# On Lorentz Transformation and Special Relativity: Critical Mathematical Analyses and Findings

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In this paper, the Lorentz transformation equations are closely examined in connection with the constancy of the speed of light postulate of the special relativity. This study demonstrates that the speed of light postulate is implicitly manifested in the transformation under the form of space-to-time ratio invariance, which has the implication of collapsing the light sphere to a straight line, and rendering the frames of reference origin-coordinates undetermined with respect to each other. Yet, Lorentz transformation is shown to be readily constructible based on this conflicting finding. Consequently, the formulated Lorentz transformation is deemed to generate mathematical contradictions, thus defying its tenability. A rationalization of the isolated contradictions is then established. An actual interpretation of the Lorentz transformation is presented, demonstrating the unreality of the space-time conversion property attributed to the transformation.

## 1 Introduction

The well-known Lorentz transformation, named after the Dutch physicist Hendrik Lorentz, is a set of equations relating the space and time coordinates of two inertial reference frames in relative uniform motion with respect to each other, so that coordinates can be transformed from one reference frame to another. Length contraction and time dilation are supposedly the principal outcome of the Lorentz transformation. Originally, Lorentz developed the transformation to explain, with other physicists (Larmor, Fitzgerald, and Pointcaré), how the speed of light seemed to be independent of the reference frame, following the puzzling results of the famous Michelson-Morley experiment [1]. These equations formed later the basis of Einstein's special relativity. Einstein [2, 3] derived Lorentz transformation on the basis of two postulates: 1 – the principle of relativity (i.e. the equations describing the laws of physics have the same form in all proper frames of reference), and 2 – the principle of the constancy of the speed of light in all reference frames.

Einstein theory of special relativity has received much criticism [4–9]. Doubts on the bases of scientific, mathematical, and philosophical contentions have been expressed. Criticism, on both academic and non-academic levels, has been mainly motivated by the unordinary physical phenomena of the time dilation and length contraction of moving objects, emerging from the purely mathematical formulation of the theory, in addition to numerous paradoxes combined with the inconsistency and ambiguity in their resolutions [10].

In this paper, the Lorentz transformation, along with the special relativity speed of light constancy principle used in its derivation, is thoroughly examined in an attempt to reach rational conclusions regarding its ever questioned tenability. Pure mathematical analysis and geometrical tools are used as the main arguments in achieving the objective of this study.

## 2 Lorentz Transformation

Consider two inertial frames of reference, $K(x, y, z, t)$ and $K'(x', y', z', t')$, in translational relative motion with parallel corresponding axes, and let their origins be aligned along the overlapped $x$- and $x'$-axes. Let $v$ be the relative motion velocity. The space and time coordinates of $K$ and $K'$ are then interrelated by the Lorentz transformation equations given in their present form by Poincaré [11], and subsequently by Einstein [2, 3] as follows:

$$\begin{aligned} x' &= \gamma(x - vt) \\ t' &= \gamma\left(t - \frac{vx}{c^2}\right) \end{aligned} \tag{1}$$

$$\begin{aligned} x &= \gamma(x' + vt') \\ t &= \gamma\left(t' + \frac{vx'}{c^2}\right) \end{aligned} \tag{2}$$

$$\begin{aligned} y &= y' \\ z &= z' \end{aligned} \tag{3}$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{4}$$

Equations (1) and (2) result in the following relativistic velocity transformation equations:

$$\begin{aligned} u' &= \frac{u - v}{1 - \frac{uv}{c^2}} \\ u &= \frac{u' + v}{1 + \frac{u'v}{c^2}} \end{aligned} \tag{5}$$

Where $c$ is the speed of light propagation in empty space, and $u$ and $u'$ are the velocity of a moving body in the $x$-direction, when measured with respect to K and K', respectively.

It is to be noted that equation (4) requires that $v$ be smaller than $c$. Also, equations (5) limit the values of $u$ and $u'$ to $c$ (i.e. if $u = c$, then $u'$ is brought to $c$ as well, and vice versa).

## 3 Lorentz Transformation Analysis

### 3.1 Constancy of the Speed of Light

Consider two inertial frames $K(x, y, z, t)$ and $K'(x', y', z', t')$ moving relative to each other with a uniform velocity $v$, and suppose at an instant of time $t_o = t'_o = 0$, the frames are overlying. Let a light beam be emitted at this time from the point of the coinciding origins in an arbitrary direction. At time $t$ in $K$, corresponding to time $t'$ in $K'$, the position vector associated with the light beam will acquire the space-time coordinates $(x, y, z, t)$ and $(x', y', z', t')$ in $K$ and $K'$, respectively. In line with the special relativity constancy of the speed of light principle [3], the light beam position vector coordinates shall satisfy the following equations, referred to as the light sphere equation transformation.

$$x^2 + y^2 + z^2 = c^2t^2, \tag{6}$$

and

$$x'^2 + y'^2 + z'^2 = c^2t'^2. \tag{7}$$

Subtracting equation (7) from equation (6), given that the $y$ and $z$ coordinates remain unaltered, leads to

$$x^2 - x'^2 = c^2t^2 - c^2t'^2. \tag{8}$$

Equation (8) exhibits only one solution, as it will be demonstrated below, readily obtained as

$$x^2 = c^2t^2, \tag{9}$$

and

$$x'^2 = c^2t'^2. \tag{10}$$

Indeed, Lorentz transformation equations (1) can lead to

$$x'^2 = \gamma^2(x^2 + v^2t^2 - 2xvt), \tag{11}$$

and

$$c^2t'^2 = \gamma^2\left(c^2t^2 + \frac{v^2x^2}{c^2} - 2xvt\right). \tag{12}$$

Eliminating the term $2xvt$ from equations (11) and (12), yields

$$x^2 + v^2t^2 - \frac{x'^2}{\gamma^2} = c^2t^2 + \frac{v^2x^2}{c^2} - \frac{c^2t'^2}{\gamma^2}. \tag{13}$$

Similarly, Lorentz transformation equations (2) bring about the following expression;

$$-x'^2 - v^2t'^2 + \frac{x^2}{\gamma^2} = -c^2t'^2 - \frac{v^2x'^2}{c^2} + \frac{c^2t^2}{\gamma^2}. \tag{14}$$

Adding equations (13) and (14) will lead to the following expression;

$$x^2\left(1 + \frac{1}{\gamma^2}\right) - x'^2\left(1 + \frac{1}{\gamma^2}\right) + v^2(t^2 - t'^2) =$$
$$= c^2t^2\left(1 + \frac{1}{\gamma^2}\right) - c^2t'^2\left(1 + \frac{1}{\gamma^2}\right) + \frac{v^2}{c^2}(x^2 - x'^2);$$

which can be simplified to

$$(x^2 - x'^2)\left(1 + \frac{1}{\gamma^2} - \frac{v^2}{c^2}\right) = c^2(t^2 - t'^2)\left(1 + \frac{1}{\gamma^2} - \frac{v^2}{c^2}\right);$$

$$x^2 - x'^2 = c^2(t^2 - t'^2), \tag{15}$$

returning actually the speed of light principle equation (8); thus validating equations (13) and (14) from the perspective of the special relativity. It should be noted that equation (8) is obtained from the Lorentz transformation equations without any restriction on the value of $\gamma$ (i.e. $\gamma$ can be replaced in the Lorentz transformation equations by an arbitrary constant, while equation (8) can still be obtained from the invalid resulting equations).

Whereas, the subtraction of equation (14) from equation (13), results in

$$(x^2 + x'^2)\left(1 - \frac{1}{\gamma^2} - \frac{v^2}{c^2}\right) = c^2(t^2 + t'^2)\left(1 - \frac{1}{\gamma^2} - \frac{v^2}{c^2}\right).$$

Now, if we assume for the time being the following equality (as suggested by the above equation)

$$x^2 + x'^2 = c^2(t^2 + t'^2), \tag{16}$$

then quations (15) and (16) will readily reduce to equations (9) and (10), namely

$$x^2 = c^2t^2$$
$$x'^2 = c^2t'^2$$

which satisfy both equations (13) and (14), as well as equation (8)—when $x^2$ and $x'^2$ are replaced with $c^2t^2$ and $c^2t'^2$, respectively—thus validating equation (16) that can also be derived from its consequent equations (9) and (10). Therefore, the verified equation (16) makes equations (9) and (10) the only solution for the constancy of the speed of light equation (8).

It should be noted that equations (9) and (10) can be evidently inferred from equations (13) and (14).

Another practical verification of equation (9) can be implemented through Fig. 1 depicting a graphical representation of Lorentz transformation equations (1), where $K$ and $K'$ are shown traveling along the overlapped time axes, $t$ and $t'$. Noticing the similar triangles within the graph, we can write,

$$\frac{vt}{\frac{vx}{c^2}} = \frac{x}{t},$$

yielding

$$x^2 = c^2 t^2.$$

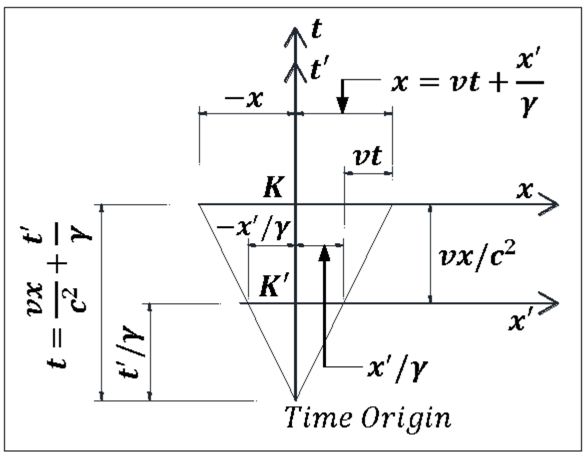


Fig. 1: Graphical representation of Lorentz transformation equations (1).

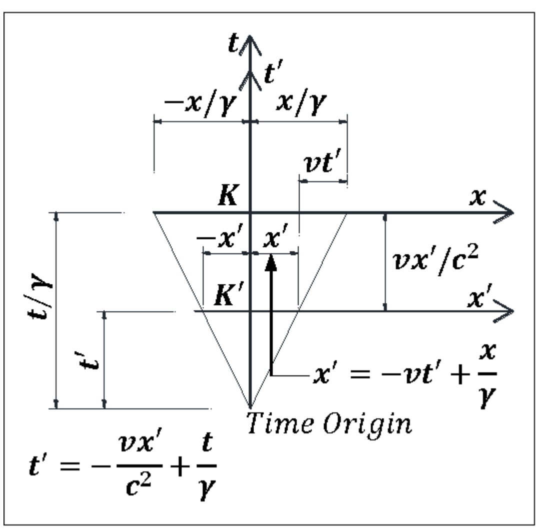


Fig. 2: Graphical representation of Lorentz transformation equations (2).

Similarly, equation (10) can be verified using Fig. 2, showing a graphical representation of Lorentz transformation equations (2), as follows;

$$\frac{vt'}{\frac{vx'}{c^2}} = \frac{x'}{t'},$$

yielding

$$x'^2 = c^2 t'^2.$$

Consequently, the light sphere equations (6) and (7) are collapsed to the line equations (9) and (10). In fact, when equations (9) and (10) are substituted into equations (6) and (7), they result in the vanishing of $y$, $z$, $y'$and $z'$, indicating that the constancy of the speed of light equations (6) and (7) are preliminarily restricted to light beam propagation along the frame axes parallel to the direction of the relative motion. This can be reconfirmed by adding equations (6) and (7), and using equation (16).

Now, dividing equation (9) by equation (10) yields

$$\left(\frac{x}{x'}\right)^2 = \left(\frac{ct}{ct'}\right)^2,$$

or

$$\frac{x}{x'} = \pm\frac{ct}{ct'}. \tag{17}$$

Assuming, for the time being, that $c > v$ (this assumption will turn out to be essential), then $x$ and $x'$ will always have the same sign (positive or negative), whether the light beam was emitted at $t_o = t'_o = 0$ in the positive or negative $x-$direction, with respect to the overlying $K$ and $K'$. It follows that

$$\frac{x}{x'} \geq 0,$$

and given that

$$\frac{ct}{ct'} \geq 0,$$

equation (17) becomes

$$\frac{x}{x'} = \frac{ct}{ct'}. \tag{18}$$

Hence, equation (18) combined with equations (9) and (10), leads to

$$c = \frac{x}{t} = \frac{x'}{t'}. \tag{19}$$

Equation (19) can also be readily obtained using Fig. 1, leading to

$$\frac{x}{t} = \frac{x'/\gamma}{t'/\gamma} = \frac{x'}{t'},$$

or Fig. 2;

$$\frac{x'}{t'} = \frac{x/\gamma}{t/\gamma} = \frac{x}{t},$$

along with equation (8). In fact, using

$$\frac{x^2}{x'^2} = \frac{t^2}{t'^2}.$$

in the expression resulting from dividing equation (8) by $x'^2$, leads to

$$\frac{t^2}{t'^2} - 1 = \frac{c^2}{x'^2}(t^2 - t'^2),$$

which yields equation (10), and equation (9) will follow when equation (10) is substituted into equation (8). Hence, equation (19) can be readily deduced.

Consequently, it can be concluded that the constancy of the speed of light for the light beam propagation in the relatively moving reference frames can be expressed by equation (19).

#### 3.1.1 Direct Inference

It will be first shown that the constancy of the speed of light postulate is certainly unviable for relatively moving inertial reference frames, without a space-time distorting transformation. In fact, assuming the space-time is preserved (i.e. cannot be modified), the coordinates $x$ and $x'$ (Fig. 3) would then be related by the following equation with respect to $K$, in accordance with the Galilean transformation;

$$x' = x - vt. \tag{20}$$

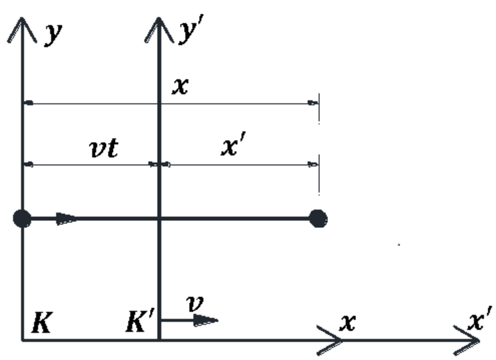


Fig. 3: $x$-coordinate with respect to $K$.

Whereas, with respect to $K'$, the same coordinates (Fig. 4) would be related by the following equation

$$x = x' + vt'. \tag{21}$$

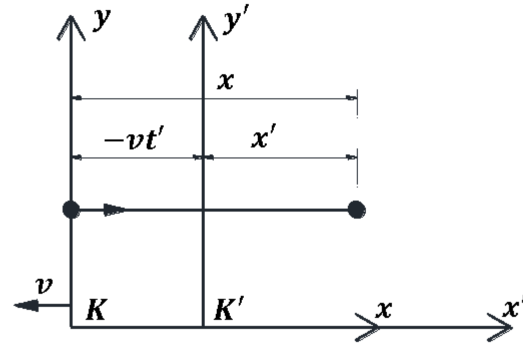


Fig. 4: $x'$-coordinate with respect to $K'$.

Substituting equation (20) into equation (21), we get

$$t = t'. \tag{22}$$

Dividing both sides of equations (20) and (21) by $c$, and applying the speed of light constancy principle as determined above ($c = x/t = x'/t'$), the following expressions are obtained;

$$t' = t - \frac{vx}{c^2}, \tag{23}$$

and

$$t = t' + \frac{vx'}{c^2}. \tag{24}$$

Substituting equation (23) in equation (24), we get

$$x = x', \tag{25}$$

and replacing equation (25) in equations (20) and (21) leads to the conflicting result of $v = 0$ at any $t > 0$, with the spatial coordinates $x$ and $x'$ being allowed to acquire non-zero values, according to equations (20) to (25).

It follows that the set of equations (20), (21), (23) and (24)—which will be referred to as (S1)—resulting from the Galilean transformation applied under the principle of the constancy of the speed of light, leads to the only conflicting solution $v = 0$, $x = x'$, and $t = t'$, binding the two reference frames together, although the relative motion of the reference frames is set as the main condition under which the equation set (S1) is derived. Consequently, the light speed constancy principle is unviable, at least in the case of no space-time distorting transformation.

On the other hand, although the equation set (S1) requires the conflicting binding of the two reference frames, it leads to the constancy of the speed of light general criteria given by equation (8), implying that the frames-binding requirement of the equation set (S1) remains applicable to equation (8).

In fact, equations (20) and (23) lead to

$$x'^2 = x^2 + v^2t^2 - 2xvt,$$

and

$$c^2t'^2 = c^2t^2 + \frac{v^2x^2}{c^2} - 2xvt.$$

Eliminating $2xvt$ from the above two equations yields

$$x^2 + v^2t^2 - x'^2 = c^2t^2 + \frac{v^2x^2}{c^2} - c^2t'^2. \tag{26}$$

Similarly, equations (21) and (24) can lead to

$$-x'^2 - v^2t'^2 + x^2 = -c^2t'^2 - \frac{v^2x'^2}{c^2} + c^2t^2. \tag{27}$$

Adding equations (26) and (27))—obtained from the equation set (S1)—and rearranging and simplifying the terms, returns equation (8):

$$x^2 - x'^2 = c^2t^2 - c^2t'^2.$$

Indeed, the addition of equations (26) and (27) results in the following expressions,

$$2(x^2 - x'^2) + v^2(t^2 - t'^2) = 2c^2(t^2 - t'^2) + \frac{v^2}{c^2}(x^2 - x'^2);$$

$$(x^2 - x'^2)\left(2 - \frac{v^2}{c^2}\right) = c^2(t^2 - t'^2)\left(2 - \frac{v^2}{c^2}\right);$$

yielding the speed of light constancy principle equation,

$$x^2 - x'^2 = c^2(t^2 - t'^2).$$

### 3.2 Lorenz Transformation Re-derivation

Assuming that the principle of the speed of light constancy must result in a space-time distorting transformation, a length conversion by a factor of $\beta$ along the direction of motion is hypothesized; the longitudinal length in one frame is scaled by a factor of $\beta$ with respect to the other frame. This length conversion can therefore be expressed with respect to $K$ and $K'$, using Figs. 3 and 4, respectively, as follows.

$$x = vt + \beta x'. \tag{28}$$

and

$$x' = -vt' + \beta x. \tag{29}$$

Where $\beta$ is a positive real number.
Rearranging equations (28) and (29), we can write

$$x' = \frac{1}{\beta}(x - vt), \tag{30}$$

and

$$x = \frac{1}{\beta}(x' + vt'). \tag{31}$$

Dividing both sides of equations (30) and (31) by $c$, and applying the speed of light constancy principle equation, as demonstrated above through equations (6) to (19) and restated here below;

$$c = \frac{x}{t} = \frac{x'}{t'}, \tag{32}$$

the following expressions are obtained.

$$t' = \frac{1}{\beta}\left(t - \frac{vx}{c^2}\right), \tag{33}$$

and

$$t = \frac{1}{\beta}\left(t' + \frac{vx'}{c^2}\right). \tag{34}$$

Solving equations (30), (31), (33) and (34) for $\beta$ results in

$$\beta = \sqrt{1 - \frac{v^2}{c^2}},$$

or

$$\frac{1}{\beta} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma. \tag{35}$$

In fact, for $x' = 0$, equations (30) and (34) yield $x = vt$, and $t' = \beta t$, respectively, reducing equation (33) to

$$\beta t = \frac{1}{\beta}\left(t - \frac{v^2t}{c^2}\right);$$

therefore

$$\beta = \sqrt{1 - \frac{v^2}{c^2}}.$$

Conversely, for $x = 0$, equations (31) and (33) yield $x' = -vt$, and $t = \beta t'$, respectively, reducing equation (34) to

$$\beta t' = \frac{1}{\beta}\left(t' - \frac{v^2t'}{c^2}\right);$$

hence

$$\beta = \sqrt{1 - \frac{v^2}{c^2}}.$$

We note that (35) is valid for $c > v$ only, thus satisfying our assumption made above in connection with the set criteria of the speed of light constancy principle.

It follows that, since $\beta < 1$, the hypothesized length conversion is a length contraction, as inferred from equations (28) and (29).

The obtained set of equations (30), (31), (33), (34), and (35) are the Lorentz transformation, representing the space-time transformation resulting from the reduced constancy of the speed of light principle given by equation (32).

It is noted that the velocity transformation equations (5) can be readily derived from the conflicting equations (20), (21), (23) and (24) — by dividing equation (20) by equation (23), and equation (21) by equation (24) — and working back from equations (5), equations (20), (21), (23) and (24) can be deduced, which implies that the Lorentz velocity transformation equations are merely invalid velocity criteria of the speed of light constancy principle, and independent of any space-time distorting transformation.

## 4 Lorentz Transformation Contradictions

The fact that the constancy of the speed of light principle is manifested as $c = x/t = x'/t'$ , as deduced from the Lorentz transformation, is sufficient to conclude the invalidity of the Lorentz transformation, explicitly and fully constructed in this paper from this fact, since the origin coordinates of the relatively moving reference frames are undetermined with respect to the each other.

In fact, for the space origin of $K'(0, 0, 0, t')$, at time $t' \neq 0$, the corresponding $K$ $x$- and $t$-coordinates shall satisfy the relation

$$\frac{x}{t} = \frac{x'}{t'}$$

that would yield

$$x = x'\left(\frac{t}{t'}\right) = 0,$$

if $t$ was determined. But, $x = 0$ results in undetermined $t$:

$$t = t'\left(\frac{x'}{x}\right) = \frac{0}{0},$$

making the above $x$-equation undetermined as well, thus leading to the set of $K$ origin coordinates

$$\left(x = \frac{0}{0}, 0, 0, t = \frac{0}{0}\right)$$

with undetermined $x$ and $t$.

And, for the time origin of $K'(x', y', z', 0)$, with spatial coordinates $\neq 0$, the corresponding $K$ $x$- and $t$-coordinates shall satisfy the relation

$$\frac{x}{t} = \frac{x'}{t'}$$

that would yield

$$t = t'\left(\frac{x}{x'}\right) = 0,$$

if $x$ was determined. But, $t = 0$ results in undetermined $x$:

$$x = x'\left(\frac{t}{t'}\right) = \frac{0}{0},$$

making the above $t$-equation undetermined as well, thus leading to the set of $K$ origin coordinates

$$\left(x = \frac{0}{0}, y = y', z = z', t = \frac{0}{0}\right)$$

with undetermined $x$ and $t$.

Hence, the constancy of the speed of light principle shall not principally be applicable at the reference frame space and time origins, restricting the time coordinate, and the spatial coordinate along the relative motion direction, from acquiring zero value (other than the set zero value at the initial overlaid-frames instant). Consequently, Lorentz transformation, implicitly incorporating equation (32) as demonstrated earlier, results in various conflicts and unresolved paradoxes.

For instance, substituting equation (33) into equation (34), returns

$$t = \gamma\left(\gamma\left(t - \frac{vx}{c^2}\right) + \frac{vx'}{c^2}\right). \tag{36}$$

Equation (36) is simplified in the following steps.

$$t = \gamma^2 t - \frac{\gamma^2 vx}{c^2} + \frac{\gamma vx'}{c^2},$$

or

$$t(\gamma^2 - 1) = \frac{vx}{c^2}\left(\gamma^2 - \frac{\gamma x'}{x}\right).$$

Using equation (32) in the above equation, we get

$$t(\gamma^2 - 1) = \frac{vx}{c^2}\left(\gamma^2 - \frac{\gamma t'}{t}\right). \tag{37}$$

With respect to equation (33), for $t' = 0$, the transformed $t$-coordinate with respect to $K$ is $t = vx/c^2$ ($t$ is undetermined with respect to equation (32), when $t' = 0$, as shown earlier, except at the initial overlaid-frames instant, the value of $t$ and $t'$ are set to zero). Therefore, for $t \neq 0$, equation (37) reduces to

$$t(\gamma^2 - 1) = t\gamma^2, \tag{38}$$

yielding the contradiction,

$$\gamma^2 - 1 = \gamma^2,$$

or

$$0 = 1,$$

which is the consequence of violating the restriction imposed by the light speed constancy principle on the coordinates (in this case setting $t' = 0$, equivalent to $t = vx/c^2$).

It follows that the transformation of $t' = 0$ to $t = vx/c^2$, for $x \neq 0$, by Lorentz transformation equation (33), is invalid, since it leads to a contradiction when used in equation (37), resulting from Lorentz transformation equations, for $t \neq 0$ (i.e. beyond the initial overlaid-frames instant satisfying $t = 0$ for $t' = 0$)—Letting $t = 0$ would satisfy equation (38), but another contradiction would emerge; the reference frames would be locked in their initial overlaid position, and no relative motion would be allowed, since in this case the corresponding coordinate to $t' = 0$ would be $t = vx/c^2 = 0$, yielding $v = 0$, as we're addressing the transformation of $t' = 0$ to $t = vx/c^2$ for $x \neq 0$.

Similar contradiction is obtained by substituting equation (34) into equation (33), using equation (32) in the resulting equation, and applying equation (34) for $t = 0$ ($t' = -vx'/c^2$).

Furthermore, substituting equation (30) into equation (31), yields

$$x = \gamma\left(\gamma(x - vt) + vt'\right);$$

$$x(\gamma^2 - 1) = \gamma v(\gamma t - t');$$

$$x(\gamma^2 - 1) = \gamma vt\left(\gamma - \frac{t'}{t}\right). \tag{39}$$

Using equation (32) in equation (39), we get,

$$x(\gamma^2 - 1) = \gamma vt\left(\gamma - \frac{x'}{x}\right). \tag{40}$$

With respect to equation (30), for $x' = 0$ (corresponding to $k'$ origin), the transformed $x$-coordinate with respect to $k$ is $x = vt$ ($x$ is undetermined with respect to equation (32) when $x' = 0$, as shown earlier, except at the initial overlaid-frame position, where the corresponding value to $x' = 0$ is $x = 0$). Therefore, for $x \neq 0$, equation (40) reduces to the following contradiction—resulting from violating the coordinate value restriction ($x' = 0$, equivalent to $x = vt$) imposed by the speed of light invariance principle.

$$x(\gamma^2 - 1) = x\gamma^2, \tag{41}$$

$$\gamma^2 - 1 = \gamma^2,$$

or

$$0 = 1.$$

It follows that the transformation of the $x'$-coordinate of $K'$ origin ($x' = 0$) to $x = vt$, at time $t > 0$, with respect to $K$ by Lorentz transformation equation (30), is invalid, since it leads to a contradiction when used in equation (40), resulting from Lorentz transformation equations, for $x \neq 0$ (i.e. beyond the initial overlaid-frames position satisfying $x = 0$ for $x' = 0$)—Letting $x = 0$ would satisfy equation (41), but another contradiction would emerge; the reference frames would be locked in their initial overlaid position, and no relative motion would be allowed, since in this case the corresponding coordinate to $x' = 0$ would be $x = vt = 0$, yielding $v = 0$, as we're addressing the transformation of $x' = 0$ to $x = vt$ for $t > 0$.

Yet, this conflicting condition of setting the spatial coordinate in the primed reference frame to zero under the speed of light invariance principle constitutes a vital strategy in the Lorentz transformation derivation, and the interpretation of the time dilation, in the special relativity formulation [2, 3].

Similar contradiction would follow upon substituting equation(31) into equation (30), using equation (32), and applying equation (31) for $x = 0$, $x' = -vt'$.

It follows that, the Lorentz transformation arrived at under the principle of the constancy of the speed of light is deemed to be refuted. Consequently, the length contraction hypothesis originally introduced as an ad hoc [3] to resolve the null result of the Michelson-Morley experiment [1] (inconsistency between experiment and theory with respect to a light ray fixed-length round trip travel time in the earth travel direction compared to that in the respective transverse direction) cannot be appropriately reconciled by the light velocity relativity principle space-time transformation.

The obtained Lorentz transformation contradictions for the particular cases of converting each of the spatial—along the relative motion direction—and time coordinates having a zero value in one reference frame to its corresponding value in the other frame, imply the general unviability of the Lorentz transformation equations.

## 5 Conflict Rationalization

In the constancy of the speed of light principle equations [3]

$$x^2 + y^2 + z^2 = c^2t^2,$$

and

$$x'^2 + y'^2 + z'^2 = c^2t'^2,$$

imposed as the governing aspect describing the space-time, $(x, y, z, t)$ and $(x', y', z', t')$ represent the space-time coordinates of an arbitrary light beam position vector in the reference frames $K$ and $K'$, respectively. Therefore, for instance, assigning the entity $x = vt$ to the $x$-coordinate of the origin of the reference frame $K'$ (i.e. transforming $x' = 0$ to $x = vt$ using the Lorentz transformation $x'$ equation) imposes a conflict with the light beam position vector $x$-coordinate, which is forced in this case to take the value of $vt$. Therefore, imposing the constancy of the speed of light equations on the space-time coordinate systems prohibits the system coordinates from taking other values than those associated with, and describing the light beam position vector. Hence, the coordinates of the origin of the moving frame are in conflict with the light beam position vector coordinates. Therefore, the Lorenz transformation equations

$$x' = \gamma(x - vt),$$

and

$$x = \gamma(x' + vt'),$$

return the equations $x = vt$ and $x' = -vt'$ for the origin of $K'(x' = 0)$ and $K(x = 0)$, respectively, which are in contradiction with the constancy of the speed of light equations in which $x$ and $x'$ represent the $x$- and $x'$-coordinates of the light beam position vector. Indeed, this justifies the appearance of the identified contradictions upon using the Lorentz transformation equations under the particular condition of $x' = 0$ (or $x = 0$), for which $x = vt$ (or $x' = -vt'$), with the constancy of the speed of light condition.

## 6 Apparent Space-time Transformation

Let's consider the classical coordinate transformation equation (20), and hypothesize a general length conversion factor $\beta$ from the perspective of the reference frame $K$, under the light speed constancy assumption:

$$x = vt + \beta x',$$

or

$$x' = \frac{1}{\beta}(x - vt). \tag{42}$$

Dividing equation (42) by $c$ and applying the constancy of the speed of light equation (32), we get

$$t' = \frac{1}{\beta}\left(t - \frac{vx}{c^2}\right). \tag{43}$$

Substituting $x = vt + \beta x'$ from equation (42) into equation (43), the following operations are performed to solve for $t$.

$$t' = \frac{1}{\beta}\left(t - \frac{v(vt + \beta x')}{c^2}\right),$$

or

$$t' = \frac{t}{\beta} - \frac{v^2 t}{\beta c^2} - \frac{vx'}{c^2},$$

then

$$\frac{t}{\beta}\left(1 - \frac{v^2}{c^2}\right) = t' + \frac{vx'}{c^2};$$

letting

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}},$$

we get

$$\frac{t}{\beta}\left(\frac{1}{\gamma^2}\right) = t' + \frac{vx'}{c^2},$$

or

$$t = \beta\gamma^2\left(t' + \frac{vx'}{c^2}\right). \tag{44}$$

Substituting equation (44) into equation (42), rearranging, and simplifying the terms, we get

$$x = \beta\gamma^2(x' + vt'). \tag{45}$$

Indeed,

$$x = vt + \beta x';$$

$$x = v\beta\gamma^2\left(t' + \frac{vx'}{c^2}\right) + \beta x';$$

$$x = v\beta\gamma^2 t' + \beta x'\left(1 + \frac{v^2\gamma^2}{c^2}\right);$$

$$x = v\beta\gamma^2 t' + \beta x'\gamma^2\left(\frac{1}{\gamma^2} + \frac{v^2}{c^2}\right).$$

Since the latter equation terms between the brackets add to unity, it reduces to equation (45).

For the particular case of $\beta = 1/\gamma$, equations (42) to (45) take the form of the known Lorentz transformation equations. In addition, the relativistic velocity transformation equations can be derived from equations (42) to (45), irrespective of the value of $\beta$.

It can be concluded from the transformation resulting from the light velocity invariance principle, that for a length factor of $\beta$ with respect to $K$, equations (42) and (43) lead to the following equations for simultaneous ($\Delta t = 0$) and co-local ($\Delta x = 0$) events, respectively;

$$x = \beta x', \tag{46}$$

and

$$t = \beta t'. \tag{47}$$

Whereas, for simultaneous ($\Delta t' = 0$) and co-local ($\Delta x' = 0$) events with respect to $K'$, we have, respectively, from equations (45) and (44),

$$x' = \frac{1}{\beta}\left(\frac{1}{\gamma^2}\right)x, \tag{48}$$

and

$$t' = \frac{1}{\beta}\left(\frac{1}{\gamma^2}\right)t. \tag{49}$$

For the case of preserved space-time where the length factor is $\beta = 1$, if we consider the events of a light ray being emitted and returned, after being reflected, to the same point ($\Delta x' = 0$) in the longitudinal direction in $K'$, it can be easily shown that, in line with the constancy of the light speed, the light ray travel time in $K$ would be

$$T = \gamma^2\left(\frac{2L}{c}\right), \tag{50}$$

where $2L$ is the round trip length. According to equation (48), the corresponding round trip length in $K'$ would be

$$2L' = \frac{2L}{\gamma^2}, \tag{51}$$

and the corresponding travel time in $K'$ becomes, according to equation (49),

$$T' = \frac{T}{\gamma^2} = \frac{\gamma^2(2L/c)}{\gamma^2} = \frac{2L}{c}. \tag{52}$$

Now, with the length factor of $\beta$ being applied in $K$, the round trip length becomes $\beta(2L)$, and the light ray round trip travel time in $K$ becomes, using equation (50),

$$T = \gamma^2\left(\frac{\beta(2L)}{c}\right) = \beta\gamma^2\left(\frac{2L}{c}\right), \tag{53}$$

whereas, according to equation (49), the corresponding travel time in $K'$ , using equation (53), becomes,

$$T' = \frac{1}{\beta}\left(\frac{T}{\gamma^2}\right) = \frac{1}{\beta}\left(\frac{\beta\gamma^2(2L/c)}{\gamma^2}\right) = \frac{2L}{c}, \tag{54}$$

while, from equation (48) the corresponding round trip length becomes

$$2L' = \frac{1}{\beta}\left(\frac{\beta(2L)}{\gamma^2}\right) = \frac{2L}{\gamma^2}. \tag{55}$$

It follows from equations (51), (52), (54) and (55) that the transformed longitudinal light ray round trip length and travel time in $K'$ are independent of the introduced length conversion factor $\beta$ in $K$, and always converted to $(2L/\gamma^2)$ and $(2L/c)$, respectively.

It becomes then obvious that the transformation (converting from $K$ coordinates to $K'$ coordinates) resulting from a length conversion factor with respect to $K$ under the application of the contradictory light velocity invariance criteria ($c = x/t = x'/t'$), simply reverses the length factor to recover the original length in $K$, and scales the recovered length down by a factor of $1/\gamma^2$ so that the local time in $K'$ is obtained. This is indeed an amazing, tricky transformation; when the introduced length conversion factor is $1/\gamma$, thus changing the travel time in $K$ from $\gamma^2(2L/c)$ to $\gamma(2L/c)$, the light velocity constancy resulting transformation would reverse the length factor, returning the original time of $\gamma^2(2L/c)$ in $K$, and apply a new length factor of $1/\gamma^2$ converting the travel time to $2L/c$ in $K'$, with a net length factor of $(1/\gamma)^{-1}(1/\gamma^2) = 1/\gamma$, giving the impression of a space-time distorting transformation, with a time dilation factor of $\gamma$ and a length contraction of $1/\gamma$, although the actual length contraction factor in $K'$ is $1/\gamma^2$!

It follows that the length conversion factor of $1/\gamma$ is nothing but a particular factor resulting in [conflicting] symmetrical transformation equations, when applying the restricted speed of light constancy principle on the classical spatial transformation equation with a length conversion factor. Otherwise, any length conversion factor $\beta$ introduced to the classical spatial coordinate equation $x = vt + x'$ (changing it to $x = vt + \beta x'$) under the restricted assumption of the constancy of the speed of light, would result in inapplicable time and space transformation equations — (42) to (45) — invariantly satisfying the basic criteria of the light velocity assumption given by equation (8).

Indeed, squaring both equations (42) and ($c\times$ equation (43)), and eliminating the similar terms from the resulting two equations, leads to

$$x^2 + v^2t^2 - x'^2\beta^2 = c^2t^2 + \frac{v^2x^2}{c^2} - c^2t'^2\beta^2. \tag{56}$$

Similar application of equations (44) and (45) will result in

$$-x'^2 - v^2t'^2 + \frac{x^2}{\beta^2\gamma^4} = -c^2t'^2 - \frac{v^2x'^2}{c^2} + \frac{c^2t^2}{\beta^2\gamma^4}. \tag{57}$$

— It should be noted that equations (56) and (57) reconfirm the equalities $x^2 = c^2t^2$, and $x'^2 = c^2t'^2$.
Adding equations (56) and (57), and simplifying and rearranging the terms, leads to

$$x^2 - x'^2 = c^2t^2 - c^2t'^2 + \frac{1}{\beta^2\gamma^2}(c^2t^2 - x^2) - \beta^2\gamma^2(c^2t'^2 - x'^2), \tag{58}$$

which reduces to the constancy of the speed of light equation (8):

$$x^2 - x'^2 = c^2t^2 - c^2t'^2.$$

Finally, the above discussion, carried out from equation (20), from the perspective of $K$, can also be repeated based on equation (21), from the perspective of $K'$, with identical results being obtained.

## 7 Conclusion

Analysis of the Lorentz transformation revealed mathematical restrictions in terms of the deduced, simplified form of the constancy of the speed of light equations residing in the transformation. The Lorentz transformation, readily reconstructed using these basic, restricted light velocity invariance equations, resulted in mathematical contradictions. The principle of the constancy of the speed of light was thus demonstrated to be an unviable assumption, and the ensuing Lorentz transformation was subject to refutation. Rationalization of the revealed contradictions was established. The actual interpretation of the Lorentz transformation demonstrated the unreal aspect of the space-time conversion attributed to the transformation.

## References


1. A.A. Michelson and E.H. Morley, "On the Relative Motion of the Earth and the Luminiferous Ether," Am. J. Sci. **34**, 333–345 (1887).
2. A. Einstein, "Zur elektrodynamik bewegter Körper," Annalen der Physik **322** (10), 891–921 (1905).
3. A. Einstein, "On The Relativity Principle And The Conclusions Drawn From It", Jahrbuch der Radioaktivitat und Elektronik **4** (1907). English translations in Am. Jour. Phys. Vol. 45, NO. 6, June 1977.
4. P. Beckmann, Einstein Plus Two. (Golem Press, Boulder, CO 1987).
5. I. McCausland, "The Persistent Problem of Special Relativity," Physics Essays **18** (2005).

6. R. Hatch, Escape from Einstein (Kneat Kompany, Wilmington, CA, 1992)

7. F. Selleri, in Open Questions in Relativistic Physics, F. Selleri, ed. (Apeiron, Montreal, 1998).

8. H. Dingle, Science at the Crossroads (Martin Brian and O'keeffe, London, 1972).

9. R. A. Monti, "Theory of Relativity: A Critical Analysis," Physics Essays: **9** (2), 238–260 (1996).

10. A. Kelly, Challenging Modern Physics: Questioning Einstein's Relativity Theories (Brown Walker Press, Boca Raton, 2005).

11. H. Poincaré, "On the Dynamics of the Electron," Comptes Rendus **140**, 1504–1508 (1905).