The model of Majorana particle travelling at the speed of light

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ABSTRACT

In this paper we present the model of Majorana particle travelling at the speed of light.

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1 Spinorial equation with Majorana condition

In this paper we present the model of Majorana particle travelling at the speed of light.

Our model is based on the Lorentz invariant equation that can be written in terms of the "left" (ξ) and "right" ($\dot{\eta}$) spinor components as follows:

$$\begin{bmatrix} \partial_0 + \partial_3 & \partial_1 - i\partial_2 \\ \partial_1 + i\partial_2 & \partial_0 - \partial_3 \end{bmatrix} \begin{bmatrix} \eta_{\dot{1}} \\ \eta_{\dot{2}} \end{bmatrix} = -\, im \begin{bmatrix} \xi^1 \\ \xi^2 \end{bmatrix}$$
$$\begin{bmatrix} \partial_0 - \partial_3 & -\partial_1 + i\partial_2 \\ -\partial_1 - i\partial_2 & \partial_0 + \partial_3 \end{bmatrix} \begin{bmatrix} \xi^1 \\ \xi^2 \end{bmatrix} = +\, im \begin{bmatrix} \eta_{\dot{1}} \\ \eta_{\dot{2}} \end{bmatrix} \tag{1.1}$$

This equation looks very similar to the free Dirac equation, but with the opposite sign in the r.h.s. of the second pair of equations. This equation was earlier considered by Guang-jiong Ni and Tsao Chang (see [2]), but for some reason Ni and Chang associated this equation with superluminal neutrinos.

In this paper we will only consider the case of *Majorana particles*, by requiring that "left" (ξ) and "right" ($\dot{\eta}$) spinor components of the particle field satisfy the following Lorentz invariant condition (known as *Majorana condition*, or *Neutrality condition*):

$$\begin{aligned} \eta_{\dot{1}} &= +\, \overline{\xi^2} \qquad & \xi^1 &= -\, \overline{\eta}_{\dot{2}} \\ \eta_{\dot{2}} &= -\, \overline{\xi^1} \qquad & \xi^2 &= +\, \overline{\eta}_{\dot{1}} \end{aligned} \tag{1.2}$$

If we will put (1.2) into equation (1.1), we will obtain:

$$\begin{bmatrix} \partial_0 + \partial_3 & \partial_1 - i\partial_2 \\ \partial_1 + i\partial_2 & \partial_0 - \partial_3 \end{bmatrix} \begin{bmatrix} \bar{\xi}^2 \\ -\bar{\xi}^1 \end{bmatrix} = -\, im \begin{bmatrix} \xi^1 \\ \xi^2 \end{bmatrix}$$
$$\begin{bmatrix} \partial_0 - \partial_3 & -\partial_1 + i\partial_2 \\ -\partial_1 - i\partial_2 & \partial_0 + \partial_3 \end{bmatrix} \begin{bmatrix} \xi^1 \\ \xi^2 \end{bmatrix} = +\, im \begin{bmatrix} \bar{\xi}^2 \\ -\bar{\xi}^1 \end{bmatrix} \tag{1.3}$$

and after expansion of the formulas and complex conjugation of the first pair of equations we obtain

$$\left.\begin{aligned} (\partial_0+\partial_3)\,\xi^2-(\partial_1+i\partial_2)\,\xi^1 &= +\,im\;\bar{\xi}^1 \\ (\partial_1-i\partial_2)\,\xi^2-(\partial_0-\partial_3)\,\xi^1 &= +\,im\;\bar{\xi}^2 \end{aligned}\right\}$$
$$\left.\begin{aligned} (\partial_0-\partial_3)\,\xi^1-(\partial_1-i\partial_2)\,\xi^2 &= +\,im\;\bar{\xi}^2 \\ -(\partial_1+i\partial_2)\,\xi^1+(\partial_0+\partial_3)\,\xi^2 &= +\,im\;\bar{\xi}^1 \end{aligned}\right\} \tag{1.4}$$

From (1.4) it is clear that, due to Majorana condition (1.2), the two pairs of equations (1.1) become equivalent to each other, hence only one of these equations is *independent*.

2 Momentum density

As usual we define the momentum density 4-vector as a sum of "left" and "right" chiral currents.

The "left" chiral current is defined as:

$$p_\mu = \frac{1}{2}\,\left(\xi^+\sigma_\mu\xi\right) \tag{2.1}$$

$$\begin{aligned} p_0 &= \tfrac{1}{2}\left(\xi^+\xi\right) = \tfrac{1}{2}\left(\overline{\xi^1}\xi^1+\overline{\xi^2}\xi^2\right) \qquad p_1 = \tfrac{1}{2}\left(\xi^+\sigma_1\xi\right) = \tfrac{1}{2}\left(\overline{\xi^2}\xi^1+\overline{\xi^1}\xi^2\right) \\ p_2 &= \tfrac{1}{2}\left(\xi^+\sigma_2\xi\right) = \tfrac{i}{2}(\overline{\xi^2}\xi^1-\overline{\xi^1}\xi^2) \qquad p_3 = \tfrac{1}{2}\left(\xi^+\sigma_3\xi\right) = \tfrac{1}{2}\left(\overline{\xi^1}\xi^1-\overline{\xi^2}\xi^2\right) \end{aligned} \tag{2.2}$$

One can easily check that $p_\mu p^\mu \equiv 0$, and that vector $p_\mu = \frac{1}{2}\left(\xi^+\sigma_\mu\xi\right)$ transforms as *covariant* vector.

Similarly, we define *contravariant* vector $\hat{p}^\mu$ as "right" chiral current:

$$\hat{p}^\mu = \frac{1}{2}\left(\dot{\eta}^+\acute{\sigma}^\mu\dot{\eta}\right) \tag{2.3}$$

$$\begin{aligned} \hat{p}^0 &= \tfrac{1}{2}\left(\dot{\eta}^+\dot{\eta}\right) = \tfrac{1}{2}\left(\overline{\eta_{\dot 1}}\eta_{\dot 1}+\overline{\eta_{\dot 2}}\eta_{\dot 2}\right) \qquad \hat{p}^1 = \tfrac{1}{2}\left(\dot{\eta}^+\acute{\sigma}^1\dot{\eta}\right) = \tfrac{1}{2}\left(\overline{\eta_{\dot 2}}\eta_{\dot 1}+\overline{\eta_{\dot 1}}\eta_{\dot 2}\right) \\ \hat{p}^2 &= \tfrac{1}{2}\left(\dot{\eta}^+\acute{\sigma}^2\dot{\eta}\right) = \tfrac{i}{2}(\overline{\eta_{\dot 2}}\eta_{\dot 1}-\overline{\eta_{\dot 1}}\eta_{\dot 2}) \qquad \hat{p}^3 = \tfrac{1}{2}\left(\dot{\eta}^+\acute{\sigma}^3\dot{\eta}\right) = \tfrac{1}{2}\left(\overline{\eta_{\dot 1}}\eta_{\dot 1}-\overline{\eta_{\dot 2}}\eta_{\dot 2}\right) \end{aligned} \tag{2.4}$$

Vector $\hat{p}^\mu$ is also isotropic: $\hat{p}^\mu\hat{p}_\mu \equiv 0$.

The momentum density 4-vector (*Dirac current*) is a sum of "left" and "right" chiral currents:

$$P_\mu = \ p_\mu + \ g_{\mu\nu}\hat{p}^\nu \tag{2.5}$$

or, equivalently

$$\begin{aligned}
P_0 &= \tfrac{1}{2}\left(\overline{\xi^1}\xi^1 + \ \overline{\xi^2}\xi^2\right) + \ \tfrac{1}{2}\left(\overline{\eta_{\dot{1}}}\eta_{\dot{1}} + \ \overline{\eta_{\dot{2}}}\eta_{\dot{2}}\right) = \ p_0 + \hat{p}^0 \\
P_1 &= \tfrac{1}{2}\left(\overline{\xi^2}\xi^1 + \ \overline{\xi^1}\xi^2\right) - \ \tfrac{1}{2}\left(\overline{\eta_{\dot{2}}}\eta_{\dot{1}} + \ \overline{\eta_{\dot{1}}}\eta_{\dot{2}}\right) = \ p_1 - \hat{p}^1 \\
P_2 &= \tfrac{i}{2}\left(\overline{\xi^2}\xi^1 - \ \overline{\xi^1}\xi^2\right) - \tfrac{i}{2}\left(\overline{\eta_{\dot{2}}}\eta_{\dot{1}} - \ \overline{\eta_{\dot{1}}}\eta_{\dot{2}}\right) = \ p_2 - \hat{p}^2 \\
P_3 &= \tfrac{1}{2}\left(\overline{\xi^1}\xi^1 - \ \overline{\xi^2}\xi^2\right) - \ \tfrac{1}{2}\left(\overline{\eta_{\dot{1}}}\eta_{\dot{1}} - \ \overline{\eta_{\dot{2}}}\eta_{\dot{2}}\right) = \ p_3 - \hat{p}^3
\end{aligned} \tag{2.6}$$

With spinorial equations (1.1) one can easily find that

$$\begin{aligned}
\partial_\mu p^\mu &= im\left(\eta_{\dot{\nu}}\ \overline{\xi^\nu} - \overline{\eta_{\dot{\mu}}}\ \xi^\mu\right) \\
\partial_\mu \hat{p}^\mu &= im\left(\eta_{\dot{\nu}}\ \overline{\xi^\nu} - \overline{\eta_{\dot{\mu}}}\ \xi^\mu\right)
\end{aligned} \tag{2.7}$$

and, according to (2.5)

$$\partial_\mu P^\mu = 2im\left(\eta_{\dot{\nu}}\ \overline{\xi^\nu} - \overline{\eta_{\dot{\mu}}}\ \xi^\mu\right) \tag{2.8}$$

Now it is easy to check that due to Majorana condition (1.2)

$$\begin{aligned}
\eta_{\dot{\nu}}\ \overline{\xi^\nu} &= \eta_{\dot{1}}\ \overline{\xi^1} + \eta_{\dot{2}}\ \overline{\xi^2} = 0 \\
\overline{\eta_{\dot{\mu}}}\ \xi^\mu &= \overline{\eta_{\dot{1}}}\ \xi^1 + \overline{\eta_{\dot{2}}}\ \xi^2 = 0
\end{aligned} \tag{2.9}$$

Consequently we conclude that, due to Majorana condition, both chiral currents p_μ and $\hat{p}_\mu$, as well as momentum density current P_μ are conserved.

3 Spinorial fields

Spinorial fields satisfying the Majorana condition can be chosen in the following form:

$$\begin{bmatrix} \xi^1 \\ \\ \xi^2 \end{bmatrix} = \begin{bmatrix} -\,1 \\ \\ +\,1 \end{bmatrix} \phi(x) \qquad \begin{bmatrix} \eta_{\dot{1}} \\ \\ \eta_{\dot{2}} \end{bmatrix} = \begin{bmatrix} +\,1 \\ \\ +\,1 \end{bmatrix} \phi(x) \tag{3.1}$$

where $\phi(x)$ is an arbitrary complex valued function.

Using (3.1) we can find that chiral currents (2.2) and (2.4) will be written as

$$\begin{aligned} p_0 &= +\,\phi\,\bar{\phi} \quad p_1 = -\,\phi\,\bar{\phi} \\ p_2 &= 0 \qquad\quad p_3 = 0 \end{aligned} \tag{3.2}$$

and

$$\begin{aligned} \hat{p}^0 &= +\,\phi\,\bar{\phi} \quad \hat{p}^1 = +\,\phi\,\bar{\phi} \\ \hat{p}^2 &= 0 \qquad\quad \hat{p}^3 = 0 \end{aligned} \tag{3.3}$$

Consequently, with our choice of spinorial fields (3.1), the spatial parts of both ciral currents p_μ and $\hat{p}_\mu$, as well as momentum density vector P_μ, are *opposite in direction to the axis* $\mathbf{e}_1$, while the momentum density 4-vector is *isotropic*: $P^\mu P_\mu = 0$. This is the first indication that our model decsribes the *particle that is travelling at the speed of light*.

Let us now use the expressions (3.1) for the spinorial fields in the equation (1.4):

$$\begin{aligned} (\partial_0 + \partial_3)\,\phi + (\partial_1 + i\partial_2)\,\phi &= -\,im\,\bar{\phi} \\ (\partial_1 - i\partial_2)\,\phi + (\partial_0 - \partial_3)\,\phi &= +\,im\,\bar{\phi} \end{aligned} \tag{3.4}$$

By adding and subtracting equations (3.4) we obtain:

$$\begin{aligned} (\partial_0 + \partial_1)\,\phi &= 0 \\ (\partial_3 + i\partial_2)\,\phi &= -2im\bar{\phi} \end{aligned} \tag{3.5}$$

The first equation in (3.5) means that the field $\phi(x)$ is travelling at the speed of light in the direction opposite to the axis $\mathbf{e}_1$. Together these equations define the evolution of the field $\phi(x)$ in 4 dimensions.

Further generalization of the equation (1.1) (by allowing the "mass term" m to be not constant, but *variable*) is presented in [1].

References

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