \rho_2)$ are the inertial masses and densities of bodies $\mathcal{G}_1$ and $\mathcal{G}_2$, respectively.

Hence, the mere existence of the object gravitational mass is inescapably associated with the surrounding medium density; due to this, it is possible to find the conditions under which the gravitational mass may be either positive or negative. In a certain medium this factor clearly manifests itself as *repulsion* or *attraction* of the bodies.

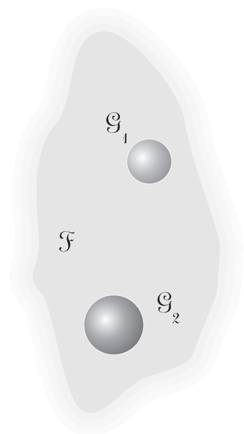


Figure 6: Gravitating medium $\mathcal{F}$ and two anomalies $\mathcal{G}_1$ and $\mathcal{G}_2$.

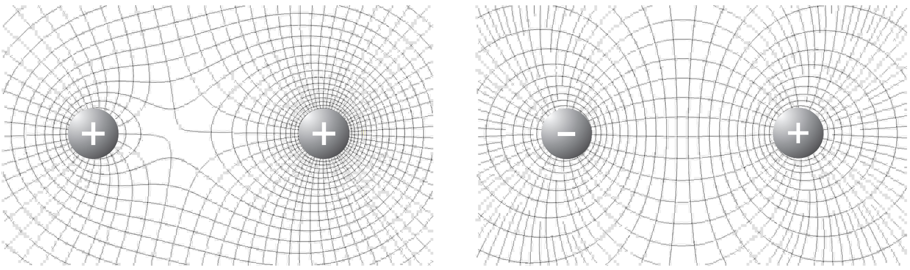
For instance, the submarine submerging or emerging process is nothing but controlling the gravitational mass value and sign by regulating the ballast; at the zero running speed, this makes the submarine moving along the Earth's gravity field line. The submarine suspension at a preset depth means that its gravitational mass is zero. The same principle is valid also for aircrafts. A dirigible or air balloon rises not because of the difference in gas densities inside and outside the balloon shell but because of the pressure gradient in the Earth's atmosphere induced by the Earth's gravity field. While the dirigible is rising, it is repulsed from the Earth due to its negative gravitational mass (the dirigible mean density is lower than the medium density) and, vice versa, its gravitational mass becomes positive while it moves down (the dirigible mean density is higher than that of the surrounding atmosphere).

The Earth's fauna also exhibits many examples of controlling the gravitational mass. Those examples demonstrate the object's ability to change its gravitational mass value and sign at a constant density of the surrounding medium.

Consider a model problem in the 3D space. Let gravitating medium $\mathcal{F}$ (ρ_0 in density) accommodate two bodies $\mathcal{G}_1$ and $\mathcal{G}_2$ with fixed densities ρ_1 and ρ_2, respectively. Assume that $\rho_1 > \rho_2$. The variable is the medium density ρ_0. For each of three domains $\mathcal{F}$, $\mathcal{G}_1$, $\mathcal{G}_2$ we can write a set of Poisson's equations for the potential:

$$\begin{cases} \Delta\mathbf{U}_{\mathcal{F}} & = -4\pi\mathbf{G}\rho_0 \ , \\ \Delta\mathbf{U}_{\mathcal{G}_1} & = -4\pi\mathbf{G}\rho_1 \ , \\ \Delta\mathbf{U}_{\mathcal{G}_2} & = -4\pi\mathbf{G}\rho_2 \ . \end{cases} \tag{21}$$

Fig. 7 presents the solution of equation set (21) as the $\mathbf{U}$ gradient lines. Two versions of the density ratio are considered. Fig. 7(a) demonstrates the field line distribution for the case when the medium density ρ_0 exceeds the densities of inclusions $\mathcal{G}_1$ and $\mathcal{G}_2$. The other version (Fig. 7(b)) is that the medium density is between the densities of bodies $\mathcal{G}_1$ and $\mathcal{G}_2$.



(a) $\rho_0 > \rho_1 > \rho_2$ (b) $\rho_1 > \rho_0 > \rho_2$

Figure 7: Distribution of gradient $\mathbf{U}_{\mathcal{F}}$ over domain $\mathcal{F}$ when the two gravitational masses are of the same sign (a) and of different signs (b).

The numerical experiment (Fig. 7(b)) showed that the *gravitational dipole* is a physically real object.

Definition 1. *The gravitational dipole is formed by two connected bodies of an arbitrary volume and shape and of different densities, which are located in a medium with a non-zero density, provided the medium density is lower than that of one body and higher than that of the other one.*

A body in equilibrium at the interface of two media

Consider a static problem: equilibrium of a wooden cube floating in water (Fig. 8). Let us estimate the depth it is submerged to. It is known that the air density (the medium above the water surface) is $\rho_{air} = 1.29\,[\mathrm{kg/m^3}]$, water density is $\rho_{water} = 1000\,[\mathrm{kg/m^3}]$, and the wooden (oak) cube density is $\rho_{cube} = 800\,[\mathrm{kg/m^3}]$. The cube face length is $a = 1\,[\mathrm{m}]$. The gravity field gradient $\underline{\mathbf{g}}$ is perpendicular to the water surface.

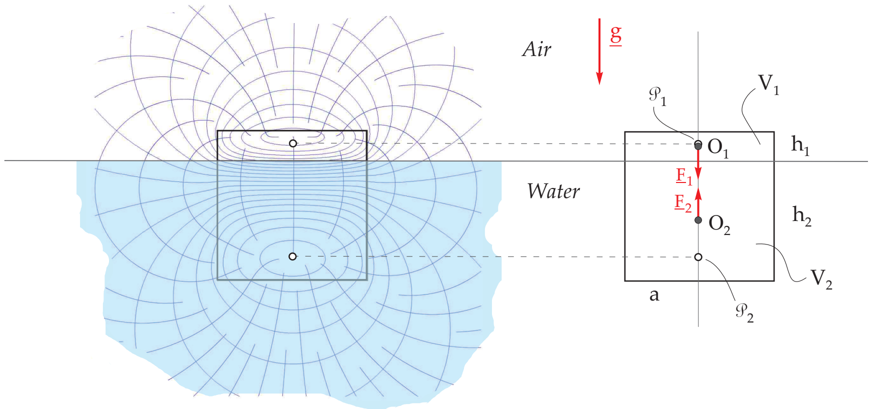


Figure 8: Equilibrium of a body at the interface of two media.

The problem shall be solved in the following way: consider the cube as an object consisting of two parts. One of those parts is an anomaly in air, the other is an anomaly in water. Each anomaly is exposed to the Earth's gravity field. Since the cube is in equilibrium, the sum of gravity forces acting on its parts is zero; taking into account that the Earth's gravity field gradient $\underline{\mathbf{g}}$ is directed to the same side both in air and water, obtain

$$\underline{F}_1 + \underline{F}_2 = 0\,, \quad \text{or} \quad (m_1 + m_2)\underline{\mathbf{g}} = 0\,.$$

Hence, we have obtained the condition for the body equilibrium: the sum of gravitational masses of the cube above-water and sub-water parts is zero:

$$m_1 + m_2 = 0\,, \tag{22}$$

where

$$m_1 = \overbrace{\rho_{\text{cube}} V_1}^{M_1} \left(1 - \frac{\rho_{\text{air}}}{\rho_{\text{cube}}}\right)\,, \quad m_2 = \overbrace{\rho_{\text{cube}} V_2}^{M_2} \left(1 - \frac{\rho_{\text{water}}}{\rho_{\text{cube}}}\right)\,.$$

Substituting m_1 and m_2 into (22) and taking into account that $V_1 = a^2 h_1$ and $V_2 = a^2 h_2$, obtain the set of equations:

$$h_1(\rho_{\text{cube}} - \rho_{\text{air}}) + h_2(\rho_{\text{cube}} - \rho_{\text{water}}) = 0\,, \quad h_1 + h_2 = a$$

and, hence,

$$h_2 = a\,\frac{\rho_{\text{cube}} - \rho_{\text{air}}}{\rho_{\text{water}} - \rho_{\text{air}}}\,.$$

Thus, the cube submersion depth in water is $h_2 \approx 0.8\,[\text{m}]$, while its above-water part height is $h_1 \approx 0.2\,[\text{m}]$. The body is in equilibrium.

In this case we can, taking into account definition 1, formulate the following axiom:

Axioma 1. *Each material body that is in equilibrium at the interface of two media is a gravitational dipole.*

6. Gravitational pole as a function of medium density

Consider again problem (21) about the interaction of two homogeneous spheres $\mathcal{G}_1$ and $\mathcal{G}_2$ in stationary gravitating medium $\mathcal{F}$. Let us supplement the problem by searching for equilibrium points and consider it in a 3D space large enough to exclude the edge effects. The bodies are motionless. To solve the problem, we shall use the finite-element method.

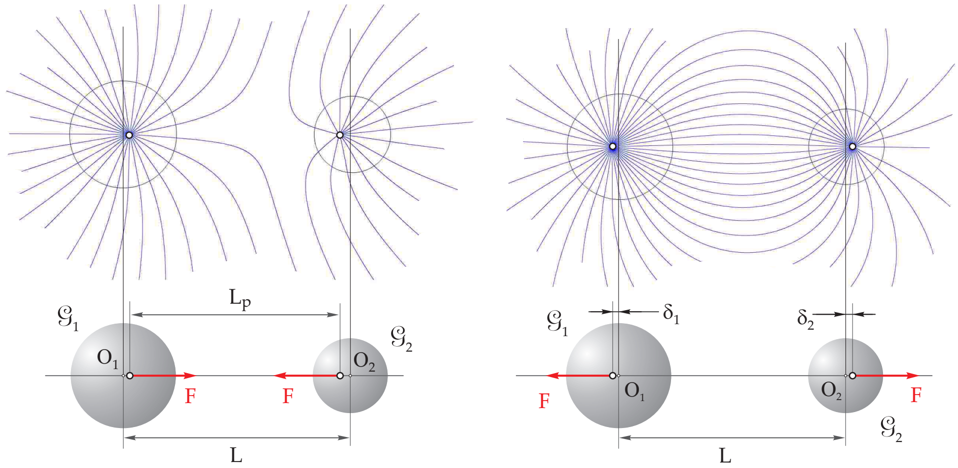
Analysis of the two-body gravity field character showed that the field (gradient) lines converge at or originate from peculiar points not coinciding with the body gravity centers. These are equilibrium points where the sum of all the forces is zero.

Designate the distance between gravity centers of homogeneous spheres $\mathcal{G}_1$ and $\mathcal{G}_2$ as L; let $L = 0.030\,[\text{m}]$. Other bodies' characteristics are listed in Table 1.

Table 1: Parameters of the gravitational pair presented in Fig. 9

	density	radius
$\mathcal{G}_1$	$\rho_1 = 7200\,[\text{kg/m}^3]$	$R_1 = 0.007\,[\text{m}]$
$\mathcal{G}_2$	$\rho_2 = 6700\,[\text{kg/m}^3]$	$R_2 = 0.005\,[\text{m}]$

The solution in the form of scalar field gradient $\mathbf{U}$ was plotted for two cases: when medium $\mathcal{F}$ density is $\rho_0 = 0$ and $\rho_0 = 7000\,[\text{kg/m}^3]$. These two solutions are presented in Fig. 9 and Tab. 2. L_p represents the distance between the gravitational equilibrium points (or gravitational poles). As Fig. 9 shows, the peculiar feature of those points is that the gradient lines either end on the poles in the case of the gravitational dipole (when gravitational masses are of different signs) or play the role of a "sink" when the gravitational masses are of the same sign.



(a) Attraction, $\rho_0 = 0$

(b) Repulsion, $\rho_0 = 7200\,[\text{kg/m}^3]$

Figure 9: Displacement of gravitational poles δ_1 and δ_2 versus density ρ_0 of medium $\mathcal{F}$. Fig. 9(a) presents the solution for the gravitational masses of the same sign. The points are located between the gravity centers of bodies $\mathcal{G}_1$ and $\mathcal{G}_2$. Fig. 9(b) illustrates the existence of opposite-sign gravitational masses. Actually, in this case a gravitational dipole is observed.

Table 2: Displacement of gravitational poles with respect to the gravity center positions for the similar-sign and opposite-sign gravitational masses $\mathcal{G}_1$ and $\mathcal{G}_1$ placed in different media at the distance $L = 0.03\,[\text{m}]$ from each other.

ρ_0, [kg/m^3]	δ_1, [mm]	δ_2, [mm]
0	0.126	0.493
7000	-0.198	-0.250

Positions of the gravitational poles in each body of the pair under consideration depend on their geometric and physical (density) parameters and also on the inter-body distance (e.g., the distance between their gravity centers).

7. Conclusions

A. We have proposed a version for generalization of the universal gravitation law; this allowed us to spread the law to interaction of bodies in a gravitating medium, i.e., to expand its field of application. The generalization obtained involves the classical universal gravitation law and the Archimedes' principle that is as ancient as the universe. Actually, Archimedes was the first who formulated the law of gravitational interaction in gravitating media by defining the buoyant force (see (15)).

B. The paper testifies that the inertial mass may seem to be equal to the gravitational mass only when they are compared in the absence of the surrounding gravitating medium, i.e., if the medium density is **strictly** zero. Evidently, the case of the gravitational and inertial masses identity is purely speculative.

C. We have shown that in our natural environment there are such objects as gravitational dipoles (bodies with negative and positive gravitational masses).

D. Comparison of the character of force interaction of electrostatically charged bodies with that of gravitational bodies have shown the existence of such a fundamental difference in these characters that they cannot be combined in one and the same theory: bodies with similar-sign electrical charge repulse each other, while bodies with similar-sign gravitational masses attract each other.

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