FITZGERALD-LORENTZ CONTRACTION: REAL OR APPARENT?

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ABSTRACT. After reviewing some classical and modern opinions on the 'notorious controversy' on the real or apparent nature of FitzGerald-Lorentz contraction, this paper analyzes a conflicting relativistic situation related to the mechanical tension of an elastic cord stretched by a hanging mass. A sliding pulley divides the cord into a vertical and a horizontal section. Thanks to a metric scale printed on the cord, FitzGerald-Lorentz contraction makes each section appear with different mechanical tension when observed in relative motion parallel to one of the sections of the elastic cord. By sliding the pulley on its horizontal support the length of the stretched cord changes while it is always stretched by the same hanging mass.

1. The notorious controversy

1. As is well known, in the year 1889 G. F. FitzGerald [4] and in 1892 H. A. Lorentz [7] proposed independently a *real* length contraction of moving objects in the direction of motion through the luminiferous aether in order to explain the negative results of the Michelson-Morley experiment. According to FitzGerald and Lorentz, the contraction was caused by changes in the intermolecular forces of moving bodies (where motion has to be understood as absolute motion). Since there were no reason to such changes, the proposal was considered as an *ad hoc* hypothesis.

2. It is also well known that FitzGerald-Lorentz contraction is an inevitable consequence of the principles of relativity that can be immediately deduced from Lorentz transformation. Now then, is that contraction real[1] or apparent? Most of the authors of books on the special theory of relativity avoid dealing with this 'notorious controversy,' as Max Born called it [3]. On this controversy Anthony P. French wrote [5, pp. 113-114]:

> This discussion should make it clear that the question "Does the FitzGerald-Lorentz contraction really take place?" has no single, unequivocal answer from a relativistic point of view. The whole emphasis is on defining what actual observations we must make if we want to measure the length of some object that may be in motion relative to us. And the prescription is simply that we measure the positions of its ends at the same instant as judged by us. What else could we possibly do? Thus the contraction, when we observe it, is not a property of matter but something inherent in the measuring process.

3. In his now classical book on Einstein's relativity Max Born wrote [3, pp. 254-55]:

> If we slice a cucumber, the slices will be larger the more obliquely we cut them. It is meaningless to call the sizes of the various oblique slices 'apparent' and call, say, the smallest which we get by slicing perpendicularly to the axis, the 'real' size. In exactly the same way a rod in Einstein's theory has various lengths

[1]As some contemporary authors (for instance [1]) claim.

> according to the point of view of the observer. One of these lengths, the statical or proper length, is the greatest but this does not make it more real than the others.

4. On the same issue, David Bohm wrote [2, Loc. 1253-71 Kindle edition]:

> One may perhaps compare this situation to what happens when two people A and B separate, while still in each other's line of sight. A says that B seems to be getting smaller, while B says that A seems to be getting smaller. Why then does not B say that A seems to be getting larger? The answer is that each is seeing *something different*, i.e. the image of the world on his retina. There is no paradox in the fact that the image of A on B's retina gets smaller at the same time that the image of B on A's retina gets smaller. Similarly, there is no paradox in saying that A will ascribe a contraction to B's ruler, while B ascribes a contraction to A's simply because each is referring to *something different* when he talks about the length of an object.

5. And finally, in a contemporary university textbook of physics we can read [6, p. 1032]:

> Does a moving object really shrink? Reality is based on observations and measurements; if the results are always consistent and if no error can be determined, then what is observed and measured is real. In that sense, the object really does shrink.

6. The question: does a moving object really shrink? reveals a certain intellectual dishonesty: motion can only be relative because absolute motion is incompatible with relativity. Thus, all we can say is that an object, when observed (measured) in relative motion, is *observed* contracted in the direction of the relative motion (once corrected the distortions due to the finite speed of light that travels from the object to the optical or biological device [9], [11], [8]).

7. In its proper reference frame the object, in fact, is not contracted. And, obviously, no object can be really contracted and really non-contracted at the same time. It can be observed simultaneously as contracted and as non-contracted by two different observers that observe it in two different ways: in relative motion in the first case (contracted) and at rest in the second one (non contracted).

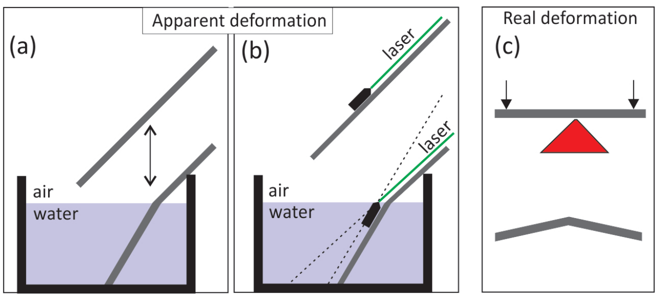

Figure 1. Apparent and real deformation. Note that in (b) the laser beam is parallel to the rod even if the laser source is submerged.

8. As a counterpoint, consider a straight rod partially and obliquely submerged in water. Due to the refraction of light, the rod seems to be bent, but evidently it is not, as can be immediately proved by removing it from the water, or even without removing it (Figure 1 (b)). Consider also other rod identical to the first one but mechanically and irreversibly deformed by the application of an appropriate mechanical effort. There is no controversy here: in the first case the deformation is only apparent; in the second the deformation is real.

9. Something changes in the atomic structure of the actually deformed rod by a mechanical effort, and that change can be experimentally proved, for instance, by means of X-ray diffraction. This is not what happens in the apparently deformed rod submerged in water, as could also be experimentally proved, for instance by the laser beam method suggested in Figure 1. We have then an experimentally testable asymmetry between an apparent and a real deformation. So it makes sense to speak of real and apparent deformations.

10. The discussion about the real or apparent nature of the FitzGerald-Lorentz contraction is further complicated by other external discussions that put into question the very existence of an objective reality beyond human observers. Though the idea that specific realities cannot exist without human observers makes it impossible the existence of human observers, because the specific history of life from which they have evolved could not have been possible without human observers. That said, and recalling Ockham's razor, should not we use the word 'real' only in the cases as the rod mechanically deformed and the word 'apparent' in the cases as the partially submerged rod? In this sense, is FitzGerald-Lorentz contraction real as the mechanically deformed rod or apparent as the submerged one? Or perhaps it is real (or apparent) in another sense. And if this were the case, in which sense?

11. Let us now compare FitzGerald-Lorentz contraction with the deformation of the submerged rod:

- FitzGerald-Lorentz contraction is real in the same sense it is real the bending of the partially submerged rod: both perceptions are not hallucinations of the observers. And both are perfectly explainable in physical terms.
- Both deformations are consequences of two particular ways of observing an object: in relative motion in the first case, and partially submerged in water in the second one.
- If we observe a partially submerged rod we can easily reconstruct its actual shape and size by a simple application of Snell law of light refraction. In the same way, if we observe a FitzGerald-Lorentz contracted object we can also reconstruct its real (proper) shape and size by means of the relativistic factor γ.
- Both deformations are reversible in the sense that by removing the rod from the water and by decreasing the relative velocity to a null valor both rods recover their original (proper) size and shape.
- By changing the inclination of the partially submerged rod, the level of deformation will also change. Similarly, by changing the relative velocity at which an object is observed, the degree of its contraction in the direction of the relative motion will also change.
- And mainly, both deformations occur without a mechanical effort acts on the deformed objects.

12. By way of illustration of the above discussion, let B be a bubble of a certain fluid F_2 in hydrostatic equilibrium with other fluid F_1 (Figure 2). In its proper frame RF_o, the bubble has a spherical shape due to the fact that the hydrostatic pressure is the same in all directions. In RF_v that moves relative to RF_o in the X_o direction, the bubble has

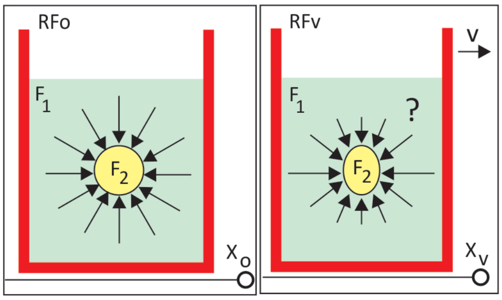

FIGURE 2. A bubble in hydrostatic equilibrium as seen from its proper frame RF_o (left) and from other reference frame RF_v that moves relative to RF_o with a velocity v in the X_o direction.

an ellipsoidal shape due to FitzGerald-Lorentz contraction in the direction of the relative motion. Will the observers in RF_v explain the shape of the bubble in terms of a real or of an apparent deformation?

13. Let me end this section by paraphrasing the following quote from a university text ([10, p. 42]):

> (Original) I need to warn you about language. I have said that a rod with length L_o as observed from its own frame has a shorter length L_v as observed from another frame. Often this result is stated as 'A rod with length L_o as observed from its own frame, appears to have a shorter length L_v as observed from another frame.' This statement is true: the rod appears to have shorter length L_v because it does have shorter length L_v. Using the term 'appears' gives the false impression that, when the rod is observed from a frame in which it moves, the rod really is of length L_o and only appears to be of length L_v. No. As observed from a frame in which it moves, the rod really does have the shorter length L_v.
>
> (Paraphrase) I need to warn you about language. I have said that a straight rod is bent when observed partially submerged in water. Often this result is stated as 'A straight rod partially submerged in water appears bent' This statement is true: the rod appears bent because it is bent. Using the term 'appears' gives the false impression that, when the rod is observed partially submerged, the rod really is straight and only appears to be bent. No. As observed partially submerged, the rod is really bent.

2. The sliding pulley

14. RF_o is the proper frame of the mechanical device schematically depicted in Figure 3 in which a load of rest mass m_o hangs from an elastic cord whose left end is attached to a fixed vertical support. The cord runs around a pulley which in turn is attached to a horizontal sliding support whose position can be set with the corresponding fixing screw. A metric scale, that consists of a certain number of black and white marks of equal length each, is printed on the cord[2] (Figure 3). At equilibrium, the mechanical tension can only be constant along the whole cord, in consequence all metric marks have to have the same length, whether they are in the horizontal or in the vertical section of the elastic cord. A

[2]A common trick we use in geology in order to illustrate the mechanical deformations of rocks.

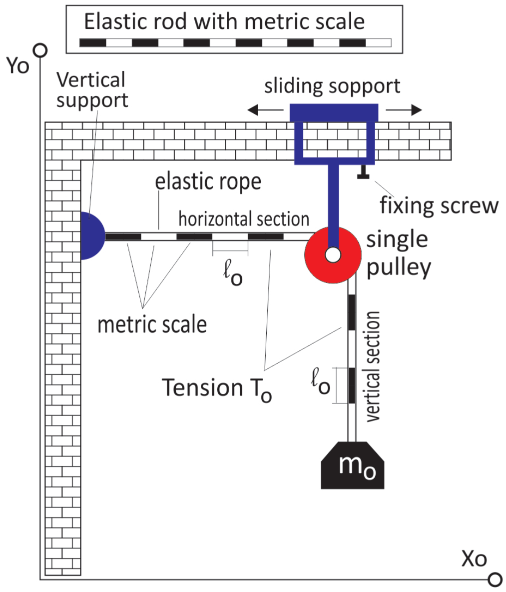

FIGURE 3. According to the laws of mechanics, at equilibrium all black and white marks of the elastic cord must of necessity have the same length.

conclusion that, being an immediate consequence of the laws of mechanics, must hold in all reference frames.

15. Let l_o be the proper length of the metric marks, and let us assume the X_o axis of RF_o is parallel to the horizontal section of the cord. By sliding its horizontal support, the pulley moves in the left-right direction, which has the effect of changing, in a complementary way, the number of the horizontal and the vertical metric marks (for the sake of simplicity we will assume the pulley always slides an integer number of marks). As expected, in RF_o the length of all metric marks remains constant and equal to l_o, be them in the horizontal of in the vertical section of the cord.

16. RF_v is an inertial reference frame whose spacetime diagram coincides with that of RF_o at a certain instant, and from which RF_o moves from left to right, in the X_o, direction with a velocity v. In consequence, and according to Lorentz transformation, in RF_v the length of the vertical marks of the metric scale is also l_o, while the horizontal marks have a contracted length $\gamma^{-1}l_o$.

17. Let n be the number of metric marks, and assume n_h of them are in the horizontal section in a certain position of the sliding pulley. Obviously the number of vertical marks in that position will be $n - n_h$. In RF_o the length L_o of the cord at equilibrium with the pending mass is always the same, independent of the pulley position:

$$L_o = n_h l_o + (n - n_h) l_o \tag{1}$$

$$= l_o(n_h + n - n_h) = n l_o \tag{2}$$

However, in RF_v the length L_v of the cord is variable, depending on the pulley position, i.e. on the number n_h of horizontal marks:

$$L_v = n_h \gamma^{-1} l_o + (n - n_h) l_o \tag{3}$$

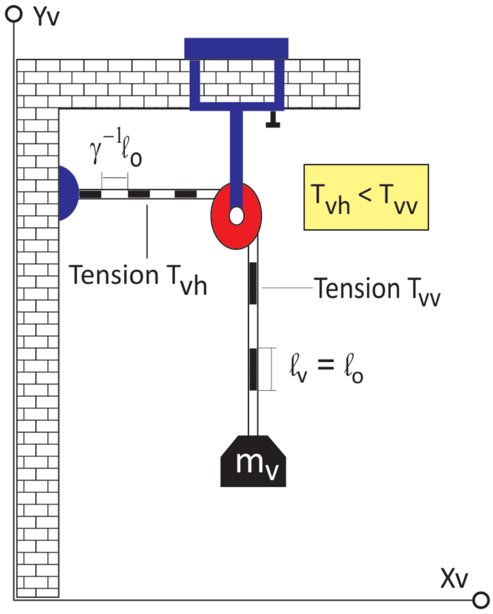

FIGURE 4. In RF_v, from which RF_o moves from left to right in the X_o direction, the length of the horizontal metric marks are contracted by a factor γ^{-1} while the vertical marks maintain its proper length l_o.

$$= l_o(n_h\gamma^{-1} + n - n_h) \tag{4}$$

$$= l_o(n - n_h(1 - \gamma^{-1})) \tag{5}$$

So, as the number n_h of horizontal marks increases the cord's length L_v decreases. Or in other words, as the pulley moves to the right the length of the cord decreases; and as it moves to the left its length increases. Obviously these changes of length disagree with what is expected from the laws of mechanics: the same hanging mass cannot originate different mechanical tensions, and then different stretchings, in the same elastic cord.

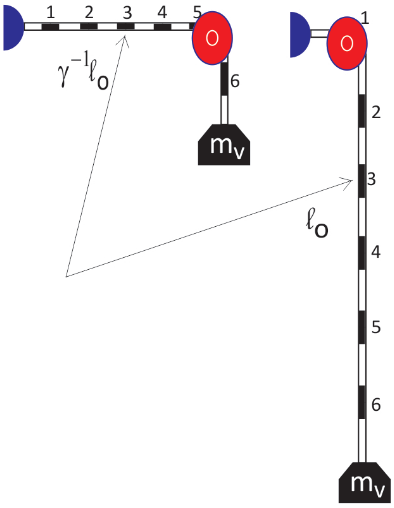

FIGURE 5. The horizontal mechanical tension of any part of the cord changes when it changes its position from the horizontal to the vertical section and vice versa.

18. By sliding the pulley towards the left or towards the right we can make any part of the rope to move from the horizontal to the vertical position and vice versa. When changing, its mechanical tension also changes, being greater when its position is vertical (Figure 5). These changes of tension are also incompatible with the laws of mechanics

19. As we have just seen, the mechanical situation observed from RF_v is incompatible with the known physical laws. On the one hand the mechanical tension of the cord has to be the same along its entire length and, therefore, the length of all its metric marks has also to be the same, be them in the horizontal or in the vertical section. On the other, the length of the cord at equilibrium with the pending mass has to be always the same, it cannot change as the pulley slides.

20. The observers in RF_v would have to conclude they observe a FitzGerald-Lorentz contraction that can only be apparent, as is apparent the bending of the rod partially submerged in water. Otherwise, they would have to explain the strange mechanical behaviour of the elastic cord: they would have to explain how it is possible for an elastic cord not to have the same mechanical tension along its entire length, and how it can be stretched in different ways by exactly the same hanging mass.

21. By symmetry, if the contraction of lengths by relative motion were only apparent, so would be for time dilation, phase difference in synchronization and mass increment with relative motion. In short, all observers in relative motion with respect to RF_o should consider the possibility their observations and measurements are distorted by relative motion in such a way they cannot get conclusions physically acceptable on what happens in RF_o. In this sense, only the observers in RF_o may conclude their observations and measurements agree with what is expected from physical laws.

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