Dual structures
in
cube nets disclosed

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Abstract: It will be shown how the well known eleven nets for three dimensional cubes, separated in 10 + 1 forms, are hiding a special dual 3-6-1-structure. Implications for space - time models in theoretical physics will be questioned.

I. Analysing the interior structure of cube nets

We look among the 35 free hexominoes at the well known given eleven cube nets of a three dimensional cube:

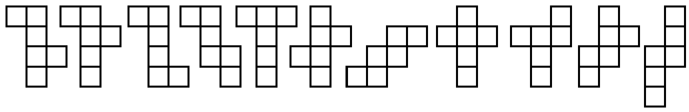

Figure 1: The eleven cube nets

We are able to identify some differences between the net constructions. Doing this we need to give numbers from 1 to 6 to the six squares of these cube nets. The numbering rule is orientated to the cube geometry: The sum of two numbers of opposed cube squares amount 7.

```
|   1||       ||       ||       ||       ||       |
|   2|| 1 2 6 || 1 2   || 1 2   || 1 2   ||   2   |
|3  6||   3   ||   3 6 ||   3   ||   3   || 1 3   |
|5   ||   5   ||   5   ||   5 6 ||   5   ||   5 6 |
|4   ||   4   ||   4   ||   4   ||   4 6 ||   4   |

|   2   || 1 2   ||   2   ||   2   || 1     |
| 1 3 6 ||   3   || 1 3   ||   3   || 2 3   |
|   5   ||   5 6 ||   5 6 || 1 5 6 ||   6 5 |
|   4   ||     4 ||     4 ||     4 ||     4 |
```

Figure 2: The cube nets with numbers

In those cube nets some cube squares are connected together, others are not. If between two cube squares a and b a direct connection exists, then we note a<>b.

For example:

```
| 1 2   |
|   3 6 |
|   5   |
|   4   |
```

Figure 3: Example cube net

The cube net in Figure 3 has the notation

1<>2, 2<>3, 3<>6, 3<>5, 5<>4

Now we can analyse the structure of the eleven cube nets.

First of all we find that the eleven cube nets are split into two groups. Ten cube nets have a four to three (4x3) structure (four net squares high and three squares wide) and only one cube net has a five to two (5x2) structure .

```
|   1|
|   2|
|3  6|
|5   |
|4   |
```

Figure 4: The five to two structured cube net

Now we look at the ten cube nets with the 4x3 structure.

We find three different substructures regarding the axis length of main direction like in the next figure:

```
|  2  ||  2  ||1    |
|1 3 6||  3  ||2 3  |
|  5  ||1 5 6||  6 5|
|  4  ||    4||    4|
```

Figure 5: Three different length of main direction

In figure 5 – left side – we see a net cube with four cube squares from 2 down to 3 down to 5 and down to 4 are build in one axis straight downward without a ‘sidestep’ between them. The maximum length of an axis has four squares. The middle cube net shows a sidestep from square 5 to 6 and after that it follows the downward direction. Square number 4 is not in the same axis as it is seen in the left cube net. Result: In the middle cube net only three squares are in the same axis straight downward. The maximum length of an axis has three squares. The right cube net in figure 5 shows only two squares as maximum length in one axis.

If we sort all ten 4x3 structured cube nets using the substructure of maximum axis length then in figure 2 we can find

6 cube nets with maximum axis length 4.
3 cube nets with maximum axis length 3.
1 cube nets with maximum axis length 2.

This finding gets more interest if we find the same interior structure by 6-3-1-separation of the ten cube nets with other criteria.

II. Duality in 6-3-1 separations

Ones more we look at figure 5. But now we do not use the axis length criterion. In this case we have to look at the maximum connections of one cube square in the whole cube net.

Figure 6 shows the same cube nets but under the perspective of the maximum connection number for one of its squares.

```
|    2    ||    2    ||1        |
|1   3   6||    3    ||2   3    |
|    5    ||1   5   6||    6   5|
|    4    ||        4||        4|
```

Figure 6: Three different maximum connection numbers

In figure 6 in the left cube net we see the square 3 with the four connections 3<>1, 3<>2, 3<>6 and 3<>5. This square has the maximum number of connections in that cube net. In the middle cube net of figure 6 we see the square 5 with three connections 5<>1, 5<>3, 5<>6. This square has the maximum number of connections in this whole cube net. The right cube net has in maximum two connections for the squares 2, 3, 6 and 5. No square has more than two connections to another square of this cube net.

This leads to the result for the ten cube nets with 4x3 structure, if we look at figure 2:

6 cube nets have in maximum three connection in a square.
3 cube nets have in maximum two connection in a square.
1 cube net has in maximum four connections in a square.

This 6-3-1 structure is a duality to the structure given in chapter 1.

But there is one more duality to this.

Now we do not look at the maximum connection number of one square in a given cube net. We look at the number of squares which have the maximum connections in a given cube net.

```
|    2    ||    2    |
|1   3   6||    3    |
|    5    ||1   5   6|
|    4    ||        4|
```

Figure 7: Two cube nets with one square that has the most specific connections

In figure 7 the left cube net has a square with four connections, but this is the only one with this maximum in that cube net. The right cube net has only one square with his specific maximum connections, too. That means that both cube nets are in the same substructure regarding this criterion.

Looking at figure 2 we find

6 cube nets with only one square holding the maximum connection number.
3 cube nets with four squares holding the maximum connection number.
1 cube net with two squares holding the maximum connection number.

This dualities are very interesting for that simple cube net geometry. It is clear that the dual 6-3-1 structures are not build up with all the same cube net constructions in every part of the six or three or one substructures.

This 6-3-1 substructures could be used for theoretical modulation in physics of space and time. The last chapter gives some questions about this.

III. Do cube nets relate to theoretical physics?

These eleven cube nets could be related to the 11-dimensional space-time in the model of the M-Theory. In this case the ten 4x3-structured cube nets could be seen as ten space dimensions and the one 5x2-strucutred cube net has his counterpart in the one time dimension. In this case the ten space dimensions must have a 3-6-1-separation and we know that some models of String Theories show

6 Calabi-Yau dimensions
3 dimensions of our space
1 special dimension linking all string subtype models using duality in M-Theory

Could a relation between cube net dualities and M-Theory or another model of theoretical physics exist?

If there is a relation then we have to use the LQG model, because one cube could be a background independent model for a time or space quantum with Planck length on every edge in the given structures above. In this case we have another interesting question. If a cube with the ten different cube nets could be seen as a useable model for a space quantum then the extra-dimensions are virtual dimensions: They are mirror images of the ten cube nets of the given 4x3 structure. Only the three space dimensions of the cube are real. This space cubes get dynamic by changing its interior cube net into another 4x3 or the time structure 5x2 and vice versa. To connect this cubes with quantum physics we only have to put a hexagon inside the cube. This inside hexagon can used as a SU(3) symmetry in relation to the given cube net.

For example:

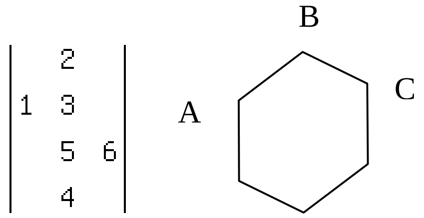

Figure 8: Cube net with inside hexagon

Figure 8 shows a related hexagon inside a cube. The corners A, B and C halves the cube edges which connect the cube squares 1<>3, 3<>5 and 5<>6. The other both connection edges 2<>3 and 5<>4 are not halved by the hexagon. Those specific correlations between the inside hexagon with the different cube nets show the way how to build on a model in LQG. Could that lead to a new approach in theoretical physics?

References

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